Abstract

The pull-in voltage of a single side clamped free-standing beam, under lateral deflection, has been investigated for application as a DC voltage reference. A 2D energy-based analytical model for the static pull-in is compared with measurements. Bifurcation diagrams are computed numerically, based on a local continuation method. A left-right comb drive electrostatic structure is used to drive the 100 µm long beam (by momentum actuation) beyond the stability border. The device can be operated in feedback or as a seesaw, by using these two sets of electrodes. The single end clamping ensures that the pull-in voltage is insensitive to technology-induced stresses.

1. Introduction

MEMS technology is gradually also penetrating into mainstream Instrumentation and Measurement applications [1]. Although most microsystems are in principle instruments designed for measuring a non-electrical quantity, and thus serving a measurement application, the I&M application area can be defined as the field, where the instrument is the purpose and not the means. MEMS technology has a huge potential for contributing to critical components in a professional instrument, such as AC-to-DC converter or internal DC reference.

The pull-in voltage of beams of different designs has been investigated with the purpose of using the micromechanical structure as an on-chip voltage reference. Such a device may find application in integrated circuits, next to the conventionally used bandgap reference, and in metrology, where Zener references are widely used as so-called “transfer standard” [6]. The operation of a Zener diode is based on avalanche breakdown and is associated with a high noise level.

As the electrostatic force in a vertical field is inversely proportional to the square of the deflection and the restoring force of the beam is, in a first approximation, linear with deflection, an unstable system results in case of a deflection, v, beyond a critical value, v_{crit}. The pull-in voltage, V_{pi}, is defined as the voltage that is required to obtain this critical deflection. For a stable equilibrium deflection the second derivative of the potential energy of the system to deflection should be positive: \( \frac{\partial^2 U_p}{\partial v^2} > 0 \), thus \( V_{pi} \) results from \( \frac{\partial^2 U_p}{\partial v^2} = 0 \). The potential energy is composed of the bending energy, U_{b}, the strain energy, U_{s} and the potential energy of the excitation (the potential energy of the electrostatic force, U_{el}).
2. Modeling the pull-in structure

The basic phenomena is the loss of stability at the equilibrium position \( \bar{x}^* \), where the elastic forces equilibrate the electrostatic ones. Two methodologies are generally used:

(A). The dynamic system approach, in which the electromechanical system is described by a set of differential equations, and an analysis of the stability of its equilibrium points is performed (indirect Lyapunov method).

(B). The variational approach, in which the equilibrium points and their stability are determined by studying the variations of the total energy \( U(\bar{x},V) = U_{\text{elastic}}(\bar{x}) + U_{\text{electric}}(\bar{x},V) \). The equilibrium points \( \bar{x}_{eq} \) are given by \( \frac{\partial U}{\partial \bar{x}}(\bar{x}_{eq}) \); these are stable if \( U(\bar{x}_{eq}) \) is a local minimum, which is determined by the eigenvalues of \( \frac{\partial^2 U}{\partial \bar{x}^2}(\bar{x}_{eq}) \). The pull-in voltage is the value of the applied voltage for which the physical equilibrium point loses its stability.

Specifics of the analytical modeling used:
1. Applied to elastic beam, clamped at one end, and actuated by an electrostatic momentum at the free end (Fig. 1).

The elastic energy is described by:

\[
U_{\text{beam}}(w_1, \phi_1) = \frac{1}{2} EI \int_0^{l_x} \left( \frac{\partial \phi}{\partial x} \right)^2 dx =
\]

\[
\frac{2EI}{l_x} \left( \phi^2 - 3\phi_1 \frac{w_1}{l_x} + \left( \frac{w_1}{l_x} \right)^2 \right)
\]

Figure 1. Single-side clamped structure.

Figure 2. Identification of the state variables used in the model.

2. The energy balance is evaluated using two parameters \((w_1, j_1)\) to fully determine the configuration (Fig. 1 and Fig. 2).
The electrostatic energy is described by:

\[ U_{electric}(w_1, \varphi_1; V) = \frac{1}{2} V^2 \left[ C(w_1, \varphi_1) + C(-w_1, \varphi_1) - 2C_0^2 \right] \]

3. A local continuation method was implemented in Mathematica for tracing the equilibrium point coordinates at increasing voltage. The approach used to solve the problem is based on sweeping of the voltage, from the initial value \( V_0 \) toward increasing positive values. For each voltage value, the stability points are computed, by approximating the general potential in Taylor series around the previously computed equilibrium point, \( x_i = \{w_{n,eq}[k - 1], \varphi_{n,eq}[k - 1]\} \). This makes it possible to trace the evolution of the equilibrium point as function of \( V \). For the computed values of \( w_{1,eq}, \varphi_{1,eq} \), the eigenvalues of the associated Hessian at that point can be computed. If any of them is <0, then the equilibrium point is unstable, and drawn with a different color. Otherwise, it is stable. As shown in Fig. 3, the predicted pull-in voltage is at \( V_{pi} = 9.5V \).

4. Experimental results

The measured pull-in voltage was about 9.1V, which is near the predicted value from Mathematica model (\( V_{pi} = 9.5V \)). This difference is mainly due to the simplifying assumptions used, the process deviations and uncertainties in the value of the Young’s modulus used in the numerical computation. Figure 5 also indicates a profound quality difference between the devices. For proper operation as a voltage reference the pull-in should be as abrupt as possible. Moreover, the effect should be reproducible. Clearly, device #1 performs much better with respect to both requirements. The origins of these effects require further analysis. Figure 6 shows the response of a selected device and Fig. 7 indicates that some devices also exhibit hysteresis. This property seriously complicates operation in a feedback loop.

3. Microstructure fabrication

An epi-poly process was used for the fabrication of 11\( \mu \)m thick single-side clamped 200 \( \mu \)m long free-standing structures, as shown in Fig. 4. A differential sense capacitor, underneath the top mass of the horizontal beam, was used for detecting the pull-in voltage. The readout circuit is based on a capacitor bridge with two active arms.
5. Conclusions

The design and fabrication of a pull-in microstructure for application in metrology has been described in this paper. The feasibility has been demonstrated. First prototypes suffer from limited reproducibility and hysteresis. Experiments from another group on a metal-silicon electrode combination show a polarity dependent drift of about 0.5%/50hrs. [1, 6]. This is not acceptable in I&M applications. The source of drift has been identified as charging of the (native) oxide of the polysilicon beam. After coating the silicon with a metal much better results are expected. The design of improved devices is forthcoming and measurements are scheduled to verify the long-term reproducibility.

6. References


[7] vdivide-it.de/mst/imsto/Europractice/Bosch/