

Cooperative Moving Path Following using Event based Control and Communication

R. Praveen Jain, Andrea Alessandretti, A. Pedro Aguiar, and João Borges de Sousa

Abstract—This paper introduces a Cooperative Moving Path Following (CMPF) approach where the robotic vehicles are required to converge to a desired geometric path specified with respect to a *moving* frame of reference, while maintaining a desired formation. This is in contrast to the existing Cooperative Path Following framework, where the robotic vehicles perform a coordinated maneuver to follow a *fixed* reference path. The results on the path following controllers are extended to incorporate the effects of moving frame of reference of the path, leading to a Lyapunov-based nonlinear control law for a moving path following motion control problem of an individual robotic vehicle. In order to achieve cooperation, the moving path following controllers are augmented with decentralized, first order consensus approach, resulting in a Cooperative Moving Path Following architecture. Furthermore, event-based control and communication methods are applied at the cooperation level to reduce the frequency of information exchange between the robotic vehicles. The advantage of the CMPF method proposed lies in its flexibility to choose different parameterizable paths and is illustrated in simulations through coordinated source seeking and convoy protection scenarios using under-actuated robotic vehicles.

I. INTRODUCTION

Formation control of robotic vehicles is a problem of practical importance and finds interesting applications in target tracking [1], convoy protection [2] and cooperative transportation [3]. One possible strategy to solve the formation control problem is to employ a two layered control structure. The lower layer is responsible for the motion control of the individual robotic vehicle, called the *Path Following* (PF) controller [4]. The higher layer is responsible for cooperation among the robots called the *Cooperative* controller. The combination of these two control subsystems forms the Cooperative Path Following (CPF) described in [5]. In the CPF framework, each robot is tasked to converge to a desired geometric path that is specified a priori and is expressed with respect to a stationary frame of reference. A desired formation is maintained through the cooperative controller which is usually a consensus controller [6].

The inherent assumption in the PF framework, that the desired geometric path is expressed with respect to a fixed reference frame is not always advantageous. It is interesting to consider the control problems where the desired path is

expressed with respect to a reference frame that is *moving* with respect to time. Such problems arise in the context of applications such as convoy protection or autonomous landing on a moving platform, where the robotic vehicle needs to follow a path that is expressed with respect to a reference frame attached to the moving platform. These problems cannot be addressed in the classical PF framework. Hence, a Moving Path Following (MPF) approach was first introduced in [7] for a single fixed wing Unmanned Aerial Vehicle (UAV), that reduces to a Path Following problem for a stationary target. A nonlinear model predictive control approach was adopted in [8] to solve the MPF problem for a constrained under-actuated vehicle. The MPF framework offers a generalization to the classical Path Following framework, thereby increasing the domain of the motion control problems that can be addressed with such a framework.

The next logical extension to the research on MPF is to consider multiple robotic vehicles following a moving path while cooperating with one another. This leads to a new multi-vehicle motion cooperation framework termed the Cooperative Moving Path Following (CMPF), that is investigated in this paper. Similar to the CPF method, the proposed CMPF strategy consists of an interconnection (or sometimes just a cascade) of two subsystems. The PF controller for an individual robotic vehicle is replaced by the MPF controller at the motion control level. The cooperation layer remains unchanged, which is usually a consensus controller. The need for a CMPF framework can be motivated through an example of source localization [9] or coordinated source seeking [10] by autonomous vehicles. In such applications, the robotic vehicles need to position itself, forming an optimal sensing geometry around a possibly moving source in order to estimate its position [11]. For autonomous vehicles such as UAVs, it is not possible to remain stationary while forming an optimal sensing geometry around the source. Hence an orbiting maneuver around the source is proposed in [12], [13] such that the multiple UAVs are distributed around the circle, centered at the assumed source position to form an equilateral polygon. This problem can be formulated as a CMPF problem and is illustrated in this paper through simulations. However, the true advantage of the CMPF framework lies in its flexibility to choose the desired geometric path that does not require to be a circular path. This allows reformulation of various motion cooperation tasks that require non-circular parameterizable path specifications as a CMPF problem. One example is the use of lemniscate path for convoy protection as shown in [7] for a single UAV. Motivated by these observations, the CMPF framework is formally introduced and

