Bright, dark, and gray spatial soliton states in photorefractive media

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Received November 22, 1994; revised manuscript received May 10, 1995

A theory based on the Kukhtarev–Vinetskii model is developed that provides the evolution equation of one-dimensional optical spatial solitons in photorefractive media. In the steady-state regime and under appropriate external bias conditions, our analysis indicates that the underlying wave equation can exhibit bright and dark as well as gray spatial soliton states. The characteristics of these self-trapped optical beams are discussed in detail.

1. INTRODUCTION

Recently two research groups reported the first successful observations of bright spatial soliton behavior in photorefractive (PR) crystals.[1,2] In their studies it was found that these PR self-trapped states can actually display a host of interesting and potentially useful characteristics. As previously noted,[3,4] this class of optical solitons can be observed provided that the PR sample has been appropriately biased (externally) and that the PR crystal has been properly oriented. In such a case, i.e., under high external bias conditions, the drift current dominates, and as a result the space-charge field[5] can induce an index waveguide by means of the Pockels effect. The latter process is then capable of counteracting the effects of diffraction, and thus the end result can be a nondiffracting optical beam or what is better known as a PR spatial soliton.

Thus far the theory[3,4] of these newly discovered transient optical solitons has proceeded by directly invoking the PR two-wave-mixing response function.[5] In this approach the self-trapping wave equation has been found to admit solitary wave solutions of the bright kind.[3,4] According to this formalism, soliton domains of this sort are possible only for a well-defined range of external electric fields. Within this same theoretical framework the dynamics of Gaussian beams were recently considered[6] under similar conditions. Other peripheral issues regarding the properties of these solitons have also been investigated in recent experimental and theoretical studies.[7,8] On the other hand, under steady-state conditions the external bias field is expected to be nonuniformly screened. In this case an alternative approach is required that is more directly based on the transport equations of Kukhtarev *et al.*[9]

In this paper we develop a theory based on the Kukhtarev–Vinetskii model that provides the evolution equation of one-dimensional spatial solitons in PR media. As will be shown, this equation takes the form of a nonlinear Schrödinger equation (NLSE) with a higher-order nonlinearity. In the steady-state regime and under proper bias conditions, our formalism predicts not only bright but also dark soliton states as well. This is in accord with recent experimental observations of dark soliton behavior in PR crystals.[10,11] Moreover, other types of spatial domains such as gray solitons are also found to be possible. The properties of these steady-state self-trapped optical beams are then discussed in detail.

2. THEORETICAL MODEL

Let us consider an optical beam that propagates in a PR material along the z axis and is permitted to diffract only along the x direction. In essence, and for simplicity, we will be dealing with a one-dimensional nonlinear diffraction theory. For illustration purposes, let the PR crystal be strontium barium niobate[12] (SBN) with its optical c axis oriented along the x coordinate. Moreover, let us assume that the optical beam is linearly polarized along x and that the external bias electric field is applied in the same direction. Under these conditions the perturbed extraordinary refractive index n_e' (along the c axis) is given by[5] $(n_e')^2 = n_e^2 - n_e^4 r_{33} E_{sc}$, where r_{33} is the electro-optic coefficient, n_e is the unperturbed extraordinary index of refraction, and $\mathbf{E}_{sc} = E_{sc}\hat{x}$ is the space-charge field induced in this PR sample. On the other hand, the electric-field component $\mathbf{E}$ of the optical beam satisfies the Helmholtz equation

$$\nabla^2 \mathbf{E} + (k_0 n_e')^2 \mathbf{E} = 0\,, \tag{1}$$

where $k_0 = 2\pi/\lambda_0$ and λ_0 is the free-space wavelength of the lightwave employed. By expressing $\mathbf{E}$ in terms of a slowly varying envelope ϕ, i.e., $\mathbf{E} = \hat{x}\phi(x, z)\exp(ikz)$, we find that Eq. (1) leads to the following paraxial equation of diffraction:

$$i\phi_z + \frac{1}{2k}\phi_{xx} - \frac{k_0}{2}(n_e^3 r_{33} E_{sc})\phi = 0\,, \tag{2}$$

where $k = k_0 n_e$ and $\phi_z = \partial\phi/\partial z$, etc.

