

# NON-NEWTONIAN HEAT TRANSFER

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**Keywords:** pipe flow, laminar regime, turbulent regime, viscous dissipation, developing flow, developed flow, polymer melts and solutions, surfactant solutions, drag and heat transfer reduction

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## Summary

This chapter is devoted to heat transfer of non-Newtonian fluids but given the vastness of this topic it focuses on pipe flow after the presentation of the required governing equations. Before presenting results and correlations for laminar and turbulent heat transfer of purely viscous and viscoelastic fluids the chapter introduces the relevant non-dimensional numbers and in particular presents the various definitions of characteristic viscosity currently used. The correlations and data are useful to design heat transfer systems based on confined flows for fully-developed and developing pipe flows and the data are based on experimental, theoretical and semi-theoretical investigations. This chapter also discusses other relevant issues such as scaling methods, the degradation of fluids and the analogy between heat and momentum for viscoelastic turbulent flows. For other flows the reader is referred to the literature and complete forms of the required governing equations are presented in Appendix 1.

## 1. Introduction



Previous chapters in this Encyclopedia have already conveyed the complex nature of

non-Newtonian fluids, often leading to remarkable flow features that contrast to those of Newtonian fluids. In particular, the previous chapters have shown that such characteristics are typical of a wide variety of fluids: concentrated and dilute polymer solutions, polymer melts, suspensions of particles, suspensions of immiscible liquids and surfactant solutions, amongst others. With few exceptions, blood and other biofluids for instance, a common denominator to all these fluids is that they tend to be synthetic, and consequently transfer of thermal energy not only takes place when they are being used, but also manufactured. Examples are the manufacture of plastic products by injection and extrusion, the production of paints, food products, chemicals or the use of "intelligent" thermal fluids in district heating and cooling systems, amongst other cases.

Investigations on heat transfer and friction with non-Newtonian fluids are as old as the studies on their rheology and fluid dynamics, and over the years there have been several reviews on the topic. As with Newtonian fluids, research and the solution of engineering problems on heat transfer and flow friction of non-Newtonian fluids can be carried out experimentally, theoretically and analytically. However, there is an important difference between Newtonian and non-Newtonian studies rooted in the non-linear nature of the latter. It emphasizes the role of experimental and, more recently, numerical methods when dealing with non-Newtonian fluids: whereas the rheological constitutive equation is known *a priori* for Newtonian fluids, for non-Newtonian fluids the rheological equation of state cannot be anticipated prior to an extensive fluid characterization, which anyway is invariably more complex than its Newtonian counterpart.

Heat transfer with non-Newtonian fluids is a vast topic given the wide range of fluids and flows of interest, and it cannot be covered in its vastness in this chapter. Internal flows are quite relevant in the scope of non-Newtonian fluid flows and heat transfer, with the pipe flow playing a leading role. Given the limited space available, it was decided to cover extensively this flow, but not all the possible combinations of heat transfer boundary conditions, with the expectation that the extensive treatment of the pipe flow case guides the reader towards other problems in this and other geometries. The review will also emphasize recent advances, especially for viscoelastic fluids. A fundamental approach will be followed to help the interested reader on the use of existing data and correlations for engineering purposes and to introduce heat transfer calculations in complex industrial non-Newtonian flows, especially for viscoelastic fluids described by differential constitutive equations, which over the last ten to fifteen years has become a reliable method for engineering purposes. In this work, a few reviews in the field will be used to some extent to limit the scope of this chapter.

Non-isothermal flows of viscoelastic fluids have captured the attention of researchers and engineers due to their industrial relevance. In recent times, there have been significant contributions in the thermodynamics of viscoelastic fluid flows leading to the development of more accurate rheological constitutive and energy equations accounting for such effects as storage of mechanical energy by the molecules, thermodynamics of non-equilibrium processes, temperature dependent properties, fluid compressibility and deformation induced anisotropy of heat conduction. The proper quantification of some of these effects is still a matter of research, but in most applications they are small, or inexistent, and can be safely ignored. In a few situations they may have to be considered, but these are rather advanced issues for this text and will not be pursued here. The interested reader is invited to further their knowledge in some of the advanced texts in the bibliography.

This chapter starts with the presentation of the relevant governing equations stating their assumptions and validity range and then proceeds to describe the required thermal fluid properties and their specificities for non-Newtonian fluids prior to addressing the applications. The focus is on pipe flow, dealing first with fully-developed laminar flow, followed by developing flow and the issues on thermal entrance length and viscous dissipation. The treatment of turbulent flow comes next and here the issue of drag and heat transfer reduction by additives is quite important. New experimental results will be presented, dealing both with polymer solutions and surfactants to shed light onto flow and heat transfer characteristics in the various flow regimes. Issues of combined forced and free convection will be briefly addressed, together with problems related to fluid degradation, solvent chemistry, solute type and the coupling between friction and heat transfer under turbulent flow conditions. Before the end some brief comments are also made on the subject of heat transfer in other fully-developed confined flows to warn the reader to the danger of generalizations to non-Newtonian fluids of arguments and conclusions based on Newtonian fluid mechanics and heat transfer.

## 2. Governing Equations



The behavior of viscoelastic fluids undergoing heat transfer processes is governed by the momentum, continuity and energy equations in addition to constitutive equations for the stress and the heat flux. For non-isothermal flows the momentum and rheological constitutive equations are affected in two ways relative to an isothermal case: dependence of fluid properties on temperature, leading to the buoyancy term amongst others and, on thermodynamic arguments the existence of extra terms in the constitutive equation, which can be traced back to the effect of temperature on the mechanisms acting at microscopic level. Such microscopic level changes, together with considerations of second law of thermodynamics for irreversible processes also affect the classical thermal energy equation. However, as mentioned in Section 1, the treatment of these extra terms is an advanced topic still under investigation and given their limited role in many engineering processes, they will not be pursued here. Another simplification in this text is the consideration of a constant and isotropic thermal conductivity, even though some non-Newtonian fluids do exhibit deviations from such idealized characteristics. A brief reference on how to deal with this problem is made later in this chapter.

In general, the fluid dynamics and heat transfer problems are coupled and the whole set of equations has to be solved simultaneously. For a general flow problem this can only be done numerically. However, when thermal fluid properties are considered to be independent of temperature (for instance, if temperature variations are small) it is possible to solve for the flow without consideration for the thermal problem (although not the other way around). In some other cases, the solution can still be obtained assuming temperature-independent properties, but a correction is introduced to compensate for the neglected effect. This is a fairly successful approach for simple geometries and simple fluids (such as inelastic fluids), but for viscoelastic fluids a more exact approach may be required for accurate results.

The general form of the governing equations is presented in Appendix 1. Since the review concerns pipe flow, the governing equations of Section 2.1 are presented in simplified form for steady pipe flow of incompressible fluids.

### 2.1. Equations for Pipe Flow

The equations of motion for laminar pipe flow of incompressible fluids are written in cylindrical coordinates with  $x$ ,  $r$  and  $\theta$  denoting the streamwise, radial and tangential coordinates and  $u$  and  $v$  denoting the streamwise and radial velocity components. For turbulent flow the governing equations to be considered are either those in Appendix 1, where all quantities are instantaneous and a solution must be obtained by Direct Numerical Simulation (DNS), or time-average quantities are being considered and extra terms originating from the linear terms must be added. These extra terms contain correlations between fluctuating terms. The equations in this section are only valid under laminar flow conditions.

- Continuity equation:

$$\frac{1}{r} \frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial x} = 0 \quad (1)$$

-  $x$  -momentum equation:

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} \right) = - \frac{\partial p}{\partial x} + \rho g_x + \left( \frac{\partial \tau_{xx}^T}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rx}^T) \right). \quad (2)$$

-  $r$  - momentum equation:

$$\rho \left( v \frac{\partial v}{\partial r} + u \frac{\partial v}{\partial x} \right) = - \frac{\partial p}{\partial r} + \rho g_r + \left( \frac{\partial \tau_{xr}^T}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rr}^T) - \frac{\tau_{\theta\theta}^T}{r} \right). \quad (3)$$

where  $\rho$  is the fluid density,  $g_x$  and  $g_r$  are components of the acceleration of gravity vector and  $p$  is the pressure. The fluid total extra stress ( $\tau_{ij}^T$ ) is the sum of a Newtonian solvent contribution having a solvent viscosity  $\eta_s$  and a polymer/additive stress contribution  $\tau_{ij}^P$  as in Equation (4).

- Constitutive equation:

$$\tau_{xx}^T = 2\eta_s \frac{\partial u}{\partial x} + \tau_{xx}^P; \quad \tau_{xr}^T = \eta_s \left( \frac{\partial u}{\partial r} + \frac{\partial v}{\partial x} \right) + \tau_{xr}^P; \quad \tau_{rr}^T = \eta_s \frac{\partial v}{\partial r} + \tau_{rr}^P; \quad \tau_{\theta\theta}^T = \eta_s \frac{v}{r} + \tau_{\theta\theta}^P \quad (4-a)$$

$$\tau_{kk} = \tau_{xx}^P + \tau_{rr}^P + \tau_{\theta\theta}^P \quad (4-b)$$

If an inelastic non-Newtonian fluid described by the Generalized Newtonian Fluid model is being considered, the above equations remain valid, with  $\tau_{ij}^P = 0$  and the solvent viscosity no longer is a constant, but is given by a function  $\eta_s = \eta_s(\dot{\gamma})$  that depends on the second invariant of the rate of deformation tensor defined as  $\dot{\gamma} \equiv \sqrt{2D_{ij}D_{ij}}$ . The polymer/ additive stress contributions to the total stress are given by

the remaining Eqs. (4).

-  $xx$  component

$$P(\tau_{kk})\tau_{xx} + \frac{\lambda}{F(\tau_{kk}, L^2)} \tau_{xx}^{\square} - \alpha(\tau_{xx}^2 + \tau_{xr}^2) = \eta_p \frac{\partial u}{\partial x} \text{ with}$$

$$\tau_{xx}^{\square} = v \frac{\partial \tau_{xx}}{\partial r} + u \frac{\partial \tau_{xx}}{\partial x} - 2\tau_{rx} \frac{\partial u}{\partial r} - 2\tau_{xx} \frac{\partial u}{\partial x} + 2\xi \left[ \tau_{xx} \frac{\partial u}{\partial x} + \tau_{rx} \left( \frac{\partial u}{\partial r} + \frac{\partial v}{\partial x} \right) \right]$$

(4-c)

-  $xr$  component

$$P(\tau_{kk})\tau_{xr} + \frac{\lambda}{F(\tau_{kk}, L^2)} \tau_{xr}^{\square} - \alpha[\tau_{xr}(\tau_{xx} + \tau_{rr})] = \eta_p \left( \frac{\partial u}{\partial r} + \frac{\partial v}{\partial x} \right) \text{ with}$$

$$\tau_{xr}^{\square} = v \frac{\partial \tau_{xr}}{\partial r} + u \frac{\partial \tau_{xr}}{\partial x} - \tau_{rr} \frac{\partial u}{\partial r} - \tau_{xx} \frac{\partial v}{\partial x} - \tau_{xr} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} \right) + \xi \left[ \frac{1}{2} \left( \frac{\partial u}{\partial r} + \frac{\partial v}{\partial x} \right) (\tau_{xx} + \tau_{rr}) + \tau_{xr} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} \right) \right]$$

(4-d)

-  $rr$  component

$$P(\tau_{kk})\tau_{rr} + \frac{\lambda}{F(\tau_{kk}, L^2)} \tau_{rr}^{\square} - \alpha(\tau_{xr}^2 + \tau_{rr}^2) = 2\eta_p \frac{\partial v}{\partial r} \text{ with}$$

$$\tau_{rr}^{\square} = v \frac{\partial \tau_{rr}}{\partial r} + u \frac{\partial \tau_{rr}}{\partial x} - 2\tau_{rr} \frac{\partial v}{\partial r} - 2\tau_{xr} \frac{\partial v}{\partial x} + 2\xi \left[ \frac{1}{2} \left( \frac{\partial u}{\partial r} + \frac{\partial v}{\partial x} \right) \tau_{xr} + \tau_{rr} \frac{\partial v}{\partial r} \right]$$

(4-e)

-  $\theta\theta$  component

$$P(\tau_{kk})\tau_{\theta\theta} + \frac{\lambda}{F(\tau_{kk}, L^2)} \tau_{\theta\theta}^{\square} - \alpha\tau_{\theta\theta}^2 = 0 \text{ with}$$

$$\tau_{\theta\theta}^{\square} = v \frac{\partial \tau_{\theta\theta}}{\partial r} + u \frac{\partial \tau_{\theta\theta}}{\partial x} - 2\tau_{\theta\theta} \frac{v}{r}$$

(4-f)

Equation (4) is a generalized equation for pipe flow containing several viscoelastic constitutive models depending on the numerical values of the parameters and on the specific forms of functions  $F(\tau_{kk}, L^2)$  and  $P(\tau_{kk})$ . The reader is referred to Appendix 1 for the form of these functions, a brief explanation of the various viscoelastic models and to Table A01-1 for the corresponding numerical values of the parameters.

- Energy equation

$$\rho c u \frac{\partial T}{\partial x} + \rho c v \frac{\partial T}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left( k r \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \tau_{rx} \left( \frac{\partial u}{\partial r} + \frac{\partial v}{\partial x} \right),$$

(5-a)

where  $k$  is the fluid thermal conductivity,  $T$  is the temperature and  $c$  is the specific heat of the fluid. These equations are valid for developing and fully-developed pipe flow in the absence of swirl. They are complex and coupled in a complex manner, but for fully-developed conditions, they can be further simplified because  $v=0$  and all  $\partial/\partial x=0$ , except for the pressure. This set of simplifications allows some analytical solutions, even though the equations remain coupled and provided fluid properties are constant. The energy Eq. (5-a) is also used in a simplified form, even for developing flow, after realization that  $\partial v/\partial x \ll \partial u/\partial r$ , i.e., that axial diffusion is far less important than radial diffusion and that radial heat convection is much weaker than axial heat convection, leading to Eq. (5-b)

$$\rho c u \frac{\partial T}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left( k r \frac{\partial T}{\partial r} \right) + \tau_{rx} \frac{\partial u}{\partial r}$$

(5-b)

Further simplification is possible after normalization of the temperature in a manner that is problem dependent, as will be seen later. These equations will be used in subsequent Sections to obtain some relevant heat transfer solutions. When an analytical solution is not possible then the problem must be solved numerically or experimentally.

### 3. Boundary and Initial Conditions

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### 3. Boundary and Initial Conditions



The solution of differential equations requires adequate boundary and initial conditions. For the velocity at a wall the no-slip condition is imposed whereby the velocity of fluid particles in contact with the wall is equal to the wall velocity. Most commonly this velocity is null ( $u_i = 0$  at walls) unless the wall is porous with flow across. For unsteady flows it is also necessary to define the initial condition for the velocity, which is a prescribed velocity profile at some location, usually at the inlet. Symmetry conditions may also be imposed if the geometry is symmetric and it is known *a priori* that the flow is symmetric. This may seem trivial, but there are many situations where geometry and boundary conditions are symmetric and steady, whereas the flow is not, in which case the full domain must be used to obtain a solution to the problem. In numerical calculations there is also a problem of setting boundary conditions at outlets. This is usually solved by putting the outlet sufficiently far from the region of interest to allow setting there a fully-developed condition, which does not affect the flow in the latter region.

For pipe flow the main boundary condition is no-slip at the wall and symmetry on axis. If the flow is fully-developed it is not necessary to consider the radial component of velocity and these conditions are enough providing the axial pressure gradient is known. Alternatively, the flow rate is given and the constant pressure gradient is calculated as a function of the flow rate after integration of the velocity profile. For developing flow it is also necessary to impose an inlet condition, which very often is a plug profile (at  $x=0$ ,  $u = cte$  and  $v=0$ ). Again, a solution is obtained for a specific value of the pressure gradient or a flow rate is given which determines  $dp/dx$ . Note that in a developing region  $dp/dx$  is not constant.

