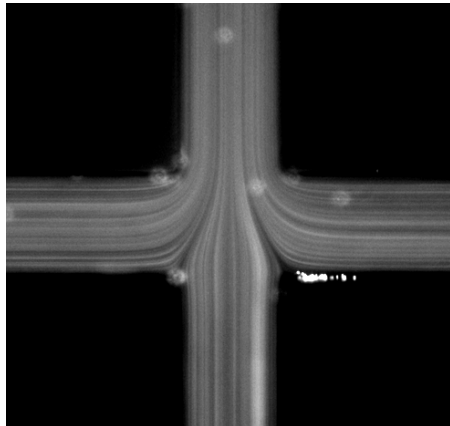


# Elastic-driven instabilities in microfluidic flow-focusing devices

**M. S. N. Oliveira<sup>1</sup>, F. T. Pinho<sup>2</sup>, M. A. Alves<sup>1</sup>**



1: Departamento de Engenharia Química, CEFT  
**Faculdade de Engenharia da Universidade do Porto**  
Rua Dr. Roberto Frias, 4200-465 Porto, Portugal  
e-mail: {mmalves, monica.oliveira}@fe.up.pt  
web: <http://www.fe.up.pt/~ceft>

2: Departamento de Engenharia Mecânica, CEFT  
**Faculdade de Engenharia da Universidade do Porto**  
Rua Dr. Roberto Frias, 4200-465 Porto, Portugal  
e-mail: [fpinho@fe.up.pt](mailto:fpinho@fe.up.pt)  
web: <http://www.fe.up.pt/~ceft>



Universidade do Porto  
Faculdade de Engenharia

**FEUP**

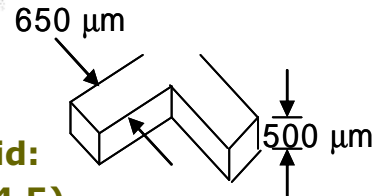
# Instabilities at the microscale: Cross-Slot

## Experimental

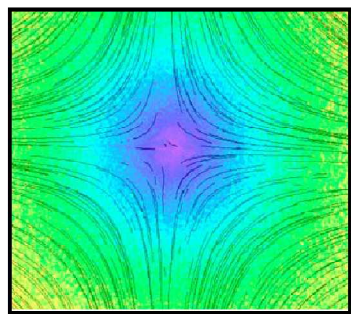
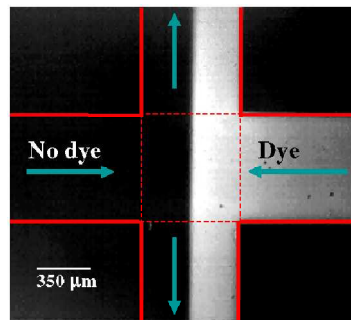
PRL 96, 144502 (2006) PHYSICAL REVIEW LETTERS

### Elastic Instabilities of Polymer Solutions in Cross-Channel Flow

P. E. Arratia,<sup>1,2</sup> C. C. Thomas,<sup>1</sup> J. Diorio,<sup>1</sup> and J. P. Gollub<sup>1,2</sup>

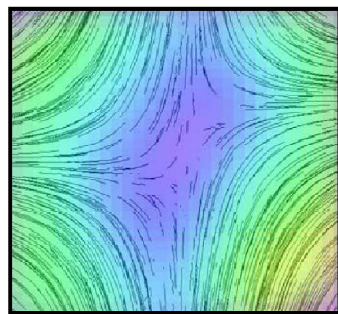
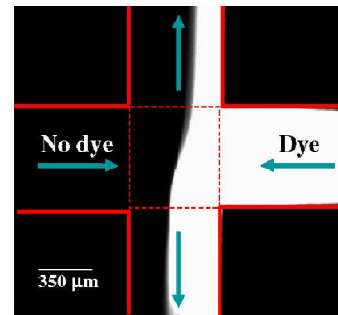


**Newtonian:**  
 $Re < 10^{-2}$



0 μm/s 1500 μm/s

**PAA Boger fluid:**  
 $Re < 10^{-2}$  ( $De=4.5$ )



0 μm/s 600 μm/s

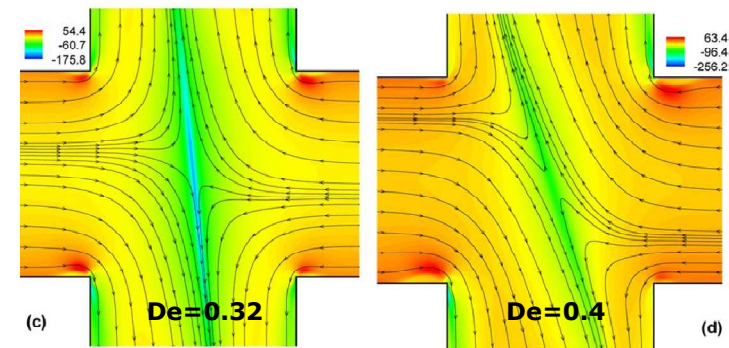
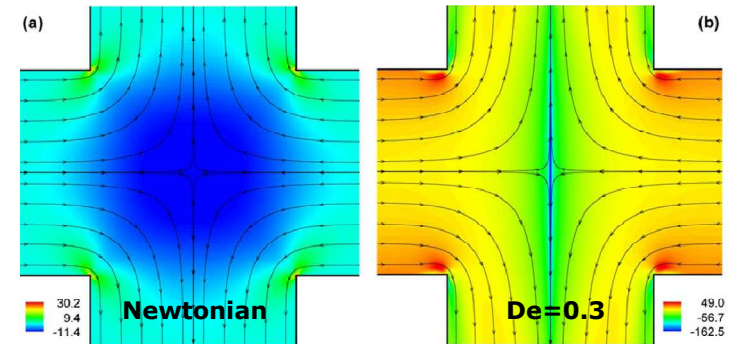
## Numerical

