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Measurement and Analysis of Frequency-Domain Volterra Kernels of Nonlinear Dynamic 3 × 3 MIMO Systems

Mahmoud Alizadeh, Student Member, IEEE, Shoaib Amin, Student Member, IEEE, and Daniel Rönnow, Senior Member, IEEE

Abstract—Multiple-input multiple-output (MIMO) frequency-domain Volterra kernels of nonlinear order 3 are experimentally determined in bandwidth-limited frequency regions. How the effect of higher nonlinear orders can be reduced and how this affects the estimated errors are discussed. The magnitude and the phase of the kernels are Kramers–Kronig consistent. The self-kernels and cross-kernels have different symmetries, and the kernels are therefore determined and analyzed in different regions in the 3-D frequency space. By analyzing the properties along certain paths in the 3-D frequency space, the block structures for the respective kernels are determined. These block structures contain the significant blocks of the general block structures for the third-order kernels. The device under test is an MIMO transmitter for radio frequency signals.

Index Terms—Amplifier distortion, concurrent dual-band, cross modulation, digital predistortion, intermodulation, multiple-input multiple-output (MIMO), power amplifier.

I. INTRODUCTION

The Volterra theory for nonlinear dynamic single-input single-output (SISO) systems was formulated in the late 19th century by Volterra [1] and further developed by Wiener [2] in the 1958s and Schetzen [3] in the 1980s. A Volterra series can be used for modeling the behavior of any time-invariant nonlinear dynamic causal system with fading memory [4]. It suffers from slow convergence and is often poor for modeling strong nonlinearities [5]. The Volterra theory has been applied in recent decades for modeling the complex behavior of (SISO) nonlinear dynamic systems [6], such as electronic devices [1], [7]–[9], electromechanical systems [10], and communication systems [11]–[13].

There are several methods for identifying the time-domain Volterra kernels from experimental data [6], [14] as well as methods for determining the SISO frequency-domain Volterra kernels (also denoted generalized frequency response functions) [15]–[21]. Parametric models can be determined using frequency-domain techniques [22]. The structural information of the block structure of the system can be obtained if the symmetry properties of the frequency-domain Volterra kernels are analyzed [23], [24].

Although much attention has been given to the Volterra theory for SISO systems, relatively little attention has been given to the Volterra theory for nonlinear dynamic multiple-input multiple-output (MIMO) systems. The Volterra theory for the latter system type has been formulated in the last decades [7], [25]–[27]. In addition to self-kernels, the properties of which are the same as those of SISO systems, MIMO Volterra series also include cross-kernels [26]–[28] that are due to the nonlinear interaction of multiple input signals. The number of kernels of an MIMO system becomes significantly higher than that of an SISO system. The cross-kernels also have lower symmetry than self-kernels.

The methods for determining the frequency-domain Volterra kernels of some block structures were reported in [25]. The general methods based on multitone signal were reported in [26], [27], and [29]. In [30], it was reported how the system parameters of an MIMO system could be determined from the frequency-domain Volterra kernels. The determination of MIMO frequency-domain Volterra kernels from experimental data has, to our knowledge, not been reported. In [31], the characterization of nonlinear multiple-input single-output (MISO) system in time domain using concurrent dual-band two-tone signals was reported. A two-tone test is a finger print method and has the drawback that it does not excite the third-order nonlinearities completely and it does not separate the effects of kernels of different nonlinear orders. To excite third-order nonlinearity completely, a signal of three or more tones is required [23].

Radio frequency (RF) amplifiers are in many cases modeled by baseband (low-pass) equivalent discrete time behavioral models [32], [33] that are used to model the behavior in a passband around the center frequency; the model parameters and the input and output signals are complex valued. The models are full Volterra models or, more commonly, reduced versions that correspond to specific block structures. The block structures determined Volterra kernels could be compared with baseband models for MIMO transmitters with input crosstalk and two input signals [34], [35].

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.
In this paper, we describe a method for determining the third-order frequency-domain Volterra kernels of an MIMO system. The input channels are excited by combinations of one-, two-, and three-tone signals, the frequencies of which are stepped. The determined Volterra kernels are analyzed along the paths in the 3-D frequency space, and the symmetry properties are used to determine the dominant block structures (Wiener, Hammerstein, or Hammerstein–Wiener). The effect on the third-order kernel from a higher nonlinear impairment that may occur in MIMO transmitters [34], [35].

This paper is organized as follows. Section II briefly describes the time- and frequency-domain Volterra kernels for the MIMO system followed by the Kramers–Kronig dispersion relationship [36]. Test signals used in this paper are briefly discussed for the identification of self-kernels and cross-kernels [36].

In the experiments in Section II-B. The methods used for the identification of self-kernels and cross-kernels are briefly discussed. A description of the experimental setup is provided in Section III. The measurement results are presented and discussed in Section IV, and the conclusion is given in Section V.

II. THEORY

We first go through the Volterra theory for MIMO systems in Section II-A. We then describe the exciting signals used in the experiments in Section II-B. The methods used for determining the kernels from experimental data are outlined in Section II-C.

A. Volterra Theory of MIMO System

The Volterra theory for MIMO nonlinear dynamic systems is described in [26] and [27]. It is an extension of the corresponding theory for SISO systems [3]. An MIMO system with R input signals and M output signals could be described by M parallel MISO systems with R input signals and one output signal. We denote the rth input signal as \( u^{(r)}(t) \) and the output signal of mth subsystem as \( y^{(m)}(t) \). The output signals of the mth subsystem is written as

\[
y^{(m)}(t) = \sum_{n=1}^{N} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \cdots \sum_{r_n=r_{n-1}}^{R} y^{(m,r_1,\ldots,r_n)}(t)
\]

where \( N \) is the maximum nonlinear order and

\[
y^{(m,r_1,\ldots,r_n)}(t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h^{(m,r_1,\ldots,r_n)}(\tau_1,\ldots,\tau_n) u^{(r_1)}(t-\tau_1) \cdots u^{(r_n)}(t-\tau_n) d\tau_1 \cdots d\tau_n
\]

where \( h^{(m,r_1,\ldots,r_n)}(\tau_1,\ldots,\tau_n) \) is the nth nonlinear order time-domain Volterra kernel of subsystem m for the input signals \( r_1,\ldots,r_n \). If \( r_1 = r_2 = \ldots = r_n = r \), the kernel is called a self-kernel, otherwise a cross-kernel. We assume that the self-kernels are symmetrized in the same way as for an SISO system [3] and that the cross-kernels are averaged [26]; hence, \( r_1 \leq r_2 \leq \cdots \leq r_n \). In the frequency-domain, the output signal \( Y^{(m)}(\omega) \) of the mth subsystem is

\[
Y^{(m)}(\omega) = \sum_{n=1}^{N} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \cdots \sum_{r_n=r_{n-1}}^{R} Y^{(m,r_1,\ldots,r_n)}(\omega)
\]

\[
Y^{(m,r_1,\ldots,r_n)}(\omega) = \frac{1}{(2\pi)^{n-1}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} H^{(m,r_1,\ldots,r_n)}(\omega - \mu_1,\ldots,\mu_{n-1}) U^{(r_1)}(\omega - \mu_1) \cdots U^{(r_n)}(\mu_n - \mu_1) d\mu_1 \cdots d\mu_{n-1}
\]

with

\[
H^{(m,r_1,\ldots,r_n)}(\omega_1,\ldots,\omega_n) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h^{(m,r_1,\ldots,r_n)}(\tau_1,\ldots,\tau_n) e^{-j(\omega_1 \tau_1 + \ldots + \omega_n \tau_n)} d\tau_1 \cdots d\tau_n
\]

being the n-dimensional Fourier transform (FT) of the nth-order Volterra kernel and \( U^{(r)}(\omega) \) is the FT of the input signal \( u^{(r)}(t) \).