For the temperature equation a similar problem exists, but now this is complicated by the infinite number of possible combinations of wall temperature and wall heat flux conditions. In general terms, the problem of the boundary conditions for temperature are similar to those for velocity, but compounded by the wider number of possibilities. It is necessary to provide also inlet and outlet conditions for steady flows and initial conditions for unsteady heat transfer. The issue of symmetry is also made more complex, because it is possible to have a symmetric flow with an asymmetric heat transfer problem, provided the velocity field is decoupled from the temperature field (a possibility if temperature differences are small). Some of the simpler more commonly used boundary conditions, for which there are analytical solutions in pipe flow, are presented below. The thermal solution is very much dependent on the boundary condition for laminar flow and less so for turbulent flow due to improved mixing of the fluid. Fluids possessing large values of the ratio between momentum and heat diffusivities, denoted Prandtl number, are also less sensitive to the type of thermal boundary condition.

For flows in pipes, the main focus in this review, the main boundary conditions are:

1. T condition: constant wall temperature;
2. H2 condition: constant wall heat flux imposed both axially and peripherally.

In all cases it is also necessary to impose an inlet temperature profile.

For other geometries, these conditions equally apply. If not all the walls are heated, then the boundary condition combines with the number of heated walls. Obviously, more complex thermal boundary conditions can also be used, such as a prescribed variable wall temperature or wall heat flux or combinations of all these.

#### 4. Integral Energy Balances



Very often an engineering problem is solved with an integral balance equation rather than the differential balance. The integral equations are also exact, but lose the detail inherent to the differential approach as they are derived from a volume integration of the differential energy equation. Depending on its formulation and on the control volume used it can also provide some level of detail. The balances are formulated in the context of control volumes, which is very useful to deal with open systems.

The unsteady macroscopic total energy balance is thus written as



$$\frac{dE_{\text{tot}}}{dt} = \dot{Q}_w + \dot{W}_m + \sum \dot{m}_{\text{in}} \left( \hat{h} + \frac{\bar{U}^2}{2} + gz \right)_{\text{in}} - \sum \dot{m}_{\text{out}} \left( \hat{h} + \frac{\bar{U}^2}{2} + gz \right)_{\text{out}} \quad (6)$$

where the total energy  $E_{\text{tot}} = U_{\text{tot}} + K_{\text{tot}} + \Phi_{\text{tot}}$  is the sum of the internal energy ( $U_{\text{tot}}$ ), the kinetic energy ( $K_{\text{tot}}$ ) and the potential energy ( $\Phi_{\text{tot}}$ ) of the fluid inside the open system,  $\dot{Q}_w$  is the rate at which heat is added to the system across the boundary and  $\dot{W}_m$  is the rate at which mechanical work is added to the system across the boundary.  $\bar{U}^2/2$  and  $gz$  designate the specific kinetic and potential energies of the flowing fluid,  $\hat{h}$  is the specific enthalpy of the flowing fluid and  $\dot{m}$  is the corresponding mass flow rate.

There is also a macroscopic mechanical energy balance given by

$$\frac{d(K_{\text{tot}} + \Phi_{\text{tot}})}{dt} = \dot{W}_m - E_c - E_v + \sum \dot{m}_{\text{in}} \left( \frac{p}{\rho} + \frac{\bar{U}^2}{2} + gz \right)_{\text{in}} - \sum \dot{m}_{\text{out}} \left( \frac{p}{\rho} + \frac{\bar{U}^2}{2} + gz \right)_{\text{out}} \quad (7)$$

where  $E_c$  is the compression term, here equal to zero because the fluid is incompressible, and  $E_v = \int_{\text{Vol}} (\boldsymbol{\tau}^T : \mathbf{D}) d\text{Vol}$  is the fluid stress work. For a purely

viscous fluid this term is always positive and is known as viscous dissipation, but for elastic fluids it also includes elastic energy work. However, even for elastic fluids  $E_v$  may turn out to reduce to the viscous dissipation as is the case in fully-developed pipe flow where elastic energy work is null.

Subtracting the mechanical energy equation from the total energy equation gives the macroscopic balance of internal energy

$$\begin{aligned} \frac{dU_{\text{tot}}}{dt} &= \dot{Q}_w + E_v + \sum \dot{m}_{\text{in}} \hat{u}_{\text{in}} - \sum \dot{m}_{\text{out}} \hat{u}_{\text{out}} \text{ or} \\ \frac{d(m_{\text{tot}} cT)}{dt} &= \dot{Q}_w + E_v + \sum \dot{m}_{\text{in}} cT_{\text{in}} - \sum \dot{m}_{\text{out}} cT_{\text{out}} \end{aligned} \quad (8)$$

where  $\hat{u}$  designates the specific internal energy of the flowing fluid. Since for liquids the specific internal energy is given by  $\hat{u} = cT$ , the product of the specific heat by the fluid temperature, one obtains the second form of Eq. (8) where  $T_{\text{in}}$  and  $T_{\text{out}}$  are the fluid bulk temperatures at the inlets and outlets, respectively.

The integral energy balance equation just derived does not substitute the differential transport equations of Section 2, instead it complements them. In fact, the integral Eq. (8) relies on knowledge of the Nusselt number, amongst other variables, in order to calculate the wall heat transfer rate  $\dot{Q}_w$ . To arrive at the Nusselt number it is

necessary to either conduct extensive and expensive experiments, or to solve the differential equations to determine the velocity profile and temperature distribution from which the bulk temperature and Nusselt number are calculated. Once this Nusselt number is known quick but accurate engineering calculations can be carried out using Newton's law of convection.

A frequent and useful application of the integral energy balance is the calculation of the outlet bulk temperature in steady pipe flow subject to a specific boundary condition such as imposed wall temperature ( $T_w$ ). In this case, the bulk outlet temperature is given by

$$T_{\text{out}} = T_{\text{in}} + \frac{hA_w (T_w - \bar{T})}{\dot{m}c} \quad (9)$$

where  $A_w$  is the wall heat transfer area,  $\bar{T}$  is the average bulk temperature of the fluid inside the pipe and  $h$  is the heat transfer coefficient that must be calculated using the adequate expression for the Nusselt number (the Nusselt number is the nondimensional heat transfer coefficient defined in section 5 and useful correlations are presented in sections 7 and 8).

Information on this quantity is known for "simple" cases and simple geometries, but as discussed in the previous section the number of possible combinations of boundary conditions and flow geometries is infinite and often one must resort to extensive numerical solutions of the differential equations in order to arrive at the correlations that allow the use of the integral equation.

## 5. Non-dimensional Numbers



In fluid mechanics and heat transfer it is very common and convenient to work with non-dimensional numbers to reduce the number of independent and dependent parameters. These numbers originate from an exercise of normalization of all governing equations and of the corresponding boundary conditions. Physical insight of the problem to identify the associated relevant physical quantities offers an alternative way to determine the important non-dimensional groups. The non-dimensional numbers, some of which are presented in this section, provide generality to the solutions regardless of whether the method used is theoretical, numerical or experimental.

One of the problems in non-Newtonian fluid mechanics and heat transfer relates with the multitude of definitions of the same non-dimensional number resulting from inherent variability of fluid properties, in particular the viscosity, typical of the non-Newtonian behavior. Throughout this text, this will be dealt with by using different symbols. The typical example is the Reynolds number, which quantifies the ratio between inertial and viscous forces in the flow. The Reynolds number depends on the fluid viscosity, here lying the main problem since the viscosity of non-Newtonian fluids usually varies across the flow domain. Even for such a simple problem as pipe flow the literature uses several characteristic viscosities in a wide variety of Reynolds number definitions. In the following subsections this issue is discussed in detail, but emphasizing pipe flow.

## 5.1 Definitions of Reynolds and Prandtl Numbers

The non-dimensional Reynolds number,  $Re$ , is proportional to the ratio between inertial and viscous forces and is defined as

$$Re = \frac{\rho \bar{U} D}{\eta} \quad (10)$$

where,  $\bar{U}$  is the average velocity in the pipe,  $D$  is the pipe diameter and  $\eta$  is the shear viscosity of the fluid. For Newtonian fluids this is often called the dynamic viscosity, an inadequate designation in the context of non-Newtonian fluid dynamics where a dynamic viscosity refers to a material property measured in oscillatory shear flow.

The non-dimensional Prandtl number,  $Pr$ , represents the ratio between momentum and thermal diffusivities. The former is also known as the kinematic viscosity,  $\nu$ , and the thermal diffusivity,  $\alpha$ , is  $k/\rho c$ . Therefore the Prandtl number is mathematically defined by the following ratio

$$Pr = \frac{\nu}{\alpha} = \frac{\nu}{(k/\rho c)} = \frac{\eta c}{k} \quad (11)$$

where  $k$  and  $c$  are the thermal conductivity and the specific heat of the fluid, respectively. It should be remembered that the kinematic viscosity is related to the shear viscosity by  $\nu = \eta/\rho$ .

The product between the Reynolds and Prandtl numbers is called the Péclet number,  $Pe$ ,

$$Pe = \frac{\rho \bar{U} D c}{k} = \frac{\bar{U} D}{\alpha} \quad (12)$$

which is independent of the fluid viscosity, but may still depend on the flow field through other fluid properties (for instance, if the thermal conductivity also depends on flow kinematics, not to mention fluid temperature).

For non-Newtonian fluids with a viscosity that is shear-rate dependent the question arises of what should be the characteristic viscosity used to calculate the Reynolds and Prandtl numbers? For variable viscosity fluids represented by the power-law model there are four definitions of Reynolds number in the specialized literature, i.e., there are four different ways of defining the characteristic fluid viscosity. Here, only a brief description of these viscosities is presented.

The generalized Reynolds number  $Re'$  is calculated in such way that the Fanning friction factor data for Newtonian and variable viscosity fluids in laminar pipe flow collapse onto a single curve  $f = 16/Re'$ . The Fanning friction factor  $f$  is defined in section 5.3. The corresponding generalized Prandtl number is also presented in Eq.

(13).

$$\eta' = K' \left( \frac{8\bar{U}}{D} \right)^{n-1}, \quad Re' = \frac{\rho \bar{U} D}{\eta'} = \frac{\rho \bar{U}^{2-n} D^n}{K' 8^{n-1}}, \quad Pr' = \frac{\eta' c}{k} = \frac{K' (8\bar{U}/D)^{n-1} c}{k}$$

(13)

Parameters  $K'$  and  $n$  are functions of a characteristic shear rate  $8\bar{U}/D$ , which is the wall shear rate for a Newtonian fluid. The coefficient  $K'$  is related to the consistency ( $K$ ) and power law ( $n$ ) indices of a power law model fitted to the viscosity of the non-Newtonian fluid by the expression

$$K' = K \left( \frac{3n+1}{4n} \right)^n$$

(14)

and appears in the equation that calculates the wall shear stress,  $\tau_w$ , for fully developed laminar pipe flow of a power law fluid,

$$\tau_w = K' \left( \frac{8\bar{U}}{D} \right)^n$$

(15)

$Re'$  is the most commonly used definition for laminar pipe flow, where the viscosity  $\eta'$  is equal to the ratio between the wall shear stress and  $8\bar{U}/D$ . Note also that in some literature  $\eta'$  is called  $\eta_{\text{eff}}$  and  $Re'$  is called  $Re_{\text{eff}}$ .

A second choice of Reynolds number,  $Re^+$ , is usually found in external flows of non-Newtonian fluids relying on the definition of a characteristic shear rate of  $\bar{U}/D$ , where  $D$  is a characteristic length scale.

$$\eta^+ = K \left( \frac{\bar{U}}{D} \right)^{n-1}, \quad Re^+ = \frac{\rho \bar{U} D}{\eta^+} = \frac{\rho \bar{U}^{2-n} D^n}{K}, \quad Pr^+ = \frac{\eta^+ c}{k} = \frac{K (\bar{U}/D)^{n-1} c}{k}$$

(16)

A third Reynolds number definition,  $Re_a$  in Eq. (18), is used for pipe flow and relies on the apparent viscosity at the wall,  $\eta_a = \tau_w / \dot{\gamma}_w$ , where  $\tau_w$  is the wall shear stress, Eq. (15), and  $\dot{\gamma}_w$  is the shear rate at the pipe wall given by Eq. (17). The corresponding Prandtl number,  $Pr_a$ , is also defined in Eq. (18).

$$\dot{\gamma}_w = \frac{3n+1}{4n} \frac{8\bar{U}}{D}$$

(17)

$$Re_a = \frac{\rho \bar{U} D}{\eta_a} \quad \text{with} \quad \eta_a = \frac{\tau_w}{\dot{\gamma}_w} = K (\dot{\gamma}_w)^{n-1} = K \left( \frac{3n+1}{4n} \frac{8\bar{U}}{D} \right)^{n-1} \quad \text{leading also to}$$

$$Pr_a = \frac{\eta_a c}{k} \quad (18)$$

If the non-Newtonian fluid has a constant viscosity, in which case it is called a Boger fluid, all the above Reynolds numbers are equivalent. The viscosity of very dilute polymer solutions is well approximated by that of the solvent,  $\eta_s$ , and this suggests another Reynolds number definition,  $Re_s$ .

$$\eta_s = \eta_{\text{solvent}}, \quad Re_s = \frac{\rho \bar{U} D}{\eta_s}, \quad Pr_s = \frac{\eta_s c}{k}. \quad (19)$$

From the above, the various Reynolds numbers commonly used in pipe flow are related as follows

$$Re_a = \frac{3n+1}{4n} Re', \quad Re^+ = \left( \frac{3n+1}{4n} \right)^n 8^{n-1} Re', \quad (20)$$

Consequently, the Prandtl numbers are also related as in

$$Pr_a = \left( \frac{3n+1}{4n} \right)^{-1} Pr', \quad Pr^+ = \left( \frac{3n+1}{4n} \right)^{-n} 8^{1-n} Pr' \quad (21)$$

Because of this diversity in the definitions of Reynolds and Prandtl numbers, great care must be exercised when using equations from the literature involving these non-dimensional groups.

## 5.2 Newton's Law of Convection and the Nusselt Number

The normalization of the boundary conditions of the energy equation introduces the concept of convection coefficient,  $h$ , so useful in engineering calculations with integral equations. The convection coefficient is defined through Newton's law of convection (Eq. 22) as the ratio between the heat rate per unit area,  $\dot{q}_w$ , and a temperature difference. The wall heat flux is positive when entering the pipe. The temperature difference is usually the difference between the undisturbed fluid temperature ( $T_\infty$ ) and the wall temperature ( $T_w$ ). In the case of duct flow, the bulk temperature of the fluid ( $T_b$ ) inside the duct is used instead of the undisturbed flow temperature.

$$h = \frac{\dot{q}_w}{T_w - T_b} = \frac{k (dT/dr)_w}{T_w - T_b} \quad (22)$$

The corresponding non-dimensional number is called the Nusselt number,  $Nu$ , and is

defined in Eq. (23). It normalizes  $h$  with the thermal conductivity and a length scale, but this is not the only possibility.

$$Nu = \frac{hD}{k} \quad (23)$$

In the literature the convection coefficient is sometimes normalized with the thermal capacity leading to the Stanton number

$$St = \frac{Nu}{RePr} = \frac{h}{\rho \bar{U} c} \quad (24)$$

or alternatively in a more sophisticated way to the Chilton-Colburn  $j$ - factor for heat transfer defined in Eq. (25). Note that the Stanton number is independent of the fluid viscosity, but for non-Newtonian fluids this implies that the Reynolds and Prandtl numbers must be consistently defined, i.e., they must rely on the same definition of characteristic viscosity. The Chilton- Colburn  $j$ - factor has its roots on the Reynolds analogy, the similarity of the fluid dynamic and heat transfer solutions in cases where the boundary layer approximation was valid for fluids with  $Pr \approx 1$ . This implied a similar dependence of the friction factor and Nusselt number on the Reynolds number, flow geometry and boundary conditions. Then, it was realized that a Prandtl number correction would make it valid over a wide range of  $Pr$ , the so-called Chilton- Colburn analogy, which is given by a simple explicit expression. The Chilton- Colburn analogy has proven useful also for other flows, such as the flow around cylinders, spheres or across packed beds, but it may require small modifications for some specific flows.

$$j_H = St Pr^{2/3} \quad (25)$$

Henceforth,  $j_H$  will be denoted simply as the Chilton-Colburn factor, but the reader should also be aware of the existence of a Chilton-Colburn  $j$ -factor for mass transfer (denoted  $j_D$ ), and the corresponding analogy, the discussion of which is outside the scope of this chapter.