In turn, the induced space-charge field E_{sc} can be obtained from the transport model of Kukhtarev *et al.*[9] Under time-independent or steady-state conditions the charge-transport equations are given by

$$\gamma_R n N_D{}^+ = s_i(I + I_d)(N_D - N_D{}^+), \qquad (3)$$

$$J = e\mu\left(nE_{sc} + \frac{K_BT}{e}\frac{\partial n}{\partial x}\right), \qquad (4)$$

$$\frac{\partial J}{\partial x} = 0 \quad \text{or} \quad J = \text{constant}, \qquad (5)$$

$$\frac{\partial E_{sc}}{\partial x} = \frac{e}{\varepsilon_0\varepsilon_r}(N_D{}^+ - N_A - n). \qquad (6)$$

In the above equations s_i is the photoexcitation cross section, J is the current density, γ_R is the carrier recombination rate, μ and e are, respectively, the electron mobility and the charge, n is the free-electron density, K_B is Boltzmann's constant, T is the absolute temperature, and ε_r is the static relative permittivity. N_D is the donor concentration, N_A is the acceptor or trap density, and $N_D{}^+$ is the ionized donor density. I_d is the so-called dark irradiance that phenomenologically accounts for (through the product s_iI_d) the rate of thermally generated electrons. $I = I(x, z)$ is the power density profile of the optical beam, which can be also expressed in terms of the envelope ϕ by use of Poynting's theorem, i.e., $I = (n_e/2\eta_0)|\phi|^2$, where $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$. Moreover, in Eqs. (3)–(6) we have ignored any z spatial dependence by assuming, as usual,[9] that the variables involved vary much more rapidly in the x direction.

Even though the space-charge field E_{sc} can be obtained in principle from Eqs. (3)–(6), this task is considerably involved. However, we can greatly simplify Eqs. (3)–(6) by keeping in mind that the following inequalities hold true in typical PR media: N_D or $N_A \gg n$ and $N_D{}^+ \gg n$. In this case, Eqs. (3)–(6) yield the following results:

$$N_D{}^+ = N_A\left(1 + \frac{\varepsilon_0\varepsilon_r}{eN_A}\frac{\partial E_{sc}}{\partial x}\right), \qquad (7)$$

$$n = \frac{s_i(N_D - N_A)}{\gamma_R N_A}(I + I_d)\left(1 + \frac{\varepsilon_0\varepsilon_r}{eN_A}\frac{\partial E_{sc}}{\partial x}\right)^{-1}. \qquad (8)$$

At this point, let us also assume that the power density $I(x, z)$ of the optical beam attains asymptotically a constant value at $x \to \pm\infty$, that is $I(x \to \pm\infty, z) = I_\infty$. In these regions of constant illumination, Eqs. (3)–(6) require that the space-charge field is also independent of x, i.e., $E_{sc}(x \to \pm\infty, z) = E_0$. If the spatial extent of the optical wave is much less that the x width W of the PR crystal, then under a constant voltage bias V, E_0 is approximately $\pm V/W$. On the other hand, if W is comparable with the wave's width, then this approximation breaks down. In this case the intensity profile of the optical beam has to be taken into account in order to estimate the appropriate correction factors (see Appendix A). From Eq. (8) the free-electron density n_0 in these regions ($x \to \pm\infty$) can be subsequently determined and is given by

$$n_0 = \frac{s_i(N_D - N_A)}{\gamma_R N_A}(I_\infty + I_d). \qquad (9)$$

On the other hand, Eq. (5) implies that the current density J is constant everywhere and therefore $n_0E_0 = nE_{sc} + (K_BT/e)(\partial n/\partial x)$, or

$$E_{sc} = \frac{n_0E_0}{n} - \frac{K_BT}{e}\frac{1}{n}\frac{\partial n}{\partial x}. \qquad (10)$$

Substitution of Eq. (8) into (10) then yields the following relation:

$$E_{sc} = E_0\frac{(I_\infty + I_d)}{(I + I_d)}\left(1 + \frac{\varepsilon_0\varepsilon_r}{eN_A}\frac{\partial E_{sc}}{\partial x}\right) - \frac{K_BT}{e}\frac{(\partial I/\partial x)}{(I + I_d)} + \frac{K_BT}{e}\frac{\varepsilon_0\varepsilon_r}{eN_A}\left(1 + \frac{\varepsilon_0\varepsilon_r}{eN_A}\frac{\partial E_{sc}}{\partial x}\right)^{-1}\frac{\partial^2 E_{sc}}{\partial x^2}. \qquad (11)$$