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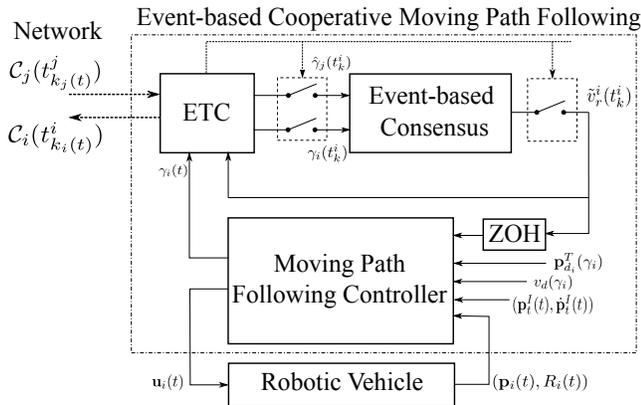


Figure 1: Event based CMPF framework

solved along the lines of CPF. Specifically, a Lyapunov-based nonlinear control law is proposed for an under-actuated robotic vehicle that steers the robot to the desired moving path at a pre-specified nominal reference speed. Then, a consensus law with event based communication proposed in [14] is used to coordinate multiple-robotic vehicles while reducing the frequency of inter-vehicle communication. The efficacy of the proposed CMPF approach is illustrated through simulations for two scenarios: i) coordinated source seeking scenario, where three Autonomous Surface Vessels (ASVs) are required to orbit around a known possibly mobile source, and ii) convoy protection using a group of UAVs that follow a moving lemniscate path. Due to the limitations of space, proofs are excluded and pointers to relevant references are provided.

The rest of the paper is organized as follows, Section II formulates the CMPF problem addressed in this paper. The main result is provided in Section III which includes the MPF control design and event based communication strategy for multi-robot cooperation. The simulation results are provided in Section IV followed by conclusion in Section V.

II. PROBLEM FORMULATION

Consider N robotic vehicles and a coordinate frame $\{R_i\}$ for all $i \in \mathcal{I} := \{1, 2, \dots, N\}$ attached to each vehicle. Origin of the frame $\{R_i\}$ describes the position of the robotic vehicle $\mathbf{p}_{R_i}^I(t) \in \mathbb{R}^n$, expressed with respect to an inertial frame of reference $\{I\}$. Here, $n = 3$ for robotic vehicles moving in a three dimensional space, and $n = 2$ for robots constrained to two dimensional space. The Event-based CMPF problem is divided into two separate control problems working together and consists of i) *Event-based Cooperative Controller*, which forms the upper layer - tasked to achieve cooperation between the robotic vehicles in an event-based manner, and ii) *Moving Path Following* controller which forms the lower layer and is responsible to guide the robots to a desired moving geometric path with a desired speed profile as illustrated in Figure 1 for a generic vehicle $i \in \mathcal{I}$.

Let $\mathbf{p}_{d_i}^T(\gamma_i) \in \mathbb{R}^n$ be a fixed reference geometric path parameterized by $\gamma_i \in \mathbb{R}$ expressed with respect to a (possibly moving) frame $\{T\}$ and $v_d \in \mathbb{R}$ be a common desired speed assignment for all $\gamma_i, i \in \mathcal{I}$. The parameter γ_i could be the

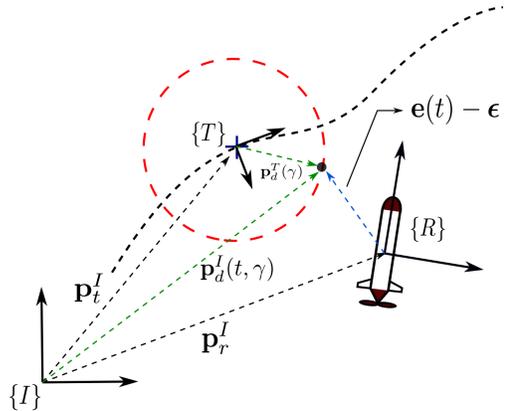


Figure 2: Coordinate frames for MPF problem

arc length along the reference path and the value of γ_i could be seen as a virtual point moving along the reference path. Then, the speed assignment v_d dictates the evolution of the virtual point over time. The position (origin) and velocity of the moving frame $\{T\}$ expressed with respect to the inertial frame $\{I\}$ is denoted as $\mathbf{p}_t^I \in \mathbb{R}^n$ and $\mathbf{v}_t^I \in \mathbb{R}^n$, respectively. Then, the desired moving path for a vehicle i at any time t satisfies $\mathbf{p}_{d_i}^I(t, \gamma_i) = \mathbf{p}_t^I(t) + \mathbf{p}_{d_i}^T(\gamma_i)$. Roughly speaking, the control objective is to steer the robotic vehicle to the vicinity of the desired moving path while achieving cooperation over the variables γ_i , henceforth referred to as cooperation variables or path variables. Figure 2 illustrates the coordinate frames and variables of interest for a single robotic vehicle.