[1. Introduction](#)

[5.3 The Friction Factor](#)

## NON-NEWTONIAN HEAT TRANSFER

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### 5.3 The Friction Factor

As just mentioned, there is analogy between heat and momentum transfer and the validity of the Chilton-Colburn analogy outside the realm of Newtonian fluids is still open to investigation, even though surprises are not expected for inelastic fluids. For Newtonian fully-developed turbulent pipe flow at Reynolds numbers well above transition ( $Re > 10\,000$ ) it results in  $f/2 \approx j_H$ , and a generalization of this equation is expected to hold for power law fluids. Therefore, some heat transfer correlations appear as a function of the friction factor, which is defined as a normalized wall shear stress. For duct flows, the literature uses either the Darcy friction factor  $f_D$  or the Fanning friction factor,  $f$ , both of which are defined in Eq. (26).

$$f_D \equiv \frac{8\bar{\tau}_w}{\rho\bar{U}^2} = 4f \quad (26)$$

The average wall shear stress is used in this definition (the overbar denotes area

average) to cope with ducts of non-circular cross-section.

## 6. Fluid Properties



The governing equations introduced various fluid properties some of which are specific to the non-Newtonian fluids, such as the relaxation time  $\lambda$  and the parameters  $\varepsilon$ ,  $\xi$  and  $L^2$  appearing in the rheological constitutive equations. The other fluid properties are also relevant in the context of Newtonian fluids, but may have specificities inherent to the non-linear nature of the non-Newtonian fluids. The most obvious choice is the shear viscosity, which often is no longer a constant but depends on time and on the deformation field. Obviously, it also depends on temperature, but this is not a non-Newtonian characteristic: all fluid properties depend on the thermodynamic state, i.e. they depend on pressure and temperature regardless of the type of fluid. Here, of concern are the dependencies of the fluid properties on the flow field.

The dependence of viscosity on the invariants of the rate of deformation field has been discussed in a previous chapter of this Encyclopedia and will not be discussed further. If the momentum transport fluid property depends on the rate of deformation field there is no reason to admit that other transport fluid properties, such as the thermal conductivity  $k$ , are independent of the rate of deformation field. The thermal conductivity of a fluid depends on its chemistry and in the case of a solution it naturally depends on the quality and quantity of additives. This dependence on concentration is weak at low additive concentrations, but becomes important for concentrated solutions. The literature shows that the thermal conductivity of a polymer solution usually decreases when the polymer concentration is raised. The values of the thermal conductivity of non-Newtonian fluids found in Tables were usually measured under static conditions, but measurements of  $k$  obtained in pure shear flow confirm its dependence on the shear rate ( $\dot{\gamma}$ ). The dependence is generally much weaker than that found for the viscosity.

Recent experimental results for aqueous solutions of carboxymethylcellulose (CMC 7H4) at concentrations of 1 500, 2 500 and 5 000 wppm and polyacrilamide (Separan AP-273) at concentrations of 1 000 and 2 000 wppm, show an increase in the thermal conductivity with an increase in the shear rate (shear-conducting effect), also confirmed by other works for carbopol solutions. However, in some other investigations the opposite effect (shear-insulating effect) has been observed and at least in one case a mixed behavior reported.

Depending on the test temperature, in the range between 20 °C and 50 °C, the thermal conductivity of the CMC solutions has been seen to increase between 20% and 70% as  $\dot{\gamma}$  varied from 100 to 900 s<sup>-1</sup>. For the Separan solutions  $k$  increased between 20% and 50% as the shear rate was raised from 50 to 600 s<sup>-1</sup>. The largest variations were observed at the higher temperatures and lower concentrations.

With low concentration solutions, say at concentrations below 1 percent by weight, and except for the rheological properties, it can be assumed that all other physical properties of the solutions are essentially equal to those of the Newtonian solvent. In Table 1, taken from the literature, the thermal conductivity for several aqueous polymer solutions are listed as a function of polymer concentration. The listed



values of  $k$  were measured under static conditions and are very similar to that of water.

For other fluids, such as solutions of surfactants or suspensions of particles and fibres at low concentrations it is also possible to estimate the thermal properties on the basis of their solvents and neglecting any shear rate dependence.

[Table 1](#). Thermal conductivities  $k$ , W/(m·K), of aqueous solutions under static conditions.

The variation of thermal conductivity with shear rate can be important, but it is often disregarded under the assumption that the variations of the viscosity and the effect of elasticity on the flow field, and hence on the thermal field, are much stronger and essentially determine the non-Newtonian nature of the heat transfer characteristics. In experimental studies the results are often correlations for the Nusselt number, in which the shear rate dependence of  $k$  is eventually taken into account as an increase or decrease in heat transfer, although in a less physically meaningful way. The problems come from the more frequent use of computers to solve engineering heat transfer problems. Here the transport equations are numerically solved and usually such dependence is not taken into account, which can result in some severe discrepancies. This is certainly an area where the future will provide more accurate answers.

Measurements of thermal conductivity can be carried out under static conditions using the same procedure and equipment as for Newtonian fluids, whereas to account for shear rate dependence it is necessary to impose a constant shear rate flow. This can be achieved in an adapted rheometer/ viscometer such as in the annular gap between two concentric cylinders.

## 7. Laminar Flow



In the context of non-Newtonian fluid mechanics and heat transfer, laminar flow is far more important and frequent than for Newtonian fluids on account of the usually higher viscosity of the former fluids. The high viscosity of the fluids also leads to faster dynamic flow development than for Newtonian fluids and consequently in this context it is often the case that the thermal behavior develops slower than the flow dynamics. This section deals with heat transfer in laminar flow for a selection of relevant flow and heat transfer conditions, each of which is presented in a specific sub-section. It includes situations of fully-developed and developing flow, imposed wall heat fluxes and imposed wall temperatures. The emphasis is on duct flow and most material presented here is in the literature and can be traced back from the bibliography and the references listed at the end.

Heat transfer of viscoelastic fluids in laminar duct flow is not too different from the corresponding heat transfer of inelastic non-Newtonian fluids having the same viscosity behavior. In fact, under fully-developed conditions and provided there is no secondary flow, the heat transfer characteristics of both fluids are exactly the same. In ducts of non-circular cross-section viscoelasticity leads to secondary flow and the heat transfer characteristics differ significantly from the corresponding Newtonian behavior even if in terms of friction factor the differences are small.

In Newtonian pipe flow the Reynolds number marking the transition between laminar and turbulent flow is typically around 2 100. For a non-Newtonian purely viscous fluid the literature considers a slightly higher transitional Reynolds number,  $Re^+$ , but the value of 2 100 can still be used as a transitional criterion. Actually, a wiser approach is to consider that both flow regimes are possible when operating in the range  $1\,500 < Re^+ < 3\,000$ , and to choose between the laminar and turbulent flow in order to arrive at the safer design.

For viscoelastic fluids the transitional Reynolds number for pipe flow is much higher than 2 100. Laminar pipe flow of viscoelastic fluids has been experimentally observed at Reynolds numbers,  $Re'$ , as high as 6 000 and this is associated with the strong interference between the additives (polymer molecules or micelles) with turbulence, as will be discussed in Section 8.

### 7.1 Fully-developed Pipe Flow of Purely Viscous Fluids

The adoption of the GNF model with the Ostwald de Waele power law, Eq. (1-7) of the appendix, has the advantage of allowing analytical solutions for some simple flow geometries as the fully-developed laminar pipe flow. Its velocity profile is given by Eq. (27) after normalization with the pipe radius ( $R$ ) and bulk velocity ( $\bar{U}$ ), where  $r$  is the radial coordinate and  $n$  is the power law index.

$$\frac{u}{\bar{U}} = \frac{3n+1}{n+1} \left[ 1 - \left( \frac{r}{R} \right)^{\frac{(n+1)}{n}} \right]. \quad (27)$$

The corresponding pressure drop is well predicted by Eq. (28), where  $f$  is the Fanning friction factor.

$$f = \frac{16}{Re'}. \quad (28)$$

This equation was extensively validated against experimentally data and remains valid for fully-developed pipe flow of viscoelastic fluids provided their viscosity follows a power law in the range of relevant shear rates. This is so because under fully-developed conditions the pipe flow dynamics is governed by the equilibrium between the axial pressure gradient and the radial shear stress gradient, the latter depending only on a single rheological property, the shear viscosity. However, under some circumstances it can be advantageous or useful to use expressions derived for particular viscoelastic constitutive equations and this will be dealt with later. Next, heat transfer solutions are presented for the power law fluid.

For the constant wall heat flux boundary condition the energy equation for fully-developed pipe flow can be readily integrated (Eq. 5-b), under the assumption of temperature-independent fluid properties, to give the temperature profile from which the normalised forced convection coefficient is obtained. The resulting Nusselt number is

$$Nu_{\infty} = \frac{8(5n+1)(3n+1)}{31n^2 + 12n + 1} \quad (29)$$

For  $n = 1$  the Newtonian value of 4.36 is recovered, while for a plug flow,  $n = 0$ , the Nusselt number reaches its highest possible value of 8. Equation (29) is equally valid for purely-viscous and viscoelastic fluids beyond the thermal entrance region.

For the constant wall temperature boundary condition the Nusselt number of a power law fluid is also obtained by integration of the energy equation although by a more laborious process than for the constant heat flux case, since it is usually necessary to determine the limiting case of a Graetz-Nusselt problem or to resort to iterative calculation methods. Data from the literature is reproduced in Table 2, which lists the variation of Nusselt number with  $n$ . Again, these values are valid for purely viscous and viscoelastic fluids in the fully-developed thermal region.

[Table 2](#). Laminar Nusselt number for constant wall temperature beyond the thermal entrance region

If the range of shear rates inside the pipe is large enough so that the viscosities in the flow vary with shear rate in a significantly more complex way than suggested by the power law, a different viscosity law must be fitted to the viscosity data and the corresponding Nusselt numbers must be determined numerically or experimentally in the absence of an analytical solution. Near the pipe axis there will always be a difference between the finite viscosity of the fluid at vanishing shear rates and the corresponding infinite viscosity of the power law fluid. If this is very localized it will not affect the balance of forces, consequently the friction factor and Nusselt number expressions derived for the power law fluid remain accurate.

[3. Boundary and Initial Conditions](#)

[7.2 Developing Pipe Flow for Purely Viscous Fluids](#)

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## 7.2 Developing Pipe Flow for Purely Viscous Fluids

In the entrance region of a pipe the velocity profile changes along the streamwise direction, due to the thickening of the shear layers that develop near the wall. Shortly after the shear layers meet on the pipe axis, at  $x = L_h$ , the velocity profile remains unchanged henceforth and the flow is considered to be dynamically fully-developed. The region comprised between  $0 < x < L_h$  is called the hydrodynamic entrance region. For engineers the two main flow characteristics of interest in this region are the corresponding additional pressure drop, compared with the fully-developed pressure drop, and the entrance length  $L_h$ .

There is experimental and numerical information in the literature on the evolution of the local friction factor in the developing flow region for pipe flow of purely viscous fluids. The latter was obtained from the numerical solution of the governing equations and the non-dimensional entrance length,  $L_h / (D Re')$  is given as a function of  $n$ . Since the dynamic flow development is essentially dominated by the

viscous shear stresses large differences are not expected between the behaviors of purely viscous and viscoelastic fluids unless the elastic stresses interfere significantly in the boundary layer growth process (there is here scope for research but certainly this requires extremely elastic fluids and/or flows at very large Weissenberg numbers). Strictly speaking the solutions for the two fluids are different and this justifies the use of viscoelastic constitutive equations to determine flow and heat transfer characteristics for fully-developed and developing flow. In fact, each viscoelastic rheological constitutive equation has a specific viscosity function in pure shear flow and a solution for this particular viscosity function is exact and includes the elastic effects whereas the solution for a power law or any GNF viscosity model will always be approximate at least for the dynamic developing region.

For power law fluids Eq. (30), taken from the literature, gives the normalized hydrodynamic entrance length as a function of the power law index,  $n$  and the Reynolds number  $Re'$ . This equation is valid in the range  $0.4 < n < 1.5$  and  $0 < Re' < 1000$ . In the limiting case of  $n = 0$  the velocity profile simply remains plug-shaped and the entrance length is null.

$$L_h/D = \left[ \left( 0.246n^2 - 0.675n + 1.03 \right)^{1.6} + (0.0567Re')^{1.6} \right]^{1/1.6} \quad (30)$$

As for the velocity, the temperature field shape also changes at the inlet of a pipe. In this thermal entrance region the Nusselt number decreases towards the fully-developed value. The higher values of Nusselt number in the entrance are a consequence of a thinner thermal boundary layer exerting less thermal resistance to the heat transfer. The thermal entrance length,  $L_t$ , can be estimated using Eq. (31) which is independent of the viscosity and hence of the power law index.

$$L_t/D = 0.04 Re Pr = 0.04 Pe = 0.04 \bar{U}D/\alpha \quad (31)$$

Since the non-Newtonian fluids are usually fairly viscous it is quite common for the flow to become dynamically fully-developed well before it becomes thermally fully-developed and so, in this case the thermal development length turns out to be almost independent of the particular shape of the viscosity function. These fluids are characterized by a large value of the Prandtl number because their thermal diffusivity  $\alpha$ ,  $\alpha = k/\rho c$ , is low in comparison with the momentum diffusivity.

So, for dynamically fully-developed, but thermally developing flow, and for a constant wall heat flux boundary condition, the local Nusselt number,  $Nu_x$ , for a power-law fluid is calculated using Eq. (32)

$$Nu_x = 1.41 \left( \frac{3n+1}{4n} \right)^{1/3} Gz^{1/3}. \quad (32)$$

For the constant wall temperature case the corresponding  $Nu_x$  is given by

$$Nu_x = 1.16 \left( \frac{3n+1}{4n} \right)^{1/3} Gz^{1/3}. \quad (33)$$

Both expressions, from the literature, can be used for purely viscous and viscoelastic fluids in pipe flow in the absence of dissipation effects. The Graetz number,  $Gz$ , is independent of the fluid rheology and is given by Eq. (34).

$$Gz = mc/kx = (RePr D/x) \cdot \pi/4 \quad (34)$$

Combining Eq. (34) with Eq. (31) it is also possible to define the thermal entrance length as a function of the inverse of the Graetz number,  $Gz_t^{-1}$ , which is simply

$$Gz_t^{-1} = 0.051 \quad (35)$$

Of more interest in engineering calculations is the average Nusselt number between the entrance and a location at a distance  $x$  from the entrance ( $\overline{Nu}$ ), but still smaller than  $L_t$ , which for laminar flow is obtained multiplying by 1.5 the local Nusselt number at  $x = L$ , i.e.  $\overline{Nu} = 1.5Nu_{x=L}$ , this is only true in the absence of viscous dissipation.

When the hydrodynamic entrance length is much smaller than the thermal entrance length it is acceptable to estimate the pressure drop assuming fully developed flow, while simultaneously the heat transfer calculation must be carried out using an equation for thermally developing flow.

### 7.3 Laminar Flow of Viscoelastic Fluids and Viscous Dissipation Effects

As already mentioned, many non-Newtonian fluids are quite viscous, as is the case with polymer melts, concentrated polymer solutions in low viscosity solvents or dilute polymer solutions in highly viscous solvents. This high viscosity leads to internal heat generation by friction, which can no longer be neglected in the energy equation. The literature deals erratically with this case, and but does not present a comprehensive readily useful set of results for power law fluids in the form of Nusselt number expressions or tables of data. However, it does so for a particular class of viscoelastic fluids represented by a single mode of the simplified ( $\xi = 0$ ) linearized Phan-Thien—Tanner model (sPTT) in the absence of a Newtonian solvent contribution. The available solutions for the sPTT fluid are for fully-developed and thermally developing pipe flow for both imposed constant wall heat flux and wall temperature and include viscous dissipation effects. They are presented next including the particular solutions for negligible internal heat generation which for the sPTT fluid corresponds to the previous sets of solutions for the power law fluid.