Under strong bias conditions E_0 will reach appreciable values, and as a result the drift component of the current will be dominant. In this case all the terms associated with the process of diffusion (K_BT/e terms) can be considered small and thus can be neglected in Eq. (11). Furthermore, if the power density $I(x, z)$ of the optical beam varies slowly with respect to x, then in typical PR media the dimensionless term $|(\varepsilon_0\varepsilon_r/eN_A)(\partial E_{sc}/\partial x)|$ is expected to be much less than unity.[13] Under these conditions the space-charge field can be determined from Eq. (11) and is approximately given by

$$E_{sc} = E_0\frac{(I_\infty + I_d)}{(I + I_d)}. \qquad (12)$$

The envelope evolution equation can now be established by insertion of Eq. (12) into Eq. (2). It proves more convenient, however, to study this equation in a normalized fashion. In doing so, let us adopt the following dimensionless coordinates and variables; i.e., let $\xi = z/(kx_0{}^2)$, $s = x/x_0$, and $\phi = (2\eta_0I_d/n_e)^{1/2}U$. x_0 is an arbitrary spatial width, and the beam power density has been scaled with respect to the dark irradiance I_d. Using Eqs. (2) and (12), we can then show that the normalized envelope U obeys the following dynamical evolution equation:

$$iU_\xi + \frac{1}{2}U_{ss} - \frac{\beta(1+\rho)U}{1 + |U|^2} = 0, \qquad (13)$$

where $\rho = I_\infty/I_d$ and $\beta = (k_0x_0)^2(n_e{}^4r_{33}/2)E_0$. To simplify the analysis, we have neglected any loss effects in Eq. (13). Under a constant voltage bias V, the relation $-\int E_{sc}dx = V$ complements Eq. (13) with a conserved quantity, i.e.,

$$\beta\int_{-\frac{W}{2x_0}}^{\frac{W}{2x_0}}\frac{ds(1+\rho)}{1 + |U|^2} = \frac{k_0{}^2x_0}{2}n_e{}^4r_{33}V. \qquad (14)$$

The dimensionless parameter β can be positive or negative depending on the sign of E_0 or the polarity of the externally applied electric field. Strictly speaking, under strong dynamical evolution conditions Eq. (14) implies that β or E_0 are not constants but instead may vary with respect to z. As shown in Appendix A, however, if the x-width W of the crystal is considerably bigger than the spatial extent of the optical beam, then the quantities E_0 and β become relatively insensitive to the wave's dynamics and hence can be treated as constants. In the latter case (i.e., when $\beta =$ constant) Eq. (13) can take the form of a NLSE with a saturable nonlinearity.[14] The saturable nature of this nonlinearity becomes more evident if one employs the transformation $U = u\exp[-i\beta(1+\rho)\xi]$, in which case

$$iu_\xi + \frac{1}{2}u_{ss} + \beta(1+\rho)\left(\frac{|u|^2}{1 + |u|^2}\right)u = 0. \qquad (15)$$

In what follows we will consider the spatial soliton solutions of Eq. (13). The various characteristics and properties of these waves will be discussed in detail.

3. SPATIAL SOLITON STATES

We begin our analysis by considering first the class of bright soliton states. In this case the optical beam intensity is expected to vanish at infinity ($s \to \pm\infty$), and thus $I_\infty = \rho = 0$. From Eq. (13) bright-type waves should therefore satisfy

$$iU_\xi + \frac{1}{2} U_{ss} - \frac{\beta U}{1 + |U|^2} = 0. \tag{16}$$

It can be directly verified that Eq. (16) can be obtained from the Lagrangian density

$$L = \frac{i}{2}(UU_\xi^* - U^* U_\xi) + \frac{1}{2} U_s U_s^* + \beta \ln(1 + |U|^2), \tag{17}$$

and that the bright-wave evolution equation (16) exhibits the following two conservation laws:

$$P = \int_{-\infty}^{\infty} ds |U|^2, \tag{18a}$$

$$Q = \int_{-\infty}^{\infty} ds[(|U_s|^2/2) + \beta \ln(1 + |U|^2)], \tag{18b}$$

where P accounts for the total power conveyed by the optical beam and Q is associated with the Hamiltonian density of Eq. (16).