The system is real valued; hence, for any kernel [3]

\[
H^{(m,r_1,\ldots,r_n)}(\omega_1,\ldots,\omega_n) = H^{(m,r_1,\ldots,r_n)}(-\omega_1,\ldots,-\omega_n)^{*}
\]

where * denotes the complex conjugate. We study a \( 3 \times 3 \) odd-order system and measure and analyze third-order kernels. We notice that, for a \( 3 \times 3 \) system, there are nine linear kernels, nine self-kernels, eighteen \( 2 \times 1 \) cross-kernels, and three \( 3 \times 1 \) cross-kernels. For the symmetrized third-order self-kernels, we have the symmetry properties

\[
H^{(3,m,r,r)}(\omega_1,\omega_2,\omega_3) = H^{(3,m,r,r)}(\omega_2,\omega_1,\omega_3)
\]

and for the other permutations of \( (\omega_1,\omega_2,\omega_3) \). For \( 2 \times 1 \) third-order cross-kernels

\[
H^{(3,m,r_1,r_2)}(\omega_1,\omega_2,\omega_3) = H^{(3,m,r_1,r_2)}(\omega_2,\omega_1,\omega_3)
\]

but different for all other permutations of \( (\omega_1,\omega_2,\omega_3) \). For third-order cross-kernels, when \( r_1 \neq r_2 \neq r_3 \)

\[
H^{(3,m,r_1,r_2,r_3)}(\omega_1,\omega_2,\omega_3) \neq H^{(3,m,r_1,r_2,r_3)}(\omega_2,\omega_1,\omega_3)
\]

and all other permutations of \( (\omega_1,\omega_2,\omega_3) \), in the general case. In the Appendix, we derive the relations between the real valued representation in (1)–(5) and a complex representation that is common for narrowband communication systems.

We use the method in [3] for an SISO system for the synthesis of a basic third-order Volterra operator from two multiplications and five linear filters but extend it to cross-kernels. In Fig. 1, a basic third-order system is shown with three input signals, \( u^{(r_1)}, u^{(r_2)}, \) and \( u^{(r_3)} \). \( H_{a,q}, \ldots, H_{c,q} \) are linear time-invariant systems and \( q \) is an index explained in the following.
The third-order kernel transform without symmetrization is [3]

\[ H_3^{(m,r,r)}(ω_1, ω_2, ω_3) = H_{a,q}(ω_1)H_{b,q}(ω_2) \times H_{c,q}(ω_1 + ω_2 + 2ω_3). \] (10)

For a self-kernel, we have \( r_1 = r_2 = r_3 = r \) and \( q = 1 \) [3]

\[ H_3^{(m,r,r)}(ω_1, ω_2, ω_3) = \frac{1}{6} \sum \text{permutations of } (ω_1, ω_2, ω_3) \] (11)

For a 2 × 1 cross-kernel, we have \( r_1 = r_2 ≠ r_3 \) and there are three permutations of the input signals \( u^{(r_1)}, u^{(r_2)} \), and \( u^{(r_3)} \) in Fig. 1, which correspond to different systems. The permutations \( [u^{(r_1)}, u^{(r_2)}, u^{(r_3)}] \) and \( [u^{(r_3)}, u^{(r_1)}, u^{(r_2)}] \) result in the same type of system, as interchanging \( H_{a,q} \) and \( H_{b,q} \) in Fig. 1 does not result in different types of system when \( r_1 ≠ r_2 ≠ r_3 \). The basic block structure of a 2 × 1 kernel is the parallel of two systems like that in Fig. 1, with input signals \( [u^{(r_1)}, u^{(r_2)}, u^{(r_3)}] \) and \( [u^{(r_3)}, u^{(r_1)}, u^{(r_2)}] \). The symmetrized kernel is

\[ H_3^{(m,r,r)}(ω_1, ω_2, ω_3) = \frac{1}{2} \left[ H_{3,1}^{(m,r,r)}(ω_1, ω_2, ω_3) + H_{3,2}^{(m,r,r)}(ω_1, ω_2, ω_3) \right] \] (12)

where \( q = 1 \) corresponds to the input signals \( [u^{(r_1)}, u^{(r_2)}, u^{(r_3)}] \) and \( q = 2 \) corresponds to the input signals \( [u^{(r_1)}, u^{(r_2)}, u^{(r_3)}] \). For a 3 × 1 cross-kernel, we have \( r_1 ≠ r_2 ≠ r_3 \) and there are six permutations of the input signals \( u^{(r_1)}, u^{(r_2)}, u^{(r_3)} \). There are three different systems: the permutations \( [u^{(r_1)}, u^{(r_2)}, u^{(r_3)}] \) and \( [u^{(r_2)}, u^{(r_1)}, u^{(r_3)}] \) result in the same type of system that we number \( q = 1 \), the permutations \( [u^{(r_1)}, u^{(r_2)}, u^{(r_3)}] \) and \( [u^{(r_3)}, u^{(r_1)}, u^{(r_2)}] \) in another type of system \( (q = 2) \), and the permutations \( [u^{(r_2)}, u^{(r_3)}, u^{(r_1)}] \) and \( [u^{(r_3)}, u^{(r_2)}, u^{(r_1)}] \) in a third system \( (q = 3) \). The general kernel has three systems in parallel and cannot be symmetrized

\[ H_3^{(m,r,r)}(ω_1, ω_2, ω_3) = H_{3,1}^{(m,r,r)}(ω_1, ω_2, ω_3) + H_{3,2}^{(m,r,r)}(ω_1, ω_2, ω_3) \] (13)

The system is causal; hence, the real and imaginary parts or amplitude and phase of the frequency-domain Volterra kernels are related to Kramers–Kronig relations that for the latter case are [23], [36]

\[ \angle H_3(σ) = -\frac{2π}{P} \int_0^∞ \ln |H_3(σ′)| dσ′ \]

\[ = -\frac{1}{P} \int_0^∞ \ln |H_3(σ′)| \ln \left( \frac{σ + σ'}{σ - σ'} \right) dσ′ \]

where \( σ \) is a path in the \( n \)-dimensional frequency space and \( P \) denotes the principal value.

B. Test Signals

We use three-tone signals and combinations of two- and one-tone signals to determine the frequency-domain Volterra kernels, as shown in Fig. 1. The input signals are bandwidth limited around the carrier frequency \( f_c = ω_c / 2π \). Therefore, we disregard the even-order terms [23]. Moreover, only the first- and third-order responses are studied, as the first-order response is the dominating one and the third-order response typically is the largest of the nonlinear terms. The test signals for the identification of self- and cross-Volterra kernels are described in the following. A three-tone signal is required for determining self-kernels. The 2 × 1 kernels require a two-tone signal in one channel and a single tone signal in another. The 3 × 1 kernels require a single-tone signal in each channel.

In the following, we give equations for a self-kernel \( H_3^{(2,2,2,2)} \). They are, of course, valid for any other self-kernel. In the same way, we give equations for the 2 × 1 cross-kernel \( H_3^{(2,2,2,3)} \) and the 3 × 1 cross-kernel \( H_3^{(2,1,2,3)} \). Naturally, these could be used for any 2 × 1 or 3 × 1 cross-kernels.

1) Three-Tone Test: To determine the third-order self-Volterra kernels of a \( 3 \times 3 \) nonlinear MIMO system [see Fig. 2(a)], the system is excited with a three-tone signal at input channel 2. The three-tone signal is given as [23]

\[ u^{(2)}(t) = A[α′ \cos(ω′t + φ′) + α'' \cos(ω''t + φ'')] \] (15)

where \( u^{(2)}(t) \) denotes the input signal at the second input of a \( 3 \times 3 \) MIMO system. In (15), \( A \) is the amplitude, \( ω′, ω'' \) and \( φ′ \) are the angular frequencies, and \( φ' \) and \( φ'' \) are the phases of the three tones, respectively, \( α', α'' \), and \( α'' \) are dimensionless constants. For simplicity, we assume that \( α' = α'' = α''' = 1 \).