The solutions depend on the product  $\varepsilon Wi^2$ , where  $Wi$  is the non-dimensional Weissenberg number ( $Wi \equiv \lambda \overline{U}/R$ ) quantifying elastic effects and  $\varepsilon$  is a non-dimensional parameter inversely proportional to the extensional viscosity. The main underlying assumptions of these results were those of constant physical properties, negligible axial heat conduction, and for thermally developing conditions the flow

field is considered to be fully-developed. The corresponding velocity profile in a pipe is given by the following equation

$$\frac{u}{\bar{U}} = 2 \frac{\bar{U}_N}{\bar{U}} \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \left\{ 1 + 16 \varepsilon Wi^2 \left( \frac{\bar{U}_N}{\bar{U}} \right)^2 \left[ 1 + \left( \frac{r}{R} \right)^2 \right] \right\} \quad (36)$$

where  $\bar{U}_N \equiv \left[ -(dP/dx) R^2 \right] / (8\eta_p)$  is the average velocity for a Newtonian fluid flowing under the same pressure gradient and  $\eta_p$  is the viscosity coefficient of the sPTT model. For pipe flow the ratio  $\bar{U}_N/\bar{U}$  is the following explicit function of the product  $\varepsilon Wi^2$ ,

$$\frac{\bar{U}_N}{\bar{U}} = \frac{\left[ \sqrt{144\varepsilon Wi^2 + 1} + 12\sqrt{\varepsilon Wi^2} \right]^{1/3} - \left[ \sqrt{144\varepsilon Wi^2 + 1} - 12\sqrt{\varepsilon Wi^2} \right]^{1/3}}{8\sqrt{\varepsilon Wi^2}}. \quad (37)$$

It is important to realize that the results for the sPTT model are identical to those reported for purely viscous fluids provided the power law parameters used in the equations of subsections 7.1 and 7.2 are adequately selected to match the viscosity function of the sPTT fluid for a given set of values of  $\varepsilon Wi^2$ . However, no such comparison can always be made for lack of solutions as is the case for developing thermal flow accounting for dissipation effects for which the authors are unaware of solutions for power law fluids.

Figure 1 compares the velocity profiles for power law fluids (Eq. 27) with the sPTT profiles of Eq. (36) for the corresponding pairs of values of  $n$  and  $\varepsilon Wi^2$ . The numerical values of the pairs  $\varepsilon Wi^2 - n$ , for which the laminar velocity profiles are similar, are listed in Table 3. The consequence in terms of the heat transfer is that both fluids will also have similar values of the Nusselt number, other conditions being identical. With viscous dissipation effects, no solutions were presented for power law fluids in Sections 7.1 and 7.2, so only the viscoelastic solution is presented next.

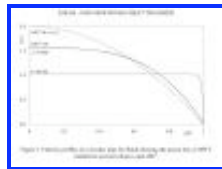


Figure 1. Velocity profiles in a circular pipe for fluids obeying the power law or SPTT models for several values  $n$  and  $\varepsilon Wi^2$ .

**Table 3.** Values of  $\varepsilon Wi^2$  and  $n$ , that correspond to similar laminar velocity profiles in a circular duct.

### 7.3.1. The Brinkman Number

Consideration of viscous dissipation effects introduces an extra non-dimensional

parameter, the Brinkman number,  $Br$ , which is defined according to the boundary condition. For a Newtonian fluid of viscosity  $\eta$  the usual definitions are

$$Br \equiv \frac{\eta \bar{U}^2}{k(T_w - T_0)} \quad (38)$$

for the constant wall temperature case, and by

$$Br \equiv \frac{\eta \bar{U}^2}{\dot{q}_w D} \quad (39)$$

for the constant wall heat flux case, where  $\dot{q}_w$  is positive when the heat flux enters the pipe of diameter  $D$ ,  $T_w$  denotes the wall temperature and  $T_0$  is the inlet bulk temperature. These definitions are used frequently for non-Newtonian fluids, but they do not take into account the variable viscosity of non-Newtonian fluids. A more correct definition is the so-called generalized Brinkman number, denoted  $Br^*$ , which allows the comparison of dissipation effects using different rheological models and fluids, because the ratio of heat generated by dissipation and the convective heat transfer at the wall becomes the same at identical values of  $Br^*$  regardless of fluid type or geometrical size.

The generalized Brinkman number directly compares the energy dissipated internally by viscous effects with the convective heat transfer at the wall. For pipe flow the energy dissipated throughout the flow per unit area is the product of the wall shear stress by the bulk velocity and is given by

$$\tau_w \bar{U} = \left( -\frac{dp}{dx} \right) \frac{\bar{U} A}{\mathcal{P}} \quad (40)$$

where  $dp/dx$  is the frictional pressure gradient and  $A$  and  $\mathcal{P}$  are the cross-section area and wetted perimeter, respectively. To generalize the Brinkman number to other geometries it is necessary to relate the pressure drop with a perimeter averaged wall shear stress as in annular ducts. Hence, the generalized Brinkman numbers are defined as

$$Br^* = \frac{\bar{U} \tau_w}{8 \dot{q}_w} \quad (41)$$

$$Br^* = \frac{\bar{U} \tau_w D}{8k(T_w - T_0)} \quad (42)$$

for the constant wall heat flux and constant wall temperature, respectively. The coefficient 8 is introduced with the sole purpose of making this definitions identical to the standard definitions of Eqs. (38) and (39) for Newtonian fluids. Two examples of the application of Eq. (41) to pipe flow of power-law and sPTT fluids are  $Br^* = 2^{n-3} (1 + 3n/n)^n K \bar{U}^{1+n} / (\dot{q}_w D^n)$  and  $Br^* = (\eta_p \bar{U}^2 / \dot{q}_w D) (\bar{U}_N / \bar{U})$ ,



respectively.

### 7.3.2. Fully Developed Flow for Viscoelastic and Purely Viscous Fluids

For dynamic and thermally fully-developed pipe flow with a constant wall temperature boundary condition and internal heat generation, the Nusselt number is independent of the Brinkman number. The Nusselt number involves the ratio of two quantities that, in this case, vary identically with the Brinkman number thus cancelling each other and leading to the following final expression for an sPTT fluid.

$$Nu_{\infty} = \frac{240(4a+3)^2}{280a^2 + 504a + 225} \quad (\text{not valid for } Br^*=0) \quad (43)$$

Parameter  $a$  in Eq. (43) is related to  $\varepsilon Wi^2$  by

$$a = \frac{3}{4} \left[ \left( \frac{\bar{U}_N}{\bar{U}} \right)^{-1} - 1 \right]. \quad (44)$$

For a power law fluid, the corresponding fully developed Nusselt number is given by

$$Nu_{\infty} = \frac{2(3n+1)(5n+1)}{n(4n+1)} \quad (\text{not valid for } Br^*=0) \quad (45)$$

an expression that is also not valid when  $Br^* = 0$ , as was the case of Eq. (43) for the sPTT fluid.

In the absence of viscous dissipation,  $Br^* = 0$ , the corresponding expression of the Nusselt number for the sPTT fluid with a constant wall temperature is

$$Nu_{\infty} = 3.658 \frac{\left(1 + \frac{4}{3}a\right) \left(1 + 4.7967a + 8.641a^2 + 6.9291a^3 + 2.0869a^4\right)}{\left(1 + 5.961a + 14.230a^2 + 17.004a^3 + 10.172a^4 + 2.4368a^5\right)} \quad (46)$$

For the constant wall heat flux case, the fully developed Nusselt number is given by the following equations, which are valid for any value of Brinkman number including  $Br^* = 0$ . Equation (47) pertains to sPPT fluids and Eq. (48) applies to power law fluids.

$$Nu_{\infty} = \left[ \frac{760a^2 + 1224a + 495}{240(4a+3)^2} + Br^* \right]^{-1}, \quad (47)$$

$$Nu_{\infty} = \left[ \frac{31n^2 + 12n + 1}{8(5n+1)(3n+1)} + Br^* \right]^{-1} \quad (48)$$

Note that the author who derived Eq. (48) did not use the generalized Brinkman number.

### 5.3 The Friction Factor

### 7.3.3. Thermally Developing Pipe Flow of Viscoelastic Fluids

## NON-NEWTONIAN HEAT TRANSFER

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**Keywords:** pipe flow, laminar regime, turbulent regime, viscous dissipation, developing flow, developed flow, polymer melts and solutions, surfactant solutions, drag and heat transfer reduction

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### 7.3.3. Thermally Developing Pipe Flow of Viscoelastic Fluids

Heat transfer for the sPTT model fluid in the thermally developing region of a pipe is considered in some detail assuming that the flow is dynamically fully-developed. Here, the interest lies in considering viscous dissipation effects, but comparisons are also made for the  $Br^* = 0$  case, which is analyzed first. Regardless of the boundary condition, for  $Br^* = 0$  the local Nusselt number ( $Nu_x$ ) decreases continuously towards a constant value corresponding to the fully-developed condition, as shown in Figure 2. This figure plots the variation of the local Nusselt number,  $Nu_x$ , as a function of the normalized distance from the pipe inlet,  $x'$  where  $x' = x/(DPrRe) = Gz^{-1} \pi/4$ . The fluid is being heated and there is viscous dissipation, ( $Br_w^* \geq 0$  because  $T_w \geq T_0$ ), while the wall temperature remains constant. The decreasing variation of  $Nu_x$  along the entrance region is due to a decreasing wall temperature gradient as the temperature profile develops and this effect exceeds the reduction in the temperature difference appearing in the denominator of the local Nusselt number (c.f. Eq. (22) in Section 5.2), an effect associated with the increase in the bulk temperature.

The fluid rheology has an effect since  $Nu_x$  increases with  $\varepsilon Wi^2$  for an sPTT fluid or inversely with  $n$  for a power law fluid. However, the increase in  $Nu_x$  is slight, on the

order of 12% when  $\epsilon Wi^2$  varies from zero to 10. In the presence of viscous dissipation, however, changes are more dramatic.

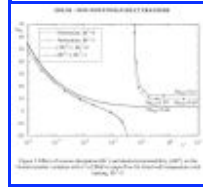


Figure 2. Effect of viscous dissipation ( $Br^*$ ) and elasticity/extensibility ( $\epsilon Wi^2$ ) on the Nusselt number variation with  $x'=x/DRePr$  in pipe flow for fixed wall temperature with heating,  $Br^* > 0$ .

To illustrate the behavior for  $Br^* > 0$  Figure 2 plots the variation of  $Nu_x$  for  $Br^* = 1$  since the variations are qualitatively the same regardless of the numerical value of  $Br^*$ . The discontinuity in  $Nu_x$  is independent of the type of fluid (Newtonian or non-Newtonian), because the continuous increase in fluid temperature due to internal heat generation eventually inverts the heat flux at the wall. This can be easily explained by inspection of the convection coefficient definition of Eq. (22) and the simultaneous variation of the bulk temperature and of the temperature gradient at the wall. The wall temperature is everywhere constant and viscous heating increases the fluid bulk temperature ( $T_b$ ), thus decreasing  $T_w - T_b$  and the wall temperature gradient,  $(dT/dr)_w$ . Since the latter decreases faster than the former,  $Nu_x$  goes through zero and becomes negative when  $(dT/dr)_w$  becomes negative. From this point forward the fluid is being cooled at the wall, but internal heat generation is stronger and makes the bulk temperature rise. When  $T_b$  equals the wall temperature the local Nusselt number goes through a discontinuity. Henceforth the bulk temperature exceeds the wall temperature,  $T_w - T_b$  becomes negative as is the case with the temperature gradient, and  $Nu_x$  becomes positive again, with the fluid being cooled instead of being heated as at the inlet of the pipe.

For  $Br^* < 0$  at constant wall temperature there is always fluid cooling ( $T_w < T_0$ ), so there is no discontinuity in the Nusselt number variation because  $(dT/dr)_w$  and  $T_w - T_b$  always remain negative, regardless of the strength of the internal heat generation. However, the variation of  $Nu_x$  is not monotonic as shown in Figure 3. Monotonicity is only observed for  $Br^* = 0$  and at an intermediate range of  $Br^*$ . For  $Br^* < 0$  the variation of  $Nu_x$  with  $x'$  depends on the way the Brinkman number affects  $(dT/dr)_w$  and  $T_w - T_b$  and Figure 3 shows the influence of Brinkman number on the local Nusselt number for Newtonian ( $\epsilon Wi^2 = 0$ ) and sPTT ( $\epsilon Wi^2 = 10$ ) fluids for  $Br^* = 0$  down to  $-100$ .

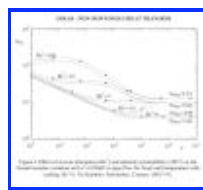


Figure 3. Effect of viscous dissipation ( $Br^*$ ) and elasticity/extensibility ( $\epsilon Wi^2$ ) on the

Nusselt number variation with  $x'=x/DRePr$  in pipe flow for fixed wall temperature with cooling,  $Br^* < 0$ . No Symbols: Newtonian; Crosses:  $\varepsilon We^2=10$ .

For this constant temperature wall case, the thermal entrance length is significantly affected by dissipation and to a lesser extent by the elastic parameter,  $\varepsilon Wi^2$ . Viscous dissipation normally imposes a substantially longer thermal development length than usually required by Eqs. (31) or (35) for negligible internal heat generation. Table 4 lists values of the thermal development length required for  $Nu_x$  to be within  $\pm 5\%$  of the fully developed Nusselt number value,  $Nu_\infty$ , for several combinations of  $Br^*$  and  $\varepsilon Wi^2$ .

**Table 4.** Thermal entrance length data,  $Gz_i^{-1} = x' 4/\pi$ , for imposed wall temperature.

In the case of wall cooling, but now with a constant wall heat flux boundary condition, viscous dissipation ( $Br^* < 0$ ) also leads to discontinuities and changes of sign in the local Nusselt number, although this is not as systematic as for fixed wall temperature with fluid heating. There are three different types of behavior depending on the value of the Brinkman number, as discussed below. Figure 4 shows the  $Nu_x - x'$  variation for the Newtonian fluid and the sPTT fluid at  $\varepsilon Wi^2 = 10$ . The figure contains plots for three different Brinkman numbers each corresponding to one of the three regions of behavior, alongside with the Newtonian curve for no dissipation ( $Br^* = 0$ ).

When there is wall cooling the heat generated by viscous dissipation is competing with the heat removed at the wall and both affect the bulk temperature,  $T_b$ , and the wall temperature,  $T_w$ , that appear in the heat transfer coefficient and in the Nusselt number (c.f. Eqs. (22) and (23)). Note that the wall temperature varies along the pipe in order to accommodate the imposed constant wall heat flux as well as the overall energy balance. When the absolute value of  $Br^*$  is lower than the absolute value of the first critical Brinkman number,  $Br_1^*$ , there is less internal heat generation than cooling at the wall and the wall temperature is everywhere lower than the bulk temperature leading to a positive Nusselt number and its monotonic decrease with  $x'$ . Since wall cooling is larger than internal heat generation both the wall and bulk temperatures decrease along the pipe. This first critical Brinkman number is negative and is equal to

$$Br_1^* = -\frac{1}{8} \quad (49)$$

meaning that the frictional heat,  $\tau_w \bar{U}$  is equal to the heat exchanged at the wall,  $\dot{q}_w$  (c.f. Eq. 41). When the absolute value of  $Br^*$  is larger than the absolute value of a second critical Brinkman number,  $Br_2^*$ , the internal heat generation is larger than the cooling at the wall so both the wall and bulk temperatures increase along the pipe. However, wall cooling forces the wall temperature to be lower than the temperature of a thin near-wall layer of fluid, in the region where internal heat generation is strong (large velocity gradients). As a consequence this wall layer cools simultaneously towards the wall and the bulk of the fluid. The fluid bulk temperature increases and the wall temperature becomes higher than the bulk temperature of the fluid ( $T_w > T_b$ ). This second critical Brinkman number is also

negative and is given by,

$$Br_2^* = -\frac{760a^2 + 1224a + 495}{240(4a + 3)^2}, \quad (50)$$

Hence, in this case the Nusselt number is once again positive at the inlet ( $T_w < T_b$ ), then it goes through a discontinuity when  $T_w = T_b$  and becomes negative when finally  $T_w > T_b$ . Since the heat flux is imposed and is always different from zero, the Nusselt number will change from positive value to negative value through a discontinuity.