We can obtain the bright solitary wave solutions of Eq. (16) by expressing the beam envelope U in the usual fashion: $U = r^{1/2} y(s) \exp(i\nu\xi)$, where ν represents a nonlinear shift of the propagation constant and $y(s)$ is a normalized real function bounded between $0 \le y(s) \le 1$. Furthermore, for bright-type solutions we require that $y(0) = 1$, $\dot{y}(0) = 0$, and that $y(s \to \pm\infty) = 0$. The positive quantity r is defined as $r = I_{\max}/I_d$, where $I_{\max} = I(0)$; i.e., r stands for the ratio of the maximum beam power density to that of the dark irradiance. Substitution of this latter form of U into Eq. (16) yields

$$\ddot{y} - 2\nu y - 2\beta \frac{y}{1 + ry^2} = 0, \tag{19}$$

where $\ddot{y} = d^2y/ds^2$, etc. By integrating Eq. (19) once and by employing the y-boundary conditions we find that

$$\nu = -(\beta/r)\ln(1 + r), \tag{20}$$

$$(\dot{y})^2 = (2\beta/r)[\ln(1 + ry^2) - y^2 \ln(1 + r)]. \tag{21}$$

Further integration of Eq. (21) leads to

$$(2\beta)^{1/2} s = \pm \int_y^1 \frac{r^{1/2} dy'}{[\ln(1 + ry'^2) - y'^2 \ln(1 + r)]^{1/2}}, \tag{22}$$

from which the normalized bright-field profile $y(s)$ can be determined. Unfortunately the nature of the integral of Eq. (22) is such that it prevents any closed-form solutions. Nevertheless, $y(s)$ can be easily obtained by use of simple numerical integration procedures. Moreover, it is straightforward to show that the quantity in the square bracket of Eq. (21) is always positive for all values of y^2 between $0 \le y^2 \le 1$. Therefore bright PR spatial solitons will be possible only when β or E_0 are positive quantities (so as $\dot{y}^2 > 0$).

To illustrate our results, we consider the following examples: Let $\lambda_0 = 0.5\ \mu m$, $x_0 = 40\ \mu m$, and $E_0 = +2 \times 10^5$ V/m. The SBN parameters are taken here to be $n_e = 2.35$ and $r_{33} = 224 \times 10^{-12}$ m/V. For this set of values, $\beta \approx 173$. Figure 1 depicts the normalized intensity profiles of such bright PR solitons for three different values of r when $\beta = 173$. This figure demonstrates that the two normalized field profiles obtained for $r = 1$ and $r = 10$ are quite similar, whereas the profile found for $r = 0.1$ is considerably broader. The question naturally arises as to what factors contribute to the FWHM of these optical beams. To answer this question, one has to consider Eq. (22). More specifically, Eq. (22) shows that the arbitrary width x_0 is no longer a variable since it has been entirely absorbed in the product $(2\beta)^{1/2}s$. In fact, in this case the new spatial scale is associated with the quantity $[2/(k_0^2 n_e^4 r_{33} E_0)]^{1/2}$, which is again independent of x_0. Whereas the value of E_0 is directly involved in the beamwidth, the parameter r tends to determine the functional form of $y(s)$, and, in doing so, it also indirectly affects the spatial width. In other words, for a given physical system the spatial beamwidth of these solitons depends on only two variables, namely, E_0 and $r = I_{\max}/I_d$.

The dependence of the intensity FWHM of these bright solitons on r and E_0 is shown in more detail in Fig. 2 for the same system parameters considered above. In

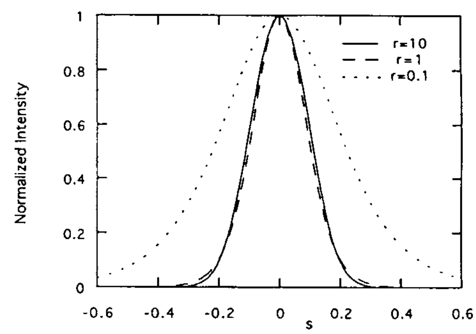

Fig. 1. Normalized intensity profiles of bright spatial solitons for $\beta = 173$, $x_0 = 40\ \mu m$, and $r = 10$, 1, and 0.1.