In this paper, we model the ASVs/UAVs using the kinematic model for an under-actuated vehicle given as,

$$\begin{aligned} \dot{\mathbf{p}}_{r_i}^I(t) &= R_{R_i}^I(t) \mathbf{v}_{r_i}^{R_i}(t) \\ \dot{R}_{R_i}^I(t) &= R_{R_i}^I(t) S(\boldsymbol{\omega}_{r_i}(t)) \end{aligned} \quad (1)$$

where $\mathbf{p}_{r_i}^I(t) \in \mathbb{R}^n$ is the position of the robotic vehicle expressed in inertial frame, $\mathbf{v}_{r_i}^{R_i}(t) = [v_{f_i}(t) \ \mathbf{0}']' \in \mathbb{R}^n$ is the linear velocity of the vehicle expressed in the robot body frame $\{R_i\}$ and $\boldsymbol{\omega}_{r_i}(t) \in \mathbb{R}^n$ is the angular velocity of the robotic vehicle. The rotation matrix $R_{R_i}^I(t)$ represents the orientation of the robot frame $\{R_i\}$ with respect to the inertial frame of reference. The control inputs for the vehicle $i \in \mathcal{I}$ is defined as $\mathbf{u}_i(t) = [v_{f_i}(t) \ \boldsymbol{\omega}_{r_i}(t)']'$. It is assumed that there exist an inner loop controller that is responsible to track the actuator references $\mathbf{u}_i(t)$ provided by the MPF controller. In the following, the notation of dependence of the variables on time are dropped when clear from the context.

We now state the MPF motion control problem as follows:

Problem 1 (Moving Path Following). Given a known trajectory of the moving frame \mathbf{p}_t^I with time derivative \mathbf{v}_t^I and the desired geometric path $\mathbf{p}_{d_i}^T(\gamma_i)$ with desired parameter speed v_d , the Moving Path Following control problem is to design a control law for $\mathbf{u}_i(t)$ that steers the vehicle along the desired moving path $\mathbf{p}_{d_i}^I(t, \gamma_i) = \mathbf{p}_t^I + \mathbf{p}_{d_i}^T(\gamma_i)$. Specifically, we wish to drive the term $\|\mathbf{p}_{r_i}^I - \mathbf{p}_{d_i}^I(t, \gamma_i)\|$ toward an arbitrarily small neighborhood of the origin as $t \rightarrow \infty$. Furthermore,

the robotic vehicle must satisfy the desired speed assignment, $\|\dot{\gamma}_i - v_d\| \rightarrow 0$ as $t \rightarrow \infty$. \square

In order to achieve the latter objective, the following condition is imposed for the dynamics of path parameter γ_i ,

$$\dot{\gamma}_i = v_d + \tilde{v}_r^i(t) \quad (2)$$

where $\tilde{v}_r^i(t) \in \mathbb{R}$ is the cooperative control actuation signal obtained from the event-based cooperative control system. Assuming that the MPF controller achieves the desired objectives on each robotic vehicle, the Event-based Cooperative control problem, stated next, has the task of synchronizing the virtual points or the path variables.

Problem 2 (Event-based Cooperative Control). Given the path variables γ_i , $i \in \mathcal{I}$, for the N robotic vehicles, and an imposed communication topology between the vehicles, the objective of the cooperative control system is to design a decentralized cooperative control actuation signal \tilde{v}_r^i together with a decentralized Event-Triggering Condition (ETC) such that: 1) the position of the virtual points (denoted by γ_i) along the reference path is synchronized, that is, $\|\gamma_i - \gamma_j\| \rightarrow 0$ for all $i, j \in \mathcal{I}$ as $t \rightarrow \infty$; and 2) there is no data transmissions between the robots while the ETC is valid. Once it breaks, an event is generated by the system, leading to information transmission and an update of the cooperative control actuation signal. \square

III. MAIN RESULTS

A. Moving path following

In this section, a globally exponentially stabilizing nonlinear control law is designed to solve the MPF motion control problem. To this end, define an error variable $\mathbf{e}_i = (R_{R_i}^I)'(\mathbf{p}_{r_i}^I - \mathbf{p}_{d_i}^I(t, \gamma_i)) + \epsilon$. The vector $\epsilon \in \mathbb{R}^n$ is a constant vector that can be made arbitrarily small. Taking the time derivative of the error variable \mathbf{e}_i , we have

$$\dot{\mathbf{e}}_i = \left(\dot{R}_{R_i}^I\right)'(\mathbf{p}_{r_i}^I - \mathbf{p}_{d_i}^I) + (R_{R_i}^I)'(\dot{\mathbf{p}}_{r_i}^I - \dot{\mathbf{p}}_{d_i}^I) \quad (3)$$

