MECHANICAL-THERMAL AND 1/f NOISE IN MEMS DEVICES

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Abstract — The meta-stability of the pull-in displacement of an electrostatically operated parallel plate is used for the capacitive measurement of the mechanical-thermal noise spectrum. Pull-in time is depending on force and is not affected by the input-referred noise of the readout circuit. Repeatedly bringing the microstructure to pull-in, while measuring the pull-in time followed by FFT enables the measurement of the mechanical noise spectrum with a non-mechanical noise set primarily by the resolution of the time measurement. The white noise level is found to be in agreement with the theory on damping. The 1/f noise 0 dB cross-over frequency is found to be independent of ambient gas pressure and reproducible for devices fabricated in the same process and the same run and is at 5.10⁻³ Hz for 1 bar.

Key Words: Mechanical-thermal noise, 1/f noise, Pull-in.

I INTRODUCTION

In a typical sensor system front-end electronics is implemented for readout and processing of the electrical signal provided by the sensing element [1]. The uncertainty (usually total noise level) of the measurement is therefore due to the combined effect of the mechanical-thermal noise in the mechanical domain [2], the electrical noise of the (resistive) mechanical sensing element and the input referred electronic noise of the readout electronics [3]. Generally, the electronic noise dominates noise performance of the system. As a consequence the details of the mechanical-thermal noise are often not considered relevant. More fundamental studies on noise and damping are hampered by the same fact, as the mechanical-thermal noise cannot be directly measured. An estimation of the mechanical-thermal noise can be made from the total sensor noise by a careful analysis or selective measurement of the electrical noise [4].

In this paper the meta-stability of the pull-in displacement of an electrostatically operated parallel plate structure is used for the capacitive measurement of mechanical-thermal noise spectrum [5]. Pull-in time is depending on force and is not affected by the input-referred noise of the readout circuit. Repeatedly bringing the microstructure to pull-in, while measuring the pull-in time followed by FFT enables the measurement of the mechanical noise spectrum with a non-mechanical noise set primarily by the resolution of the time measurement. The white noise level is found to be in agreement with the theory on damping. Moreover, long-term measurements show that MEMS devices, as all the phenomena in nature, suffer from 1/f noise. The origins of this 1/f noise have not been identified and additional measurements are needed to confirm and correctly interpret the long-term measurements.

II DYNAMIC PULL-IN: META-STABLE REGION

When an overdamped MEMS structure, with an elastic spring $k$, a zero-voltage actuation capacitance $C_0$, and an initial capacitor gap $d_0$, is actuated with a voltage slightly higher than the pull-in voltage ($V_{pi} = \frac{8}{27}\sqrt{\frac{k}{C_0}}$ for one-degree-of-freedom devices [6]), it exhibits a displacement characteristic defined by a meta-stable region [5]. The time spent in the meta-stable region is very sensitive to any external force and thus this time can be an excellent measure of any external force. A simplified analytical expression for the time spent in the meta-stable region is [7]:

$$t_m = \ln\left(\frac{(k(a^2-1)(d_0+3\Delta x)+3ma_\text{ext})^2}{(d_0k(a^2-1)+3ma_\text{ext})^2}\right) \frac{b + \text{Root}}{4k(a^2-1)(a+1)} ,$$

where $m$ is the mass, $b$ is the damping coefficient, $a_\text{ext}$ is the applied external acceleration and $\Delta x$ is the small displacement covered by the structure during the meta-stable region. It should be noted
that this region only occurs for devices with a quality factor lower than $Q<1.2$ and for voltages $V=\alpha V_{pi}$, with $1<\alpha<1.01$ [7].

By exploiting this sensitive region in the absence of an external force, the equivalent noise force can be measured and compared to the estimated value. The fabricated structure and experimental setup are explained in detail in the next sections.

III NOISE MEASUREMENTS

III.1 FABRICATED STRUCTURE

For the experimental measurements of the dynamic pull-in transition, the microstructure shown in Fig. 1, fabricated in the Bosch epi-poly process [8], was used.

