FAILURE CRITERIA FOR FIBRE-REINFORCED POLYMER COMPOSITES

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Introduction

The subject of failure criteria for fibre reinforced plastic composites has attracted numerous researchers over the last four decades [1]-[8]. The number and different types of approaches that have been proposed clearly demonstrates that failure criteria for fibre reinforced plastic composites it is still today an important research topic.
Although it is clear that important progresses have been made, it does not appear that there is any criterion universally accepted by designers as adequate under general load conditions. An evidence of this is recent publication of a special edition of *Composites Science and Technology* entirely dedicated to failure theories of fibre reinforced plastic composites [9], [10] (Figure 1), and the survey performed by C.T. Sun on the industrial use of failure criteria [11], Figure 2.

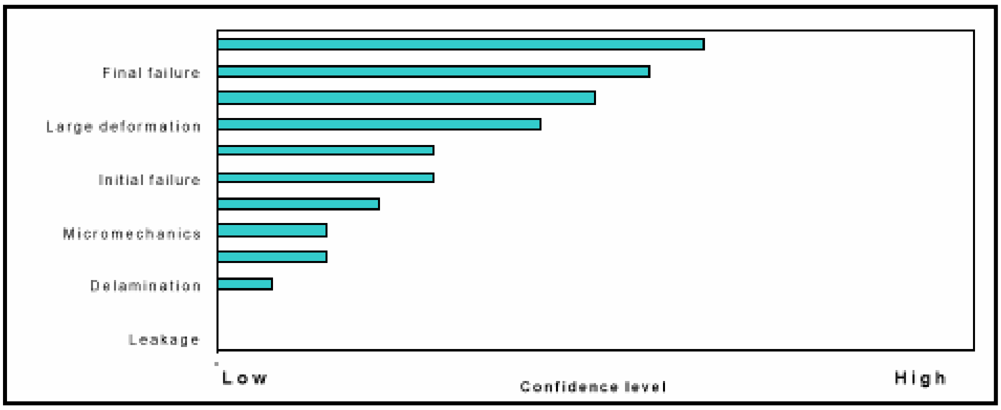

Figure 1- Confidence level displayed by the WWFE theories (from [10])

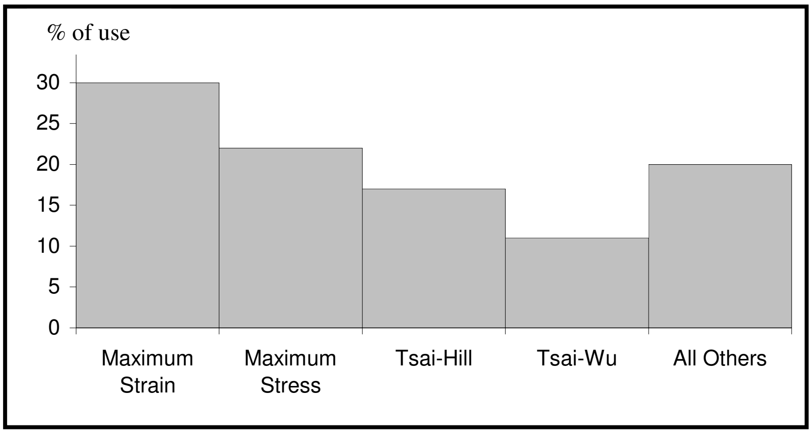

Figure 2: Industrial usage of composites failure criteria (after [11])

From Figure 1 and Figure 2 it is clear that the confidence levels of the failure theories used in the World-Wide Failure Exercise need to be improved for some damage mechanisms (e.g. delamination), and that there is no consensus in the industry on the most adequate failure criterion.
Besides failure criteria, there are other issues worth investigation, such as the inclusion of residual thermal stresses, in-situ strengths, non-linear behaviour in shear, stiffness degradation models for laminate failure, final failure definition and delamination failure.