When excited with \( u^{(2)}(t) \), the output of the second subsystem can be written as

\[ y^{(2)}(t) = y^{(2)}_1(t) + y^{(2)}_3(t) \] (16)

where \( y^{(2)}_1(t) \) denotes the linear output components of the second subsystem and \( y^{(2)}_3(t) \) describes the third-order output components. Because we are interested in determining the third-order self-Volterra kernels of a system, the third-order output components of the second subsystem are

\[ y^{(2)}_3(t) = \sum_{k=1}^K A_k |H_{3,k}| \cos(ω_k t + Δ_k + \angle H_{3,k}) \] (17)
where $K$ is the number of all third-order products, and $A_k$, $H_{3,k}$, $\omega_k$, and $\Delta_k$ are amplitude, kernel response, frequency, and phase, respectively [23]. In this paper, the analysis of self-kernel is restricted to $H_{3}(2,2,2)^{(2,2,2,2)}(-\omega', \omega'', \omega''')$; this gives $A_k = 3/2 A^3$.

2) 2 + 1 Tone Test: To analyze the third-order $2 \times 1$ cross-kernels, a two-tone signal in one channel and a single-tone in another channel are used [see Fig. 2(b)]. The excitation signals are

$$u^{(2)}(t) = A[a' \cos(\omega' t + \phi') + a'' \cos(\omega'' t + \phi'')]$$
$$u^{(3)}(t) = Aa'' \cos(\omega'' t + \phi''')]$$

(18)

where $u^{(2)}(t)$ and $u^{(3)}(t)$ denote the two- and single-tone signals at the second and third inputs of the MIMO system, respectively. In this test, we excite $u^{(2)}(t)$ and $u^{(3)}(t)$ at equal powers as in three-tone test; therefore, $a' = a'' = \sqrt{3}/2$ and $a''' = \sqrt{3}$. The corresponding third-order output component of the second subsystem becomes

$$y_3^{(2)}(t) = \sum_{r_1=1}^{3} \sum_{r_2=1}^{3} \sum_{r_3=1}^{3} y_3^{(2,r_1,r_2,r_3)}(t).$$

(19)

Note that, in this paper, we restrict the analysis of $2 \times 1$ cross-kernels to $H_{3}^{(2,2,2,2)}(-\omega', \omega'', \omega''')$ and $H_{3}^{(2,2,2,3)}(\omega', \omega'', -\omega''')$. The third-order cross components at the output of second subsystem are

$$y_3^{(2,2,2,3)}(t) = \sum_{k}^{3} \left(\frac{3}{2}\sqrt{3}\right) A^3 |H_{3,k}| \cos(\omega_k t + \Delta_k + \angle H_{3,k})$$

(20)

$k$ indicates the terms corresponding to the frequencies at $(-\omega', \omega'', \omega''')$ and $(\omega', \omega'', -\omega''')$.

3) 1+1+1 Tone Test: $3 \times 1$ cross-kernels can be determined by exciting three channels each with a single-tone signal, as shown in Fig. 2(c). The three single-tone signals are

$$u^{(1)}(t) = Aa' \cos(\omega' t + \phi')$$
$$u^{(2)}(t) = Aa'' \cos(\omega'' t + \phi'')$$
$$u^{(3)}(t) = Aa''' \cos(\omega''' t + \phi''')$$

(21)

where $u^{(1)}(t)$, $u^{(2)}(t)$, and $u^{(3)}(t)$ are three single-tone signals applied to input channels 1, 2, and 3, respectively, and $a' = a'' = a''' = \sqrt{3}$. Under these excitation signals, the corresponding third-order output components of the second subsystem can be described as

$$y_3^{(2)}(t) = \sum_{r_1=1}^{3} \sum_{r_2=1}^{3} \sum_{r_3=1}^{3} y_3^{(2,r_1,r_2,r_3)}(t).$$

(22)

Notice that, when a $3 \times 3$ MIMO system is excited by single-tone signals in each channel, the resulting third-order output components of $m$th subsystem will contain a self-component and 17 cross components. Therefore, in this paper, the analysis is restricted to a $3 \times 1$ cross-kernel of the form $H_{3}^{(2,1,2,3)}(-\omega' , \omega'', \omega''')$, $H_{3}^{(2,1,2,3)}(\omega' , -\omega'', \omega''')$ and $H_{3}^{(2,1,2,3)}(\omega' , \omega'', -\omega''')$. Similarly, the corresponding third-order cross components in the output of the second subsystem are

$$y_3^{(2,r_1,r_2,r_3)}(t) = \sum_{k}^{3} \left(3\sqrt{3}\right) A^3 |H_{3,k}| \cos(\omega_k t + \Delta_k + \angle H_{3,k})$$

(23)

where $k$ indicates the terms at $(-\omega', \omega'', \omega''')$, $(\omega', -\omega'', \omega''')$, and $(\omega', \omega'', -\omega''')$.

C. Identification of Kernels

We identify the Volterra kernels by a standard least-squares (LS) technique for identifying the amplitude and phase of spectral components of known frequencies of a sampled time-domain signal [37].

For the self-kernels and cross-kernels, we form the equation systems for each test case

$$y = X\theta$$

(24)

that are solved in the LS sense as described in [37]. In (24), $y$ is a column vector that contains the measured and sampled output signals of the respective test case

$$y = [y^{(m)}(0) \ y^{(m)}(T_s) \ \cdots \ y^{(m)}(N-1)T_s]^T$$

(25)

where $N$ is the number of samples and $T_s$ is the sampling time. $\theta$ contains the estimated kernels

$$\begin{bmatrix}
Aa' \hat{H}_1^{(mr_1)}(\omega') \\
Aa'' \hat{H}_1^{(mr_2)}(\omega'') \\
\vdots \\
\frac{3}{2} A^3 a' a'' a''' \hat{H}_3^{(mr_1,r_2,r_3)}(\omega', \omega', -\omega''')
\end{bmatrix}$$
where $\hat{H}_1$ and $\hat{H}_3$ are estimated kernels, $a'$ and $a''$, and $a'''$ are different for the test cases in II-B. $\hat{H}_1$ is dimensionless, and $\hat{H}_3$ is in $V^{-2}$. The matrix $X$ has the size $N \times 12$ and is the regression matrix of the input signals and their multiplications [23]. For simplicity, we use only the three terms that contain all $\omega'$, $\omega''$, and $\omega'''$ in the following analysis.

The LS estimation technique has previously been used in [23] and [31] for both frequency- and time-domain Volterra kernels for SISO and concurrent dual-band power amplifiers, respectively. The only requirement is to know the frequencies of each spectral component. Because the input signals are generated at the known frequencies ($\omega'$, $\omega''$, and $\omega'''$) and the kernels are at known frequencies, this requirement is satisfied. The identified kernels at each output frequency will also contain contributions from higher odd-order nonlinear terms [23] that is written in the form of polynomial model in $A^2$

$$\hat{H}_1(Q) = H_1 + \beta_{1,3}H_3A^2 + \cdots + \beta_{1,Q}H_Q(A^3)^{Q-1}, \quad Q \geq 1 \tag{26}$$

$$\hat{H}_3(Q) = H_3 + \beta_{3,5}H_5A^2 + \cdots + \beta_{3,Q}H_Q(A^3)^{Q-3}, \quad Q \geq 3 \tag{27}$$

where $Q$ is the maximum nonlinear order, and $\beta_{1,Q}$ and $\beta_{3,Q}$ are polynomial coefficients. The third-order Volterra kernels can be identified at different amplitude levels and contributions from higher nonlinear orders can be reduced by fitting a polynomial as in (27). The estimated third-order kernel is the constant term, independent of $A^2$ [23].

