Use of interferometric techniques for measuring the displacement field in the plane of a part-through crack existing in a plate

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Abstract
An experimental procedure based on holographic interferometry coupled with computer-aided image processing techniques was used to estimate the stress intensity factor (SIF) of a part-through crack in a plate. The loading consisted of a uniform remote tensile stress, leading to an opening and edge rotation displacement field along the crack plane. The evaluation of the crack opening displacements and edge rotations, carried out after the numerical interpretation of the fringe pattern, made possible an estimation of the SIF along the crack line, following the theoretical basis of the Line-Spring Model developed by Rice and Levy. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction
In structural engineering, some fabrication processes are responsible for the generation of cracks. This is, for example, the case of welding and plastic forming. In machine members submitted to cyclic loads, defects are generated and grow with the persistence of the mechanical disturbance. This is a fatigue problem, which may lead to component failure if the crack reaches a critical size. This structural problem is well understood, thanks to reliable and precise models especially developed to explain this phenomenon.

Fracture Mechanics predicts that an existing crack in a stressed component will propagate if the Stress Intensity Factor (SIF) reaches a critical value. When no information is available about the material failure mode, a conservative procedure consists of considering it as brittle and applying linear elastic fracture mechanics concepts define a criterion to assess if a cracked component under load survives or not. This prediction may be estimated with the Critical SIF ($K_{\text{IC}}$), which is related to the maximum value for the applied remote stress in a cracked component [1]. The problem of integrity estimation in components containing part-through cracks is of great interest in applied fracture mechanics, as it represents the assessment of realistic cases in structural engineering. This is the case of pressure vessels, where it is accepted that they may continue operating even if containing part-through cracks, provided that critical values for the SIF are not reached. The same cannot apply, however, if through cracks exist, which will relieve the pressure of the contained fluid, leading to potentially hazardous situations.

Raju and Newman [2], using a 3D Finite Element mesh, accurately modelled a numerical investigation for the calculation of SIF in part-through cracks with generalised geometry. This procedure, though accurate, is a hard and expensive computational task. Bearing this in mind, Rice and Levy [3] developed an efficient and reliable tool simulating the elastic behaviour of the remaining ligament along a part through crack. This method was given the name line spring model (LSM).

Remarkable improvements of the LSM were achieved. The work developed by Parks et al. and Delale and Erdogan [4,5], following the work of Rice and Levy [3], led the model to meet the requirements of compatibility with moderately thick plates and shells (based on the Reissner–Mindlin transverse shear deformation). Furthermore, the displacement field and constitutive relations were improved in order that mode II and III loads could be considered [6].

The method proposed in this paper leads to the calculation of the SIF along the crack plane, provided that the crack opening and rotation edges are known. In the finite element approach, these displacements along the crack line are obtained when the LSM is combined with a flat or curved shell. Actually, the LSM is a powerful tool, included in

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many finite element codes to assess the structural integrity of plates or shells containing cracks. For example, the ABAQUS and ADINA R & D [7,8] finite element packages have available the LSM as part of their fracture mechanics facilities.

2. Characterisation of the displacement field along a part-through crack plane

Basically, the LSM involves replacing the remaining ligament along a part-through crack by a set of thin plane strain plates having a side notch. This stack of plates reconstructs approximately the profile of the part through crack as described by Rice and Levy [3]. Considering a simple structure composed of a plate with a through crack and the set of Line Spring Elements previously described, it is possible to build-up an approximate model of the plate with a part through crack. For this model Rice and Levy [3] presented solutions for the displacement field along the crack plane involving the opening and the edge rotation, both for remote uniform stressing and bending. Such displacements result from the solution of a system of linear equations, where the coefficients are stiffness factors associated with the opening and edge rotation of each line spring element inserted along the crack plane. Such stiffness factors were calculated by Gross and Srawley [9] and used by Rice and Levy [3]. A more elaborate investigation led to increased precision for the line spring as an approach to the part-through crack problem [4,5].