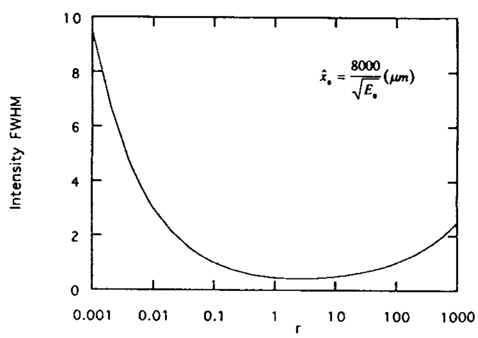

Fig. 2. Intensity FWHM of bright spatial photorefractive solitons versus r in units of $\hat{x}_0 = 8000(E_0)^{-1/2}$, where $\hat{x}_0$ is in units of micrometers and E_0 is in units of volts per meter. The system parameters are taken here to be $\lambda_0 = 0.5\ \mu m$, $n_e = 2.35$, and $r_{33} = 224$ pm/V.

this figure the FWHM is measured in units of $\hat{x}_0 = 8000(E_0)^{-1/2}$, where $\hat{x}_0$ is in micrometers and E_0 is expressed in units of volts per meter. Three regions of different behavior can be immediately recognized in this plot. In particular, for a given value of E_0 the FWHM increases when $r \leq 0.1$ and when $r \geq 100$, whereas it remains relatively constant in the region $0.1 \leq r \leq 100$. We can intuitively understand the tendency of the FWHM to increase for large values of r (i.e., $r > 10^2$) by keeping in mind that, in this regime, the nonlinearity reaches oversaturation. Conversely, for small values of r ($r \leq 0.1$), the FWHM tends again to increase because the nonlinearity is now approaching the Kerr limit. As a result, the system obeys a NLSE, and the intensity FWHM varies like $r^{-1/2}$. The FWHM plateau between $10^{-1} \leq r \leq 10^2$ is also quite interesting, as one expects that (in this region) absorption losses will play a small role in the overall dynamics of these solitary beams. As previously mentioned, these PR bright spatial solitons are always possible as long as E_0 is a positive quantity. The only restriction arises from the fact that E_0 must be appreciable enough that one can justify the neglect of all the diffusion terms in Eq. (11). Note that, for a given applied voltage bias V, we can obtain the value of E_0 for bright solitons by following the procedure outlined in Appendix A.

The low-amplitude case ($r \ll 1$ or $|U|^2 \ll 1$) also deserves special consideration. In this limit Eq. (16) is given by

$$iU_\xi + {}^1\!/_2 U_{ss} - \beta U + \beta |U|^2 U = 0, \qquad (23)$$

which is actually a modified version of the fully integrable NLSE.[15] This correspondence can be quickly established through the transformation $U = u \exp(-i\beta\xi)$. Therefore Eq. (23) can be formally solved in terms of the so-called inverse scattering transform,[15,16] and all the knowledge regarding the NLSE carries on in this case. The fundamental bright soliton solution of Eq. (23) is given by $U = r^{1/2} \operatorname{sech}[(\beta r)^{1/2} s]\exp\{i\beta[(r/2) - 1]\xi\}$, from which its intensity FWHM can be obtained directly and is given by $\mathrm{FWHM} = 1.76[2/(k_0{}^2 n_e{}^4 r_{33} E_0 r)]^{1/2}$. This result is in agreement with Fig. 2 (when $r \ll 1$) and explains why the FWHM behaves like $r^{-1/2}$ in this region. Furthermore it is interesting to note that, had one retained the diffusion term $(K_B T/e)(\partial I/\partial x)(I_d + I)^{-1}$ in Eq. (11), then in this low-amplitude regime Eq. (23) would have taken the form

$$iU_\xi + {}^1\!/_2 U_{ss} - \beta U + \beta |U|^2 U + \gamma(|U|^2)_s U = 0, \qquad (24)$$

where $\gamma = (K_B T/2e)(k_0{}^2 x_0 n_e{}^4 r_{33})$. For relatively broad optical beams this term is known to dominate the diffusion process and is responsible for beam self-bending effects.[6] Equations similar to Eq. (24) have been previously studied extensively within the context of nonlinear fiber optics,[16] in which the term $\gamma(|U|^2)_s U$ stood for the effects arising from intrapulse Raman scattering.[17,18] Similar procedures[17,18] can therefore be employed here to describe the beam self-deflection process.