Lamina Failure Criteria

The failure criteria proposed to predict lamina failure could be divided in two main groups:

a) Failure criteria not associated with failure modes

This group includes all polynomial and tensorial criteria, using mathematical expressions to describe the failure surface as a function of the material strengths. Generally, these expressions are based on the process of adjusting an expression to a curve obtained by experimental tests.
The most general polynomial failure criterion for composite materials is *Tensor Polynomial Criterion* proposed by Tsai and Wu [1]. This criterion may be expressed in tensor notation as:

$$F_i \cdot \sigma_i + F_{ij} \cdot \sigma_i \cdot \sigma_j + F_{ijk} \cdot \sigma_i \cdot \sigma_j \cdot \sigma_k \geq 1$$

where i, j, k = 1, ... , 6 for a 3-D case. The parameters F_i, F_{ij} and F_{ijk} are related to the lamina strengths in the principal directions. For practical proposes, and due to the large number of material constants required, the third-order tensor F_{ijk} is usually neglected [5]. Therefore, the general polynomial criterion reduces to a general quadratic expression given by:

$$F_i \cdot \sigma_i + F_{ij} \cdot \sigma_i \cdot \sigma_j \geq 1$$

where i, j = 1, ... , 6. Considering that the failure of the material is insensitive to a change of sign in shear stresses, all terms containing a shear stress to first power must vanish: $F_4 = F_5 = F_6 = 0$. Then, the explicit form of the general expression is:

$$F_1\sigma_1 + F_2\,\sigma_2 + F_3\,\sigma_3 + 2F_{12}\,\sigma_1\,\sigma_2 + 2F_{13}\,\sigma_1\,\sigma_3 + 2F_{23}\,\sigma_2\,\sigma_3 + F_{11}\,\sigma_1^{\,2} + \\ + F_{22}\,\sigma_2^{\,2} + F_{33}\,\sigma_3^{\,2} + F_{44}\,\sigma_4^{\,2} + F_{55}\,\sigma_5^{\,2} + F_{66}\,\sigma_6^{\,2} \geq 1$$

Several other quadratic criteria have been proposed, differing in the way in which the tensor stress components are determined. Other popular and well-known quadratic failure criteria include those proposed by Tsai-Hill [2], Azzi-Tsai [12], Hoffman [13] and Chamis [14]. These quadratic criteria can be represented in terms of the general Tsai-Wu quadratic criterion varying the parameters F_i and F_{ij} in order to ensure a good fit of the failure surface to the experimental results. These failure criteria are summarized in Table 1.