The time derivative of $\dot{\mathbf{p}}_{d_i}^I(t, \gamma)$ satisfies

$$\dot{\mathbf{p}}_{d_i}^I = \mathbf{v}_t^I + \frac{\partial \mathbf{p}_{d_i}^T}{\partial \gamma_i}(v_d + \tilde{v}_r^i) \quad (4)$$

Using the time derivative of $\dot{\mathbf{p}}_{d_i}^I$ and the robot dynamics (1) in (3) results in

$$\begin{aligned} \dot{\mathbf{e}}_i &= (R_{R_i}^I S(\omega_{r_i}))'(\mathbf{p}_{r_i}^I - \mathbf{p}_{d_i}^I) + \\ & (R_{R_i}^I)' \left(R_{R_i}^I \mathbf{v}_{r_i}^{R_i} - \mathbf{v}_t^I - \frac{\partial \mathbf{p}_{d_i}^T}{\partial \gamma_i}(v_d + \tilde{v}_r^i) \right) \\ &= -S(\omega_{r_i})(\mathbf{e}_i + \epsilon) + \\ & (R_{R_i}^I)' \left(R_{R_i}^I \mathbf{v}_{r_i}^{R_i} - \mathbf{v}_t^I - \frac{\partial \mathbf{p}_{d_i}^T}{\partial \gamma_i}(v_d + \tilde{v}_r^i) \right) \\ &= -S(\omega_{r_i})\mathbf{e}_i + \Delta \mathbf{u}_i \\ & - (R_{R_i}^I)' \mathbf{v}_t^I - (R_{R_i}^I)' \frac{\partial \mathbf{p}_{d_i}^T}{\partial \gamma_i}(v_d + \tilde{v}_r^i) \end{aligned} \quad (5)$$

where

$$\Delta = \begin{bmatrix} 1 & -\epsilon_2 \\ 0 & \epsilon_1 \end{bmatrix} \quad \text{or} \quad \Delta = \begin{bmatrix} 1 & 0 & \epsilon_3 & -\epsilon_2 \\ 0 & -\epsilon_3 & 0 & \epsilon_1 \\ 0 & \epsilon_2 & -\epsilon_1 & 0 \end{bmatrix}$$

for the case of horizontal plane (2D), or for the general case (3D), respectively. We consider a realistic situation where \mathbf{e}_i is not precisely known, instead only an estimate of \mathbf{e}_i , denoted as $\hat{\mathbf{e}}_i$ is known. Let $\tilde{\mathbf{e}}_i = \hat{\mathbf{e}}_i - \mathbf{e}_i$ be the estimation error and assume that ϵ is selected such that Δ is invertible and the term $|\frac{\partial \mathbf{p}_{d_i}^T}{\partial \gamma_i}|$ is bounded. Then, the following result holds.

Proposition 1 (Moving Path Following). *Consider the robotic vehicle (1) in closed loop with the feedback control law*

$$\mathbf{u}_i = \Delta^+ \left(-K_{p_i} \hat{\mathbf{e}}_i + (R_{R_i}^I)' \mathbf{v}_t^I + (R_{R_i}^I)' \frac{\partial \mathbf{p}_{d_i}^T}{\partial \gamma_i}(v_d + \tilde{v}_r^i) \right) \quad (6)$$

where Δ^+ is the Moore-Penrose pseudo inverse, K_{p_i} is a known positive definite gain matrix and ϵ chosen such that the matrix Δ in (5) is invertible. Then, the origin $\mathbf{e}_i = 0$ of the closed loop system (5) with control law (6) is Input-to-State Stable (ISS) with respect to the estimation error $\tilde{\mathbf{e}}_i$, and the cooperative control actuation signal \tilde{v}_r^i .

Proof. See [8], [14] for the proof. \square

Remark 1. The constant vector ϵ is introduced in order to have direct control over the angular velocity of the robotic vehicle. Setting $\epsilon = 0$ in (5) reveals that the control input ω_{r_i} does not appear in the error dynamics and the ability to steer the robot towards the moving path is lost.

Remark 2. Note that in the absence of estimation errors and the cooperative control input, the error $\|\mathbf{e}_i\|$ converges exponentially to zero as $t \rightarrow \infty$. Therefore, the moving path following error $\|\mathbf{p}_{r_i}^I - \mathbf{p}_{d_i}^I\| \rightarrow \|\epsilon\|$ as $t \rightarrow \infty$. Consequently, the robotic vehicle converges to an arbitrary small neighborhood of the origin.