The case of dark spatial solitons can be analyzed in a similar fashion. These waves exhibit an antisymmetric field profile (with respect to x), and, moreover, they are embedded in a constant intensity background, that is, I_∞ and ρ are finite quantities. Therefore from Eq. (13) dark-type waves should evolve according to

$$iU_\xi + \frac{1}{2} U_{ss} - \beta(1 + \rho)\frac{U}{1 + |U|^2} = 0. \qquad (25)$$

To obtain stationary waves, we let $U = \rho^{1/2} y(s)\exp(i\nu\xi)$, where $y(s)$ is a normalized odd function of s, i.e., $y(s \to \pm\infty) = \pm 1$, $y(0) = 0$, and all the derivatives of y vanish at infinity. Substitution of this form of U into Eq. (25) yields

$$\ddot{y} - 2\nu y - 2\beta(1 + \rho)\frac{y}{1 + \rho y^2} = 0, \qquad (26)$$

from which one can readily deduce that

$$\nu = -\beta. \qquad (27)$$

Equation (26) can be integrated once and leads to

$$(\dot{y})^2 = (-2\beta)\left[(y^2 - 1) - \frac{(1 + \rho)}{\rho}\ln\left(\frac{1 + \rho y^2}{1 + \rho}\right)\right], \qquad (28)$$

from which the dark envelope $y(s)$ can be obtained by numerical integration. It can be readily shown that the quantity in the square bracket of Eq. (28) remains positive for all values of $y^2 \leq 1$, and thus β or E_0 must be negative so that $\dot{y}^2 > 0$. In other words, the polarity of the external bias field must be reversed or negative if the dark PR spatial solitons are to be observed. Figure 3 illustrates the normalized field profile of such a dark PR soliton in the case in which $x_0 = 40\ \mu\mathrm{m}$, $\lambda_0 = 0.5\ \mu\mathrm{m}$, $E_0 = -1.2 \times 10^5$ V/m, $\rho = 1$, and $\beta = -103$. The SBN parameters are taken to be the same as those considered in the previous examples. Again, for a given physical system the spatial extent of these dark waves depends only on two variables, namely, β and ρ or E_0 and I_∞. In the low-amplitude regime, i.e., when $\rho \ll 1$ or $|U|^2 \ll 1$, Eq. (25) takes the form of a NLSE:

$$iU_\xi + {}^1\!/_2 U_{ss} - \beta(1 + \rho)(1 - |U|^2)U = 0, \qquad (29)$$

the dark soliton solutions of which are given by $U = \rho^{1/2} \tanh\{[(-\beta)\rho(1 + \rho)]^{1/2} s\}\exp[-i\beta(1 - \rho^2)\xi]$. It is worth pointing out that dark optical solitons have been recently observed experimentally in PR media.[10,11] Again, we can obtain the value of E_0 from the bias voltage V by following the results of Appendix A.

Another interesting class of solitary waves can also be obtained from Eq. (25). As will be later shown, these correspond to gray solitary states.[19] In this case the wave power density attains a constant value I_∞ at infinity, resulting in a finite ρ. Thus this family of waves is also

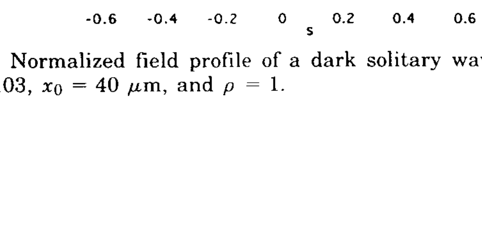

Fig. 3. Normalized field profile of a dark solitary wave when $\beta = -103$, $x_0 = 40\ \mu\mathrm{m}$, and $\rho = 1$.

expected to evolve according to Eq. (25). To obtain these solutions, let us express U in the following fashion:

$$U = \rho^{1/2} y(s) \exp\left[i\left(\nu\xi + \int^{s} \frac{J \mathrm{d}s'}{y^2(s')} \right) \right], \qquad (30)$$

where in this section J is a real constant to be determined. For this class of stationary states the normalized amplitude $y(s)$ is an even function of s and satisfies the condition $y(s \to \pm\infty) = 1$. All the derivatives of y are also zero at infinity. Moreover, we will assume that $y^2(s = 0) = m$ (i.e., the intensity is finite at the origin) and that $\dot{y}(0) = 0$. Substitution of Eq. (30) into Eq. (25) then yields the following ordinary differential equation:

$$\ddot{y} - 2\nu y - \frac{J^2}{y^3} - 2\beta(1 + \rho)\frac{y}{1 + \rho y^2} = 0 . \qquad (31)$$