In the range  $|Br_2^*| < |Br^*| < |Br_1^*|$  the heat generated by viscous dissipation is larger than the wall heat flux, but the wall temperature remains lower than the bulk temperature. Hence, the Nusselt number remains positive and decreases along the pipe even though not monotonically as shown in Figure 4.

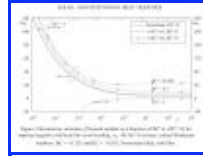


Figure 4. Streamwise variation of Nusselt number as a function of  $Br^*$  at  $\varepsilon Wi^2=10$  for imposed negative wall heat flux (wall cooling,  $\dot{q}_w < 0$ ):  $Br^* < 0$  crosses, critical Brinkman numbers  $Br_1^* = -0.125$  and  $Br_2^* = -0.203$ ; Newtonian fluid, solid line.

The last case pertains to the imposed wall heat flux for fluid heating,  $\dot{q}_w > 0$ , in the presence of viscous dissipation,  $Br^* > 0$  for which Figure 5 plots some  $Nu_x$  versus  $x'$  profiles at three different values of the Brinkman number. Here, both the internal heat generation and the wall heat transfer contribute to increase all temperatures along the pipe. The wall temperature is always greater than the bulk temperature, so the Nusselt number is always positive and decreases monotonically with  $x'$ . At low  $Br^*$ ,  $Nu$  slightly increases with  $\varepsilon Wi^2$  because of the corresponding increased degree of shear-thinning. However, at large  $Br^*$  the viscous dissipation overwhelms the shear-thinning effect and the effect of  $\varepsilon Wi^2$  on  $Nu$  becomes small and reversed, i.e.,  $Nu$  now decreases with  $\varepsilon Wi^2$ . The internal heat generation, on the other hand, strongly decreases the Nusselt number in this case. The variation of the Nusselt number in the entrance region of the pipe can also be approximated by an equation of the type

$$Nu_x = b_1 \left( \frac{b_2 + 1}{b_3} \right)^{b_4} Gz^{b_4} \quad (51)$$

where parameters  $b_1, b_2, b_3$  and  $b_4$ , listed in Table 5, are functions of  $Br^*$  and  $\varepsilon Wi^2$ .

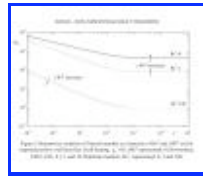


Figure 5. Streamwise variation of Nusselt number as a function of  $Br^*$  and  $\epsilon Wi^2$  ( $n$ ) for imposed positive wall heat flux (wall heating,  $\dot{q}_w > 0$ ).  $\epsilon Wi^2$  represented: 0 (Newtonian), 0.001, 0.01, 0.1, 1 and 10. Brinkman number,  $Br^*$ , represented: 0, 1 and 100.

**Table 5.** Values of the coefficients for Eq. (51) -  $Nu_x$ , fluid heating with constant heat flux at the wall, valid for  $79 < Gz < 2.6 \times 10^5$ .

The thermal entrance length for imposed wall heat flux is shorter than its counterpart for constant wall temperature. Nevertheless, there is also a dependence on  $Br^*$  and  $\epsilon Wi^2$ , with the former generally increasing  $L_t$  and the latter reducing it slightly. An exception to this behavior takes place in the range  $|Br^*| < |Br_2^*|$  where  $L_t$  is smaller than in the absence of dissipation. Regarding the effect of  $\epsilon Wi^2$ , when  $\epsilon Wi^2$  increases from 0 to 10 there is on average a reduction in  $L_t$  of about six percent. Values of the thermal entrance length are listed in Table 6 in the form of  $Gz_t^{-1} = x' 4/\pi$  for several combinations of  $Br^*$  and  $\epsilon Wi^2$ .  $L_t$  is defined again as the length required for  $Nu_x$  to be within  $\pm 5\%$  of  $Nu_\infty$ .

**Table 6.** Thermal entrance length data,  $Gz_t^{-1} = x' 4/\pi$ , for imposed wall heat flux.

All equations in section 7 can be used indistinctively for sPTT and power law fluids provided there is an equivalence between parameter  $a$  (or  $\epsilon Wi^2$ , c.f. Eqs. (37) and (44)) for the sPTT fluid and the exponent  $n$  of the power law fluid. This is so because the flow is hydrodynamically fully developed. Such equivalence was achieved by equalizing Eqs. (43) and (45) or Eqs. (47) and (48) because the resulting mathematical expressions are the same, leading to Eqs. (52) and (53), valid for  $1/3 < n < 1$  ( $\infty > a > 0$ ), that permit to find  $a$  knowing  $n$  and to find  $n$  knowing  $a$  respectively

$$a = \frac{3\sqrt{2} \left[ \sqrt{8775n^4 + 7320n^3 + 2098n^2 + 232n + 7} - 3\sqrt{2} (55n^2 - 16n - 7) \right]}{20(87n^2 - 8n - 7)} \tag{52}$$

$$n = \frac{\sqrt{(1000000a^4 + 3047040a^3 + 3490416a^2 + 1782000a + 342225)} + 8(20a^2 + 72a + 45)}{15(232a^2 + 264a + 63)} \tag{53}$$

The error associated with using an expression derived for the sPTT fluid for a power law fluid, or vice-versa, is small, not exceeding 4% in the general case. This is of the same order as the error in the corresponding comparison for  $Br^*=0$ . For fully developed thermal flow the comparison of Nusselt numbers is even better; for  $n = 0.5$  the use of Eq. (46) with  $a$  calculated by Eq. (52) provides a value of  $Nu$  within

0.25% of the corresponding value for power law fluids of the literature, c.f. Table 2.

## 8. Turbulent Flow

Even though a large proportion of non-Newtonian fluids flow in the laminar regime, turbulent flow is by no means less relevant. In fact, when the concentration of the additives is low and the solvent has a low viscosity, or the characteristic flow length scale of relevance is large, the flow takes place in the turbulent regime. In the previous section it was seen that the thermal behavior of non-Newtonian laminar fluid flows is essentially determined by the fluid viscous characteristics, with fluid elasticity imparting modifications in the presence of secondary flows or under unsteady flow conditions. The impact of these modifications on the primary flow is not very large in the fully developed laminar regime. However, this is not the case under turbulent flow conditions, where viscoelasticity has a dramatic effect in reducing both drag and heat transfer by as much as 80%, as will be discussed in this section. Therefore, it now becomes essential to distinguish between purely viscous and viscoelastic fluid flows in heat transfer, the latter often known as drag reducing fluids.

Whereas for purely viscous fluids the description of flow and heat transfer relies on relatively simple equations inspired by the treatment for Newtonian fluids and in some cases taking advantage of the analogy between friction and heat transfer, for viscoelastic fluids there are distinct regimes depending on the type of additive, its concentration, flow Reynolds number and pipe diameter.

[7.2 Developing Pipe Flow for Purely Viscous Fluids](#)

[8.1. Fully-developed Pipe Flow of Purely Viscous Fluids](#)



## NON-NEWTONIAN HEAT TRANSFER

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### 8.1. Fully-developed Pipe Flow of Purely Viscous Fluids

Most non-Newtonian fluids exhibit some form of elasticity because the components that impart a variable viscosity, such as polymer molecules and micelles, are also capable of storing energy elastically. However, the degree of flexibility of the internal structures, their characteristic scales and the corresponding flow characteristic scales determine the fluid response and not all viscoelastic fluids exhibit a viscoelastic behavior in the turbulent regime. Fluids exhibiting purely viscous behavior in turbulent flow are usually suspensions of particles or aqueous solutions of some polymer additives like carbopol. Incidentally, aqueous solutions of carbopol do exhibit elasticity in oscillatory shear flow, but behave as purely viscous fluids in turbulent pipe flow, an example that shows the complexity of non-Newtonian fluid dynamics and heat transfer.

#### 8.1.1. Friction Factor

According to the literature, the Fanning friction factor for a purely-viscous fluid in a turbulent pipe flow is well predicted by the following equation

$$\frac{1}{\sqrt{f}} = \frac{4.0}{n^{0.75}} \log_{10} \left[ Re' f^{1-(n/2)} \right] - 0.40/n^{1.2} \quad (54)$$

valid for  $(PrRe^2)f > 5 \times 10^5$ . Explicit forms of this expression can also be found in the bibliography, for instance, Eq. (55) encompasses the experimental data to within  $\pm 7.5\%$ .

$$f = \frac{F(n)}{Re^{+1/(3n+1)}} \quad (55)$$

where

$$F(n) = \frac{2 \left( \frac{2}{7^7} \right)^{1/(3n+1)}}{\left[ \frac{(3n+1)}{4n} \right]^n} \quad (56)$$

On account of the strong diffusive nature of turbulence, the hydrodynamic entrance length for turbulent pipe flow of purely-viscous fluids is almost the same as that observed for Newtonian fluids, i.e., about ten to fifteen pipe diameters. Hence, for most engineering purposes, the fully-developed flow condition can generally be assumed and considerations on developing flow will not be made here.

### 8.1.2. Heat Transfer

Equation (57) taken from the bibliography gives the heat transfer coefficient for turbulent pipe flow of purely viscous fluids via the Stanton number,  $St$ . This expression was derived on the basis of the analogy between heat and momentum transfer, so the friction coefficient  $f$  is calculated using the equations of the previous subsection. The correlation is a function of  $Re'$ , used to determine  $f$  via Eq. (54), and  $Pr_a$ , the latter being a function of  $\eta_a$ . Equation (57) is valid for  $(PrRe^2f) > 5 \times 10^5$ ,  $0.5 < Pr_a < 600$  and correlates the experimental data to within  $\pm 25\%$  in the flow index range  $0.39 < n < 1.0$ .

$$St_a = \frac{Nu}{RePr} = \frac{f/2}{1.20 + 11.8 \sqrt{f/2} (Pr_a - 1) Pr_a^{-1/3}} \quad (57)$$

A simpler empirical equation, based on the apparent viscosity, is also reported in the bibliography and correlates the available experimental data to within  $\pm 5\%$ , so more accurately than Eq. (57). Its use is recommended for  $n$  ranging from 0.2 to 0.9 and  $3\,000 < Re_a < 90\,000$ .

$$St_a = \frac{Nu}{RePr} = 0.0152 Re_a^{-0.155} Pr_a^{-2/3} \quad (58)$$

As for turbulent Newtonian flows the above expressions are valid for both constant wall heat flux and constant wall temperature, since the flow turbulence reduces significantly the influence of the thermal boundary condition.

The thermal entrance length for turbulent pipe flow of purely-viscous fluids is similar to that for Newtonian fluids, i.e., about twenty to fifty pipe diameters.

## 8.2. Turbulent Flow of Viscoelastic Fluids

Additives that impart viscoelastic behavior to otherwise Newtonian fluids interact with turbulent events and consequently delay the full establishment of turbulent flow even though the onset of flow instabilities occurs at similar Reynolds numbers as for Newtonian fluids. Hence it is often reported in the literature that turbulent flow of viscoelastic fluids is established at Reynolds numbers of around 6 000 and

correlations for integral quantities in laminar flow can be used up to higher Reynolds numbers than for Newtonian fluids.

Turbulent duct flows of viscoelastic fluids are characterized by large reductions in friction and heat transfer relative to the behavior of Newtonian fluids at the same Reynolds number. In particular, pipe and channel flows of these fluids have been extensively investigated as shown by the extent of the literature exposed in the cited bibliography. Fluids that exhibit these remarkable characteristics are polymer solutions and solutions of surfactants and the effects are strong even when the amount of additive is very small, of the order of 10 ppm for some very effective additives. At low additive concentrations the drag and heat transfer reductions depend on the Reynolds number, the pipe diameter and the additive concentration in solution. When the additive concentration is increased, the reductions of heat transfer and drag also increase up to a point when there is saturation of the effect. Above this critical concentration, whose value depends on the additive, the heat transfer and friction coefficients cease to depend on concentration and pipe diameter and depend only on Reynolds and Prandtl numbers. This is the so-called asymptotic regime and here it is not necessary to use any parameter representative of the viscoelasticity of the fluid, such as the Weissenberg number, in order to quantify the friction factor and Nusselt number. The dependence on the pipe diameter in the non-asymptotic regime is quite remarkable as it eluded for quite sometime classical dimensional analysis arguments, but there are now methods for scaling results in this regime.

The fluids exhibiting these remarkable characteristics are polymer solutions and solutions of surfactants. Even though they share qualitatively the same type of flow and heat transfer characteristics, when it comes to a quantitative description they follow different correlations. For this reason, they are dealt with separately in this Section. First, the asymptotic regime is dealt with for polymer solutions and then for surfactants, after which the non-asymptotic regime is discussed.

### 8.2.1 Asymptotic Regime for Polymer Solutions: Friction Characteristics

Even though the physics behind drag and heat transfer reductions in turbulent flow of polymeric solutions is not yet fully understood, the phenomena are well characterized and accurate correlations exist to quantify both the friction factor and Nusselt number for fully-developed flow in the asymptotic regime. Experimental data show that the hydrodynamic entrance length for viscoelastic polymeric solutions increases with the polymer concentration reaching a maximum value of about  $100 \cdot D$ . This is considerably longer than for purely viscous non-Newtonian and Newtonian fluids.

The asymptotic expression for the minimum friction factor  $f$  is known as the Maximum Drag Reduction Asymptote (MDRA) or Virk's asymptote. It is an implicit expression on  $f$ , given by Eq. (59) and depends on the apparent Reynolds number,  $Re_a$ . There is a simpler Blasius-like expression, given by Eq. (60), which is a good approximation of this implicit equation. Both equations are valid only for dynamically smooth walls. Studies on wall roughness effects for non-Newtonian fluids are very limited in scope and are not discussed in this chapter. Note that Virk's expression is not an asymptote in the true meaning of the term, but an average curve of data from a wide variety of polymer solutions for different smooth pipes at maximum drag reduction

Virk's asymptote has been validated by data from a wide variety of dilute and semi-dilute polymer solutions and this is seen in Figure 6-a), where the Fanning friction factor is plotted against the Reynolds number. Figure 6-b) compares the Blasius type form of Virk's asymptote with experimental friction factor data for

polyacrylamide solutions based on Separan AP-273 and includes data for the non-asymptotic regime when the polymer concentration is of 10 ppm. This particular set of data lie in between Virk's MDRA and the Newtonian curve.

$$\frac{1}{\sqrt{f}} = 19.0 \log(Re_a \sqrt{f}) - 32.4 \quad (59)$$

$$f = 0.58 Re_a^{-0.58} \quad (60)$$

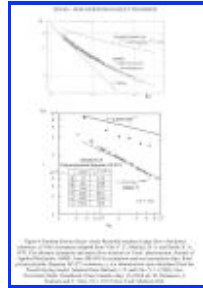


Figure 6. Fanning friction factor versus Reynolds number in pipe flow of polymer solutions: a) Virk's asymptote adapted from Virk, P. S., Mickley, H. S. and Smith, K. A. 1970. The ultimate asymptote and mean flow structure in Toms' phenomenon. *Journal of Applied Mechanics ASME*, June 488-493 b) asymptotic and non-asymptotic data, fluid: polyacrylamide (Separan AP-273) solutions,  $t_p$  is a characteristic time calculated from the Powell-Eyring model. Adapted from Hartnett, J. P. and Cho, Y. I. (1998). *Non-Newtonian fluids. Handbook of heat transfer chap. 10*, (third ed. W. Rohsenow, J. Hartnett and Y. Cho), 10.1–10.53 New York: McGraw-Hill.