III. EXPERIMENT

The experimental setup is shown in Fig. 3. The vector signal generators (VSGs) were three Rohde & Schwarz SMBV100A. The analog-to-digital converter was 14-b SP-Device ADQ214 digitized at 400-MHz sampling rate. The device under test (DUT) was composed of three RF amplifiers, Mini-Circuits ZHL-42, with typical linear gain of 31.5 dB, and with 1-dB compression point at an output of 30 dBm, which were coupled with a coupler on the input signals (see Fig. 3). The coupler causes crosstalk, which is a well-known effect in MIMO transmitters [35]. The coupler was designed in microstrip technology using FR4 substrate (50Ω). There were three transmission lines of width 15 mm and length 65 mm. The coupling between an outer and the inner channel was measured using a vector network analyzer (VNA) to be $-13.5$ dB, and between the two outer channels, it was $-21.5$ dB.

Three VSGs were synchronized in carrier frequency, reference frequency, and baseband digital clock. The generated signals were a three-tone signal, a $2 + 1$-tone signals, and three single-tone signals, respectively, as mentioned in (15), (18), and (21) and as shown in Fig. 2(a)–(c), respectively. Two types of tones were investigated in this paper. In the first type, one of the tones was stepped, whereas the other two tones were fixed; it is termed the one-stepped-two-fixed type. The second type is termed the two-stepped-one-fixed type and has two stepped tones and one fixed. Table I shows the frequencies of the test signals. In each type of tones, 1, 2, and 3 set of test signals were performed for self-kernel, $2 \times 1$ and $3 \times 1$ cross-kernels analysis, respectively. The power of the signals stepped from $-5$ to $-15$ dBm in the steps of 0.5 dB. The output RF signals were downconverted into an intermediate frequency using mixers, Mini-Circuits ZX05-42MH-S, and thereafter filtered by bandpass filters. The measured signals were compensated by a constant gain in the postprocessing part to compensate for losses in the used components. To improve the performance of the measurement system, 100 coherent averages were performed and then time aligned with the input signal in the postprocessing part [33].

IV. RESULTS

In Section IV-A, we describe how the kernels are identified from experimental data. In Section IV-B, we show how a self-kernel, a $2 \times 1$ cross-kernel, and a $3 \times 1$ cross-kernel are analyzed to determine the corresponding block structures.

A. Kernel Estimation

A third-order kernel identified from experimental data at one amplitude level may be affected by contributions from higher nonlinear order kernels. As an example, Fig. 4 shows the experimentally determined real and imaginary parts of $H_3^{(2,2,2,2)}$ data versus input signal power ($A^2$). Also shown are LS fitted polynomials as in (27) of different maximum nonlinear order of $Q$. A maximum nonlinear order of at least 7 is needed to well fit the experimental data.

In Fig. 5, as an example the self-kernel $H_3^{(2,2,2,2)}$ is shown along a path in the 3-D frequency space given by $(-f_c + 7.2$, $f_c + f_a$, $f_c - f_a)$ MHz, where $f_a$ is stepped from $-10$ to $+10$ MHz. The kernels are analyzed in detail in the following. The kernel is shown as estimated using different maximum nonlinear orders, $Q$, in (27). The error bars in Fig. 5 are obtained from the errors of the LS fit and are hence due to random errors in the experimental data. Using a higher $Q$ in (27) results in a more pronounced structure of the curve. There is also a systematic shift in magnitude
TABLE I
SUMMARY OF THE PERFORMED EXPERIMENTS, FOR THE SELF-KERNEL $r_1 = r_2 = r_3 = 2$, FOR THE $2 \times 1$ CROSS-KERNEL $r_1 = r_2 = 2$, AND $r_3 = 3$ AND FOR THE $3 \times 1$ CROSS-KERNEL $r_1 = 1$, $r_2 = 2$, AND $r_3 = 3$. THE CENTER FREQUENCY IS $f_c = 2.14$ GHz, $f_{st}$ IS STEPPED FROM $-10$ TO $+10$ MHz IN THE STEPS OF $25$ kHz AND $f_{sh}$ IS A FREQUENCY SHIFT OF $-7.5$, $-2.5$, $+2.5$, AND $+7.5$ MHz. THE CONSTANTS ARE IN MHz.

<table>
<thead>
<tr>
<th>Type</th>
<th>Kernel</th>
<th>$f'$, $u(r_1)$</th>
<th>$f''$, $u(r_2)$</th>
<th>$f'''$, $u(r_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>self</td>
<td>$f_c + 0.3 + f_{sh}$</td>
<td>$f_c + 0.1 + f_{sh}$</td>
<td>$f_c + f_{st}$</td>
<td></td>
</tr>
<tr>
<td>$2 \times 1$</td>
<td>$f_c + 0.3 + f_{st}$</td>
<td>$f_c + 0.1 + f_{sh}$</td>
<td>$f_c + f_{st}$</td>
<td></td>
</tr>
<tr>
<td>cross</td>
<td>$f_c + 0.3 + f_{sh}$</td>
<td>$f_c + 0.1 + f_{sh}$</td>
<td>$f_c + f_{st}$</td>
<td></td>
</tr>
<tr>
<td>$3 \times 1$</td>
<td>$f_c + 0.3 + f_{sh}$</td>
<td>$f_c + 0.1 + f_{sh}$</td>
<td>$f_c - 0.1 + f_{sh}$</td>
<td></td>
</tr>
<tr>
<td>cross</td>
<td>$f_c + f_{st}$</td>
<td>$f_c + f_{st}$</td>
<td>$f_c - f_{st}$</td>
<td></td>
</tr>
<tr>
<td>1-swept–1-fixed</td>
<td>$f_c + 0.3 + f_{sh}$</td>
<td>$f_c + f_{st}$</td>
<td>$f_c - f_{st}$</td>
<td></td>
</tr>
<tr>
<td>self</td>
<td>$f_c + 0.3 + f_{sh}$</td>
<td>$f_c + f_{st}$</td>
<td>$f_c - f_{st}$</td>
<td></td>
</tr>
<tr>
<td>$2 \times 1$</td>
<td>$f_c + 0.3 + f_{sh}$</td>
<td>$f_c + f_{st}$</td>
<td>$f_c - f_{st}$</td>
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<tr>
<td>cross</td>
<td>$f_c + 0.3 + f_{sh}$</td>
<td>$f_c + f_{st}$</td>
<td>$f_c - 0.1 + f_{sh}$</td>
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</tr>
<tr>
<td>$3 \times 1$</td>
<td>$f_c + 0.3 + f_{sh}$</td>
<td>$f_c + f_{st}$</td>
<td>$f_c - 0.1 + f_{sh}$</td>
<td></td>
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<tr>
<td>cross</td>
<td>$f_c + f_{st}$</td>
<td>$f_c - f_{st}$</td>
<td>$f_c - f_{st}$</td>
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</tr>
</tbody>
</table>

Fig. 4. Real and imaginary parts of $\hat{H}^{(2,2,2,2)}_3$ versus $x^2$ at frequencies ($-f_c + 7.2, f_c - 10, f_c + 10$) MHz. Also shown are fitted curves for different values of $Q$ that include the effects of higher nonlinear orders.

Fig. 5. Magnitudes of the estimated self-kernel $H^{(2,2,2,2)}_3$ along the frequency path ($-f_c + 7.2, f_c + f_{st}, f_c - f_{st}$) MHz, using $Q = 7$. Also shown are the phases obtained from the magnitudes in Fig. 5 using the Kramers–Kronig relations in (14).

