Energy spectra in forced homogenous isotropic turbulence of FENE-P solutions

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Abstract — We present results of Direct Numerical Simulations of statistically stationary (forced) homogeneous isotropic turbulence at the highest Reynolds number in existence ($Re_\lambda = 400$) for viscoelastic fluids described by the Finitely Extensible Nonlinear Elastic constitutive equation, closed with the Peterlin approximation (FENE-P), at various Deborah numbers. The DNS data show that at Deborah numbers in the range 0.2 to 0.6, the energy spectra have simultaneously the characteristic -5/3 slope of the inertial subrange at low wave numbers, followed a region of -3 slope at higher wave numbers, in the so-called inertio-elastic subrange that starts at the Lumley scale, and below the high wavenumber dissipative scales. Dimensional arguments that justify these flow regimes are also presented.

1. Introduction

Experiments by Vonlanthen *et al.* [1] of homogeneous isotropic turbulence of viscoelastic polymer solutions have shown the energy spectra to change from the classical -5/3 slope, characteristic of the inertial subrange, to a slope of -3 when elastic effects become significant. These experiments concerned decaying turbulence, and the long chain polymer molecules suffered from irreversible mechanical degradation, therefore the reduction of turbulence with time was accompanied by a reduction of fluid elasticity that led to a progressive decrease of the negative slope back to -5/3. Simultaneously, both the initial and the evolving Reynolds numbers of these experiments were rather low.

The observation of the -3 slope in the energy spectra was also reported to take place by Valente *et al.* [2] in their Direct Numerical Simulations (DNS) of forced homogeneous isotropic turbulence of viscoelastic fluids that were described by the Finitely Extensible Nonlinear Elastic rheological constitutive equation closed with the Peterlin approximation (the so-called FENE-P model). This is a coarse-grained model of polymer solutions derived from polymer kinetic theory in which ensemble of polymer molecules are represented by dumbbells with a nonlinear spring [3]. It captures the essential features of viscoelastic behaviour of polymer solutions, even though the model without the Peterlin approximation has a more faithful description, but brings a significant computational cost overhead.

However, the DNS of Valente *et al.* [2] were characterised by low to moderate Reynolds number, higher than in the experiments of Vonlanthen *et al.* [1], and also relied on the use of hyperviscosity to extend the dynamic range of the spectra. Therefore, some of the features of the effect of elasticity upon the energy spectra, that require simulations at high Reynolds number, were not so clearly observed, in particular the coexistence of the two subranges at adequate Deborah number flows. This is exactly the situation in this work where we report DNS of statistically stationary (forced) homogeneous isotropic turbulence at highest Reynolds
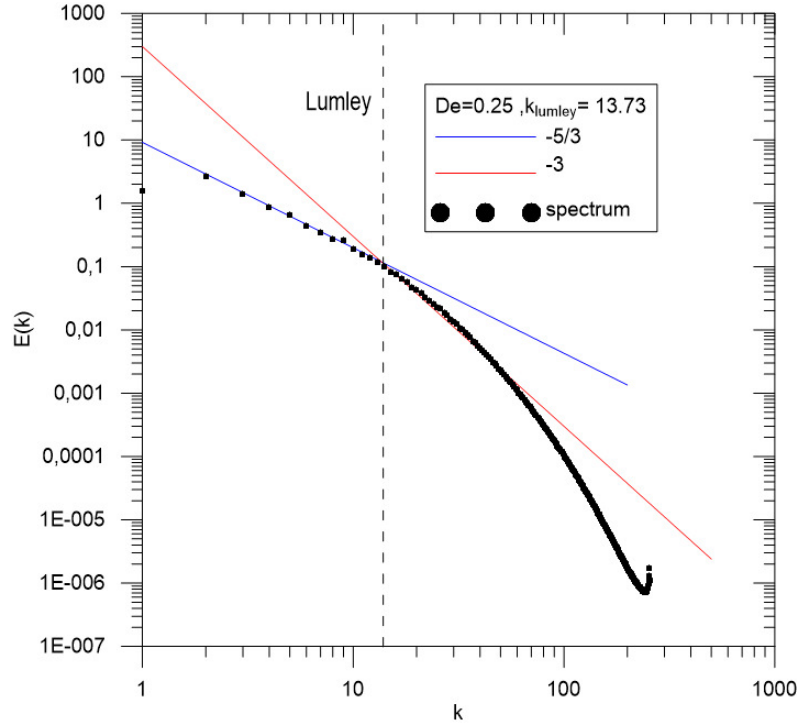
Figure 1: Energy spectrum for $De = 0.25$ and $Re_\lambda = 400$ and fitted energy spectra with -5/3 and -3 slopes. The Lumley wave number is $k_{Lumley} = 13.7m^{-1}$.