![Fabricated micromachined device](image)

The device has four folded beams, 340 $\mu$m long and 2.5 $\mu$m wide, connected to two rigid central bars of about 1mm length. Parallel plate capacitors with a 2.25 $\mu$m gap are used for the actuation of the movable mass. The displacement measurement involves sensing the changes of various sets of differential capacitors. Stoppers, located on the end of the rigid bars, 2 $\mu$m apart, prevent the electrodes to touch after pull-in is reached. The main device characteristics are presented in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ($m$)</td>
<td>4.27 $\mu$g</td>
</tr>
<tr>
<td>Mechanical spring ($k$)</td>
<td>1.2930 N/m</td>
</tr>
<tr>
<td>Initial gap ($d_0$)</td>
<td>2.25 $\mu$m</td>
</tr>
<tr>
<td>Damping coefficient ($b$)</td>
<td>$1.92\times10^{-4}$ Ns/m</td>
</tr>
<tr>
<td>$C_{d0}$ (zero-displacement actuation capacitor)</td>
<td>141 fF</td>
</tr>
<tr>
<td>$C_{s0}$ (zero-displacement sensing capacitor)</td>
<td>611 fF</td>
</tr>
</tbody>
</table>

The Nyquist’s theorem [2], relates the spectral density of the fluctuation force to mechanical resistance (damping coefficient):

$$F_{noise} = 4k_BTb \sqrt{\frac{N}{\sqrt{Hz}}}.$$  \hspace{1cm} (2)

where $k_B$ is Boltzmann’s constant ($1.38\times10^{-23}$ J/K) and $T$ is the absolute temperature. The theorem is still valid if the equivalent resistance depends on frequency. For the MEMS structure used in the experiments, and considering air at atmospheric pressure and ambient temperature as the surrounding gas, the value of the damping coefficient is the one presented in Table 1 [7]. The total noise force for this situation is $1.78\times10^{-12}$ N/$\sqrt{Hz}$.

III.2 MECHANICAL-THERMAL NOISE THEORY

The random thermally excited vibration of charge carriers that caused noise in an electrical resistor is also applicable to gas damping. The random movement of the gas at a certain temperature that is surrounding the mechanical structure leads to random fluctuations in the energy transfer between structure and damping gas, which is generally referred to as thermal-mechanical noise [2, 9].

Table 1. Main parameters of the microdevice

The basic idea is to use the high sensitivity of the meta-stable region during a pull-in event by measuring the pull-in time (time taken by the structure to move from the zero-displacement position, to the position where the electrode hits the counter-electrode).

A 5 Hz square-wave signal with stability better than 20 $\mu$V/day was applied to the MEMS device, with amplitude of $\alpha V_{pi}$. This gives as many as 5 samples per second. A data-acquisition board (DAQ) with a sampling frequency of 100 kHz was used to acquire the response of the device. The front-end amplifier of the readout electronics has a
measured input-referred electronic white noise level at 188 nV/√Hz. Noise bandwidth is 4.8 kHz, which results in a total noise of the readout circuit of about 13mV.

The total pull-in time is obtained by measuring the time elapsed from the moment the square signal is applied, until the structure passes through 75% of the gap. At 75% of the gap the voltage rate change is higher than 60 mV, which allows disregarding the readout electronic noise. The uncertainty in the measured pull-in time is the readout noise, which is set by the sampling clock and is equal to 10 µs.

Measurements during one day on a fabricated device with air at atmospheric pressure as the surrounding medium and α=1.0008 are presented in Fig. 2. There is no external acceleration applied, which means that the changes on the time during samples are due to the mechanical-thermal noise of the device.

Since the sensitivity of the sensor to any external acceleration is known, the measured time-changes can be directly translated to the equivalent noise force. By taking the derivative of (1) the sensitivity to acceleration is obtained: $S = 0.016$ s/m.s⁻². The average value of the pull in time $T_{av,pi}=12$ ms and the standard deviation is: $\sigma_{pi}= 112$ µs, which translates to the mechanical domain into: $F_n=\sigma_{pi}.m/S=2.9\times10^{-11}$ N (for 500 samples).