PRL 99, 164503 (2007) PHYSICAL REVIEW LETTERS

### Purely Elastic Flow Asymmetries

R. J. Poole,<sup>1</sup> M. A. Alves,<sup>2</sup> and P. J. Oliveira<sup>3</sup>

**UCM model**  
**2D, Creeping Flow**



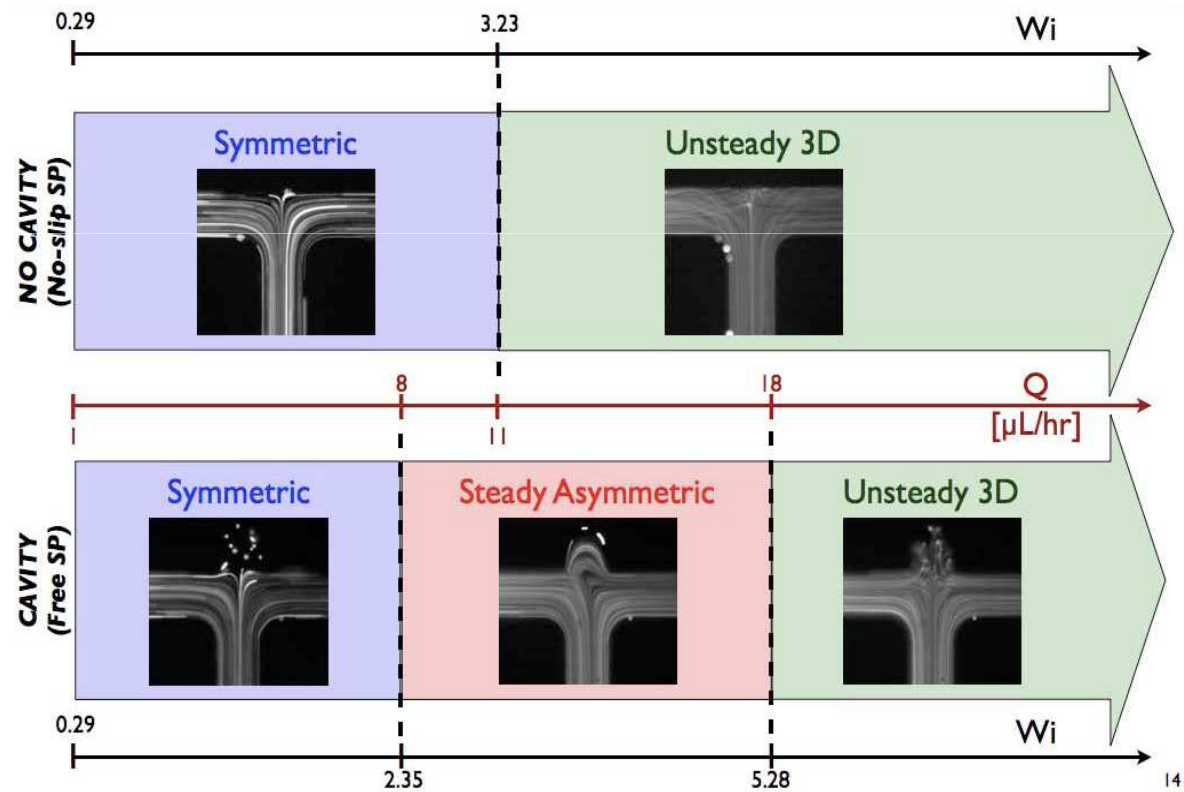
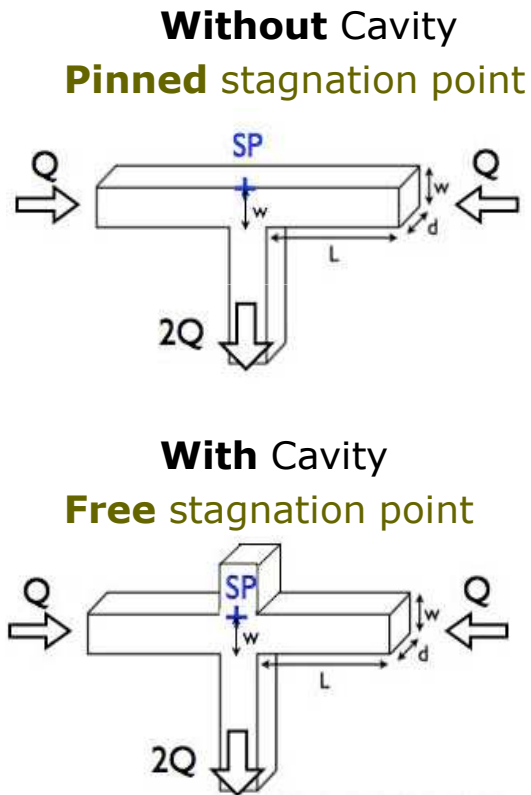
# Instabilities at the microscale: T-Channel

J. Non-Newtonian Fluid Mech. 163 (2009) 9–24

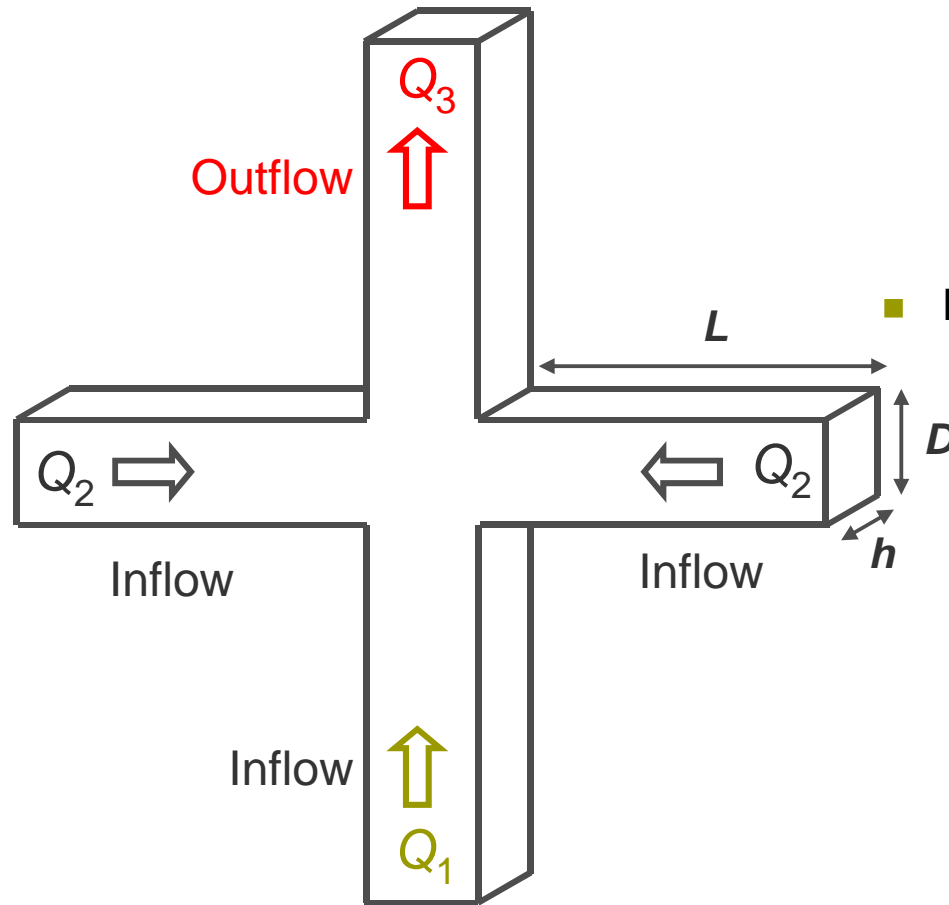
Investigating the stability of viscoelastic stagnation flows in T-shaped microchannels

J. Soulagès<sup>a,\*</sup>, M.S.N. Oliveira<sup>b</sup>, P.C. Sousa<sup>b</sup>, M.A. Alves<sup>b</sup>, G.H. McKinley<sup>a,\*</sup>

0.075 wt. % PEO in glycerol/water solution (60/40 wt. %)