Fig. 6. Estimated phases of the self-kernel $H^{(2,2,2,2)}_3$ along the path ($-f_c + 7.2, f_c + f_{st}, f_c - f_{st}$) MHz, using $Q = 7$. Also shown are the phases obtained from the magnitudes in Fig. 5 using the Kramers–Kronig relations in (14).

that gives the best compromise between random errors and magnitude shift, i.e., a tradeoff between precision and accuracy as described in [23].

The magnitude of the seventh-order kernel in Fig. 5 is used to calculate the corresponding phase using (14). The experimental data are in a finite frequency range and the magnitude outside this range is set to zero. The obtained phase is shown in Fig. 6 together with the experimentally determined phase. Fig. 6 shows that our data are Kramers–Kronig consistent, which is a necessary condition to correctly determine frequency-domain kernels. We see in Figs. 5 and 6 that the structure in the magnitude has a corresponding structure in the phase. We therefore analyze only the magnitude in the following.

B. Analysis of Volterra Kernels

The basic block structures for the third-order kernels are given in Fig. 1 and (10)–(13). Here, we show how the experimentally determined third-order kernels can be analyzed to determine which blocks give the dominating contribution to a kernel’s frequency dependence. We extend the technique to characterize the kernels along different frequency paths in [23] and [24] to the analysis of cross-kernels.
For self-kernels $H_{3}^{(m,r,r,r)}$, the symmetry properties in (7) yields that the kernel is identical in the regions in the vicinity of $(-f_c, f_c, f_c)$, $(f_c, -f_c, f_c)$, and $(f_c, f_c, -f_c)$ shown in Fig. 7. We present data along the paths in the vicinity of $(-f_c, f_c, f_c)$ in Section IV-B1.

For $2 \times 1$ cross-kernels $H_{3}^{(m,r,r,r)}$, the kernel is identical along the paths in the vicinity of $(f_c, f_c, f_c)$ and $(f_c, -f_c, f_c)$ but not $(f_c, f_c, -f_c)$ [see (8)]. The $2 \times 1$ kernel is, thus, identical in two of the three regions in Fig. 7. In Section IV-B2, we present data along the paths in the vicinity of $(f_c, f_c, f_c)$ and $(f_c, f_c, -f_c)$.

For $3 \times 1$ cross-kernels $H_{3}^{(m,r,r,r)}$, the kernel can be different in the vicinity of $(f_c, f_c, f_c)$, $(f_c, -f_c, f_c)$, and $(f_c, f_c, -f_c)$, respectively [see (9)], and we present data for all three frequency regions in Fig. 7 in Section IV-B3.

For the $2 \times 1$ and $3 \times 1$ cross-kernels, we present data along the paths for which the self-kernel is identical in the three frequency regions in Fig. 7. The paths of the type one-stepped-two-fixed in Fig. 7(a) are obtained from the tests with only one frequency being stepped, whereas the paths of the type two-stepped-one-fixed in Fig. 7(b) are obtained from tests with two stepped frequencies.

A path in the 3-D frequency space that is a straight line is described by

$$ f_i = f_c + f_{co} + \gamma_i f_{sh} + \lambda_i f_{st} \quad (28) $$

where $i = 1, 2, 3$ refers to the frequency axis, $f_c$ is the carrier frequency, $f_{co}$ is a frequency distance from the carrier, and $f_{sh}$ is a shift in frequency, i.e., different values of $f_{sh}$ give different paths within the same type of paths. $\gamma_i = 1$ when a frequency is shifted; otherwise, $\gamma_i = 0$. $f_{st}$ is stepped from $-10$ to $+10$ MHz, $\lambda_i = 1$ when a frequency is stepped; otherwise, $\lambda_i = 0$. The carrier frequency $f_c = 2.14$ GHz, $f_{co}$, $i = 1, 2, 3$, is chosen as 0.3, 0.1, and $-0.1$ MHz, respectively. The frequency shift $f_{sh}$ gets four different values of $-7.5, -2.5, +2.5,$ and $+7.5$ MHz to have four paths with a shift of $\pm5$ MHz. Fig. 7(a) shows all paths of type one-stepped-two-fixed and Fig. 7(b) shows all paths of type two-stepped-one-fixed, in the vicinity of $(-f_c, f_c, f_c)$, $(f_c, -f_c, f_c)$, and $(f_c, f_c, -f_c)$, respectively.

![Fig. 7](image)

Fig. 7. Frequency paths in three regions in the vicinity of $(-f_c, f_c, f_c)$, $(f_c, -f_c, f_c)$, and $(f_c, f_c, -f_c)$, respectively, along which we analyze the determined Volterra kernels. (a) Frequency paths of one-stepped-two-fixed tones as described in the legends. (b) Frequency paths of two-stepped-one-fixed tones as described in the legends.

Fig. 8. Magnitudes of the self-kernel $H_{3}^{(2,2,2,2)}$ along the given frequency paths. The horizontal lines indicate the frequency shifts of the largest structure.

1) Self-Kernel $H_3^{(2,2,2,2)}$: The self-kernel was determined in the vicinity of $(f_c, f_c, f_c)$ from a three-tone test. We first analyze the kernel along one-stepped-two-fixed paths, $(f_1, f_2, f_3) = (-f_c - 0.3 - f_{sh}, f_c + 0.1 + f_{sh}, f_c + f_{sh})$, where $f_1$ and $f_2$ are fixed frequencies for each path and $f_3$ is stepped. The magnitude of the self-kernel $H_3^{(2,2,2,2)}$ along these paths is shown in Fig. 8. The dominating structures of the curves are minima in magnitude located around $-7.2, -2.2, +2.8,$ and $+7.8$ MHz relative to $f_c$, i.e., there are shifts of $\pm5$ MHz between the curves. The general frequency-domain kernel of the block structure in Fig. 1 is obtained from (10) and (11). Along the paths of the type one-stepped-two-fixed, the self-kernel $H_3^{(2,2,2,2)}$ is

$$ H_3 = \frac{1}{6} \left[ H_{a,1}(-f_c - 0.3 - f_{sh})H_{b,1}(f_c + 0.1 + f_{sh}) \right. $$

$$ \times H_{d,1}(-0.2)H_{c,1}(f_c + f_{st}) $$

$$ + [H_{a,1}(f_c + 0.1 + f_{sh})H_{b,1}(f_c + f_{sh})] $$

$$ \times H_{d,1}(2f_c + 0.1 + f_{sh} + f_{st})H_{c,1}(-f_c - 0.3 - f_{sh}) $$

$$ + [H_{a,1}(f_c + f_{sh})H_{b,1}(f_c + 0.1 + f_{sh})] $$

$$ \times H_{d,1}(f_c - 0.3 - f_{sh})H_{c,1}(f_c + f_{sh}) $$

$$ \times H_{d,1}(-0.3 - f_{sh} + f_{st})H_{c,1}(f_c + 0.1 + f_{sh}) $$

$$ \times H_{c,1}(f_c - 0.2 + f_{st}). \quad (29) $$

![Fig. 8](image)
Only terms that are the functions of $f_{st}$ can give a frequency dependence of $H_3$ along paths; other terms in (29) are constant along this type of paths. The terms that are functions of $(f_c + f_{sh})$ will not give any shift when $f_{sh}$ is shifted, as shown in Fig. 8. Only terms that are the functions of $f_{sh}$ and $f_{st}$ can give a frequency dependence and frequency shift. The $H_{a,1}$, $H_{b,1}$, $H_{c,1}$, and $H_{e,1}$ terms can thus not contribute to the frequency shifts and dependence seen in Fig. 8. The term $H_{d,1}(-0.3 - f_{sh} + f_{st})$ gives a shift of $f_{sh}$ (or $+5$ MHz), which is in agreement with the shift of the structure in Fig. 8. This term corresponds to $H_{d,1}$ being excited at baseband frequencies. The term $H_{d,1}(2f_c + 0.1 + f_{sh} + f_{st})$ would give a shift of $-f_{sh}$ (or $-5$ MHz), which is not seen in Fig. 8. This term corresponds to $H_{d,1}$ being excited at $2f_c$, where the relative bandwidth is smaller than 0.5% and hence has small frequency dependence. The frequency argument of term $H_{d,1}(-0.3 - f_{sh} + f_{st})$ becomes zero at $f_{st} = f_{sh} + 0.3$. The main structures in Fig. 8 therefore occur when the frequency argument of $H_{d,1}$ is close to zero, i.e., at baseband frequencies.