Table 1: Polynomial Failure Criteria

	Tsai-Wu	Tsai-Hill*	Azzi-Tsai*	Hoffman	Chamis*†
F_1	$\frac{1}{\sigma_{1T}^u - \sigma_{1C}^u}$	0	0	$\frac{1}{\sigma_{1T}^u - \sigma_{1C}^u}$	0
F_2	$\frac{1}{\sigma_{2T}^u - \sigma_{2C}^u}$	0	0	$\frac{1}{\sigma_{2T}^u - \sigma_{2C}^u}$	0
F_3	$\frac{1}{\sigma_{3T}^u - \sigma_{3C}^u}$	0	0	$\frac{1}{\sigma_{3T}^u - \sigma_{3C}^u}$	0
F_{12}	$\frac{-1}{2\sqrt{\sigma_{1T}^u\sigma_{1C}^u\sigma_{2T}^u\sigma_{2C}^u}}$	$-\frac{1}{2}\left(\frac{1}{\sigma_1^{u\,2}} + \frac{1}{\sigma_2^{u\,2}} - \frac{1}{\sigma_3^{u\,2}}\right)$	$\frac{-1}{\sigma_1^{u\,2}}$	$-\frac{1}{2}\left(\frac{1}{\sigma_{1T}^u\sigma_{1C}^u} + \frac{1}{\sigma_{2T}^u\sigma_{2C}^u} - \frac{1}{\sigma_{3T}^u\sigma_{3C}^u}\right)$	$\frac{-K_{12}}{\sigma_1^u \cdot \sigma_2^u}$
F_{13}	$\frac{-1}{2\sqrt{\sigma_{1T}^u\sigma_{1C}^u\sigma_{3T}^u\sigma_{3C}^u}}$	$-\frac{1}{2}\left(\frac{1}{\sigma_3^{u\,2}} + \frac{1}{\sigma_1^{u\,2}} - \frac{1}{\sigma_2^{u\,2}}\right)$	0	$-\frac{1}{2}\left(\frac{1}{\sigma_{3T}^u\sigma_{3C}^u} + \frac{1}{\sigma_{1T}^u\sigma_{1C}^u} - \frac{1}{\sigma_{2T}^u\sigma_{2C}^u}\right)$	$\frac{-K_{13}}{\sigma_1^u \cdot \sigma_3^u}$
F_{23}	$\frac{-1}{2\sqrt{\sigma_{2T}^u\sigma_{2C}^u\sigma_{3T}^u\sigma_{3C}^u}}$	$-\frac{1}{2}\left(\frac{1}{\sigma_2^{u\,2}} + \frac{1}{\sigma_3^{u\,2}} - \frac{1}{\sigma_1^{u\,2}}\right)$	0	$-\frac{1}{2}\left(\frac{1}{\sigma_{2T}^u\sigma_{2C}^u} + \frac{1}{\sigma_{3T}^u\sigma_{3C}^u} - \frac{1}{\sigma_{1T}^u\sigma_{1C}^u}\right)$	$\frac{-K_{23}}{\sigma_2^u \cdot \sigma_3^u}$
F_{11}	$\frac{1}{\sigma_{1T}^u \cdot \sigma_{1C}^u}$	$\frac{1}{\sigma_1^{u\,2}}$	$\frac{1}{\sigma_1^{u\,2}}$	$\frac{1}{\sigma_{1T}^u \cdot \sigma_{1C}^u}$	$\frac{1}{\sigma_1^{u\,2}}$
F_{22}	$\frac{1}{\sigma_{2T}^u \cdot \sigma_{2C}^u}$	$\frac{1}{\sigma_2^{u\,2}}$	$\frac{1}{\sigma_2^{u\,2}}$	$\frac{1}{\sigma_{2T}^u \cdot \sigma_{2C}^u}$	$\frac{1}{\sigma_2^{u\,2}}$
F_{33}	$\frac{1}{\sigma_{3T}^u \cdot \sigma_{3C}^u}$	$\frac{1}{\sigma_3^{u\,2}}$	0	$\frac{1}{\sigma_{3T}^u \cdot \sigma_{3C}^u}$	$\frac{1}{\sigma_3^{u\,2}}$
F_{44}	$\frac{1}{\sigma_{23}^{u\,2}}$	$\frac{1}{\sigma_{23}^{u\,2}}$	0	$\frac{1}{\sigma_{23}^{u\,2}}$	$\frac{1}{\sigma_{23}^{u\,2}}$
F_{55}	$\frac{1}{\sigma_{13}^{u\,2}}$	$\frac{1}{\sigma_{13}^{u\,2}}$	0	$\frac{1}{\sigma_{13}^{u\,2}}$	$\frac{1}{\sigma_{13}^{u\,2}}$
F_{66}	$\frac{1}{\sigma_{12}^{u\,2}}$	$\frac{1}{\sigma_{12}^{u\,2}}$	$\frac{1}{\sigma_{12}^{u\,2}}$	$\frac{1}{\sigma_{12}^{u\,2}}$	$\frac{1}{\sigma_{12}^{u\,2}}$

σ_1^u, σ_2^u, σ_3^u: normal strength of the lamina in the 1, 2 and 3 directions.
σ_{23}^u, σ_{13}^u, σ_{12}^u: shear strengths of the material in the 23, 31 and 12 planes.
* σ_1^u, σ_2^u, σ_3^u: σ_{1C}^u, σ_{2C}^u, σ_{3C}^u or σ_{1T}^u, σ_{2T}^u, σ_{3T}^u depending on the sign of σ_1, σ_2 and σ_3 respectively.
† K_{12}, K_{13} and K_{23}: strength coefficients depending on the material.