B. Event-based cooperative control

Distributed consensus algorithms have been at the core of coordinated motion control problems. In a multi-robot system, cooperation is achieved by exchange of certain variables of interest (states, measurements, etc.) over the communication network with the neighboring robots within the communication range. The distributed consensus algorithms then enable the robots to reach an agreement over the variables of interest. In the context of CMPF, the cooperation variables γ_i are the variables of interest that needs to be agreed upon i.e., $\|\gamma_i - \gamma_j\| \rightarrow 0$ for all $i, j \in \mathcal{I}$. When an agreement is reached, the robotic vehicles are said to be cooperating by maintaining a desired formation. In order to avoid constant communication over the network, we utilize event-based control and communication results of [14] to achieve cooperation with reduced transmissions over the communication network. A state-dependent, decentralized Event Triggering Condition (ETC) is designed that specifies the event time instants at

Algorithm 1 Event-based Communication and Control for Agent i

Input: Information received: $C_j(t_{k_j}^j) \forall j \in \mathcal{N}_i$ and γ_i .

Output: Event-based control input: $\tilde{v}_r^i(t_k^i)$ and communication packet $C_i(t_k^i)$.

- 1: **Initialization:**
 - 2: Define $k \leftarrow 0$; $t_k^i \leftarrow t$; $\tilde{v}_r^i(t_k^i) \leftarrow 0$; $\gamma_i(t_k^i) \leftarrow \gamma_i$.
 - 3: Transmit $C_i(t_k^i)$
 - 4: **for** $t = h\tau_s$ where $h \in \mathbb{Z}_{\geq 0}$ **do**
 - 5: Estimate $\hat{\gamma}_j^i(t)$ for all $j \in \mathcal{N}_i$ using (10).
 - 6: Compute ETC using (9).
 - 7: **if** $\xi_i^2 > \frac{\sigma_i}{8} [L\hat{\gamma}_i^i]^2$ and $|\xi_i^2 - \frac{\sigma_i}{8} [L\hat{\gamma}_i^i]^2| > \varrho$ **then**
 - 8: $k \leftarrow k + 1$; $t_k^i \leftarrow t$; $\gamma_i(t_k^i) = \gamma_i(t)$
 - 9: Compute $\tilde{v}_r^i(t_k^i)$ using (7)
 - 10: Reset $\xi_i(t_k^i)$ to zero
 - 11: Transmit $C_i(t_k^i)$
 - 12: **end if**
 - 13: Output $\tilde{v}_r^i(t) \leftarrow \tilde{v}_r^i(t_k^i)$
 - 14: **end for**
-

which information needs to be transmitted by a robot i while ensuring ISS of the overall networked cooperation system.

Recall the dynamics of the cooperation variables that satisfy $\dot{\gamma}_i = v_d + \tilde{v}_r^i(t) \forall i \in \mathcal{I}$. In a CMPF framework, cooperation is achieved by employing the well studied first order consensus law [6] for the cooperative control actuation signal $\tilde{v}_r^i(t) = -\sum_{j \in \mathcal{N}_i} (\gamma_i(t) - \gamma_j(t))$ where \mathcal{N}_i denotes the neighbors of the robotic vehicle i . Implementation of this control law however requires continual availability of the cooperation variables of the neighbors. This leads to constant communication between the robotic vehicles over the network which is not desirable. Hence, we propose the use of an event-based cooperative control system, where the robotic vehicles transmit the information over the network at discrete event times.

Let t_k^i for all $k \in \mathbb{Z}_{\geq 0}$ denote the time instants, also referred to as event time, at which agent i transmits the necessary information over the network and updates its control input $\tilde{v}_r^i(t)$ for all $t \in [t_k^i, t_{k+1}^i)$. Furthermore, assume that the following conditions hold.

Assumption 1. The i^{th} robot communicates only with its neighbors $j \in \mathcal{N}_i$. Furthermore, it is able to transmit successfully at its event time t_k^i , and vice versa, with no time delays.

Assumption 2. The time on the robotic vehicles are synchronized. This is achieved in practice by using the GPS time on-board each robotic vehicle.

In order to meet the objective of reducing the frequency of communication among the agents, the following piecewise constant event-based control law, inspired from [15]

$$\tilde{v}_r^i(t) = -\sum_{j \in \mathcal{N}_i} (\hat{\gamma}_i^i(t_k^i) - \hat{\gamma}_j^i(t_k^i)) \quad (7)$$

is used, where the notation $[\cdot]_i$ is used to denote the i^{th} component of a vector and $\hat{\gamma}_i^j$ denotes the estimate of the

path variable of the agent j computed on agent i for all $j \in \mathcal{N}_i$. Define the measurement error variable $\xi_i(t) := \sum_{j \in \mathcal{N}_i} (\hat{\gamma}_i^i(t_k^i) - \hat{\gamma}_j^i(t_k^i)) - \sum_{j \in \mathcal{N}_i} (\hat{\gamma}_i^i(t) - \hat{\gamma}_j^i(t))$, then by definition $\xi_i(t_k^i) = 0$. The following proposition states the event-based control and communication strategy used to reduce the frequency of information exchange over the network.