Slightly different asymptotes have been presented in the literature pertaining to individual data sets. Some of these expressions are presented next in Table 7 together with the corresponding range of validity and the fluids used to obtain them, but their use is not limited to those fluids even though there is a lower experimental uncertainty associated with them.

[Table 7](#). Other asymptotic expressions for the friction factor  $f$  for polymer solutions.

### 8.2.2. Asymptotic Regime for Polymer Solutions: Heat Transfer Characteristics

In recent years it has been found that polymeric solutions have very long thermal entrance lengths, something that was overlooked for a long time in experimental research. The thermal entrance length increases especially for concentrated polymer solutions and/or large Reynolds numbers, i.e., at conditions that foster large heat transfer reductions. In these conditions the thermal length reaches values of the order of 400 to 600 diameters, which are much greater than the 20 diameters found for turbulent Newtonian flow and purely viscous non-Newtonian fluids or even for the dynamic flow development of viscoelastic fluids, the latter requiring a length of the order of 100 diameters. The reasons for this behavior are not yet fully understood, but some authors argue that the momentum eddy diffusivity is several times higher than the heat eddy diffusivity and this difference justifies the discrepancy.

Figure 7, taken from the bibliography, plots the Chilton-Colburn factor,  $j_H = StPr^{2/3}$ , for several Separan AP-273 solutions in both the asymptotic and non-asymptotic regimes as a function of  $Re_a$ . The measurements were made at  $x/D = 430$ , i.e., close to thermally fully developed conditions, and the minimum asymptotic heat transfer coefficient, approximated by the experimental data, is

correlated by Eq. (61) valid for  $Re_a > 6\,000$ .

$$j_H = StPr^{2/3} = 0.03Re_a^{-0.45} \quad (61)$$

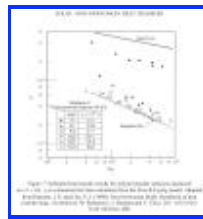


Figure 7. Turbulent heat transfer results for polyacrylamide solutions measured at  $x/D = 430$ .  $t_p$  is a characteristic time calculated from the Powell-Eyring model.

Adapted from Hartnett, J. P. and Cho, Y. I. (1998). Non-Newtonian fluids. Handbook of heat transfer chap. 10, (third ed. W. Rohsenow, J. Hartnett and Y. Cho), 10.1–10.53 New York: McGraw-Hill.

Figure 8, taken from the bibliography, plots the local Nusselt number in the turbulent entrance region for a 200 ppm aqueous solution of Separan AP-273. As the distance from the entrance increases the Nusselt number decreases to the fully developed value, the expected behavior for a boundary layer type flow. This plot clearly shows the very long lengths required to attain a constant Nusselt number for viscoelastic fluids.

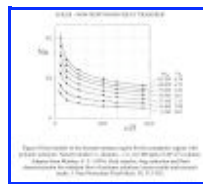


Figure 8. Heat transfer in the thermal entrance region for the asymptotic regime with polymer solutions: Nusselt number vs. distance,  $x/D$ , for 200 ppm of AP-273 solution. Adapted from Matthys, E. F. (1991). Heat transfer, drag reduction and fluid characterization for turbulent flow of polymer solutions: recent results and research needs. J. Non-Newtonian Fluid Mech. 38, 313-342.

Since the thermal entrance length is significantly larger for viscoelastic fluids than for Newtonian fluids, it is necessary to account for this developing effect in short pipes in order to correctly design thermal equipment. This is the case of tube heat exchangers, where entry effects contribute significantly to the overall heat transfer. Here, the correct average heat transfer coefficient can be considerably higher than the fully developed value. To account for this effect Eq. (62) should be used to quantify the local heat transfer coefficient, where the local Colburn number depends on the axial position  $x/D$ .

$$j_H = StPr^{2/3} = 0.13(x/D)^{-0.24} Re_a^{-0.45} \quad (62)$$

This equation is valid for  $x/D < 450$  and for  $x/D \geq 450$  Eq. (61) must be used instead.

As for the friction factor, the above correlations for the normalized heat transfer coefficient at maximum heat transfer reduction should be regarded as envelopes of experimental data. Again, the literature provides several alternative expressions, for both thermally fully-developed and developing pipe flow. They are valid regardless of the fluid used, even though they were obtained with specific sets of fluids. These

conditions and the corresponding equations are presented next in Table 8 for the sake of completeness.

[Table 8](#). Other correlations for the Nusselt number in the asymptotic regime for polymer solutions

### 8.2.3. Asymptotic Regime for Surfactant Solutions: Friction Characteristics

A surfactant is a material that is added to a solvent to reduce its surface tension. Its molecules are small, significantly smaller than those of the polymer additives that make the polymer solutions discussed in the previous subsection, but they aggregate to form large structures in the bulk solution when mixed with an appropriate counter-ion. These aggregates are called micelles and they form because the molecules of the surfactants usually have hydrophobic and hydrophilic parts and in the micelles the hydrophobic tails of the molecules go inside the aggregates whereas the hydrophilic tails stay in contact with the solvent. The micelles have different shapes and sizes depending on their chemical composition and also on the history of deformation. They induce a complex viscoelastic rheological behavior and interact with the dynamics of turbulence leading to higher drag and heat transfer reductions than those of polymeric solutions. However, the main advantage of the surfactants over the polymeric solutions is their high resistance to mechanical degradation. The structures that are responsible for drag reduction are the micelles, which breakdown under severe stress, but since the individual surfactant molecules do not break down, the micelles can reform reversibly. The high resistance to mechanical degradation of the surfactant molecules means that their drag and heat transfer reduction capabilities remain unscathed for a long time. In contrast, in the polymer solutions they are the molecules that directly interact with the turbulence dynamics and because these exist as very large macro-molecules they breakdown easily. Molecular scission is an irreversible process and this is the reason for the much lower resistance of polymer solutions to mechanically induced degradation. The high resistance of surfactant solutions to mechanical degradation coupled with their large drag and heat transfer reductions holds promise of interesting applications in industry such as thermal fluids in district heating and cooling systems.

The asymptotes of drag and heat transfer reduction for surfactant solutions are different from those of polymer solutions, but there are other important behavioral differences between these two types of complex fluids. Experimental data have shown that the dynamic and thermal entrance lengths for surfactants are of the same order of magnitude and of the order of several hundred diameters (some experiments indicate that even after three hundred diameters the fully developed condition is not yet fully established). Other recent experimental work points to the existence of distinct types of drag reduction for surfactants, but the DNS investigations of polymer solutions also indicate the existence of different drag reduction regimes for the latter fluids. In fact, drag and heat transfer reduction for fluids based on polymers and surfactants is still a rich area of research, and what is clear at this moment is that both exhibit complex specific behaviors due to their inherent chemical and physical differences. The phenomenological similarities found in the nineteen seventies and nineteen eighties fostered a wealth of experimental and numerical research with both types of fluids that has been providing a deeper understanding, but also the realization that the differences of behavior are as many as the similarities.

According to the bibliography the asymptotic Fanning friction factor in the fully-developed turbulent regime is well correlated by Eq. (63). This implicit correlation was obtained with fresh and contaminated cationic surfactant Ethoquad T13/27 solutions at various concentrations and can be approximated by the Blasius-type expression (64) over the range  $6\,000 < Re_a < 80\,000$  and  $4.0 < Pr_a < 6.5$ .

$$\frac{1}{\sqrt{f}} = 23.9 \log_{10} (Re_a \sqrt{f}) - 40 \quad (63)$$

$$f = 0.18 Re_a^{-0.50} \quad (64)$$

There is only one extensive research paper on the dynamic and thermal flow development for surfactant solutions, where the authors have tried to derive correlations for both the local friction factor (and heat transfer). This investigation concentrated essentially on the first one hundred diameters nearer the pipe inlet ( $5 < x/D < 125$ ) and showed that the fluid dynamic and thermal developments seem coupled through a constant turbulent Prandtl number. The local friction factor and Nusselt number normalized by their local values at  $x/D = 117$  and  $x/D = 130$ , respectively  $f_{rel} = f/f_{@x/D=117}$  and  $Nu_{rel} = Nu/Nu_{@x/D=130}$ , follow the same curve when represented as a function of  $x/D$ , as shown in Figure 9. The solid line represents a double exponential equation that correlates the experimental data for heat transfer given as Eq. (66). The figure also suggests that at the beginning of the pipe ( $x/D < 30$ ) the relative local friction factor may be lower than the relative local Nusselt number, but this is due to experimental limitations according to the authors, i.e., the true  $f_{rel} = f/f_{@x/D=117}$  at  $x/D < 30$  should be closer to the plotted  $Nu_{rel} = Nu/Nu_{@x/D=130}$  than shown here.

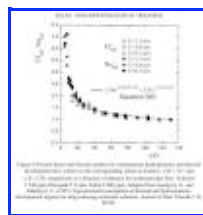


Figure 9. Friction factor and Nusselt number for simultaneous hydrodynamic and thermal development flow relative to the corresponding values at locations  $x/D = 117$  and  $x/D = 130$ , respectively as a function of distance for surfactant pipe flow. Solution: 2 300 ppm Ethoquad T13 plus NaSal 2 000 ppm. Adapted from Gasljevic, K. and Matthys E. F. (1997). Experimental Investigation of thermal and hydrodynamic development regions for drag-reducing surfactant solutions. Journal of Heat Transfer 119, 80-88.

#### 8.2.4. Asymptotic Regime for Surfactant Solutions: Heat Transfer Characteristics

The asymptotic normalized heat transfer in the fully-developed turbulent regime is given by Eq. (65) derived by the same authors who arrived at Eq. (63) for the friction factor. Equation (65) was also validated against data for various fresh and contaminated cationic surfactant solutions of Ethoquad T13/27 at various concentrations and is valid for  $12\,000 < Re_a < 80\,000$  and  $4.0 < Pr_a < 6.5$ .

$$StPr^{2/3} = 0.164 Re_a^{-0.649} \quad (65)$$

As discussed at the end of the previous subsection, in the entrance region there is simultaneous dynamic and thermal flow development and the relative Nusselt number  $Nu_{rel} = Nu/Nu_{@x/D=130}$  obeys the following expression

$$Nu_{rel} = 1.56e^{-0.0035(x/D)} + 5.31e^{-0.133(x/D)} \quad (66)$$

valid for  $5 < x/D < 125$  and  $1 < \bar{U} < 8$  m/s with a maximum deviation of  $\pm 5\%$ . There is clearly scope here for more research in order to come up with a more general relationship, but nevertheless a simpler expression is already available (Eq. 67), correlating all data with a maximum deviation of  $\pm 10\%$ .

$$Nu_{rel} = 8.66(x/D)^{-0.47} \quad (67)$$

[7.3.3. Thermally Developing Pipe Flow of Viscoelastic Fluids](#)

[8.2.5. Non-asymptotic Regime for Polymer and Surfactant Solutions](#)



## NON-NEWTONIAN HEAT TRANSFER

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**Keywords:** pipe flow, laminar regime, turbulent regime, viscous dissipation, developing flow, developed flow, polymer melts and solutions, surfactant solutions, drag and heat transfer reduction

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### 8.2.5. Non-asymptotic Regime for Polymer and Surfactant Solutions

Between the Newtonian and the asymptotic behaviors lies a zone where the drag and heat transfer reductions depend not only on the Reynolds number but also on the polymer concentration and Weissenberg number. This is the so-called non-asymptotic regime. The non-asymptotic regime has also been extensively investigated especially for polymeric solutions, initially from a pure experimental point of view and over the last fifteen years mostly using direct numerical simulation (DNS) aimed at understanding the physics of drag reduction and its relation with molecular conformation. Over the last twenty years there have also been many investigations with surfactants in this flow regime.

The difficulty of measuring viscoelastic material properties with dilute polymer solutions and surfactants, together with the complexity of their flow characteristics under drag reduction conditions, contributed to the absence of a clear well established method of calculation and prediction of flow and heat transfer characteristics in the non-asymptotic regime. The absence of data on elastic material properties does not allow the quantification of non-dimensional elastic numbers, such as the Weissenberg number, and this led to some difficulties in the interpretation of results pertaining to this regime.

Earlier authors reported the existence of a diameter effect, i.e., the friction factor not only depended on the Reynolds number, but continued to depend on a dimensional

quantity, the pipe diameter. On the trail of these findings some methods were devised for scaling purposes, which were validated by experiments and here a simple methodology published in the early nineteen nineties is presented. This scaling method was validated only for polymer solutions. Investigations into surfactant and polymer drag reduction in the late nineteen nineties provided an alternative that is theoretically better, but is of more difficult application as it is compounded by two different scaling procedures. This second method is also discussed here, but it still requires research to enhance its applicability to engineering. Underlying both methods is their exclusive reliance on viscous fluid properties, a clear indication of their limitations since they are incomplete as far as a connection with fluid viscoelasticity is concerned.

The first scaling method for the non-asymptotic regime starts by assuming prior knowledge of friction factor data  $f_1$  at various values of the Reynolds number  $Re_1$  in a particular pipe of diameter  $D_1$  for a specific fluid and the objective is the determination of the corresponding  $f_2$  versus  $Re_2$  behavior in another pipe of diameter  $D_2$ . This method is based on the validity of an equation for the friction factor having the mathematical form of the Colebrook-White expression relating friction factor, Reynolds number and roughness. Drag reduction or drag increase is accounted for by the term of the kernel  $N$  of the log-function that bring the roughness contribution into the equation, i.e., drag reduction is transformed into a relative negative roughness. Such equation is written here as Eq. (68). Using the  $f_1$  data, Eq. (68) is used to calculate the kernel of the log-function for the pipe with diameter  $D_1$  and this kernel is multiplied by the ratio  $D_2/D_1$  to quantify the new value of the kernel corresponding to pipe diameter  $D_2$ .

$$\frac{1}{\sqrt{f}} = 2.0 \log_{10} \left[ \frac{Re \sqrt{f}}{2.51} + N \right] \quad (68)$$

With the new value of the kernel the set of data  $f_2$  is obtained with Eq. (68), or equivalently using Eq. (69) with the corresponding values of the Reynolds number calculated by Eq. (70). This method is recommended when pipe diameters are larger than 13 mm and should not be used when the pipe diameter is less than 5 mm.

$$\frac{1}{\sqrt{f_2}} = 2 \log \frac{D_2}{D_1} + \frac{1}{\sqrt{f_1}} \quad (69)$$

$$Re_2 = Re_1 \frac{D_2}{D_1} \sqrt{\frac{f_1}{f_2}} \quad (70)$$

The second method relies on observations of the non-asymptotic behavior for a wide variety of fluids having different conformations these leading to different scaling laws. In turbulent flow in the non-asymptotic regime, the conformation of molecules in polymer solutions can vary from a coiled state to full extension for flexible polymers, or can be in a fairly extended state when the polymer molecules are semi-rigid or rigid. Identically, depending on the state of deformation and chemical composition of surfactant solutions, micellar structures can have a variety of shapes and sizes, from spherical to long elongated cylinders. To each of these typical situations corresponds a different scaling law if the flow condition does not correspond to an asymptotic behavior, but a specific fluid can also reach asymptotic conditions due to the variation of a flow parameter (flow rate or pipe diameter) in which case the expressions of the previous section become valid. One of the non-

asymptotic scaling laws correlates drag reduction, defined in Eq. (71), with the flow bulk velocity and this law appears applicable to polymer solutions whose molecules are far from the fully-extended state and to some surfactant solutions. The other non-asymptotic scaling law is based on a correlation between the wall shear stress,  $\tau_w$ , and the flow bulk velocity,  $\bar{U}$ , and is valid for many surfactant solutions and some polymer solutions as well, the latter usually made from semi-rigid and rigid polymers.

$$DR = \frac{f_{\text{solvent, turbulent}} - f}{f_{\text{solvent, turbulent}}} \quad (71)$$

Depending on flow conditions, the same fluid can behave in such a way that both non-asymptotic scaling laws and the asymptotic behavior will be valid at different flow rates/ pipe diameters and here lies one of the difficulties of drag reduction scaling. This discussion on drag reduction is equally valid for heat transfer reduction, because there is coupling between the two mechanisms, even though the turbulent Prandtl number is not of the order 1 as found for Newtonian fluids, and as discussed in the next section. Note also that these descriptions are to be understood in the absence of mechanical degradation. Mechanical degradation obviously complicates matters significantly because it not only decreases the capability of fluids to reduce drag and heat transfer and consequently transforms an asymptotic behavior into a non-asymptotic behavior, but it also can make this variation time-dependent.