Using the boundary conditions of y at infinity, we find that

$$J^2 = -2(\nu + \beta), \qquad (32)$$

and by further integrating Eq. (31) we obtain

$$\nu = \frac{(-\beta)}{(m - 1)^2}\left[\frac{m(1 + \rho)}{\rho} \ln\left(\frac{1 + \rho m}{1 + \rho} \right) + (1 - m) \right], \qquad (33)$$

and

$$(\dot{y})^2 = 2\nu(y^2 - 1) + \frac{2\beta}{\rho}(1 + \rho)\ln\left(\frac{1 + \rho y^2}{1 + \rho} \right) + 2(\nu + \beta)\left(\frac{1 - y^2}{y^2} \right). \qquad (34)$$

Given a physical system and a set of parameters β, ρ, m, the quantities J and ν can be determined from Eqs. (32) and (33). The set (β, ρ, m) has to be judiciously selected so that $(\dot{y})^2$ is positive for all values of y^2 and that $J^2 > 0$. Subsequently, the normalized amplitude $y(s)$ can be readily obtained by numerical integration of Eq. (34). It can be shown that these solitary waves are possible only when $m < 1$ and $\beta < 0$. Therefore this class of solutions is related to the dark family, and, in fact, they represent a generalization of the so-called gray solitons previously found[19] in connection with the NLSE. Unlike their bright/dark counterparts, their phase is no longer constant across s but instead varies in a rather involved fashion; i.e., it follows the term $\exp\{i \int \mathrm{d}s'[J/y^2(s')]\}$ of Eq. (30). Figure 4 shows the normalized intensity profile of a gray spatial PR soliton when $\rho = 5$, $x_0 = 40\ \mu\mathrm{m}$, $m = 0.4$, and $\beta = -34.5$.

Finally, it is interesting to compare these steady-state spatial solitons with those previously obtained on the basis of the two-wave-mixing response function.[3,4,6] First of all, Eqs. (16) and (25) now predict both bright and darklike soliton states, unlike the evolution equation of Refs. 3, 4, and 6 that exhibits only bright solitary waves. Even within the subset of bright stationary waves, significant differences also exist. In particular, we have found that bright solitons are always possible whenever $E_0 > 0$, and thus E_0 does not have to lie within a narrow range of values, as previously predicted. The only restriction in this case arises from the fact that the applied bias field strength E_0 must be appreciable enough that the drift current dominates and thus the diffusion terms in Eq. (11) can be neglected. Moreover, we have shown that the spatial extent of these waves is not arbitrary (as in the case of transient solitons), but instead it is uniquely determined for a given physical system by E_0 and r (or ρ). The ratio of the maximum power density of these steady-state waves with respect to the dark irradiance[20] was also found to play a key role.

4. CONCLUSIONS

In conclusion, a theory based on the Kukhtarev–Vinetskii model has been developed that provides the evolution equation of optical spatial solitons in photorefractive media. Under strong external bias conditions and in the steady-state regime, our analysis indicates that the underlying wave equation takes the form of a NLSE with a higher-order nonlinearity. Subsequently, this equation was found to exhibit a variety of solitary wave solutions, which include bright, dark, and gray stationary states. Moreover, it has been shown that the bright family of solutions is possible only when $E_0 > 0$, whereas the dark branch requires the polarity of the external bias field to be reversed. The dependence of the spatial extent of these waves on relevant parameters was also considered in detail. In closing, we point out that there are a number of additional issues that merit further investigation. These include the dynamical behavior and stability of these PR solitons as well as their response to loss and diffusion effects, topics we hope to address in the future.

APPENDIX A

To evaluate E_0 in terms of V, W, r, or ρ, we first simplify Eq. (13) by introducing new dimensionless variables, i.e., $\eta = (|\beta|)^{1/2}s = \alpha x$ and $\zeta = |\beta|\xi$, where $\alpha = (k_0^2 n_e^4 r_{33}|E_0|/2)^{1/2}$. For SBN:60 and at $\lambda_0 = 0.5\ \mu\mathrm{m}$, $\alpha \approx 7.34 \times 10^2(|E_0|)^{1/2}\ \mathrm{m}^{-1}$, provided that E_0 is expressed in volts per meter. Using these transformations, we see that Eq. (13) takes the form

$$iU_\zeta + \frac{1}{2}U_{\eta\eta} \mp (1 + \rho)\frac{U}{1 + |U|^2} = 0 . \qquad (A1)$$

The upper sign in the minus-or-plus symbol corresponds to $\beta > 0$, whereas the lower one corresponds to negative β's. As previously noted, the bright solitons of Eq. (A1) are given by $|U|^2 = ry^2(\eta)$ and the dark ones by $|U|^2 = \rho y^2(\eta)$. If we define $\hat{\eta}$ as a half-spatial extent of these solitary waves, that is, $y^2(\hat{\eta}) = 1 \times 10^{-4}$ for bright waves and $y^2(\hat{\eta}) = 0.9999$ for dark domains, then it is straightforward to show (by use of their x symmetry) that