Although presenting some noticeable features, such as invariance under rotation of co-ordinates and transformation according to established tensorial relations, these criteria do not take into account the different damage mechanisms that promote laminate failure. In fact, these criteria take into account the lack of isotropy of composite laminates in terms of macromechanical variables (stresses) using appropriate constitutive equations, but do not account for the lack of homogeneity of these materials. It is clear that the lack of homogeneity govern the type of failure.
Furthermore, there are some other issues worth noticing when using some polynomial criteria, such as the fact that it is predicted that failure under biaxial *tensile* stresses depends on the *compressive* strengths. This is unacceptable from the physical point of view.
In order to deal with the non-homogeneous character of composites a second group of criteria has been proposed:

b) Failure criteria associated with failure modes

These criteria consider that the non-homogeneous character of composites leads different failure modes of the constituents. The criteria are established in terms of mathematical expressions using the material strengths, and consider the different failure modes of the constituents. These criteria have the advantage of being able to predict failure modes, being therefore adequate to be used in a progressive damage analysis.
The majority of the criteria proposed identify the following failure modes:
- Fibre fracture.
- Transverse matrix cracking.
- Shear matrix cracking.

Failure criteria associated with failure modes can be further sub-divided in two sub-groups:

b.1) Non-interactive: do not take into account interactions between stresses/strains acting on a lamina. This fact typically leads to errors in the strength predictions when multiaxial states of stress occur in a structure. Typical examples of non-interactive criteria are:

Maximum Strain criterion

This criterion considers that the composite fails when the strain exceeds the respective allowable, being a simple and direct way to predict failure of composites. Three different conditions of failure are considered in correspondence with a maximum strain in fibre direction, matrix or transversal direction and for shear strains.

- Fibre: $\varepsilon_1 \geq \varepsilon_{1T}^u$ or $|\varepsilon_1| \geq \varepsilon_{1C}^u$
- Matrix: $\varepsilon_2 \geq \varepsilon_{2T}^u$ or $|\varepsilon_2| \geq \varepsilon_{2C}^u$
- Shear: $|\varepsilon_{12}| \geq \varepsilon_{12}^u$

Maximum Stress criterion

This criterion considers that the composite fails when the stress exceeds the respective allowable. As in the previous case, is a simple and direct way to predict failure of composites and no interaction between the stresses acting on the lamina is considered.
Three different conditions of failure are considered:

- Fibre: $\sigma_1 \geq \sigma_{1T}^u$ or $|\sigma_1| \geq \sigma_{1C}^u$
- Matrix: $\sigma_2 \geq \sigma_{2T}^u$ or $|\sigma_2| \geq \sigma_{2C}^u$
- Shear: $|\sigma_{12}| \geq \sigma_{12}^u$

b.1) Interactive: take into account interactions between stresses/strains acting on a lamina. Examples of interactive failure criteria are:

- Hashin-Rotem [15]

This criterion involves two failure mechanisms, one associated with fibre failure and the other with matrix failure, distinguishing between tension and compression.

Fibre failure in tension: ($\sigma_1 > 0$)

$$\sigma_1 = \sigma_{1T}^u$$

Fibre failure in compression: ($\sigma_1 < 0$)

$$-\sigma_1 = \sigma_{1C}^u$$

Matrix failure in tension: ($\sigma_2 > 0$)

$$\left(\frac{\sigma_2}{\sigma_{2T}^u}\right)^2 + \left(\frac{\sigma_{12}}{\sigma_{12}^u}\right)^2 = 1$$

Matrix failure in compression: ($\sigma_2 < 0$)

$$\left(\frac{\sigma_2}{\sigma_{2C}^u}\right)^2 + \left(\frac{\sigma_{12}}{\sigma_{12}^u}\right)^2 = 1$$

- Hashin [3]

Hashin later proposed a failure criterion for fibrous composites under a three-dimensional state of stress. For the matrix failure mode, a quadratic approach was chosen because a linear criterion underestimates the material strength, and a polynomial of higher degree would be too complicated to deal with. Furthermore, the effect of the shear stress is now taken into account in the tensile fibre mode:

Fibre failure in tension: ($\sigma_1 > 0$)

$$\left(\frac{\sigma_1}{\sigma_{1T}^u}\right)^2 + \frac{\sigma_{12}^2 + \sigma_{13}^2}{(\sigma_{12}^u)^2} = 1 \quad \text{or} \quad \sigma_1 = \sigma_{1T}^u$$

Fibre failure in compression: ($\sigma_1 < 0$)

$$-\sigma_1 = \sigma_{1C}^u$$

Matrix failure in tension: ($(\sigma_2 + \sigma_3) > 0$)

$$\left(\frac{\sigma_2 + \sigma_3}{\sigma_{2T}^u}\right)^2 + \frac{\sigma_{23}^2 - \sigma_2\sigma_3}{(\sigma_{23}^u)^2} + \frac{\sigma_{12}^2 + \sigma_{13}^2}{(\sigma_{12}^u)^2} = 1$$