The calculation of the SIF for a plate with a side notch as represented in Fig. 1, is given by Rice and Levy [3]:

\[ K_1 = \sqrt{h}(\sigma g_1 + m g_b), \]

(1)

where \( \sigma \) and \( m \) are, respectively, the membrane and the bending stress along the crack line. The parameters \( g_1 \) and \( g_b \) depend on the local crack depth and were calculated by Gross and Srawley [9]:

\[ g_1 = \xi^{1/2}(1.99 - 0.41\xi + 18.7\xi^2 - 38.48\xi^3 + 53.85\xi^4), \]

(2)

\[ g_b = x^{1/2}(1.99 - 2.47\xi + 12.97\xi^2 - 23.17\xi^3 + 24.8\xi^4), \]

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where \( \xi = l/l_0 \) is the local depth of the part-through crack at the \( x \)-coordinate (see Fig. 1) and \( h \) is the plate thickness. These expressions allow the calculation of \( K_1 \) at any \( x \)-coordinate along the crack line.

The computation of the stress field \( \sigma \) and \( m \) along the crack plane is achieved after the solution of a system of equations. The dimension of this system of equations depends on the number of divisions along the crack line, which determines the solution accuracy. The solution algorithm appears as a system of equations with a global matrix resulting from the assembly of each line-spring element constitutive equation:

\[ \begin{bmatrix} \delta \\ \theta \end{bmatrix} = \frac{1 - \nu^2}{E} \begin{bmatrix} 2\alpha_{el} & \frac{12\alpha_{elb}}{h} \\ \frac{12\alpha_{elb}}{h} & \frac{72\alpha_{elb}}{h^2} \end{bmatrix} \begin{bmatrix} h\sigma \\ \frac{mh^2}{6} \end{bmatrix}. \]

(3)

where \( E \) and \( \nu \) are Young’s modulus and Poisson’s ratio of the plate, which is assumed to have a linear elastic behaviour. The solution algorithm is explained in detail in Ref. [3]. The parameters \( \alpha_{el} \), \( \alpha_{elb} \), and \( \alpha_{elb} \) are polynomial expansions referring to the stiffness properties of each edge-notched plate and given as follows:

\[ \alpha_{el} = \xi^2(1.98 - 0.94\xi + 18.65\xi^2 - 33.7\xi^3 + 99.26\xi^4 - 211.9\xi^5 + 436.84\xi^6 - 460.48\xi^7 + 289.98\xi^8), \]

(4a)
\[ \alpha_{th} = \xi^2 (1.98 - 1.91\xi + 16.01\xi^2 - 34.84\xi^3 + 83.93\xi^4 \\
- 153.65\xi^5 + 256.72\xi^6 - 244.67\xi^7 + 133.55\xi^8), \] (4b)

\[ \alpha_{ob} = \xi^2 (1.98 - 1.98\xi + 14.43\xi^2 - 31.26\xi^3 + 63.56\xi^4 \\
- 103.36\xi^5 + 147.52\xi^6 - 127.69\xi^7 + 61.5\xi^8). \] (4c)

The procedure to compute the SIF needs only the determination of the displacement field along the crack line. This involves not only the extensional opening but also the edge rotation along the crack line, which is carried out with the solution of the system of equations above. Using the experimental technique described in Section 3 below leads to a hybrid procedure, by which the displacement field along the crack line is first evaluated. Thereafter, the data referring to the displacement field are inserted in Eq. (3), leading to the definition of the stress field along the crack line.

The advantage of this hybrid method consists of avoiding the need for a refined mesh and lengthy computer calculation in order to obtain the displacement field along the crack plane. This will be described in Section 3.