According to the literature, the  $DR - \bar{U}$  scaling was found to be applicable to fluids for which the temperature profile followed the three-layers model, and it was assumed that when this happened the velocity profile also followed the three-layers model. In this case the drag reduction is always the same at the same bulk velocity regardless of the pipe diameter. The  $\tau_w - \bar{U}$  scaling was found to be applicable to fluids for which the slope of the temperature profile increased with flow velocity like an opening fan, hence called fan-type development of temperature profiles. Now, the wall shear stress was found to be constant at the same bulk velocity regardless of pipe diameter.

Hence, the identification of which scaling is appropriate for a given fluid requires extensive measurements of the flow and thermal characteristics, i.e., as with the first scaling method described above, this compounded second scaling method requires prior knowledge of the fluid behavior, but now things are considerably more complex. Whereas in the first scaling method it sufficed to know the  $f-Re$  data for a particular pipe diameter, to use the second scaling method a fairly extensive experimental program must be conducted.

Predictive scaling laws are an essential part of engineering and this is an area where there is clearly scope for more research, because the more accurate methods are far from practical and yet they are essential especially for applications involving very large pipes.

### 8.3. Coupling Between Heat and Momentum Transfer Mechanisms for Viscoelastic Fluids

The classical definition of drag reduction in Eq. (71) uses as a basis of comparison the friction coefficient of the solvent at the same Reynolds number. It considers that ideally  $f$  could be reduced to that experienced by a zero viscosity fluid, i.e. no friction, when in reality the lower limit of the friction factor is that of the solvent in laminar flow at the same Reynolds number. This suggests an alternative definition,

the so-called *TRD* (turbulence reduction-drag) given by

$$TRD = \frac{f_{\text{solvent, turbulent}} - f}{f_{\text{solvent, turbulent}} - f_{\text{solvent, laminar}}} \quad (72)$$

Identically, for heat transfer an alternative to *HTR* (heat transfer reduction) is *TRH* (turbulence reduction-heat) both defined below.

$$HTR = \frac{Nu_{\text{solvent, turbulent}} - Nu}{Nu_{\text{solvent, turbulent}}} \quad (73)$$

$$TRH = \frac{Nu_{\text{solvent, turbulent}} - Nu}{Nu_{\text{solvent, turbulent}} - Nu_{\text{solvent, laminar}}} \quad (74)$$

In many situations the use of one or another does not make a difference, but in a few cases *TRD* and *TRH* provide a much better alternative as they show better the coupling and scaling features of heat and momentum turbulent transport reduction.

In the asymptotic regime the friction factor and Nusselt number depend on Reynolds number, whereas in the non-asymptotic regime they also depend on pipe diameter for some fluids. However, *DR* and *HTR* do not depend on pipe diameter and are a unique function of the bulk velocity the same applying to *TRD* and *TRH*. This uniqueness is a clear suggestion that the diameter effect for heat transfer is the same as the diameter effect for friction; therefore the scaling procedures developed for friction are probably equally valid for the Nusselt number, regardless of whether the fluids are polymer or surfactant solutions.

In the asymptotic regime there is an analogy between momentum and heat transfer for polymer and surfactant solutions, which is not as simple as the Reynolds analogy for Newtonian fluids based on a unity turbulent Prandtl number. Instead, it is characterized by a constant turbulent Prandtl number of around 5 to 6 and the ratio *TRH/TRD* is independent of the Reynolds number and equal to 1.06 to 1.07, whereas *HTR/DR* is a decreasing function of the Reynolds number.

In the non-asymptotic regime the ratio *TRH/TRD* is a unique decreasing function of *TRD* well modeled again by the assumption of a constant turbulent Prandtl number of around 6, and this is valid for both polymer and surfactant solutions. Note however, that these values were obtained with experimental data for a limited range of fluids. A wider analysis is required which is not expected to change qualitatively the conclusions, only the numerical values of some physical quantities.

It is also an established fact that momentum and heat transfer reductions are intimately coupled and related. For a time several authors suggested there could exist drag reduction without heat transfer reduction or that departure from the asymptotic behavior due to fluid degradation affected heat transfer reduction earlier than drag reduction. However, the use of the proper parameters in the data analysis and accurate measurements showed the strong coupling between heat and momentum transfer also at onset conditions and in the neighborhood of asymptotes.

In spite of these analogies between drag and heat transfer reductions, the amount of *DR* and *HTR* (or *TRD* and *TRH*) for polymer solutions and surfactants are different. Very recent experimental and numerical research results suggest that the constant turbulent Prandtl number takes on different numerical values of 6 for polymer solutions and of about 9 for surfactants.

Finally, dynamic as well as thermal entrance lengths differ also in accordance with the momentum-heat transfer coupling, with the development of the thermal boundary layer requiring significantly longer distances (a few hundred pipe diameters) than the development of the dynamic boundary layer (around one hundred diameters), both for non-degraded and recovering surfactant solutions as well as for polymer solutions.

Over the next years, it is expected that DNS simulations will shed light over some of the remaining controversies, but this also depends on the choice of adequate constitutive equations to describe the correct rheology and consequently experimental research will also be required.

## 9. Heat Transfer in Other Fully-developed Confined Flows



Investigations of heat transfer in other fully-developed confined flows, such as in rectangular ducts, and the corresponding developing flows are quite limited. The interested reader should consult the literature and should also be aware that the analogy with Newtonian heat transfer is dangerous, because it is usually not valid. As an example, it suffices to say that in fully-developed laminar flow of viscoelastic fluids in a rectangular duct the friction factor is essentially determined by the viscous characteristics and so it can be well predicted by correlations for power law fluids. Simultaneously, there is a significant heat transfer enhancement associated with the weak secondary flow generated by fluid elasticity and the use of Nusselt number correlations for power law fluids is not recommended, i.e, the secondary flows are not strong enough to affect the drag characteristics, but have an important impact on heat transfer.

## 10. Combined Free and Forced Convection



Experimental work on the combined effects of free and forced convection in non-Newtonian pipe and duct flows are scarce and these authors are not aware of the existence of validated correlations. Under laminar flow conditions, and for viscoelastic fluids, buoyancy can increase the Nusselt number by as much as fifty percent especially in non-circular duct flows because of the presence of secondary flows associated with gradients of normal stresses. In contrast, for purely viscous and for Newtonian fluids the increase is small, of about twelve percent. So, this secondary flow of elastic nature strongly increases heat transfer, but has a minor effect on the friction factor. Some other times buoyancy can decrease the heat transfer with viscoelastic fluids by counteracting the secondary flow induced by the viscoelastic normal stress differences. The enhanced heat transfer in laminar flow of viscoelastic fluids in rectangular ducts has been reported in the literature.

In turbulent pipe flow of viscoelastic fluids it appears that buoyancy effects are still non-negligible even though they are not as important as in laminar flow due to the higher fluid velocities. A paper published in 2000 analyzed effects of buoyancy and showed that its neglect can sometimes result in severe errors. The top-bottom temperature profile asymmetries inside a pipe generated by the free convection effect lead to variations in the local Nusselt number that can reach a value over 2. The experimentalist and the engineers should be aware of this and if it is necessary to avoid free convection effects a small wall heat flux should be set. To minimize the impact of buoyancy effects it is also suggested that the relation  $GrPr/Re < 3$ , where  $Gr$  stands for Grashof number, should be respected.

## 11. Some Considerations on Fluid Degradation, Solvent Effects and

## Applications of Surfactants



Since non-Newtonian fluids, and in particular viscoelastic fluids, are made from complex and long molecules or structures usually having an organic basis, the issues of mechanical and chemical/ bacteriological degradation are quite important. The polymeric solutions are particularly vulnerable to mechanical degradation due to scission of flexible macromolecules and this process is irreversible. Molecular scission arises when the fluids are subjected to high shear rates, for example while flowing through narrow tubes at high flow rates, and in particular when flowing through the narrow gaps of centrifugal pumps, partially closed valves or filters. As macromolecules breakdown into smaller molecules the viscosity of the fluids decrease, the capacity of the molecules to store elastic energy is severely reduced and the fluids lose their elasticity. This is the so-called mechanical degradation. The level of molecular disruption depends on the intensity and duration of the disturbance, which can render the viscoelastic solution into a purely viscous fluid or even into a Newtonian fluid after a long time, when the internal structure has been broken in much smaller pieces.

The effect is stronger the higher the flow rate and unsurprisingly it is more often reported under turbulent flow conditions, where the molecules are extended and consequently more vulnerable to scission. In turbulent flow the effect can be so severe that a solution of a highly flexible polymer, such as polyethylene oxide, can lose 50% of its drag reducing capability in a single pass through a few meters of pipe. The repeated flow of such solutions in a recirculating system can quickly degrade the fluid beyond recognition in just a few minutes. This is easily detected as an increase in the drag and heat transfer coefficients. The fluid degradation is mirrored in changes in fluid properties, as shown in Figure 10, where the variation with flow time of the steady shear viscosity of a polyacrylamide solution (Separan AP-273) in Chicago tap water is plotted. The solution flowed inside a degradation test flow loop for nearly three days and the zero shear rate viscosity has dropped by more than one order of magnitude. Simultaneously, the elasticity of the solution measured by its relaxation time dropped by two orders of magnitude (the relaxation time,  $t_p$ , is listed inside the Figure). Indeed, elasticity is more sensible to mechanical degradation than viscous properties and it is this abiding loss of elasticity that ultimately causes the loss of drag and heat transfer reduction capability documented in Figure 11. Here, DR ( $DR = (f_N - f_p) / f_N \times 100$ ) and HTR ( $HTR = (U_{o,N} - U_{o,p}) / U_{o,N} \times 100$ ) are plotted as a function of time for two polymeric solutions. In Figure 11 a) a solution of 200 ppm of Separan AP-273 in deionized water circulated in a degradation test loop at a constant flow rate of  $9.53 \text{ kg min}^{-1}$  at  $25^\circ\text{C}$ .  $f$  and  $U_o$  denote the friction and overall heat transfer coefficients, respectively with subscripts p and N referring to polymer and Newtonian fluids at the same Reynolds number. Initially HTR is larger than DR, at 78% and 65% respectively, after about 60 hours of shear both have similar magnitudes of about 60% and after 400 hours of shear HTR stabilized at about 20%, with DR consistently higher at 25%.

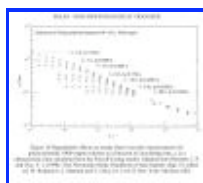


Figure 10. Degradation effects on steady shear viscosity measurements for

polyacrylamide 1000-wppm solution as a function of circulating time,  $t_p$  is a characteristic time calculated from the Powell-Eyring model. Adapted from Hartnett, J. P. and Cho, Y. I. (1998). Non-Newtonian fluids. Handbook of heat transfer chap. 10, (third ed. W. Rohsenow, J. Hartnett and Y. Cho), 10.1–10.53 New York: McGraw-Hill.

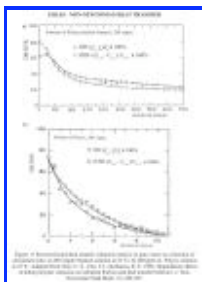


Figure 11. Percent friction/heat transfer reduction relative to pure water as a function of circulation time: a) 200 wppm Separan solution at 25 °C; b) 200 ppm wt. Polyox solution at 25 °C. Adapted from Choi, U. S., Cho, Y I. and Kasza, K. E. 1992. Degradation effects of dilute polymer solutions on turbulent friction and heat transfer behavior. *J. Non-Newtonian Fluid Mech.* 41, 289-307.

The sensitivity of the polymer solutions to degradation depends on various factors: the type of molecule (rigid and semi-rigid molecules resist more than flexible molecules), their size (smaller molecules are more resistant) and their configuration state (elongated molecules are more susceptible to scission than coiled molecules), which also depends on the quality of the solvent. It has been suggested that each polymer solution has a threshold value of wall shear stress for degradation, frequently expressed as a friction velocity,  $U^* = \sqrt{\tau_w / \rho}$ .

Figure 11b) shows the results for a degradation experiment identical to the one referred above, but this time using a different polymer, a 200 wppm polyox solution also at 25 °C and it is clear that it degrades significantly faster than the Separan solution of Figure 11 a).

A demonstration of the large impact of the solvent quality on the rheology of polymer solutions, and by implication on its degradation, is shown in Figure 12, where the shear viscosity of four aqueous solutions of 1000 wppm polyacrylamide are plotted. NaOH or NaCl were added to two of the solutions. The solution prepared with distilled water has a zero shear rate viscosity 25 times higher than the equivalent viscosity for the solution prepared with tap water. All the viscosity curves tend to collapse at large shear rates to the second Newtonian plateau, where the viscosity approaches that of the solvent. The addition of 100 wppm NaOH to tap water doubles the zero shear rate viscosity, whereas the addition of 4 percent NaCl reduces the viscosity of the polyacrylamide solution between four and twenty five times depending on the shear rate value. So, the chemistry of the solvent has a large effect on the rheology of the polymer solution and for this reason one must continuously check the fluid properties, in particular, when renewing batches of fluid using tap water, the composition of which is not under our control.

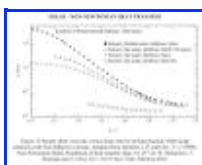


Figure 12. Steady shear viscosity versus shear rate for polyacrylamide 1000-wppm solutions with four different solvents. Adapted from Hartnett, J. P. and Cho, Y. I.

(1998). Non-Newtonian fluids. Handbook of heat transfer chap. 10, (3<sup>rd</sup> ed. W. Rohsenow, J. Hartnett and Y. Cho), 10.1–10.53 New York: McGraw-Hill.

Degradation can also be caused by thermal and chemical processes. At high temperatures the chemical bonds are weaker and molecules break down irreversibly more easily. At relatively low temperatures, thermal degradation is usually not a problem, but may turn into a relevant issue at high temperature applications such as in district heating and cooling systems. For instance, it has been referred in the literature that polyacrylamide solutions degrade in just a few hours at 90 °C, while it shows little degradation at 7 °C for several hundred hours.

Being made from organic material polymer solutions are usually an easy prey for bacteria. A sign of bacteriological degradation is the appearance of algae in suspension, which can occur in just a few hours in a settling tank (the authors have experience of finding some of their polymer solutions greenish with suspended algae just after overnight hydration in the absence of a biocide). The solution to this problem is the addition of an acid at preparation, often known as a biocide.

As was already mentioned in Section 8.2.3, surfactant solutions are more resistant to degradation than polymer solutions because their basic chemical unit is a small molecule, whereas the rheology is essentially defined by the micelles, large structures made from many individual molecules. Extensive field tests have shown that the continuous use of surfactant solutions through cycles of destruction and reforming of micelles over periods of time in excess of three years does not degrade the fluids. This requires that the operation temperature of the solution falls within the range for which the fluid and its shear-induced-structures were designed. Consequently if the surfactant solution is taken to extremely high temperatures, there is the risk that individual molecules breakdown and the fluid becomes also irreversibly degraded.

From what has just been said, it is obvious that polymer solutions are not really appropriate for applications in recirculation flow systems and at high temperatures, but are more adequate for once-through applications, where cost is not an important decision parameter, such as in sewage systems for increased throughput during floods, or during maintenance of pumping stations of oil pipelines. In contrast, surfactants are suitable for recirculating systems due to their high mechanical resistance provided their capability to form micelles exists in the correct range of temperatures for the application in view and also after the adequate safeguards are taken to protect the environment (surfactants are often by-products of some chemical industrial activity and also harmful to the environment). These safeguards require special chemical formulations, which some manufacturing companies are willing and able to do. For district heating and cooling applications, and depending on the particular size and operations capabilities of a system, the range of temperature is a window of about 50°C, somewhere between 20°C and 120°C.