# Flow focusing geometry



Cross-slot geometry  
with **3 inlets** and **1 outlet**

## Operational Variables

$$Q_1, Q_2 \downarrow$$

$$Q_3 = 2 \times Q_2 + Q_1$$

## Dimensionless Variables

$$FR = Q_2 / Q_1$$

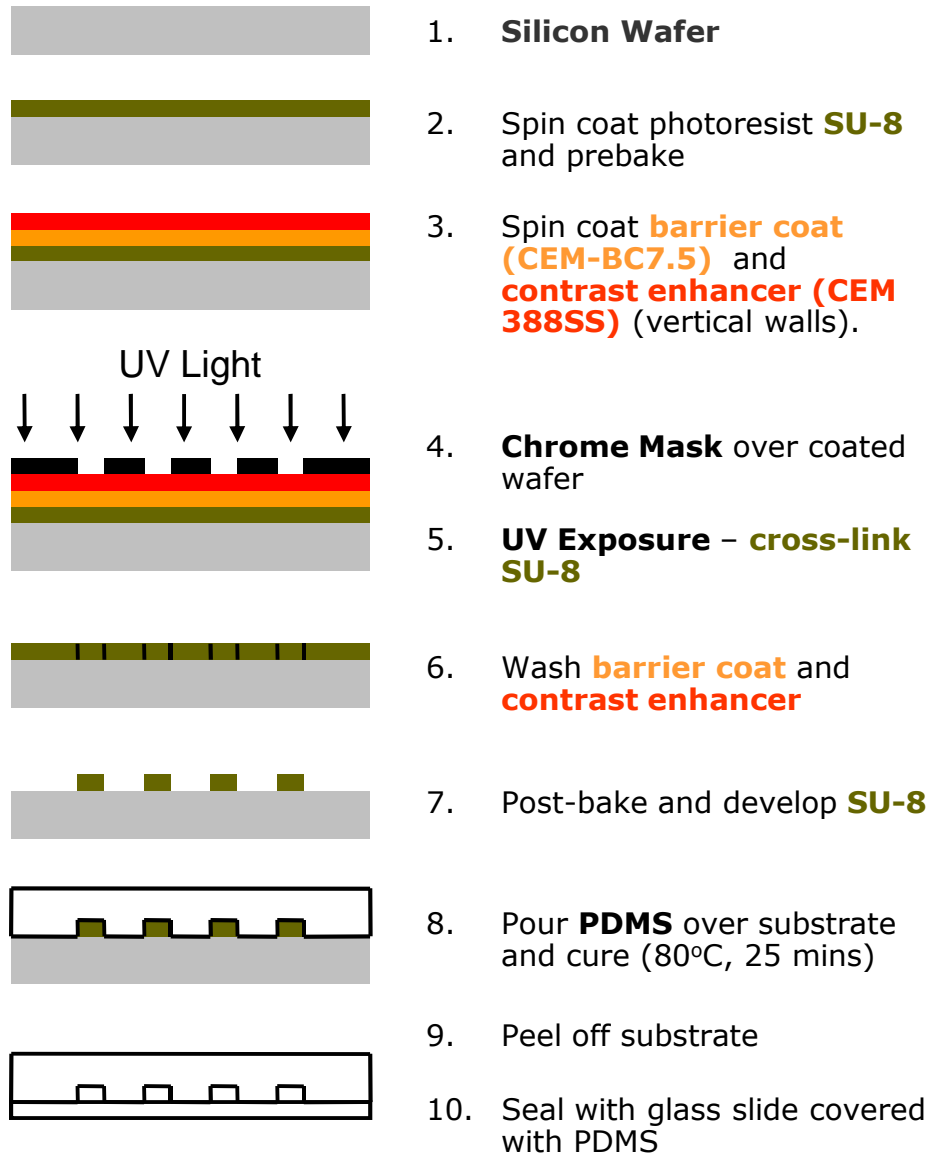
$$VR = U_2 / U_1 (= FR)$$

$$\left. \begin{aligned} Re &= \frac{\rho U_2 D}{\eta_0} \\ De &= \lambda \frac{U_2}{D} \end{aligned} \right\} El = \frac{De}{Re}$$

## Channel dimensions:

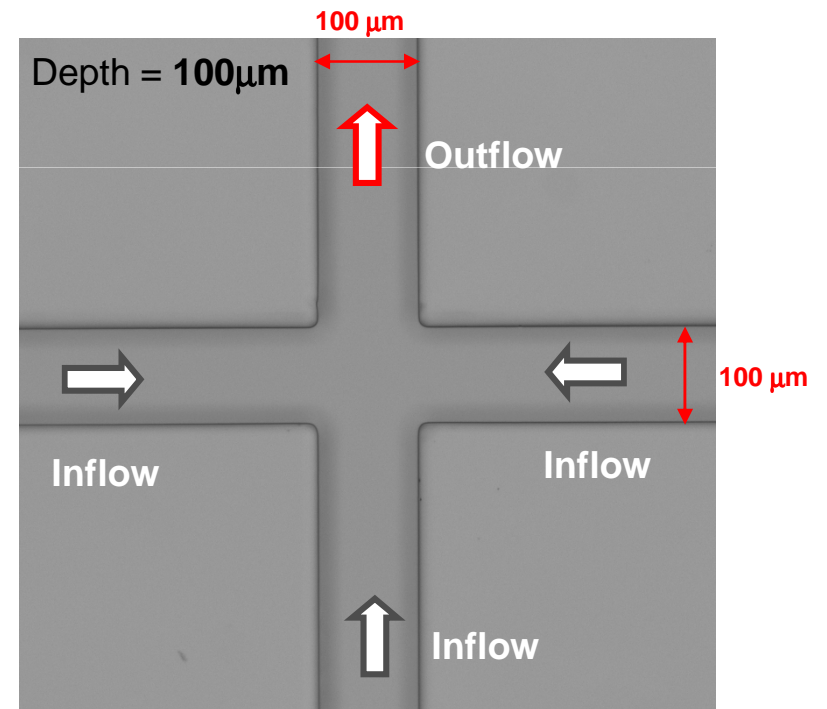
- kept constant for all experiments and numerical calculations
- Equal dimensions for all inlet/outlet arms

