A PHYSICAL SIMULATION OF OBJECTS’ BEHAVIOUR BY FINITE ELEMENT METHOD, MODAL MATCHING AND DYNAMIC EQUILIBRIUM EQUATION

Raquel Ramos Pinho, João Manuel R. S. Tavares

FEUP – Faculty of Engineering, University of Porto, Portugal
LOME – Laboratory of Optics and Experimental Mechanics

ICCAM 2004
Leuven / Belgium
Introduction

Modeling and Simulating Objects Deformation:

The simulation of the deformation between object’s shapes is done in this work, attending to the objects physical attributes, by solving the Dynamic Equilibrium Equation (DEE) and using the Finite Element Method with Modal Matching.

Main Objectives:

- Simulate the displacement field between two objects represented in images (2D/3D);
- Compare the integration methods results: Central Difference, Newmark’s and Mode Superpostion.
Introduction

Applications Areas:
- Computer graphics;
- Medical imaging;
- Virtual reality…

Examples:
- Objects 3D reconstruction from 2D images (slices);
- Haptic forces feedback;
- Quantification of the involved deformation;
- Object identification …

Demands:
- Interactivity;
- Accuracy;
- Speed;
- Physical behavior…
Introduction

Approach:

- The objects are represented in images;
- Objects’ physical behavior is simulated by using the Finite Element Method;
- Models’ nodes are matched by using Modal Analysis;
- Temporal deformation simulation is done by the Dynamic Equilibrium Equation (DEE) resolution.
- To solve the DEE we used the Central Difference, the Newmark’s or the Mode Superposition methods;
- To accelerate the simulation we stop the resolution when an estimated shape gets close enough to the target shape (distance predefined by the user);
- To measure the difference between shapes we sum the distances between each intermediate shape’s node and its final position in the target shape.
Fundaments

- **Finite Element Method (FEM):**
  - Objects expected behavior is simulated by the selection of an elastic virtual material;
  - **Finite element used**: Sclaroff’s Isoparametric Element:
    * Based on Gaussian interpolants;
    * Models are independent of the nodes order;
    * Modeled object behaves like an elastic membrane (2D) or a rubbery blob (3D);

- Models’ nodes matched by **Modal Analysis**: pairs of nodes with similar displacements in each modal space are considered good candidates to be matched;

- **Damping** matrix used: linear combination of the Mass and Stiffness matrices (*Rayleigh’s Damping*).
A Physical Simulation of Objects’ Behaviour by Finite Element Method, Modal Matching and Dynamic Equilibrium Equation

Resolution of the DEE

Contents

Introduction
Fundaments
Resolution of the Dynamic Equilibrium Equation
Implementation
Experimental Results
Conclusions

Input
Data (pixels) as nodes of a finite element model

Physical models built using FEM
Mass and stiffness matrices (M and K) of each model are assembled

Eigenmodes are computed
Eigenmodes are computed
Resolution of the generalized eigenvalue/vector problem

Modal Matching
Displacements are analyzed in each modal space

Physical Deformation Simulation

Dynamic Equilibrium Equation Resolution

\[ M \ddot{U} + C \dot{U} + KU = R \]
Displacement field is obtained with the Central Difference, the Newmark’s or the Mode Superposition methods

Estimations

Damping matrix, C
Implicit applied charges on models’ nodes, R
Initial displacement \( U^0 \), and velocity \( \dot{U}^0 \)
Resolution of the DEE

Contents

Introduction
Fundaments
Resolution of the Dynamic Equilibrium Equation
Implementation
Experimental Results
Conclusions

A Physical Simulation of Objects’ Behaviour by Finite Element Method, Modal Matching and Dynamic Equilibrium Equation
Integration Methods used:

- **Direct Integration Methods:**
  - Central Difference Method:
    - Explicit method;
    - First order accuracy;
    - Conditionally stable;
  - Newmark’s Method (we used it as a):
    - Implicit method;
    - Second order accuracy;
    - Unconditionally stable;

- **Indirect Integration Method:**
  - Mode Superposition Method:
    - Transforms the original system into a set of uncoupled equations (solving for the modal displacements and not for the nodal ones);
    - Involved computational effort may be reduced using just some of the vibration modes.
Resolution of the DEE

Estimates:

- Applied charges on unmatched nodes (to apply on objects that don’t have all nodes successfully matched);

- Initial Displacement and Velocity:
  - Initial displacement considered proportional to the total displacement;
  - Initial velocity considered proportional to the initial displacement.
Implementation

This approach was implemented on an previously existing software platform that can be used to develop and test image and computer graphics algorithms.

**Features:**

- Programming Language: C++;
- Development tool: *Microsoft Visual C++*;
- Operating systems: *Microsoft Windows*;
- Modular architecture;
- Some public libraries incorporated (e.g. *Newmat*, *VTK*).
Experimental Results

Comparing the Results obtained by the considered Integration Methods:

An example…

One object with 124 nodes from a real pedobarography image, a second object obtained by applying a rigid transformation to the first one, all nodes successfully matched
Experimental Results

Contents

Introduction
Fundaments
Resolution of the Dynamic Equilibrium Equation
Implementation
Experimental Results
Conclusions

Initial
Central Difference
Newmark
Mode Superposition and Central Difference (75% modes)
Mode Superposition and Newmark (75% modes)

Target
As the simulation evolves more the obtained results differ.
Best approach to the target surface obtained by the Mode Superposition with Newmark’s method, using different vibration modes %

- Results obtained by the Newmark’s method and by the Mode Superposition method do not differ significantly if a considerable % of modes is used.
Depending on the application purposes, the Computational effort of the Mode Superposition method may be reduced without a considerable effect on the results.
Experimental Results

Another example…

Two surfaces obtained from real images, and 90 of their 140 nodes successfully matched

Simulation of the non rigid deformation using the Newmark’s method (obtained in 24 seconds)
Experimental Results

A 2D example...

Two real isobars’ contours with 28 and 32 nodes, where 21 node are successfully matched

Intermediate shapes obtained (in less than one second)
**Conclusions**

Comparing the Results obtained by the considered Integration Methods:

- Results obtained by the Newmark’s and the Mode Superposition methods do not differ significantly when a considerable % of modes is used - both have second order precision;

- In some applications it may be more interesting to apply the Mode Superposition method (when a few number of nodes is used) but generally it is more suitable to apply the Newmark’s method as we used it.

**Simulation Results:**

- Results can be obtained in real time, but the computational effort depends on the number of used nodes;

- Results are coherent with the expected behavior of physical objects (depending on the virtual material attributes).