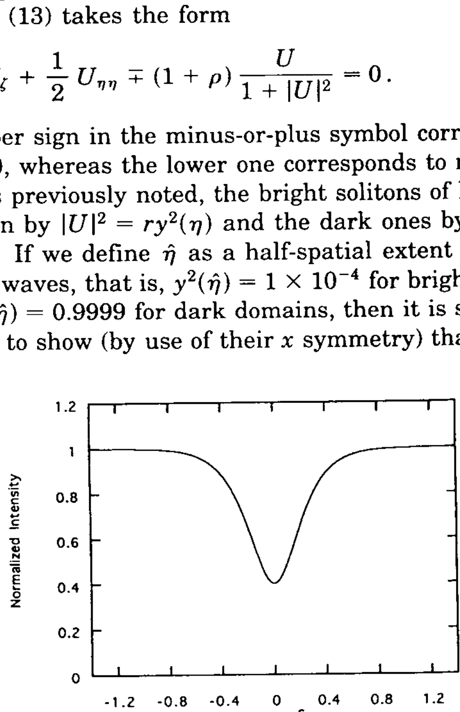

Fig. 4. Normalized intensity versus s for a gray spatial soliton state when $x_0 = 40\ \mu\mathrm{m}$, $\rho = 5$, $\beta = -34.5$, and $m = 0.4$.

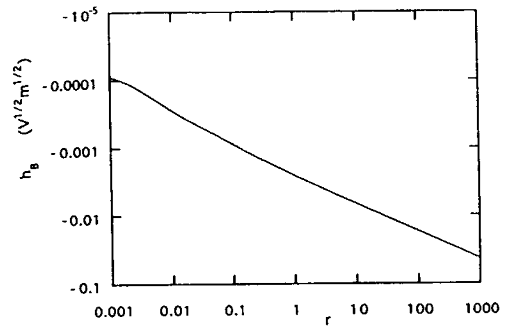

Fig. 5. Correction factor h_B for bright photorefractive solitons versus r.

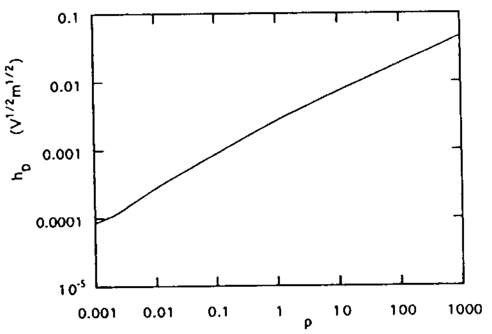

Fig. 6. Correction factor h_D for dark photorefractive solitons versus ρ.

$$|V| = 2\frac{|E_0|}{\alpha}\left[\int_0^{\hat{\eta}} \frac{(1+\rho)\mathrm{d}\eta}{1+|U|^2} + \left(\frac{\alpha W}{2} - \hat{\eta}\right)\right]. \quad \text{(A2)}$$

Equation (A2) can also be rewritten as $|V| = W|E_0| + h(|E_0|)^{1/2}$, where the correction factor h depends on r or ρ and is defined as follows:

$$h = \frac{2}{7.34 \times 10^2}\left[\int_0^{\hat{\eta}} \frac{(1+\rho)\mathrm{d}\eta}{1+|U|^2} - \hat{\eta}\right]. \quad \text{(A3)}$$

The correction factors for both bright and dark solitary waves have been evaluated by use of their functional forms and are plotted in Figs. 5 and 6 versus r and ρ, respectively. Given r or ρ, the value of E_0 can be obtained from the previous quadratic equation and is given by

$$|E_0| = \left(\frac{-h + \sqrt{h^2 + 4W|V|}}{2W}\right)^2. \quad \text{(A4)}$$

It can be easily shown that, for $[h^2/(|V|W)] \ll 1$, i.e., when W is big enough, E_0 is approximately given by V/W.

ACKNOWLEDGMENTS

This study was supported in part by JNICT of Portugal. The authors thank the reviewers for useful suggestions.

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Note added in proof: Since this manuscript was submitted a similar treatment of bright and dark solitons in biased PR media was presented by Segev *et al.*[21]

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