# Fabrication process: soft lithography

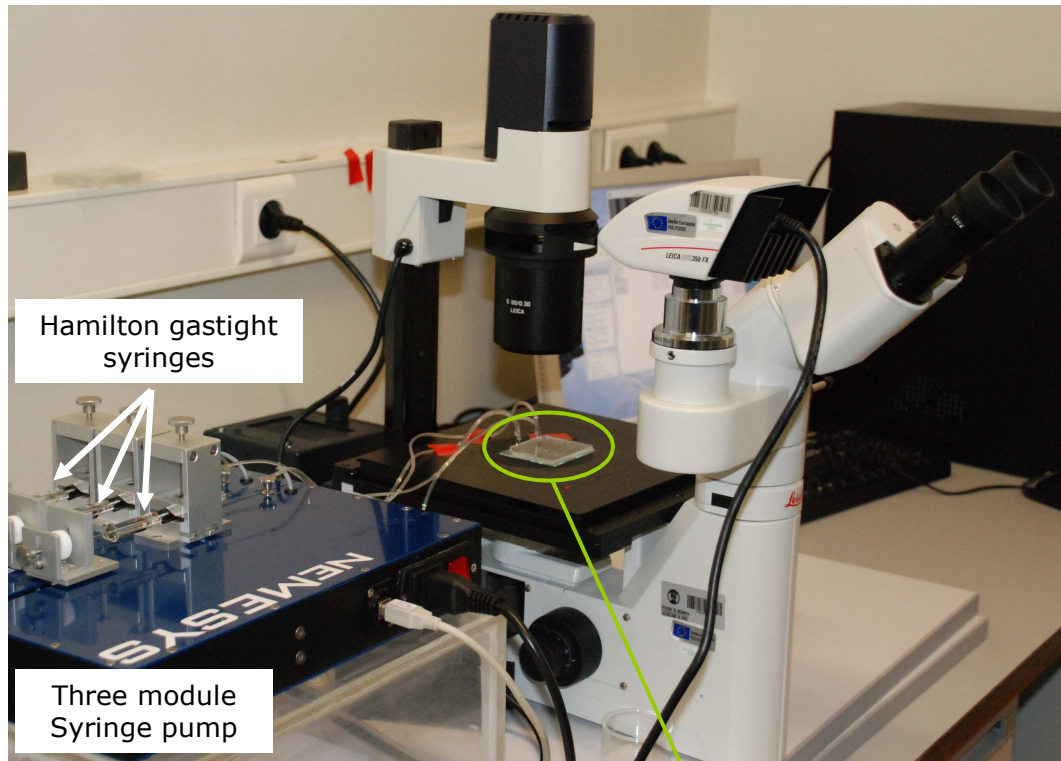


## □ **Microchannels:**

- Planar geometry
- Square cross-section
- Nearly vertical walls
- Accuracy of channels to within 5%



# Experimental set-up



## Visualizations

### **Microscope:**

Leica, DMI LED

### **Camera:**

Leica, DFC 350X

### **Objective lens:**

10× objective  
NA=0.3 ( $\delta z=30\mu\text{m}$ )

### **Illumination:**

Mercury lamp ( $\lambda=532\text{nm}$ )

### **Filter Cube:**

Emission filter: BP 530-545 nm  
Dichroic mirror: 565 nm  
Barrier filter: 610-675 nm

### **Tracers:**

1.1 $\mu\text{m}$  Nile red fluorescent particles (Ex/Em: 520/580 nm)

### **Additives:**

SDS (0.1 wt.%)

## Flow

### **Syringe pump:**

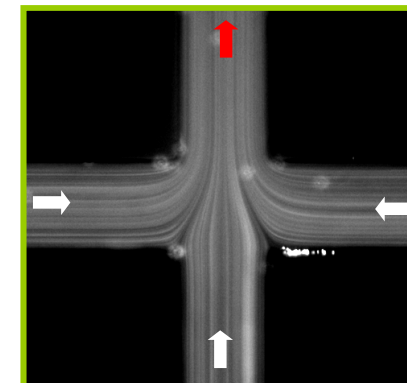
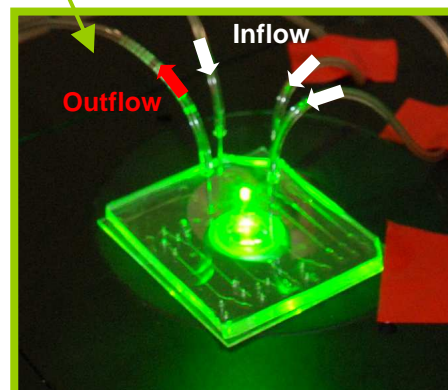
CETONI, neMESYS  
with 3 independent modules

### **Flow rate:**

$0.001 \leq Q \leq 3 \text{ ml/h}$

### **Syringes:**

Hamilton Gastight syringes



# Shear Rheology

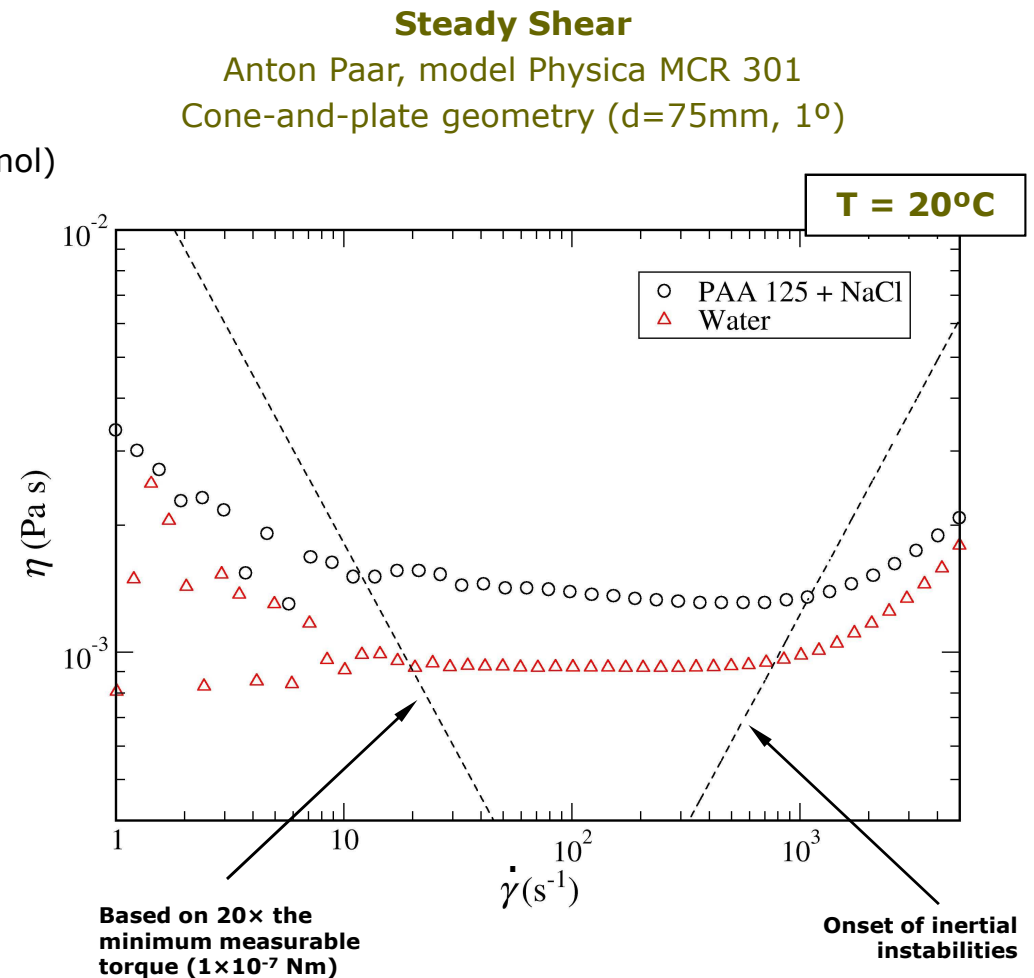
- Viscoelastic Fluid:

Aqueous solution of **PAA** ( $M_w = 18 \times 10^6$  g/mol)

- **PAA 125 + NaCl**  
125 ppm PAA  
+ 1% NaCl (wt)  
+ 0.1% SDS (wt)

- Newtonian Fluid:

- **Water**  
Water  
+ 0.1% SDS (wt)