Proposition 2 (Event-based Cooperative Control). *Given the dynamics of the cooperation variable (2), let each robotic vehicle i transmit the information packet*

$$C_i(t_k^i) := (t_k^i, \gamma_i(t_k^i), \tilde{v}_r^i(t_k^i)) \quad (8)$$

at its event time t_k^i to the neighboring agents $j \in \mathcal{N}_i$. Then, the event based cooperative control law (7) for all $t \in [t_k^i, t_{k+1}^i)$, under the Assumptions 1 - 2, makes the system ISS with respect to the cooperation variable measurement error $\xi_i(t)$, provided the Event Triggering Condition (ETC)

$$\xi_i^2 \leq \frac{\sigma_i}{8} \left(\sum_{j \in \mathcal{N}_i} (\hat{\gamma}_i^i(t) - \hat{\gamma}_j^i(t)) \right)^2 \quad (9)$$

where

$$\begin{aligned} \hat{\gamma}_i^i(t) &= \gamma_i(t) \\ \hat{\gamma}_j^i(t) &= \gamma_j(t_{k_j}^j) + (t - t_{k_j}^j)(v_d + \tilde{v}_r^j(t_{k_j}^j)) \end{aligned} \quad (10)$$

is satisfied for all $0 < \sigma_i < 1$ and $t_{k_j}^j$ denotes most recent event time on agent $j \in \mathcal{N}_i$. The next event time t_{k+1}^i is triggered when the ETC (9) is violated and is defined as

$$t_{k+1}^i = t_k^i + \min\{\tau_k^i, \tau_{\text{ub}}\} \quad (11)$$

with $\tau_k^i = \min\left\{t - t_k^i > 0 : \xi_i^2 \geq \frac{\sigma_i}{8} \left(\sum_{j \in \mathcal{N}_i} (\hat{\gamma}_i^i - \hat{\gamma}_j^i)\right)^2\right\}$ and $\tau_{\text{ub}} > 0$ is an upper bound to the inter-event time specified as a design parameter.

Proof. See Section IV of [14]. \square

The algorithm for event-based control and communication strategy adopted with sampling period τ_s is given in Algorithm 1. In order to implement the event-based consensus controller, the following modification is made to the ETC, $\xi_i^2 > (\sigma_i/8)[L\hat{\gamma}_i^i]^2$ and $|\xi_i^2 - (\sigma_i/8)[L\hat{\gamma}_i^i]^2| > \varrho$, with a small parameter ϱ (in our case $\varrho = 10^{-4}$). The second condition allows some slack for the ETC in order to prevent event-generation due to the numerical issues when the consensus has been achieved. The main result of Event-based Cooperative Moving Path Following is given by the following theorem.

Theorem 1 (Event-based CMPF). *The decentralized event-based control and communication method given by (7) with the ETC (9), along with the moving path following controller (6), collectively termed as Event-based CMPF controller, makes the overall system (1) and (2) ISS with respect to the estimation error of the MPF error variable $\tilde{e}_i(t)$, and cooperation variable measurement error $\xi_i(t)$ for all $i \in \mathcal{I}$.*

Proof. The proof directly follows from the results of Proposition 1 and 2 that are two ISS subsystems of Event-based