The second type of paths, two-stepped-one-fixed, is $(f_1, f_2, f_3) = (-f_c - 0.3 - f_{sh}, f_c + f_{st}, f_c - f_{st})$, where $f_2$ and $f_3$ are stepped and $f_1$ is fixed. Fig. 9 shows the kernel $H_3^{(2,2,2,2)}$ along these paths. Each curve has two clear structures that are shifted. The first is located at $-7.2$, $-2.2$, $+2.8$, and $+7.8$ MHz relative to $f_c$ and is shifted $+5$ MHz; the second one is located at $+7.2$, $+2.2$, $-2.8$, and $-7.8$ MHz relative to $f_c$ and is shifted $-5$ MHz. The self-kernel along this type of paths is obtained from (10) and (11)

$$H_3 = \frac{1}{6} [H_{a,1}(-f_c - 0.3 - f_{sh})H_{b,1}(f_c + f_{st}) + H_{d,1}(f_c + f_{st})H_{b,1}(-f_c - 0.3 - f_{sh}) + H_{d,1}(-0.3 - f_{sh} + f_{st})H_c(-f_c - f_{st}) + H_{d,1}(f_c + f_{st})H_d(2f_c)H_{c,1}(-f_c - 0.3 - f_{sh}) + H_{d,1}(f_c - f_{st})H_{b,1}(-f_c - 0.3 - f_{sh}) + H_{e,1}(-f_c - 0.3 - f_{sh})H_{b,1}(f_c - f_{st}) + H_{d,1}(-0.3 - f_{sh} - f_{st})H_{c,1}(f_c + f_{st}) + H_{e,1}(-f_c - 0.3 - f_{sh})].$$

(30)

The only terms in (30) that may both be frequency dependent when $f_{st}$ is stepped and have structures that are shifted when $f_{sh}$ is changed are the terms $H_{d,1}(-0.3 - f_{sh} + f_{st})$ and $H_{d,1}(-0.3 - f_{sh} - f_{st})$. The former causes a shift of $f_{sh}$ (or $+5$ MHz), which is seen in one of the structures in Fig. 9 at $-7.2$, $-2.2$, $+2.8$, and $+7.8$ MHz. The latter causes a shift of $-f_{sh}$ (or $-5$ MHz), which is seen in Fig. 9 in the structure at $+7.2$, $+2.2$, $-2.8$, and $-7.8$ MHz. In the same way as for Fig. 8, the terms $H_{a,1}$, $H_{b,1}$, $H_{c,1}$, and $H_{e,1}$ cannot cause the type of shifts and frequency dependence seen in Fig. 9. Thus, the data in Figs. 8 and 9 show that the frequency dependence of $H_3^{(2,2,2,2)}$ is due to a term of the type $H_{d,1}$ in Fig. 1 that is excited at baseband frequencies. These baseband effects are attributed to bias modulation or thermal or thermo electric memory effects [38].

The functions $H_3(\cdot)$, $H_b(\cdot)$, and $H_c(\cdot)$ in Fig. 1 can be interpreted as the models of the input matching networks and $H_b(\cdot)$ describes the output matching network [39]. The blocks $H_a$, $H_b$, $H_c$, and $H_d$ do not contribute to the frequency dependence of $H_3^{(2,2,2,2)}$ for the DUT. Therefore, the simplified model that describes the main behavior of $H_3^{(2,2,2,2)}$ becomes

$$H_3^{(2,2,2,2)}(w_1, w_2, w_3) = \frac{1}{3} \{H_{d,1}(w_1, w_2) + H_{d,1}(w_1, w_3) + H_{d,1}(w_2, w_3)\} \quad (31)$$

and the corresponding block structure is shown in Fig. 10. The block structure of the self-kernel corresponds to an input–output relationship $y_3(t) = u(t) \times H_{d,1}[u^2(t)]$. A time-domain complex baseband model can be derived from the block structure in Fig. 10, using standard techniques [32], [33]. The complex baseband equivalent third-order terms are

$$y_3^{(2,2,2,2)}(n) = u^{(2)}(n) \sum_{m=0}^{M} h_3(m) \cdot |u^{(2)}(n - m)|^2 \quad (32)$$

where $u^{(2)}(n)$ is the baseband discrete time equivalent of $u^{(2)}(t)$, $h_3(m)$ is the model parameter, and $M$ is the memory length of $H_{d,1}(\cdot)$ for the physical system $M \to \infty$. The model in (32) is known as the envelope-memory polynomial model [40]. It can be explained by a Hammerstein–Wiener structure of a cascaded system $N\cdot H\cdot N$, where $N$ is a static nonlinearity and $H$ is a linear dynamic system [23]. We did not include the term $u^{*}(n) \cdot (u(n-m))^2$ in (32), as this term corresponds to exciting $H_d(\cdot)$ at $2f_c$. It was found by analyzing (29) and (30) that this term does not contribute to the frequency dependence and hence memory effects.

2) Cross-Kernel $H_3^{(2,2,2,3)}$. We analyze $H_3^{(2,2,2,3)}$ in the vicinity of $(-f_c, f_c, f_c)$ and $(f_c, f_c, -f_c)$ along the paths of the types one-stepped-two-fixed and two-stepped-one-fixed. To simplify the expressions, we omit the terms corresponding to $H_{a,q}$, $H_{b,q}$, $H_{c,q}$, and $H_{e,q}$ in Fig. 1 in the following analysis. These terms are analyzed in the same way as for the self-kernel, and it was found that they did not contribute to
the frequency dependence of the analyzed kernel. We use the expressions for the paths in the 3-D frequency space in (10) and (12) to obtain model expressions for the Volterra kernel. The first paths of type one-stepped-two-fixed are \((-f_c - 0.3 - f_{sh}, f_c + 0.1 + f_{sh}, f_c + f_{st})\) in the vicinity of \((-f_c, f_c, f_c)\). These paths give

\[
H_3 = \frac{1}{2} \{ H_{d,1}(-0.2) + H_{d,2}(2f_c + 0.1 + f_{sh} + f_{st}) \\
+ H_{d,2}(-0.3 - f_{sh} + f_{st}) \},
\]

(33)

Only \(H_{d,2}(-0.3 - f_{sh} + f_{st})\) gives a shift of \(f_{sh}\) (or +5 MHz) and hence describes the dominating structure in the curves in Fig. 11(a). The \(H_{d,1}\) term does not contribute to the frequency dependence seen in Fig. 11(a). The term \(H_{d,2}(2f_c + 0.1 + f_{sh} + f_{st})\) is excited at \(2f_c\) and does not cause any frequency dependence, by analogy with the self-kernel.

In a similar way, analyzing the kernel along paths \((f_c + f_{st}, f_c + 0.1 + f_{sh}, -f_c + 0.1 - f_{sh})\) in the vicinity of \((f_c, f_c, -f_c)\) gives the same result that \(H_{d,2}\) gives the frequency dependence seen in Fig. 11(b).