# Extensional Rheology

## Extensional Rheology

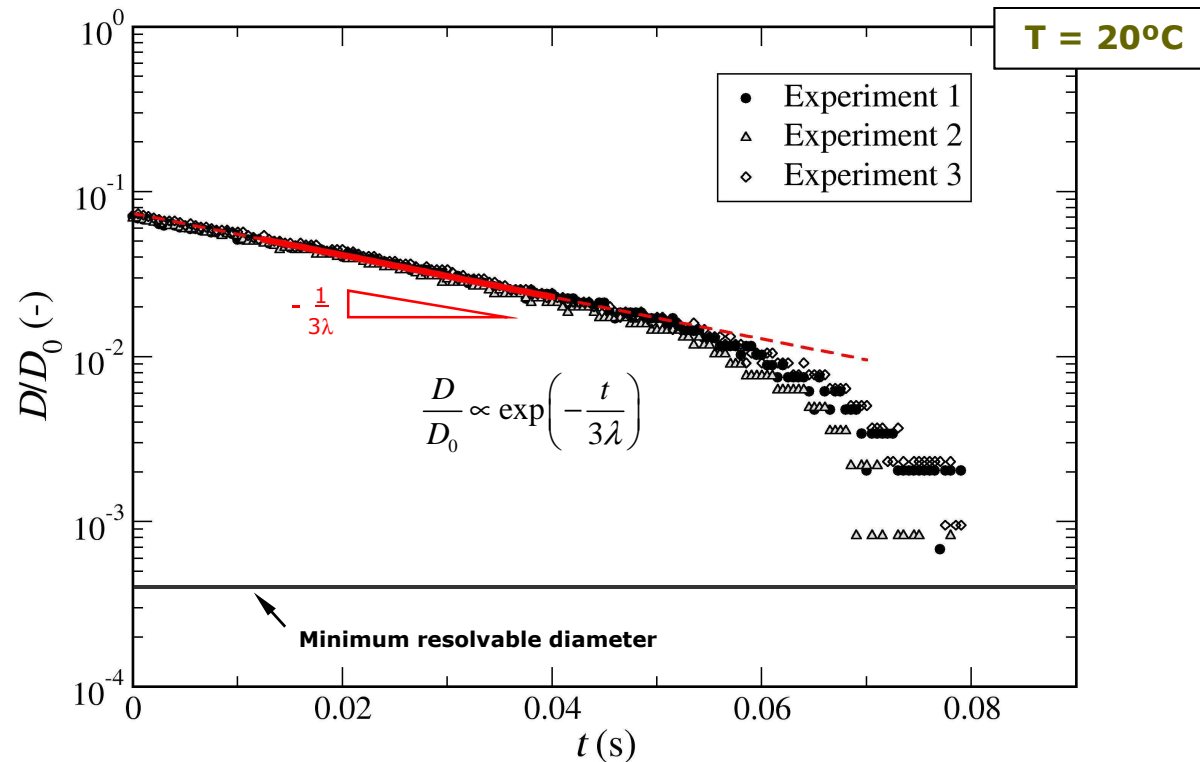
Haake CaBER 1, Thermo Electron Corporation  
( $D_p = 6$  mm)

**Aspect ratios:**  $\Lambda_i = D_i/D_p = 0.33$   
 $\Lambda_f = D_f/D_p = 1.56$

**Imposed strain:**  $\varepsilon = \ln(\Lambda_f/\Lambda_i) = 1.53$

## Fluid properties

PAA 125 + NaCl		
Zero-shear rate viscosity	$\eta_0$ [Pa s]	0.00131
CaBER relaxation time	$\lambda_{\text{CaBER}}$ [ms]	12.4 $\pm$ 0.2
Density	$\rho$ [kg/m <sup>3</sup> ]	1005





# Governing equations

## □ Isothermal Incompressible Flow

$$\nabla \cdot \mathbf{u} = 0$$

Conservation of mass

$$-\nabla p + \nabla \cdot \boldsymbol{\tau} + \eta_s \nabla^2 \mathbf{u} = \mathbf{0}$$

Conservation of momentum

## □ Constitutive Equations

$$\left[ 1 + \frac{\lambda \varepsilon}{\eta_p} \text{tr}(\boldsymbol{\tau}) \right] \boldsymbol{\tau} + \lambda \overset{\nabla}{\boldsymbol{\tau}} = 2\eta_p \mathbf{D}$$

Simplified PTT model

$$\boldsymbol{\tau} + \lambda \overset{\nabla}{\boldsymbol{\tau}} = 2\eta_p \mathbf{D}$$

$\varepsilon = \mathbf{0}$

Oldroyd-B model

$$\boldsymbol{\tau} + \lambda \overset{\nabla}{\boldsymbol{\tau}} = 2\eta \mathbf{D}$$

$\eta_s = \mathbf{0}$

**UCM model**

$$\boldsymbol{\tau} = 2\mu \mathbf{D}$$

$\lambda = \mathbf{0}$

Newtonian fluid

# Numerical method

---

- **Finite Volume Method**, using a time-marching algorithm\*
  - Governing equations discretized in time over a small time step,  $\delta t$
  - Differential equations integrated over control volumes, CV

\*Oliveira et al., JNNFM,79 (1998) 1-43
  
- **Log-conformation approach**\*\* to solve the equivalent form of the constitutive equation containing an evolution equation of the conformation tensor.

\*\*Fattal and Kupferman, JNNFM, 123 (2004) 281-285  
Afonso et al., JNNFM, *submitted* (2008)
  
- Implicit first-order Euler scheme for time-derivative discretization.
  
- Central differences for discretization of diffusive terms.
  
- **CUBISTA**\*\*\* high-resolution scheme for discretization of the advective terms of the momentum equations.

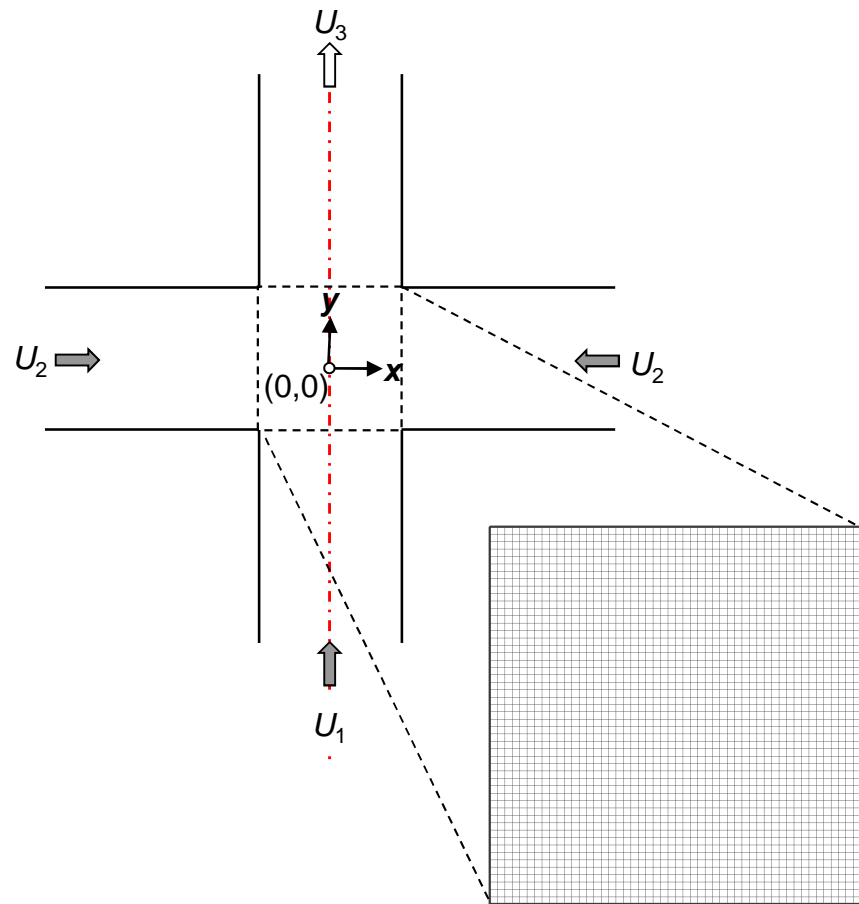
\*\*\*Alves et al., Int. J. Num. Methods in Fluids, 122 (2003) 47-75
  
- Pressure-velocity and velocity-stress coupling ensured at the CV faces – SIMPLEC algorithm\*\*\*\*

\*\*\*\*Patankar and Spalding, Int. J. Heat and Mass Transfer, 15 (1972) 1787-806
  
- The ensemble of all control volumes defines the computational mesh.