Matrix failure in compression: ($(\sigma_2 + \sigma_3) < 0$)

$$\left[\left(\frac{\sigma_{2C}^u}{2\sigma_{23}^u}\right)^2 - 1\right]\frac{\sigma_2 + \sigma_3}{\sigma_{2C}^u} + \left(\frac{\sigma_2 + \sigma_3}{2\sigma_{23}^u}\right)^2 + \frac{\sigma_{23}^2 - \sigma_2\sigma_3}{(\sigma_{23}^u)^2} + \frac{\sigma_{12}^2 + \sigma_{13}^2}{(\sigma_{12}^u)^2} = 1$$

- Puck

Two different types of failure or fracture are considered: inter-fibre fracture (matrix cracking) and fibre fracture.
The most noticeable difference between this criteria and the ones proposed by Hashin is that three modes of matrix cracking are considered, differing in the angle between the fracture plane and the lamina, as well as in the type of load which causes the fracture, as shown in Figure 2.

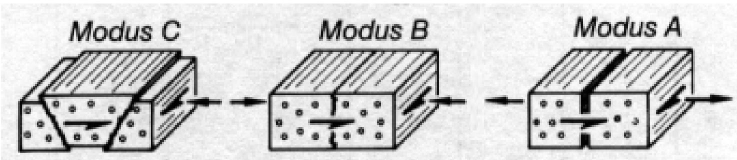

Figure 2- Inter-Fibre Fracture modes A, B and C (from [16])

Other criteria worth investigation are: Cuntze [17], Yamada-Sun [18], Koop-Michaeli [19], Kroll-Hufenbach [20], Sun-Tao [11], Zinoviev [21], Gosse [22], and Hart-Smith [5].

Laminate Strength Analyses

The failure criteria previously described deal with lamina failure. In order to predict laminate strength, the progressive accumulation of damage leading to final failure needs to be taken into account. Clearly, this is a difficult task, since failure mechanisms in laminates are a great deal more complicated than those in a unidirectional composite under in-plane loading. New damage mechanisms, such as delamination, and complex interactions between intralaminar and interlaminar damage mechanisms may occur in a laminate. The effects of delamination are usually treated separately from intralaminar damage mechanisms, although recent work has taken into consideration all the damage mechanisms in the failure analysis of a skin-stiffener composite structure [23].
Experimental evidence [24] has shown that the failure in a laminated composite is very often progressive in nature, occurring by a process of damage accumulation. Therefore, the

progressive loss of lamina stiffness must be taken into account as a function of the type of damage predicted. The typical procedure to predict the strength of a laminate when intralaminar damage mechanisms are dominant is:

1. Lamina strain and stress analyses.
2. Lamina failure criteria.
3. Stiffness degradation models (as a function of type of failure predicted at lamina level).
4. Laminate failure criterion.

The stiffness degradation (point 3) is usually performed using a reduction of ply elastic properties, typically reducing E_1 for fibre failures and both E_2 and G_{12} for matrix transverse of shear cracking [25]-[26]. This reduction may be sudden [25] or progressive [26]. For transverse matrix cracking, the progressive degradation of elastic properties has a good physical basis, since it represents the progressive accumulation of transverse cracks until the crack density saturation (CDS) is achieved. The reduction of the transverse elastic properties can also be a function of the stress state [25]. This consideration is also reasonable, since a matrix crack under compressive stresses can still carry some load.

A number of procedures have been proposed to determine ultimate laminate failure (point 4). A common procedure is to assume ultimate laminate failure when fibre fracture occurs in any lamina. This procedure is inadequate when stress concentrations are present, like in access holes and bolted joints, since localised fibre fracture, actually *relieving* stress concentrations, occur without laminate failure [27]. Furthermore, in matrix dominated laminates, such as $(\pm 45°)_s$ laminates, failure may occur without fibre fracture.

It is then clear that the guidelines for implementation of lamina failure criteria should be based on a study taking into account no only lamina failure criteria, but also stiffness degradation models and laminate failure criterion.