During a pull-in event, the high-frequency components in the force noise are time-integrated and do not contribute to variations in the displacement. Frequencies lower than $2/f_{pi}$ are present during the full pull-in movement and are the cause of the pull-in time noise [5]. For $f_{pi}=12ms$, the bandwidth to be considered for the noise is: $B_n=(\pi/2).(2/f_{pi})= 166$ Hz. This results in a predicted equivalent input total force noise: $1.78\times10^{-12} \times \sqrt{166} = 2.3.10^{11}$ N. If the uncertainty of 10 µs of the DAQ system is considered as noise, the predicted white noise level increases to: $10.10^{-6}.m/S+ 2.3.10^{-11}= 2.56.10^{11}$ N, which is in good agreement with the measured value.

Figure 3 presents the FFT of the measured pull-in time of Fig. 2 (0 dB= 1 N/√Hz). The 1/f noise is clearly visible, but additional measurements are needed to conclusively attribute the cross-over frequency to the thermal-mechanical 1/f noise.

III.4 RC-NETWORK

Since the noise of the readout electronics is negligible, the 1/f noise could also originate from noise in the power supply or the DAQ. In order to check whether the 1/f is indeed due to the thermal-mechanical noise, the sensor and readout electronics were replaced by a RC-network (low-pass filter) with $R=11k\Omega$ and $C=0.47\mu$F. The measured setup was kept the same with voltage amplitude of $V=5V$. Figure 4 presents the measurement series and the corresponding spectrum after FFT.

Since the noise of the RC-network is defined by the thermal noise in the resistor, the mechanism is similar to the mechanical-thermal noise force. Thus the results are comparable and shown in Fig. 4b. The predicted total current noise is:

$$i_{\text{noise}} = \sqrt{\frac{V_{\text{noise}}^2}{R^2} + \frac{\kappa B T}{R^2 C}} = 8\times10^{-12} [A] \ F_{\text{noise}} .$$

The sensitivity of the RC-network transition time to voltage changes, when the voltage goes from $V_1$ to $V_2$ is given by:

$$\frac{\partial t}{\partial V} = - \frac{(V_2-V_1)RC}{(V-V_1)(V-V_2)} . \ (3)$$

For $V_1= 0.2V$ and $V_2= 4.5$, the sensitivity is about $S_1=9.5$ ms/V. The measured time noise can
be related to the resistor current noise $i_{\text{noise}} = \frac{\sigma_{\text{rr}}}{R}$, resulting in a measured current noise of \(9.8 \times 10^{-8}\) A. Basically, this is the total uncertainty of the 10µs resolution (\(10^{10} / 11000 \times 9.5 \times 10^{-8}\)) which means that the noise measured with the RC-network is due exclusively to the DAQ and power supply. The difference in the corner frequencies between sensor and RC-network suggests that the 1/f of the MEMS device is not caused by the power supply or the DAQ.

III.5 VARIATION OF PRESSURE CONDITIONS

A few more measurements were performed to confirm the 1/f noise of the mechanical structure. The FFT of measurements at different pressure conditions (different damping values) are presented in Fig. 5. It should be noticed that the lower the pressure, the lower is the noise per square root of hertz. Nevertheless, as lower pressures have smaller pull-in times, the considered bandwidth for the noise increases, resulting in a higher value for the mechanical-thermal noise. The fact that the 1/f curve is the same for the three different pressures and quite different from measurements on a RC time constant suggests that the 1/f measured noise originates from the mechanical structure.

IV CONCLUSIONS

The white noise level is found to be in agreement with the theory on damping. The 1/f noise 0 dB cross-over frequency is found to be independent of ambient gas pressure and reproducible for devices fabricated in the same process and the same run and is at \(5.10^{-3}\) Hz for 1 bar. 1/f noise is known to be, amongst others, defect related. The 1/f noise 0 dB cross-over frequency (-100 dB for practical reasons) could, therefore, be a suitable parameter to analyze the roughness of the surfaces moving in the gas. However, this is at the moment highly speculative and more work is needed to analyze the origins of the 1/f mechanical noise.

REFERENCES