# Numerical mesh and boundary conditions

## Mesh Characteristics:

- Two-dimensional
- Structured, orthogonal and non-uniform
- Dimensions equal to the experiments



## Boundary Conditions:

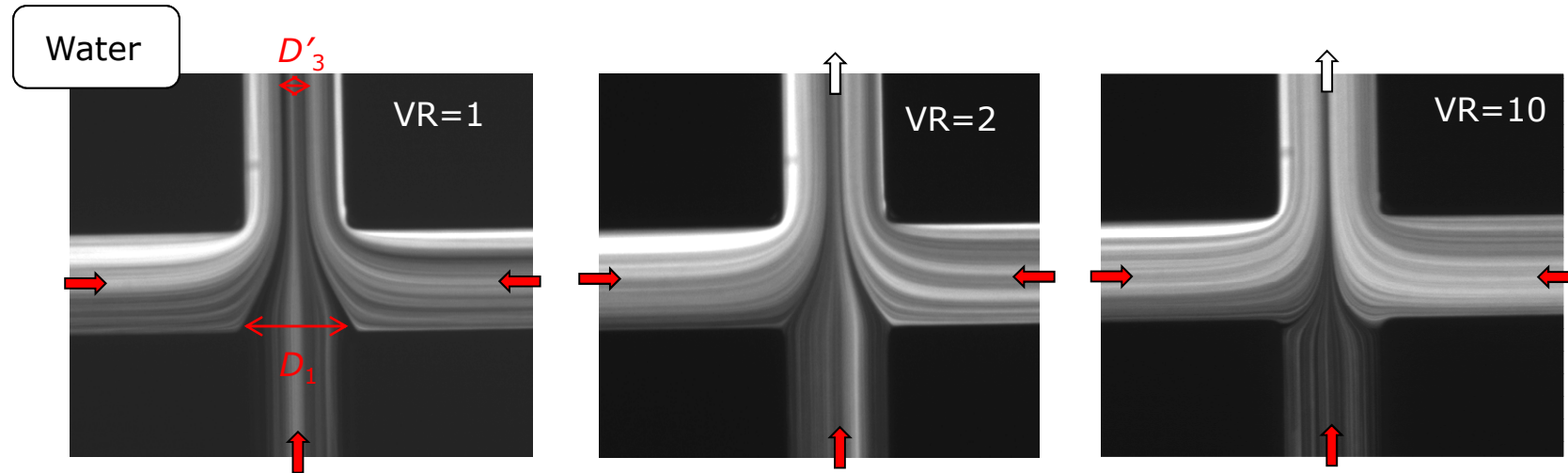
- INLET
  - Fully-developed velocity profile
  - Null stress components
- OUTLET
  - Vanishing streamwise gradients of velocity and stress components
  - Constant pressure gradient
- WALLS
  - No-slip conditions

### Standard Mesh used

(more refined meshes also used)

Geometry	NCells	$\Delta x_{\min}, \Delta y_{\min}$ $\Delta z_{\min}$
<b>2D</b>	23001	0.02D

# Converging Flow

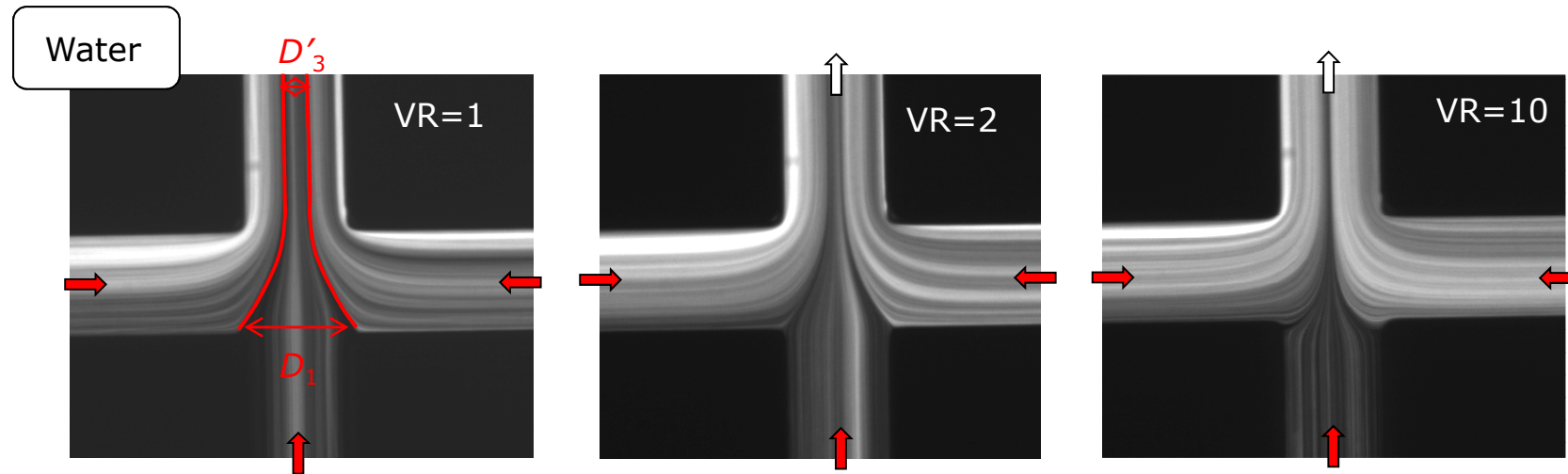


- Two opposing lateral streams shape a third inlet stream
- Converging flow region
- Separation streamlines naturally evolve to a nearly hyperbolic shape
- Hencky strain controlled by operating parameters:

$$\varepsilon_H = \ln\left(\frac{D_1}{D_3}\right) = \ln\left[\frac{3}{2}(1 + 2 \cdot VR)\right]$$