1.2.2.1.3 Other approaches for laminate failure

Methods based on Fracture Mechanics have also been proposed to predict laminate fracture. This type of approach has been successfully used to predict laminate failure in the presence of stress concentrations, and can accurately simulate hole size effects in laminates (characterized by a strength decrease for larger hole sizes in laminates without finite width effects). Methods based on Fracture Mechanics require more experimental information than the method previously described. However, since virtually all composite structures contain stress concentrations, e.g. joints, it is considered that methods based on fracture mechanics should also be investigated.

One approach is based on the Whitney-Nuismer [28] failure criterion for unloaded holes and the parameters considered are the unnotched tensile strength and a characteristic dimension. Two approaches were proposed, the point stress and the average stress methods. In the first method it was assumed that failure occurs when the direct stress in the direction of the load at a distance d_{0t} away from the hole, measured in the tension plane, is equal to or greater than the strength of the unnotched material. The second method considered that failure occurs when the average stress over some distance a_{0t} equals the unnotched material strength. These distances were considered to be a material property. These criteria were formulated for the case of uniaxial tension where combined stresses play an inconsequential role in the failure process. In cases where this cannot be assumed, these criteria must be recast.

Other approach is the use damage zone models (DZM), where damage around the hole is represented by an equivalent crack with cohesive forces acting at the crack surfaces [29]. This crack represents matrix cracking and delamination in the traction case, and fibre microbuckling and delamination in the compression case. A linearly decreasing relation between the cohesive stress and the crack opening, v, is assumed, representing the increase in the extent of damage with increasing load. The stress at the crack tip is assumed equal to the unnotched laminate strength, as shown in Figure 4.

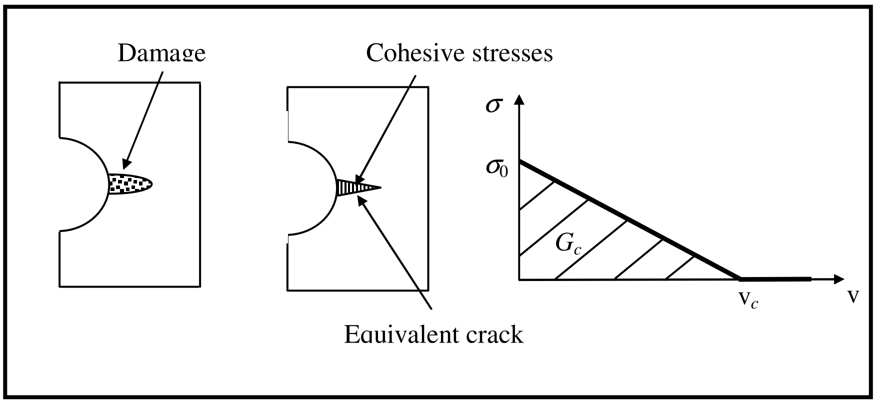

Figure 4- Damage zone model

Another alternative to predict laminate failure is the use of continuum damage models. These models typically only deal with matrix cracking and need to be further developed to be used as a design tool.

1.2.2.1.4 Delamination

Delamination is one of the predominant forms of failure in laminated composites due to the lack of reinforcement in the thickness direction. Delamination as a result of impact or a manufacturing defect can cause significant reductions in the compressive load-carrying capacity and bending stiffness of a structure. The stress gradients that occur near geometric discontinuities such as ply drop-offs, stiffener terminations and flanges, bonded and bolted joints, and access holes promote delamination initiation, trigger intralaminar damage mechanisms, and may cause a significant loss of structural integrity. Without including the delamination failure mode, the predictive capabilities of progressive failure analyses will remain limited

The analysis of delamination is commonly divided into the study of the initiation and the analysis of the propagation of an already initiated area. Delamination initiation analysis is usually based on stresses and use of criteria such as the quadratic interaction of the interlaminar stresses in conjunction with a characteristic distance [30]. This distance is a function of specimen geometry and material properties, and its determination always requires extensive testing.