In the second type of paths, two-stepped-one-fixed, the paths in the vicinity of \((-f_c, f_c, f_c)\) are \((f_1, f_2, f_3) = (-f_c - 0.3 - f_{sh}, f_c + f_{st}, f_c - f_{st})\). These paths give

\[
H_3 = \frac{1}{2} \{ 2H_{d,1}(-0.3 - f_{sh} + f_{st}) \\
+ H_{d,2}(2f_c) + H_{d,2}(-0.3 - f_{sh} - f_{st}) \}.
\]

(34)

\(H_{d,1}(-0.3 - f_{sh} + f_{st})\) gives a shift of \(f_{sh}\) (or +5 MHz), which is seen in Fig. 12(a) as the weaker structure at \(-7.2, -2.2, +2.8\), and \(+7.8\) MHz, respectively. The term \(H_{d,2}(-0.3 - f_{sh} + f_{st})\) gives shifts of \(-f_{sh}\) (\(-5\) MHz), which is seen in the more pronounced structure in Fig. 12(a) with positions at \(+7.2, +2.2, -2.8\), and \(-7.8\) MHz, respectively. Both the terms \(H_{d,1}\) and \(H_{d,2}\) contribute to the frequency dependence in Fig. 12(a).

Fig. 12(b) shows the results for the kernel along the \((f_1, f_2, f_3) = (f_c + f_{st}, f_c - f_{st}, -f_c + 0.1 - f_{sh})\) in the vicinity of \((-f_c, f_c, -f_c)\). By analyzing in a similar way, it gives that only \(H_{d,2}\) is significant along this path.

The analysis showed that a block structure with two systems in parallel with \(H_{d,1}\) and \(H_{d,2}\) is required to model the experimental data for \(H_3^{(2,2,2,3)}\) in the vicinity of both \((-f_c, f_c, f_c)\) and \((f_c, f_c, -f_c)\). In Fig. 13, such a block structure is shown.

The magnitude level of the cross-kernel \(H_3^{(2,2,2,3)}\) in Fig. 11 is \(-12\) dB lower than the magnitude level of the self-kernel \(H_3^{(2,2,2,2)}\) in Fig. 8. The difference cannot be explained by analyzing only \(H_3^{(2,2,2,3)}\). We know, however, that it is due to the crosstalk in the DUT that did not cause any significant frequency dependence, as seen in the experimental data. This result agrees with the effect of coupler attenuation measured by the VNA. In Fig. 13, we have added blocks for \(H_{c,1}\) and \(H_{c,2}\) that would model the effect of the crosstalk. For the DUT, they have only attenuations independent of frequency.

We can compare the block structure in Fig. 13 to behavioral models in the literature. Fig. 13 can be compared only to some terms in these models, as they have several cross terms of different nonlinear orders. A time-domain baseband behavioral
model can be derived from the block structure in Fig. 13 using standard techniques [32], [33]

\[
y_3^{(2,2,3)}(n) = \sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} h_{3,1}(m_1, m_2) |u^{(2)}(n - m_1)|^2 \times u^{(3)}(n - m_2) + u^{(2)}(n) \sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} h_{3,2}(m_1, m_2) \times u^{(2)}(n - m_1) u^{(3)}(n - m_1 - m_2) \sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} h_{3,3}(m_1, m_2) u^{(2)}(n - m_1 - m_2) |u^{(3)}(n - m_1 - m_2)|^2
\]

where \(h_{3,1}(m_1, m_2)\) corresponds to the multiplication of the signals after \(H_{c,1}(\cdot)\) and \(H_{d,1}(\cdot)\) in Fig. 13 and \(h_{3,2}(m_1, m_2)\) and \(h_{3,3}(m_1, m_2)\) correspond to the cascade of \(H_{c,2}(\cdot)\) and \(H_{d,2}(\cdot)\). \(M_1\) is the memory length of \(H_{d,1}(\cdot)\) and \(H_{d,2}(\cdot)\) and \(M_2\) is the memory length of crosstalk seen in \(H_{c,1}(\cdot)\) and \(H_{c,2}(\cdot)\). For the physical system \(M_1 \rightarrow \infty\) and \(M_2 \rightarrow \infty\), the truncation could be different, i.e., \(M_1 \neq M_2\). As above, the terms that excite \(H_{d}(\cdot)\) at \(2f_c\) are not included in (35). For the special case \(m_1 = m_2\), the first term in (35) corresponds to the \(2 \times 2\) parallel Hammerstein model [41].

The second and third terms in (35) resemble the \(y_3^{(2,2,2,3)}\) kernel of the generalized memory polynomial for nonlinear crosstalk [34].

3) Cross-Kernel \(H_{3}^{(2,1,2,3)}\): We analyze the cross-kernel \(H_{3}^{(2,1,2,3)}\) in the vicinity of \((-f_c, f_c, f_c)\), \((f_c, -f_c, f_c)\), and \((f_c, f_c, -f_c)\), respectively. We omit \(H_{a,q}, H_{b,q}, H_{c,q}\); these terms were analyzed in the same way as for the self-kernel and were found not to cause any frequency dependence. We use the expressions for the paths in the 3-D frequency space in (10) and (13) to obtain model expressions for the Volterra kernel along these paths.

The first paths of type one-stepped-two-fixed, \((f_1, f_2, f_3) = (-f_c, -0.3 - f_{sh}, f_c + 0.1 + f_{sh})\), give

\[
H_3 = H_{d,1}(-0.2) + H_{d,2}(2f_c + 0.1 + f_{sh} + f_{da}) + H_{d,3}(-0.3 - f_{sh} + f_{da})
\]

Only the term \(H_{d,3}(-0.3 - f_{sh} + f_{da})\) gives shifts of \(f_{sh}\) (or +5 MHz), as shown in Fig. 14(a).

Similarly, the behavior of the kernel seen in Fig. 14(b) is along the paths \((f_c + 0.3 + f_{sh}, -f_c, -0.1 - f_{sh}, f_c + f_{da})\) in the \((f_c, -f_c, f_c)\) zone, where the dominating term corresponds to \(H_{d,2}\). Fig. 14(c) shows the kernel along the paths \((f_c + f_{st}, f_c + 0.1 + f_{sh}, f_c + 0.1 - f_{sh})\) in the \((f_c, f_c, -f_c)\) zone, where by analysis it gives that \(H_{d,3}\) is dominating.
The first paths of type two-stepped-one-fixed are \((-f_c - 0.3 - f_{sh}, f_c + f_{st}, f_c - f_{st})\). The model of \(H_3^{(2,1,2,3)}\) is
\[
H_3 = H_{d,1}(-0.3 - f_{sh} + f_{st}) + H_{d,2}(2f_c + f_{st}) + H_{d,3}(-0.3 - f_{sh} - f_{st}).
\]  
(37)

The term \(H_{d,1}(-0.3 - f_{sh} + f_{st})\) gives the shifts of \(f_{sh}\) (or \(5\) MHz) seen in the weaker structure in Fig. 15(a), whereas the term \(H_{d,3}(-0.3 - f_{sh} - f_{st})\) gives the shifts of \(-f_{sh}\) (or \(-5\) MHz) seen in the stronger structure.

In a similar analysis, the kernel gives a frequency dependence on \(H_{d,1}\) and \(H_{d,2}\) along the paths \((f_c + f_{st}, f_{c} - 0.1 - f_{sh}, f_c - f_{st})\) in the vicinity of \((f_c, f_c - f_{c})\) and a frequency dependence on \(H_{d,2}\) and \(H_{d,3}\) along the paths \((f_c + f_{st}, f_c - f_{st}, f_c + 0.1 - f_{sh})\) in the vicinity of \((f_c, f_c, -f_{c})\), which are shown in Fig. 15(b) and (c), respectively.

Thus, the kernel \(H_3^{(2,1,2,3)}\) is described by the block structure in Fig. 16(a) and (c) in the \((f_c, f_c, f_c)\) frequency zone. In the vicinity of \((f_c, f_c, f_c)\), the block structures in Fig. 16(a) and (c) describe the behavior. The block structures in Fig. 16(b) and (c) describe the behavior in the vicinity of \((f_c, f_c, -f_c)\). To the best of our knowledge, there are no reports in the literature on complex baseband models for \(3 \times 1\) systems.