### 3. Experimental technique to evaluate the displacement field in a part-through cracked plate under uniform tension.

This section briefly describes the experimental procedure used to assess the displacement field in the neighbourhood of a part through crack and to estimate the SIF along the crack plane in a uniformly stressed plate. This was achieved by the use of an interferometric technique usually referred to as ESPI (Electronic Speckle Pattern Interferometry). The interference between two wave fronts (reference and object beams), allows the recording of the phase of the light diffused by an unloaded object, in the form of a hologram. When the object is loaded, and a new hologram is recorded, it is possible to obtain the deformed object shape by the

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**Fig. 2.** Experimental optical set-up for in-plane displacement measurements.

**Fig. 3.** ESPI set-up for measurement of displacements in the direction normal to the object surface (out-of-plane displacement field).
correlation of both holograms. The correlation results in an interferogram, where a set of interferometric fringes is obtained, representing equal displacements curves (displacement isocurves). The displacement difference between two adjacent fringes is half of the wavelength $\lambda$ of the laser light used in the experiment ($\lambda \approx 514$ nm, for the green radiation argon laser). Fig. 2 shows a schematic of the experimental set-up for measuring the in-plane displacement field.

The object surface is illuminated with two beams with an equal incident angle, which leads to the definition of a sensitivity vector along the object surface and normal to the plane defined by the double-beam direction. The mirror mounted on a piezoelectric transducer is used to modulate the phase of the interferometric patterns that, in combination with image processing techniques, allows the calculation of the spatial phase distribution. This can be used to measure the displacement field associated to the crack opening edges, referred as $\delta$ in Eq. (3).

In order to measure the out-of-plane displacements, a different set-up is necessary, as represented in Fig. 3.

A simple numerical processing of these displacements leads to the rotation $\theta$ of the crack edges referred in Eq. (3). This is simply carried out by approaching $\theta$ as the $\gamma$-derivation of the transverse displacement $w$ (a valid procedure for a Kirchhoff plate), by the estimation of the $\Delta w/\Delta x$ ratio between two consecutive fringes close to the crack line. The specimen consisted of a polymeric plate uniformly stressed from both ends by equal traction forces. This was achieved by using a gravity load stretching a pair of identical pieces of string looped around precision sheaves fitted with needle bearings. Fig. 4 shows the test specimen inserted in an out-of-plane displacement set-up and part of the load system.

Fig. 4. (a) Specimen in an out-of-plane displacement set-up. (b) Top and side views of the experimental set-up directly assembled on an optical table. The views show the stretched plate and loading device. The load results on a balanced action from a set of gravity masses.
Using the illumination ESPI set-up to measure the in-plane displacements, the interferometry pattern represented in Fig. 5 was obtained. The results correspond to the phase map obtained by image processing after phase calculation with a phase shift technique. The interferometric fringe pattern was obtained by applying a low traction load of about 10 N along the horizontal direction. This value was enough to generate a neat fringe pattern, also, ensuring to keep the stress state of the plate test-specimen material close to a linear elastic behaviour.

To assess the displacement field along the transverse direction to the test specimen surface, the set-up shown in Fig. 3 was used. A plane mirror was used to guide the light beam towards the test specimen surface and back to the CCD lens. This procedure avoids the need to change the position of the object between the two measurements.

Fig. 6 represents the interferometric fringes obtained for out-of-plane displacements corresponding to the same load condition as in the previous measurements.

4. Experimental calculation of the SIF for an uniformly stressed plate

The experimental procedure described above allows the evaluation of the in-plane and out-of-plane displacement field along the crack line. Both displacements give rise to the computation of the pair \( \{ \delta, \theta \} \), the extensional displacement \( \delta \) and the crack edge rotation \( \theta \) for each line-spring element located by the \( x \)-coordinate along the crack line. Having substituted each pair \( \{ \delta, \theta \} \) in Eq. (3) and after inversion gives the pair \( \{ \alpha, m \} \), for each non-dimensional...
coordinate \( \xi = l(x)/h \). With this pair \( \{ \sigma, m \} \), representing the membrane and the bending stresses along the remaining ligament with the local depth \( l(x) \), the stress intensity factor at each coordinate \( x \) is given by Eq. (1).