number in existence for viscoelastic fluids ($Re_\lambda = 400$) in order to show that in a certain range of Deborah numbers, the energy spectra have simultaneously the characteristic -5/3 slope at low wave numbers (k), typical of the inertial subrange, followed by the characteristic -3 slope at higher wave numbers, in the inertio-elastic subrange that starts at the Lumley scale, and below the high wave number dissipative scales.

2. Results

A total of 12 simulations were carried out, including 2 Newtonian, in two different meshes. Figure 1 presents the energy spectrum $E(k)$ corresponding to a simulation at $Re_\lambda = 400$, where λ is the Taylor micro-scale, and $De = 0.25$ of a FENE-P fluid with $L^2 = 100^2$, ratio of viscosities $\beta = 0.8$ and solvent kinematic viscosity $\nu^{[s]} = 0.003 m^2/s$. The Reynolds number based on the Taylor micro-scale is defined as

$$Re_\lambda = \sqrt{\frac{2K}{3}} \frac{\lambda}{\nu^{[s]}}. \tag{1}$$

and the Deborah number

$$De = \frac{\tau_p \sqrt{K}}{\ell}, \tag{2}$$

involves the fluid relaxation time (τ_p), the turbulent kinetic energy (K) and the integral length scale (l), which is defined as

$$\ell = \left(\frac{\pi}{2K}\right) \sum_{k_{min}}^{k_{max}} \frac{E(k)}{k}, \tag{3}$$

The Taylor micro-scale is defined on the basis of solvent quantities, i.e., as

$$\lambda = \sqrt{\frac{10\nu^{[s]}K}{\varepsilon^{[s]}}} \tag{4}$$

where $\varepsilon^{[s]}$ is the solvent dissipation.

The computational box for this case has $N^3 = 768^3$ collocation points with the power input $P = 10m^2s^{-3}$ concentrated in the first two wave numbers. The governing equations and numerical method are described in Ferreira *et al.* [4]. Briefly, classical pseudo-spectral methods are used for the spatial discretisation of the momentum equation, a 3-stage, 3rd-order Runge-Kutta scheme is used for time advancement and de-aliasing with the 2/3 rule is used. The differential conformation equation has discontinuities, precluding the use of spectral methods, and instead is solved by a central difference algorithm proposed by Vaithianathan *et al.* [5] to ensure positive-definiteness and symmetry of the conformation tensor.

It is clear that the energy spectrum has both a -5/3 and a -3 slope, and the lines are not just fits but have been calculated for the correct power input of the simulation and assuming, through dimensional arguments, that the energy spectrum is of the form $E(k) = C_k \epsilon^{2/3} k^{-5/3}$ and $E(k) = C_{ke} \tau_p^{-2} k^{-3}$ in the inertial and inertio-elastic subranges, respectively. In order for the lines to fit the computed energy spectra a coefficient had to be used, which agrees with the Kolmogorov constant in the inertial subrange and is a single valued linear function of the Deborah number in the inertio-elastic subrange. The two power laws intersect at the Lumley wave number, corresponding to a condition in which the turbulent time scale and the polymer relaxation time are similar.

3. Acknowledgements

References

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