V. CONCLUSION

We propose a method for determining the frequency-domain Volterra kernels in a \(3 \times 3\) MIMO system. A three-tone test, a \(2+1\)tone test, and a \(1+1\)tone test were performed to determine the self-kernels, \(2 \times 1\) cross-kernels, and \(3 \times 1\) cross-kernels, respectively. The DUT was three RF amplifiers with input crosstalk. Due to different symmetries, the self-kernel and \(2 \times 1\) cross-kernels were determined in one, two, and three frequency zones, respectively. We show how the effects of higher nonlinear order terms can be compensated for and how Kramers–Kronig analysis can be used for consistency check. Errors were estimated from LS fits. In particular, we show how the kernels can be analyzed along certain paths to determine the block structures that describe the main features of the kernels. Time discrete complex baseband models were determined from the block structures. The self-kernel was found to correspond to the envelope memory polynomial model, which can also be derived from a Hammerstein–Wiener structure. The \(2 \times 1\) kernel was found to be a more general form of the \(2 \times 2\) parallel Hammerstein model. The presented method could thus be used to determine digital predistortion algorithms for RF MIMO transmitters.

The method for determining and analyzing the third-order MIMO Volterra kernels could be modified to the second-order kernels. Also, kernels of higher nonlinear orders (fourth, fifth, \ldots) could be determined; the analysis of the kernels along the paths in the multidimensional frequency space could be made, although the paths could not be plotted but represented mathematically. The method for analyzing kernels could also be used for mechanical nonlinear dynamic MIMO systems.

APPENDIX

We derive the relationships between the time and frequency domain Volterra kernels in the real valued representation used above and in the complex baseband representation used in communication theory. We use the method for SISO systems in [32] and extend it to the third-order MIMO systems. In addition to [32], we also give the relations for the frequency domain kernels. In the derivations, it is assumed that the system is excited only in a passband around \(f_c\). We use \(\gamma\) to denote the complex baseband representation.

For real valued signals and systems, the third-order term in time domain is given by (2) with \(n = 3\). The frequency domain representation is given by (4) and (5). We write the input signals on complex form [32]
\[
u^i(t) = (1/2) [\tilde{u}^{i3} + \tilde{u}^{i3} (r) t e^{-j\omega t}]
\]  
(38)

where \(i = 1, 2, 3\) and \(\tilde{u}^{i3}(r)\) is the complex signal. We put (38) into (2) with \(n = 3\) and get
\[
\gamma_3^{(m_{r1}, r_{r2}, r_{r3})}(t) = \frac{1}{2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h_3^{(m_{r1}, r_{r2}, r_{r3})}(t_1, t_2, t_3) \{e^{j\omega t} \times [\tilde{u}^{i3}(r) (t - t_1) \tilde{u}^{i3} (r) (t - t_2) \tilde{u}^{i3} (r) (t - t_3) e^{j\omega (t_1 - t_2 + t_3)} + \tilde{u}^{i3} (r) (t - t_2) \tilde{u}^{i3} (r) (t - t_3) e^{j\omega (t_1 - t_2 + t_3)} + \tilde{u}^{i3} (r) (t - t_3) \tilde{u}^{i3} (r) (t - t_3) e^{j\omega (t_1 - t_2 + t_3)}] + e^{-j\omega t} [\ldots] + e^{3j\omega t} [\ldots] + e^{-3j\omega t} [\ldots] \}.
\]  
(39)

The terms multiplied by \(e^{-3j\omega t}\) are the complex conjugate of the \(e^{3j\omega t}\) terms. In the following, we omit the \(e^{3j\omega t}\) terms, since they give contributions at \(3f_c\) and not \(f_c\). Comparing (39) with (38), we see that the complex output signal around \(f_c\) is
\[
\gamma_3^{(m_{r1}, r_{r2}, r_{r3})}(t) = \frac{1}{2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h_3^{(m_{r1}, r_{r2}, r_{r3})}(t_1, t_2, t_3) \times [\tilde{u}^{i3} (r) (t - t_1) \tilde{u}^{i3} (r) (t - t_2) \tilde{u}^{i3} (r) (t - t_3) e^{j\omega (t_1 - t_2 + t_3)} + \tilde{u}^{i3} (r) (t - t_2) \tilde{u}^{i3} (r) (t - t_3) e^{j\omega (t_1 - t_2 + t_3)} + \tilde{u}^{i3} (r) (t - t_3) \tilde{u}^{i3} (r) (t - t_3) e^{j\omega (t_1 - t_2 + t_3)}] + e^{-j\omega t} [\ldots] + e^{3j\omega t} [\ldots] + e^{-3j\omega t} [\ldots] \}.
\]  
(40)
For the self-kernel, \( r_1 = r_2 = r_3 = r \) and \( h^3_{5}(mr,r,r) \) is symmetric under all permutations of \((r_1, r_2, r_3)\) [see (7)] and the three terms in (40) are identical. The real valued kernel can be approximated by the complex kernel [32]

\[
h^3_{5}(mr_1,r_2,r_3) = (1/2)h^3_{5}(mr_1,r_2,r_3) e^{j\omega_0(r_1 + r_2 - r_3)}.
\]

(41)

The FT, (5), of (41) gives

\[
\tilde{H}^3_{5}(m, r, r)(\omega_1, \omega_2, \omega_3) = 2H^3_{5}(m,r,r)(\omega_1 + \omega_0, \omega_2 + \omega_0, \omega_3 - \omega_0)
\]

that is, the complex valued frequency domain kernel is proportional to the real valued kernel in the vicinity of \((\omega_1 + \omega_0, \omega_2 + \omega_0, \omega_3 - \omega_0)\).

For the 3-D cross kernel, \( r_1 = r_2 \neq r_3 \) and \( h^3_{5}(mr_1,r_2,r_3) \) is symmetric under permutations of \((r_1, r_2)\) and two different terms in (40) will contribute to the output signal. For these two terms, the real valued kernel can be approximated by

\[
h^3_{5}(mr_1,r_2,r_3)(r_1, r_2, r_3) = (1/2)h^3_{5}(mr_1,r_2,r_3) e^{j\omega_0(r_1 + r_2 - r_3)}
\]

(42)

or

\[
h^3_{5}(mr_1,r_2,r_3)(r_1, r_2, r_3) = (1/2)h^3_{5}(mr_1,r_2,r_3) e^{j\omega_0(r_1 - r_2 + r_3)}
\]

(43)

For the 3-D cross kernel, \( r_1 \neq r_2 \neq r_3 \) and \( h^3_{5}(mr_1,r_2,r_3) \) is not symmetric and the three terms in (40) will all give different contributions to the output signal. The real valued kernel can be approximated by

\[
h^3_{5}(mr_1,r_2,r_3)(r_1, r_2, r_3) = (1/2)h^3_{5}(mr_1,r_2,r_3) e^{j\omega_0(r_1 + r_2 - r_3)}
\]

(44)

\[
= (1/2)h^3_{5}(mr_1,r_2,r_3) e^{j\omega_0(r_1 + r_2 - r_3)}
\]

(45)

The FT, (5), of (45) gives

\[
\tilde{H}^3_{5}(m, r, r)(\omega_1, \omega_2, \omega_3) = 2H^3_{5}(m,r,r)(\omega_1 + \omega_0, \omega_2 + \omega_0, \omega_3 - \omega_0)
\]

that is, there are three different complex baseband frequency domain kernels for the \(3 \times 1\) cross-kernel, each corresponding to one region in the 3-D frequency space in Fig. 7.

### REFERENCES


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