Delamination propagation, on the other hand, is usually predicted using Fracture Mechanics. The Fracture Mechanics approach avoids the difficulties associated with the stress singularity at a crack front. Two main approaches have been proposed:

- The virtual crack closure technique (VCCT), based on the assumption that when a crack extends by a small amount, the energy absorbed in the process is equal to the work required to close the crack to its original length [31].
- The use of decohesion finite elements placed between the composite material layers [32]. Decohesion elements combine a stress based formulation with a Fracture Mechanics based formulation and are used to define the non-linear constitutive law of the material at the interface between laminae. This approach has been used to simulate delamination onset and growth in laminated composites [23], [32], Figure 5.

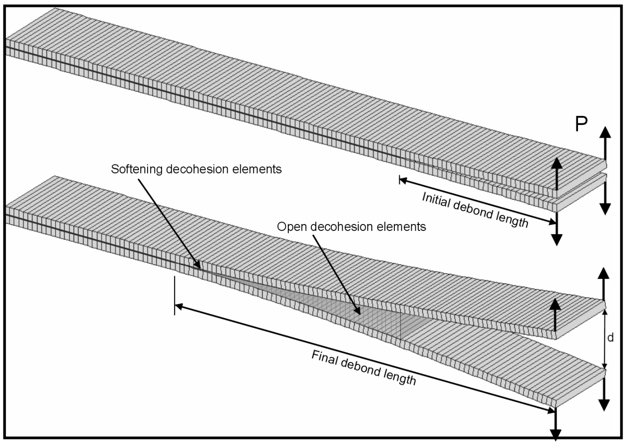

Figure 5- Simulation of delamination growth in a DCB test specimen (from [23])

1.2.2.1.5 Main difficulties and possible approach to tackle them

The main difficulties associated with a procedure to predict failure in composites and the possible approaches to deal with them are presented in Table 2.

Table 2: Main difficulties

DIFFICULTY	POSSIBLE APPROACH
1. Effect of residual thermal stresses There is no consensus whether residual thermal stresses should be taken into account in the analyses.	The effect of residual thermal stresses and their possible relaxation due to viscoelastic behavior of the polymer matrix and/or moisture absorption can be assessed by micromechanical models available in the literature [33].
2. In-situ properties In-situ lamina transverse and shear strengths can be up to 2.5 times higher than those measured using a single lamina.	It is clear that matrix cracking is influenced by adjacent plies. It should be noticed that there are micromechanical models based on Fracture Mechanics that can simulate this effect in terms of strain to first matrix cracking, and crack density saturations [33]. However, these models are quite complex, valid for simple loading conditions and laminate configurations, and require complex experimental data, being therefore unsuitable to use for design purposes. A possible solution is to use in-situ strengths, measured from laminates.
3. Non-linear behaviour in shear and transverse loading	This effect is important for cross and angle-ply laminates. It can be dealt with using higher-order polynomials or spline functions [34].
4. Laminate ultimate failure Fibre fracture in a ply not suitable as a failure criterion for matrix dominated laminates, or when stress concentrations occur.	Use predicted load-drops as ultimate loads, or a criterion based on component stiffness.
5. Reduction of elastic properties The reduction of elastic properties should be a function of type of damage predicted.	The existing micromechanical models to predict reduction of elastic properties are quite complex and valid only for matrix transverse cracking [33]. The reduction of the elastic properties to zero as a function of type of damage predicted is a possible solution.
6. Lamina shear strength The lamina shear strength increases with increasing compressive transverse stresses.	Use failure criteria that include an interaction term between the compressive transverse stress and the shear strength [11].
7. Effects of delamination Failure loads even on simple specimens under tension may be affected by the presence of delaminations.	The strength of a laminate is affected by the stacking sequence due to the interlaminar stresses at free-edges leading to delamination: the ultimate load of a $(\pm 30^o)_{2s}$ laminate is approximately 30% higher than the ultimate load of a $(+30^o_2/-30^o_2)_s$ laminate. A possible solution is the use of cohesive zone models.
8. Prediction of fiber compressive failure	For some fiber types, the failure criteria used to predict fiber compressive failure are not directly related with the physical phenomena leading to failure. Local (micro) instability is often the starting point of compressive

	failure, followed by shear failure of the fibers in the case of carbon fibers. The stresses acting perpendicular to the fiber axis, although not important for the failure of the fiber, can be important for bucking onset.

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