Since at high shear the micelles of surfactant solutions are reversibly destroyed, this can be used to our advantage to increase the thermal efficiency of systems. In fact, heat and drag reductions due to the viscoelasticity imparted by the micellar structures are desirable characteristics when the fluids are transported over long distances, but not when they enter the heat exchangers where they should transfer as much energy as possible. This calls for the destruction of micelles at the entrance of equipments, which can be accomplished by subjecting the fluids to high shear, passing them through filters or narrow gaps. Obviously, this loss should not offset the gains during fluid transport associated with the lower drag. The use of surfactant solutions in district heating and cooling systems is not yet widespread. As far as we are aware, at the time of writing there are some examples in Japan and at least one library cooling system in Halifax, Canada.



## 12. Conclusion

This chapter on heat transfer of non-Newtonian fluids presents the governing equations required to solve a heat transfer problem, but given the extension of the topic it then focuses on pipe flow. It presents experimental, theoretical and semi-theoretical results as well as useful correlations and data to design heat transfer systems for developing and fully-developed pipe flow in the laminar and turbulent regimes of purely viscous and viscoelastic fluids. It also discusses some related problems such as the definition of non-dimensional numbers for non-Newtonian fluids, the analogy between heat and momentum for viscoelastic flows, the degradation of fluids and scaling methods.

## Appendix

[Appendix-01](#)

## Acknowledgements

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## Glossary

**Asymptotic regime** : An asymptotic regime is a limiting physical behavior and here it refers to conditions of maximum drag reduction and maximum heat transfer reduction, in which the friction factor and Nusselt number are the lowest at that specific Reynolds number, i.e., regardless of the additive concentration. Outside the asymptotic regime the friction factor and Nusselt number are higher than in the asymptotic regime and depend on the Reynolds number as well as on pipe diameter and additive concentration.

**Boger fluid** : A viscoelastic fluid having a constant shear viscosity, this is achieved by dissolving a small amount of a high molecular weight polymer in a high viscosity Newtonian solvent.

**Bulk temperature** : Refers to the well-mixed temperature of a fluid. Here, it can be the space-average temperature at a cross-section or the well-mixed temperature of the fluid inside a duct, depending on the context. The space average temperature at a cross-section of area  $A$  of a duct is given by  $\bar{T} = \int_A uT dA / (\bar{U}A)$  for a fluid with constant fluid properties, where  $\bar{U}$  is the corresponding bulk (space-average) velocity.

**Conformation** : This describes the geometric structure of a polymer referring to changes in orientation due to rotation of molecules about single

bonds. Since our macromolecules have thousands or even millions of bonds, small changes in orientation in single bonds add up to modify the geometric appearance of a large molecule due to the interaction between the macromolecule and the surrounding flow, very much like a long string of rope.

- Constitutive equation** : A relation between physical quantities specific to a material in a specific context. In this chapter we refer to rheological and thermal constitutive equations. The rheological equation relates the stress tensor with other tensors associated with material deformation, such as the rate of deformation tensor, thus defining material properties linked to fluid deformation and stresses, such as the viscosity or the relaxation time. The thermal constitutive equation relates the heat flux vector with thermal quantities such as the temperature gradient as in Fourier's law.
- Dilute polymer solutions** : Solution of polymer molecules in a solvent at a small enough concentration to avoid minimize interference between polymer molecules.
- District heating and cooling systems** : This is a system to distribute thermal energy generated at a centralized location to residential and commercial clients. Each client can heat or cool their space depending on their corresponding thermal loads, with the centralized thermal energy production allowing for higher efficiencies and better pollution control than the widespread philosophy of localized machines for heating and cooling purposes. In addition district heating and cooling systems facilitate the management of a variety of simultaneous energy transformation methods, such as centralized co-generation in a plant burning fossil fuels, geothermal heating and solar heating, amongst others.
- DNS** : Stands for Direct Numerical Simulation and it refers to the numerical simulation of the exact instantaneous governing equations for turbulent flow, i.e., resolving all the spatial and time scales of the flow no matter how small.
- GNF** : Stands for Generalized Newtonian Fluid model and it refers to a constitutive equation for fluids devoid of elasticity which contains a single material function, the shear viscosity.
- Hydrophilic** : Refers to a molecule, or part of a molecule, that can be charge polarized and easily capable of forming an hydrogen bond and consequently to dissolve more readily in water. Hydrophilic molecules are also called polar molecules and they have difficulty to dissolve in oils. So, they have a natural tendency to combine with water.
- Hydrophobic** : Refers to a molecule, or part of a molecule, which does not charge polarize. They are also called non-polar molecules and they dissolve easily with other neutrally charged molecules, such as oils, and do not combine with water, a charged molecule. Molecules which have both hydrophilic and hydrophobic parts combine with both water and oils, and consequently are used for cleaning.
- Inelastic** : A fluid devoid of any elasticity is called an inelastic fluid. It is a purely viscous fluid, but the term inelastic is used when we want to emphasize the lack of elasticity.
- Invariants of the rate of deformation** : In algebra, the invariants of a square tensor  $\mathbf{A}$  ( $n \times n$ ) are the coefficients of the polynomial characteristic of the tensor. The polynomial characteristic of a tensor is the polynomial equation

$\det(\mathbf{A} - \lambda \mathbf{I}) = 0$ , where  $\mathbf{I}$  stands for the identity tensor and  $\lambda$  are the eigenvalues of the tensor. For three-dimensional tensors, the polynomial characteristic is a cubic equation with the following coefficients (up to signs): 1 is the coefficient of  $\lambda^3$ , the first invariant of  $\mathbf{A}$  ( $I_{\mathbf{A}}$ ) is the coefficient of the  $\lambda^2$  term, the second invariant of  $\mathbf{A}$  ( $II_{\mathbf{A}}$ ) is the coefficient of the  $\lambda$  term and the third invariant ( $III_{\mathbf{A}}$ ) is the independent term. The first invariant is actually the trace of  $\mathbf{A}$  ( $I_{\mathbf{A}} = \text{tr}\mathbf{A}$ ), the second invariant is  $II_{\mathbf{A}} = \left[ (\text{tr}\mathbf{A})^2 - \text{tr}\mathbf{A}^2 \right] / 2$  and the third invariant is  $III_{\mathbf{A}} = \det \mathbf{A}$ . The

important property of the invariants of a tensor is that they do not change with rotation of the coordinate system (we say they are objective quantities), so any scalar valued function of the invariants of a tensor, like the viscosity of a GNF fluid, is itself objective.

- Mechanical degradation** : Degradation of a fluid by mechanical scission of its molecules.
- Micellar structures** : These are aggregates of surfactant molecules where their hydrophobic parts are inside the micelle and the hydrophilic heads are outside, in contact with the surrounding solvent, usually water. Micelles have different forms depending on temperature, surfactant concentration, pH and ionic strength.
- Non-equilibrium processes** : In a physical process there is usually interaction of the system with its surroundings, but very often this interaction can be neglected or takes place so quickly that for some purposes we may consider it as a succession of equilibrium states, thus simplifying enormously the modeling of the process. In some other instances we cannot neglect the interaction in which case we are faced with a non-equilibrium process, the treatment of which is far more complex, but will provide answers which the simplification of equilibrium process can not provide. An example of a non-equilibrium process is dissipation of energy even under steady state conditions.
- Non-Newtonian fluids** : Fluids that do not obey the rheological constitutive equation for a Newtonian fluid also known as Newton's law of viscosity.
- Purely viscous fluid** : Fluids devoid of elasticity, i.e., fluids that respond instantaneously to a change in the rate of deformation, where the constant of proportionality only depends on invariants of the rate of deformation tensor.
- Relaxation time** : The response time of a viscoelastic fluid when there is a sudden change in imposed stress.
- Rheology** : The science of deformation of matter and its relation to the stress developed by the material.
- Secondary flows** : When a flow is due to forcing in a specific direction the main flow may induce gradients of stresses which lead also to flow in a secondary direction. A typical example is turbulent flow in a square duct induced by a pressure gradient along the axis. The so-called turbulent normal stresses due to the main flow have gradients in the cross-stream directions which create a weaker secondary flow in the cross-section. The same happens in laminar flow of viscoelastic fluids in a square duct, where the elastic normal stresses due to shear have gradients leading to a secondary flow in the plane

normal to the main flow.

**Storage of mechanical energy** : As explained under “viscoelastic fluids”, these fluids are able to store as elastic energy some of the energy supplied to create a flow and return this energy as flow when the applied stresses are removed or changed. This energy is stored in the molecules, which assume certain equilibrium configurations leading to internal stresses.

**Surfactants** : Chemicals that lower the interface tension between two liquids, facilitating their mixing and lower the surface tension of a liquid, thus allowing better wetting of a surface by the liquid. Surfactants are usually made of molecules having simultaneously hydrophilic and hydrophobic tails, thus forming micelles.

**Viscoelastic fluids** : Fluids that in addition to their viscous response, typical of a purely viscous fluid, are capable of storing some of the energy as elastic energy to be given back. Upon cessation of the stress forcing, the fluid is then capable of some reverse deformation using the stored elastic energy. Fluid viscoelasticity is also seen in various other ways, such as a finite time in response to changes in stress, which is the relaxation time of the fluid, the appearance of normal stresses in pure shear flow or very large normal stresses in extensional flows.

## Nomenclature

$A$	: Duct cross-section area
$a$	: Parameter related to $\bar{U}_N/\bar{U}$ , $a = 3/4 \left[ (\bar{U}_N/\bar{U})^{-1} - 1 \right]$
$c$	: Specific heat of the fluid
$D$	: Pipe diameter
$D$	: Rate of deformation tensor
$f$	: Fanning friction factor, $2\bar{\tau}_w / (\rho\bar{U}^2)$
$f_D$	: Darcy friction factor, $8\bar{\tau}_w / (\rho\bar{U}^2)$
$g$	: Gravitational acceleration vector
$g_i$	: Gravitational acceleration, $i$ direction
$h$	: Convection heat transfer coefficient
$I_D$	: First invariant of tensor $D$
$II_D$	: Second invariant of tensor $D$
$III_D$	: Third invariant of tensor $D$
$K$	: Consistency index of power law model
$k$	: Fluid thermal conductivity
$L^2$	: Extensibility parameter
$L_h$	: Hydrodynamic entrance length
$L_t$	: Thermal entrance length
$\dot{m}$	: Mass flow rate
$N$	: Number of modes in constitutive equation
$n$	: Power index of power law model

$P$	: Duct wetted perimeter
$p$	: Pressure
$q$	: Heat flux vector in tensor notation
$q_l$	: Heat flux vector in Einstein notation
$\dot{q}_w$	: Constant wall heat flux, positive when heating the fluid
$R$	: Pipe radius
$r$	: Radial coordinate in cylindrical coordinates
$T$	: Temperature
$t$	: Time
$T_0$	: Inlet bulk temperature
$T_b$	: Bulk temperature of the fluid
$t_p$	: Characteristic time of a viscoelastic fluid calculated using the Powel-Eyring model.
$T_w$	: Wall temperature
$T_\infty$	: Temperature at free-stream conditions
$U$	: Heat transfer coefficient
$\bar{U}_N$	: Bulk velocity for a Newtonian fluid flowing under the same pressure gradient $\bar{U}_N \equiv \left[ -(dP/dx) R^2 \right] / (8\eta_p)$
$\bar{U}$	: Flow bulk velocity
$u$	: Velocity component along the duct axis
$\mathbf{u}$	: Velocity vector
$u_i$	: Velocity vector component, $i$ direction
$v$	: Velocity component along the $r$ coordinate
$x$	: Axial distance along the duct
$x'$	: Normalized axial coordinate, $x' = x / (D Re Pr) = \pi Gz^{-1} / 4$
$x_i$	: Position vector component, $i$ direction

### Greek symbols

$\alpha$	: Thermal diffusivity of the fluid or dimensionless mobility factor of the Giesekus equation
$\beta$	: Volume coefficient of expansion in the Grashof number or ratio between the solvent viscosity and the zero shear rate viscosity of a polymer solution, $\beta = \eta_s / (\eta_s + \eta_p)$
$\varepsilon$	: PTT model parameter
$\dot{\gamma}$	: Shear rate
$\eta$	: Shear viscosity of the fluid
$\eta_p$	: Polymer viscosity
$\eta_s$	: Solvent viscosity
$\eta_{s,v}$	: Bulk viscosity of the solvent
$\lambda$	: Relaxation time
$\nu$	: Kinematic viscosity

$\theta$	: Circumferential coordinate in cylindrical coordinates
$\rho$	: Fluid density
$\tau$	: Viscous stress tensor
$\tau_{ij}$	: Viscous stress tensor component, $i$ plane $j$ direction
$\tau_w$	: Wall shear stress
$\tau_P$	: Polymer/addictive stress tensor contribution
$\square$	: Gordon- Schowalter derivative of the stress tensor
$\tau_{ij}$	
$\xi$	: Parameter that accounts for the slip between the molecular network and the continuum medium in PTT model

**Overlines**

$-$	: Averaged quantity
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**Superscripts**

$\mathbf{p}$	: Total polymer extra stress
$\mathbf{T}$	: Total extra stress or transpose of a tensor
'	: Viscosity calculated from $\eta = K \left[ \frac{(3n+1)}{4n} \right]^n \left( \frac{8\bar{U}}{D} \right)^{n-1}$ , Eq. (13)
+	: Viscosity calculated from $\eta = K \left( \frac{\bar{U}}{D} \right)^{n-1}$ , Eq. (16)

**Subscripts**

$\mathbf{a}$	: Viscosity calculated from $\eta = K \left[ \left( \frac{8\bar{U}}{D} \right) \frac{(3n+1)}{4n} \right]^{n-1}$ , Eq. (18)
$\mathbf{s}$	: Viscosity calculated from $\eta = \eta_{\text{solvent}}$ , Eq. (19)
$\mathbf{w}$	: Wall
$\mathbf{x}$	: Local value
$\mathbf{FD}$	: Fully developed value

**Named dimensionless groups**

$Br$	: Classical Brinkman number, Eqs. 38 and 39
$Br^*$	: Generalized Brinkman number, Eqs 41 and 42
$Gr$	: Grashof number, $g\beta(T_w - T_\infty)D^3/\nu^2$
$Gz$	: Graetz number, $\dot{m}c/kx = (RePr D/x) \cdot \pi/4$
$Gz_t$	: Graetz number at the thermal entrance length, $\dot{m}c/kL_t = (RePr D/L_t) \cdot \pi/4$
$jH$	: Chilton-Colburn factor, $StPr^{2/3}$
$Nu$	: Nusselt number, $hD/k$
$Pe$	: Péclet number, $\bar{U}D/\alpha$
$Pr$	: Prandtl number, $\nu/\alpha$
$Re$	: Reynolds number, $\rho\bar{U}D/\eta$

$St$  : Stanton number,  $Nu/(RePr)$

$Wi$  : Weissenberg number,  $\lambda\bar{U}/R$

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## Biographical Sketches



**Fernando Manuel Coutinho Tavares de Pinho**, Researcher at CEFT/FEUP and Associate Professor at Mechanical Engineering Department, Faculdade de Engenharia da Universidade do Porto, Portugal. Graduated in Mechanical Engineering at Faculdade de Engenharia da Universidade do Porto (FEUP) in 1984, obtained an MSc in Thermal Engineering from the same University in 1987 and did a PhD at Imperial College, London in 1990. In 1990-2000 and 2000-04 he was Assistant and Associate Professor at FEUP, respectively. In 2004 he was awarded a higher doctorate (Agregação) by Universidade de Coimbra and moved to the Mechanical Engineering Department at Universidade do Minho. He returned to FEUP in 2008 where he is currently an Associate Professor. In 1991-92 and 1993-94 he was also technical director of the Thermal Engineering Unit of INEGI, a successful private institution carrying out R&TD with industry. His research interests are in experimental, numerical and theoretical fluid mechanics of complex fluids, including microfluidics, fluid rheology, turbulence modeling and heat



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### 8.1. Fully-developed Pipe Flow of Purely Viscous Fluids

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