The usual presentation of the SIF (1) consists of normalising it by \( K_\infty \), which refers to the remote stress loading:

\[
K_\infty = \sqrt{h}(\sigma_\infty g_1).
\]  

For a uniformly stressed plate with a remote tensile stress \( \sigma_\infty \)

\[
K_\infty = \sqrt{h}(m_\infty g_2).
\]  

For a uniformly bent plate with a remote bending moment \( m_\infty \).

The polymer test-specimen used in the present work had a part-through crack cut with the help of a paper cutter. The crack profile was approximately elliptical, having as geometric parameters \( l_0/h = 0.5 \) (being \( h = 6 \text{ mm} \) the plate thickness) and \( h/a = 0.43 \) (where \( 2a = 28 \text{ mm} \) is the crack length).

The calculation of parameters \( \delta \) and \( \theta \) was achieved, respectively for in-plane and out-of-plane displacement fields, from the interfringe pitch, given closely by \( \lambda/2 \) (with the argon laser light wavelength \( \lambda = 514 \text{ nm} \) \( = 514 \times 10^{-9} \text{ m} \)). Figs. 7 and 8 show the procedure used in the computation of the extensional displacement \( \delta \) and edge rotation \( \theta \) of each line-spring element.

In the present example, the value obtained for \( \delta \) was \( \delta = 9.766 \times 10^{-5} \text{ mm} \).

To compute the crack edge rotation, a numerical derivation of the out-of-plane displacement along the direction normal to the crack line was carried out, as sketched in Fig. 8. Essentially the process consists of generating a fringe pattern from the interference between a pair of images; the first one refers to the unloaded test specimen, while the second one is for the out-of-plane displacement field in the test specimen. This results in a hill or elevation in the neighbourhood of the crack, as a consequence of the edge rotation from the bending stresses. The experimental set-up presented in Fig. 3 leads to the display of a fringe pattern only referred to the out-of-plane displacement field. The fringe pitch refers to the elevation (transverse displacement) between a pair of consecutive fringes. Therefore, it is possible to evaluate the derivative of the surface displacement following any direction. A calculation of particular interest consists of the crack edge rotation, following the y-direction, as shown in Fig. 8. Such a calculation was carried out as shortly explained in the same figure.

From the values of \( \delta \) and \( \theta \), the pair \( \{ \sigma, m \} \) can be obtained by inversion of Eq. (3) and thereafter, the SIF from Eq. (1). For the test specimen analysed here, this parameter refers to the maximum crack depth, which corresponds to \( x = 0, \xi = 0.5 \) and \( h/a \approx 0.43 \). The Young’s modulus of the test specimen material was \( E = 210 \text{ MPa} \), which leads to the results for the SIF \( K = 2.58 \text{ N mm}^{-3/2} \) from Rice and Levy [3] and the value of \( K = 2.65 \text{ N mm}^{-3/2} \) by the present procedure.

In normalised values, the present result gives a value of \( K = 0.44 \times K_\infty \) and the theory (Rice and Levy)
$K = 0.43 \times K_\infty$. Such close agreement shows how optical techniques applied to this field of material science are accurate, provided that a carefully designed test rig is available for the experiments. The researchers had to repeat the experience for several times until a neat and coherent fringe pattern was achieved.

5. Conclusions and discussion

An innovative procedure to determine the SIF of part-through crack in plates is presented. The experiment is based on interferometric techniques with enhanced definition from the most recent developments in computer-aided image processing techniques. The method proved to be quite accurate, but also very sensitive to any mechanical imperfection inherent to the set-up. The main parasitic displacement field came from rigid body displacements, which demanded the need for several experimental repetitions in order to eliminate such interference.

The reported results were calculated only at the maximum depth of the crack (the maximum SIF value), although it is possible to extend the analysis to other points along the crack line. Such a procedure is expected to be tedious, as two set-ups are necessary. The authors are developing an optical strain gage, which simply applied normally to the crack line, will measure the crack opening displacement.

References