- Approximately constant strain rate at the centerline

# Converging Flow



- Two opposing lateral streams shape a third inlet stream
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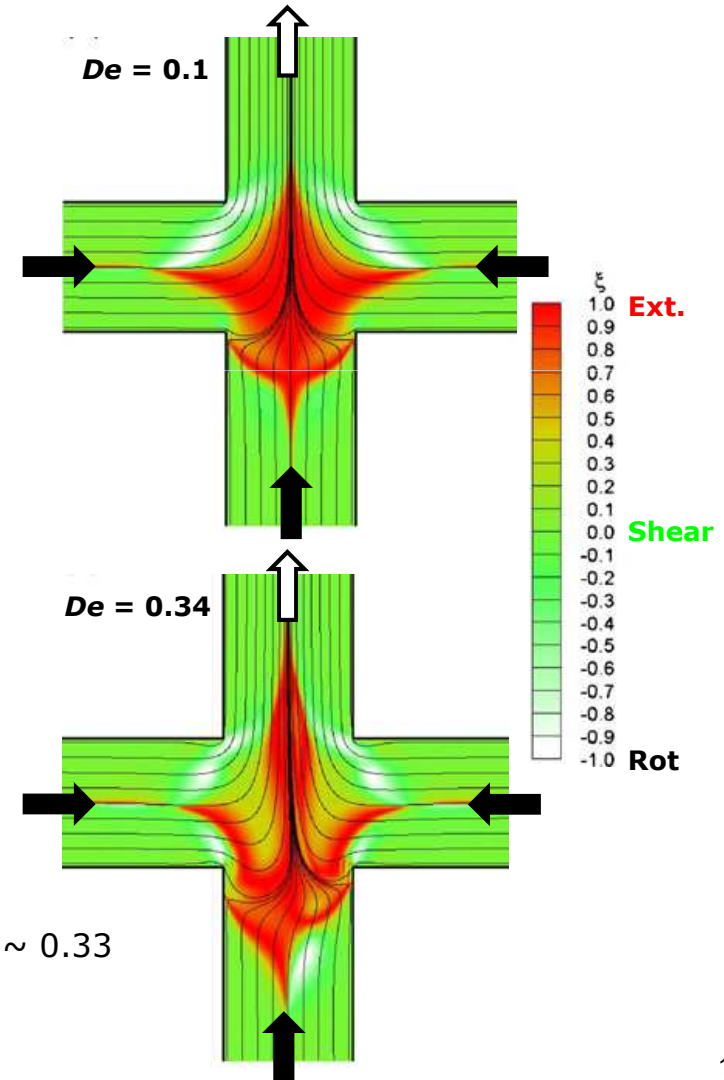
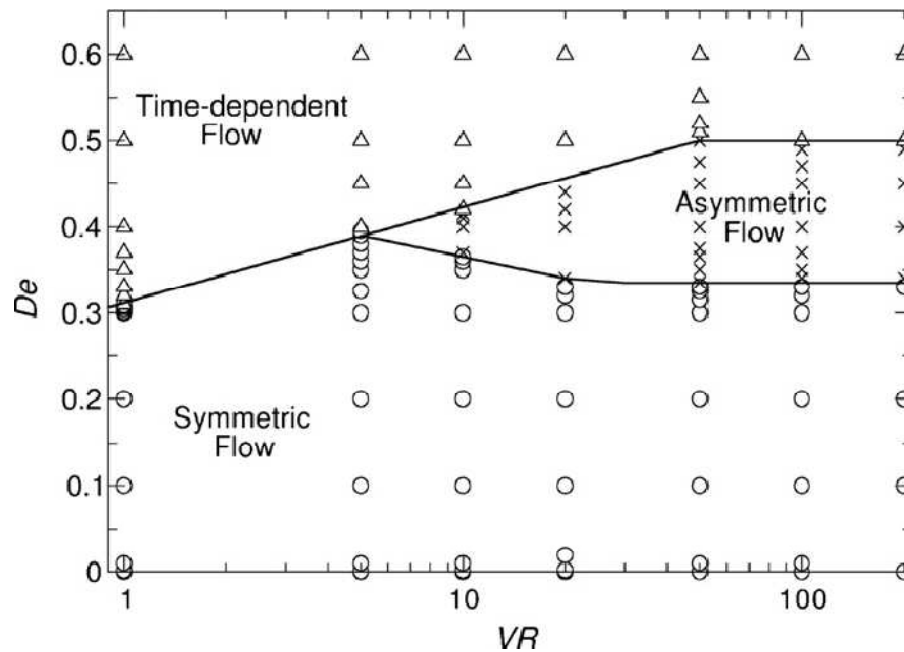
# Instabilities at the microscale

J. Non-Newtonian Fluid Mech. 160 (2009) 31–39

## Purely elastic flow asymmetries in flow-focusing devices

M.S.N. Oliveira<sup>a,\*</sup>, E.T. Pinho<sup>b</sup>, R.J. Poole<sup>c</sup>, P.J. Oliveira<sup>d</sup>, M.A. Alves<sup>a</sup>

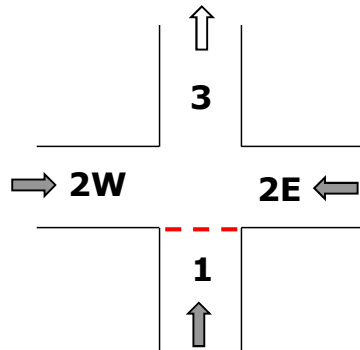
### UCM model 2D, Creeping Flow



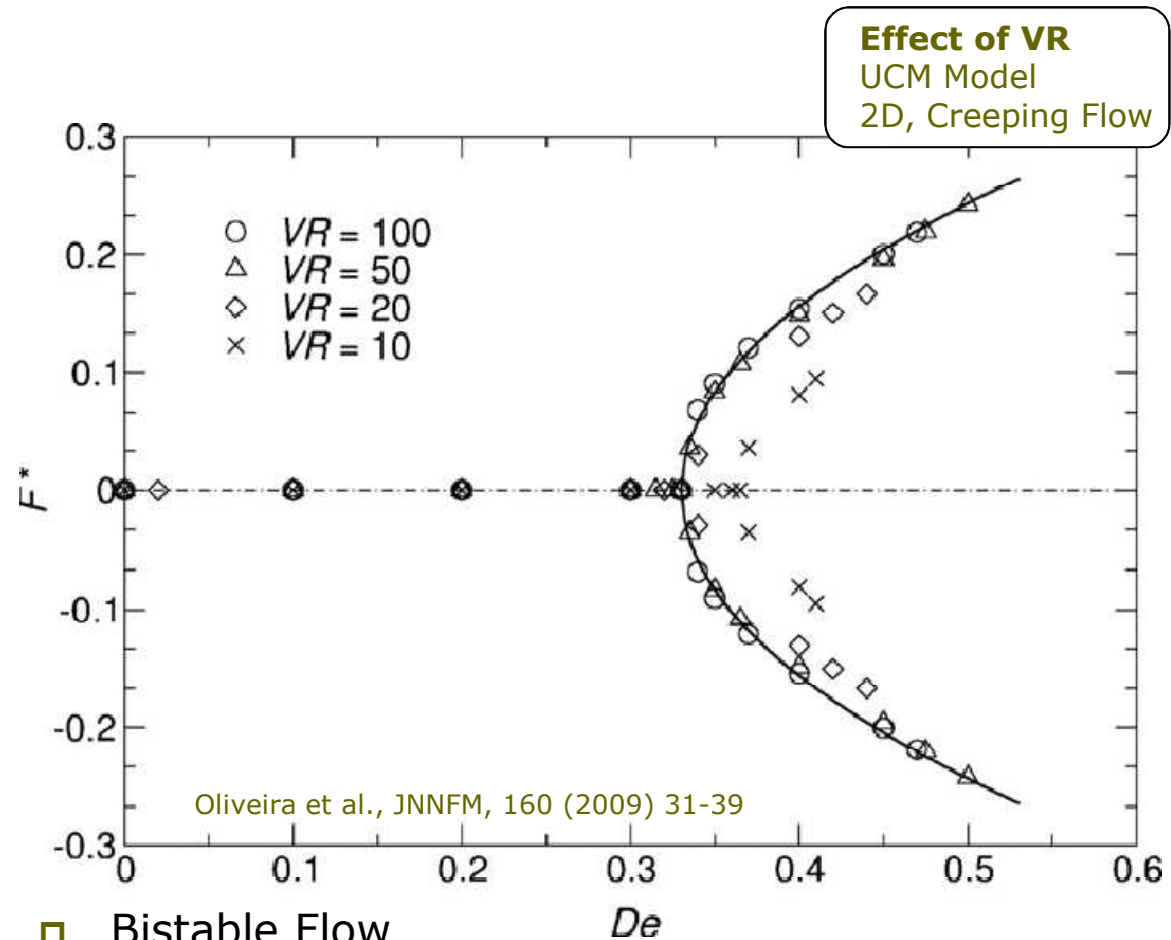
- For **low** VR: asymmetric flow not observed
- For **high** VR: **symmetric** → **asymmetric** → **oscillatory**
  - The **critical**  $De_c$  becomes approximately **constant**:  $De_c \sim 0.33$
  - **Bistable** flow