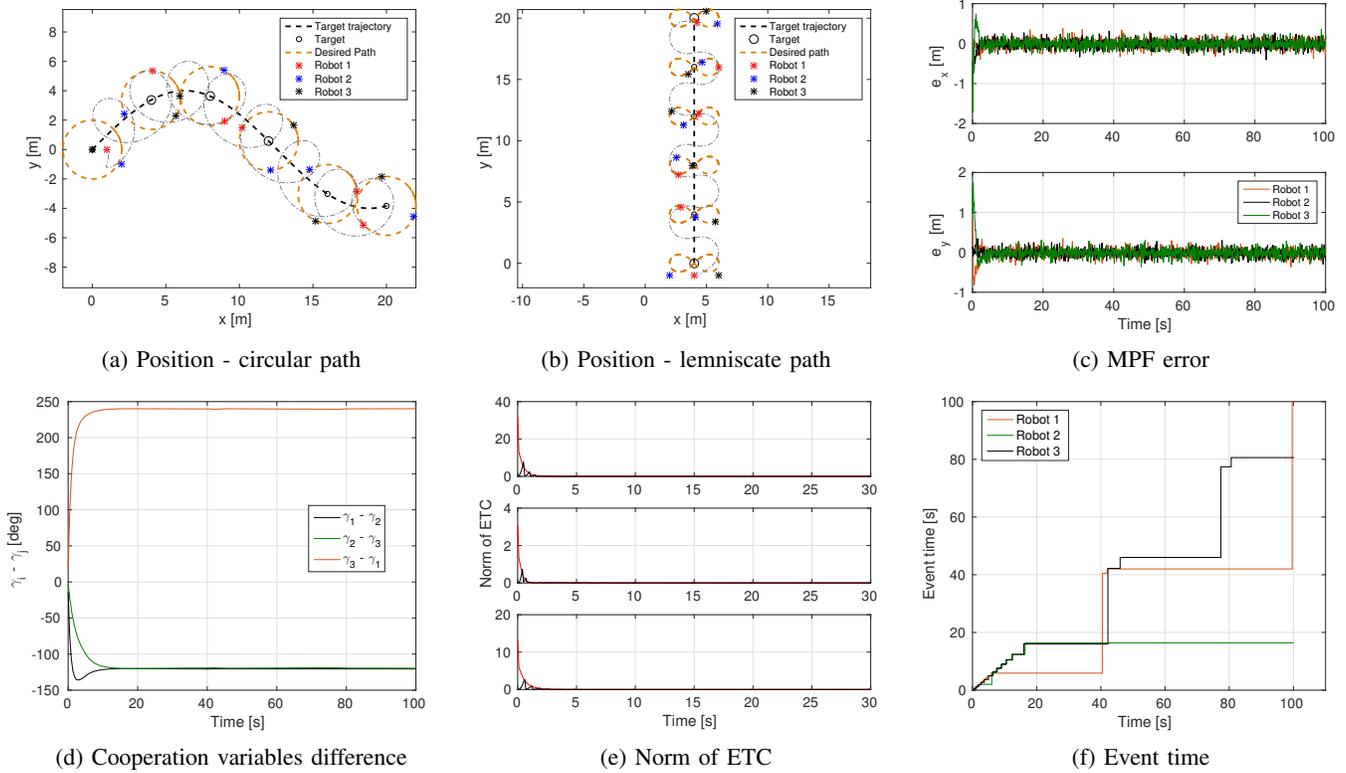


Figure 3: Event-based Cooperative Moving Path Following for coordinated source seeking and convoy protection scenarios

Table I: Event time for source localization scenario

| | ASV-1 | ASV-2 | ASV-3 |
|------------------|-------|-------|-------|
| Max τ_k [s] | 57.60 | 83.6 | 31.40 |
| Min τ_k [s] | 0.3 | 0.1 | 0.6 |
| Num Events | 16 | 14 | 16 |
| Periodic | 1000 | 1000 | 1000 |
| % Comms | 1.6 | 1.4 | 1.6 |

CMPF. The result is a direct consequence of the fact that the cascade connection of two ISS systems result in an ISS system [16]. \square

IV. SIMULATION RESULTS

The event based CPMF framework is illustrated through simulations using VirtualArena [17] with two scenarios in 2D namely, i) coordinated source seeking scenario where we consider three ASVs orbiting the target, and ii). Three UAVs in a convoy protection scenario. In the coordinated source seeking scenario, a circular path centered at the source position is specified that moves with time. A lemniscate path is specified for convoy protection scenario as described in Table II. The simulation is setup such that robotic vehicle 1 is able to communicate with 2 and 3. The desired speed of the cooperation variable was set as $v_d = 0.5$ and simulation is conducted for a simulation time of 100 seconds with sampling period 0.1 seconds. The MPF control parameters were set as follows: $K_{p_i} = \text{diag}(1, 1)$, $\epsilon = [0.2 \ 0]^T$. In the scenarios considered, it is required to uniformly distribute the vehicles along the desired path. This requires $\|\gamma_i - \gamma_j\| \rightarrow \|\gamma_{ij}^{\text{off}}\|$, where γ_{ij}^{off} is

the desired offset between cooperation variables. The results of the event-based cooperative control presented in Proposition 2 are applied over a transformed cooperation variable $\tilde{\gamma}_i := \gamma_i - \gamma_i^{\text{off}}$, where γ_i^{off} is the desired offset in the position of vehicle i along the desired path $\mathbf{p}_{d_i}^T(\gamma_i)$. The cooperative control objective $\|\tilde{\gamma}_i - \tilde{\gamma}_j\| \rightarrow 0$ implies $\|\gamma_i - \gamma_j\| \rightarrow \|\gamma_{ij}^{\text{off}}\|$. The desired offset $\gamma_i^{\text{off}} = (2\pi/N)(i - (N - 1))$ was specified. The tuning parameter in ETC was set to $\sigma_i = \sigma = 0.8$.

Figure 3a and 3b shows the position of three robotic vehicles for coordinated source seeking and convoy protection scenarios respectively. As can be seen, the robots successfully follow the desired moving path while maintaining a desired formation specified in terms of γ_i^{off} . The objective that $\|\tilde{\gamma}_i - \tilde{\gamma}_j\| \rightarrow 0$ or equivalently $\|\gamma_i - \gamma_j\| \rightarrow \|\gamma_{ij}^{\text{off}}\|$ is shown in Figure 3d. The MPF error is shown in Figure 3c where it can be seen that the signal is bounded around the origin for a zero mean, normally distributed estimation error $\tilde{\mathbf{e}}_i(t)$ with covariance of $\text{diag}(0.1, 0.1)$ introduced in the simulations. In the absence of noise, the MPF error variables $\mathbf{e}_i(t)$ converge to 0 and hence $\|\mathbf{p}_{r_i}^I - \mathbf{p}_{d_i}^I(t, \gamma_i)\| \rightarrow \|\epsilon\|$ for all $i \in \mathcal{I}$.

The norm of the ETC (in red) and the cooperative control measurement error variable ξ_i (in black) is shown in Figure 3e where events are generated when the norm of ETC is violated. The event-times are shown in Figure 3f. It can be noticed that the communication takes place more frequently during the transients. Once an agreement has been reached, the communication between the robotic vehicles ceases. For

Table II: Target position and desired path

| | Circular Path | Lemniscate Path |
|---|---|--|
| Target Position $\mathbf{p}_t^I(t)$ | $(0.2t, 4 \sin(0.05t))'$ | $(4, 0.2t)'$ |
| Desired Path $\mathbf{p}_{d_i}^T(\gamma_i)$ | $(2 \cos(\gamma_i), 2 \sin(\gamma_i))'$ | $\left(\frac{2 \cos(0.5\gamma_i)}{1+\sin(0.5\gamma_i)^2}, \frac{2 \sin(0.5\gamma_i) \cos(0.5\gamma_i)}{1+\sin(0.5\gamma_i)^2} \right)'$ |

example, it can be seen that the robot 1 does not communicate between 40 seconds to almost 100 seconds of simulation time. It is also evident that the frequency of communication between the robotic vehicles is significantly reduced. In order to appreciate the results, Table I provides the quantitative data for event times for the coordinated source seeking scenario. For ASV-1, without event-based communication there would be 1000 instances of communication for a simulation of 100 seconds with sampling period of 0.1 seconds. With event-based communication, ASV-1 has 16 instances of event generation and hence 1.6 % of communication is required which is a significant reduction. Similar results are obtained for the other two robots. Figures 3c-3f are shown for the coordinated source seeking scenario. The results for convoy protection are similar and hence not shown. From the figures, it can be seen that a CMPF approach allows specification of generic paths that can be parametrized using a variable such as γ . Consequently, such a framework allows to reformulate many motion cooperation applications as a CMPF problem. Note that when the path is stationary, the CMPF problem reduces to the CPF problem.

V. CONCLUSION

In this paper, a Cooperative Moving Path Following framework has been introduced that allows reformulation of coordinated motion control problems arising in applications such as convoy protection, source seeking, etc. into a generic framework. A solution to the CMPF was presented along the lines of CPF framework that consist of a cascade of Moving Path Following controller - responsible for motion control of an individual robotic vehicle, and an event-based cooperative controller - responsible for achieving cooperation. A decentralized event-based control and communication scheme was used which aims to reduce the frequency of communication between the robotic vehicles through a suitable defined ETC and communication of computed cooperative control input in addition to the path variables. The efficacy of the proposed solution was demonstrated in simulations. The flexibility of the CMPF method was shown by considering two scenarios namely, coordinated source seeking and a convoy protection scenario. Each of these scenarios, used a different path that is moving with respect to a target.

Future works involve the experimental validation of the proposed method and a formal comparison of the CMPF method with the existing motion cooperation approaches. The investigation of further applications and the analysis of the effects of communication losses and delays on the performance of the event based cooperative controller are other possible research directions. Self-triggered control approach presented in [18], in the context of CMPF could be another possible research direction.

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