# Effect of $VR$ on the degree of asymmetry

$$F^* = \frac{F_W - F_E}{F_3}$$



$F^*=0$  (**symmetric** flow)  
 $F^*$  progressively deviates from  $F^* = 0$   
as the flow becomes increasingly  
asymmetric.



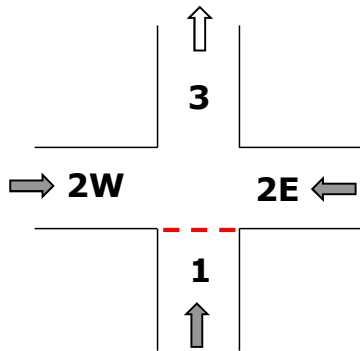
- Bistable Flow
- For high  $VR$ :
  - $De_c$  **independent of  $VR$**
  - Evolution of asymmetry independent of  $VR$
  - Supercritical pitchfork bifurcation:  $F^* = a\sqrt{De - De_c}$

$$= 0.59\sqrt{De - 0.33}$$

# Effect of $\beta$ on the degree of asymmetry

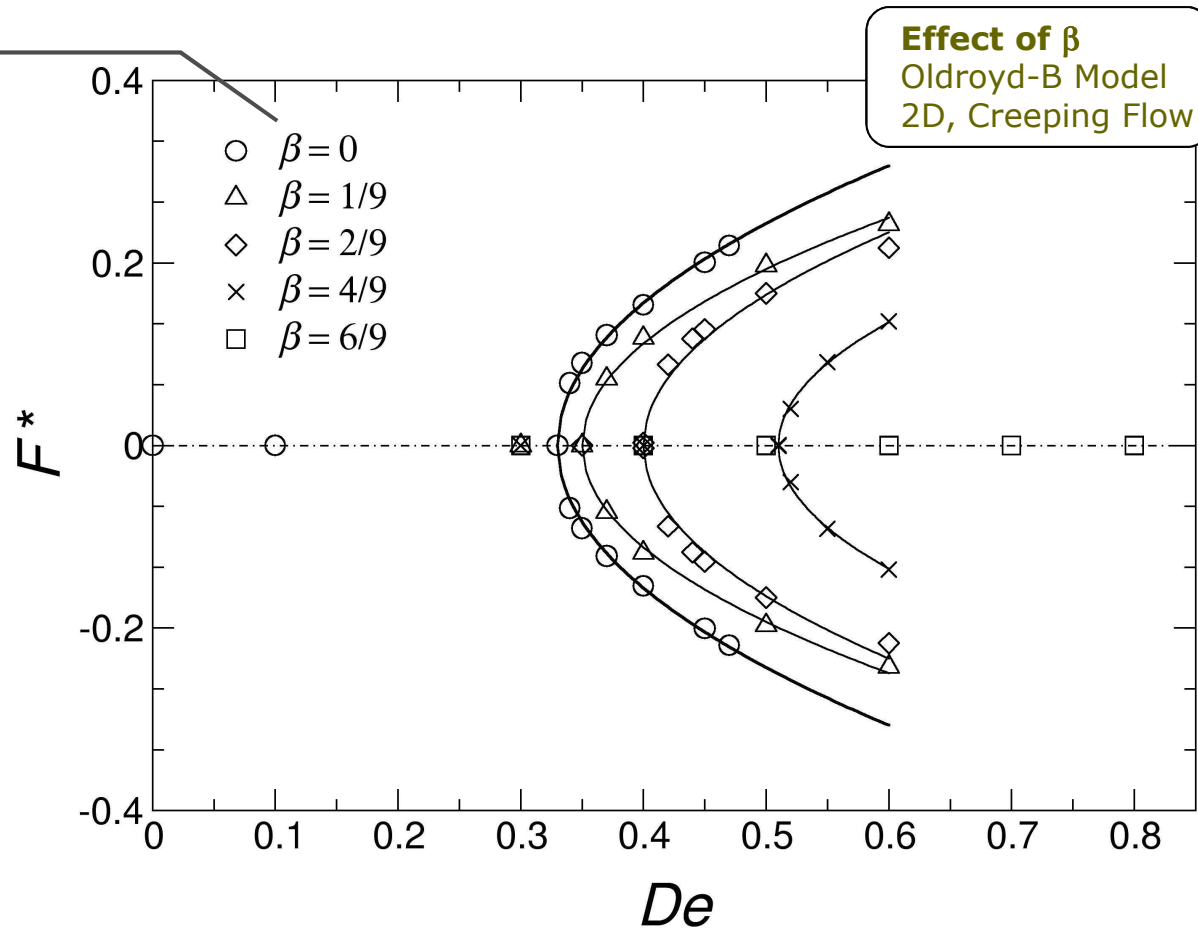
$$\beta \equiv \frac{\eta_s}{\eta_0} = \frac{\eta_s}{\eta_s + \eta_p}$$

$$F^* = \frac{F_W - F_E}{F_3}$$



$F^*=0$  (**symmetric** flow)

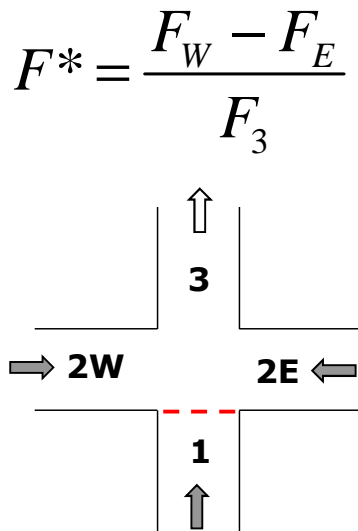
$F^*$  progressively deviates from  $F^* = 0$  as the flow becomes increasingly asymmetric.



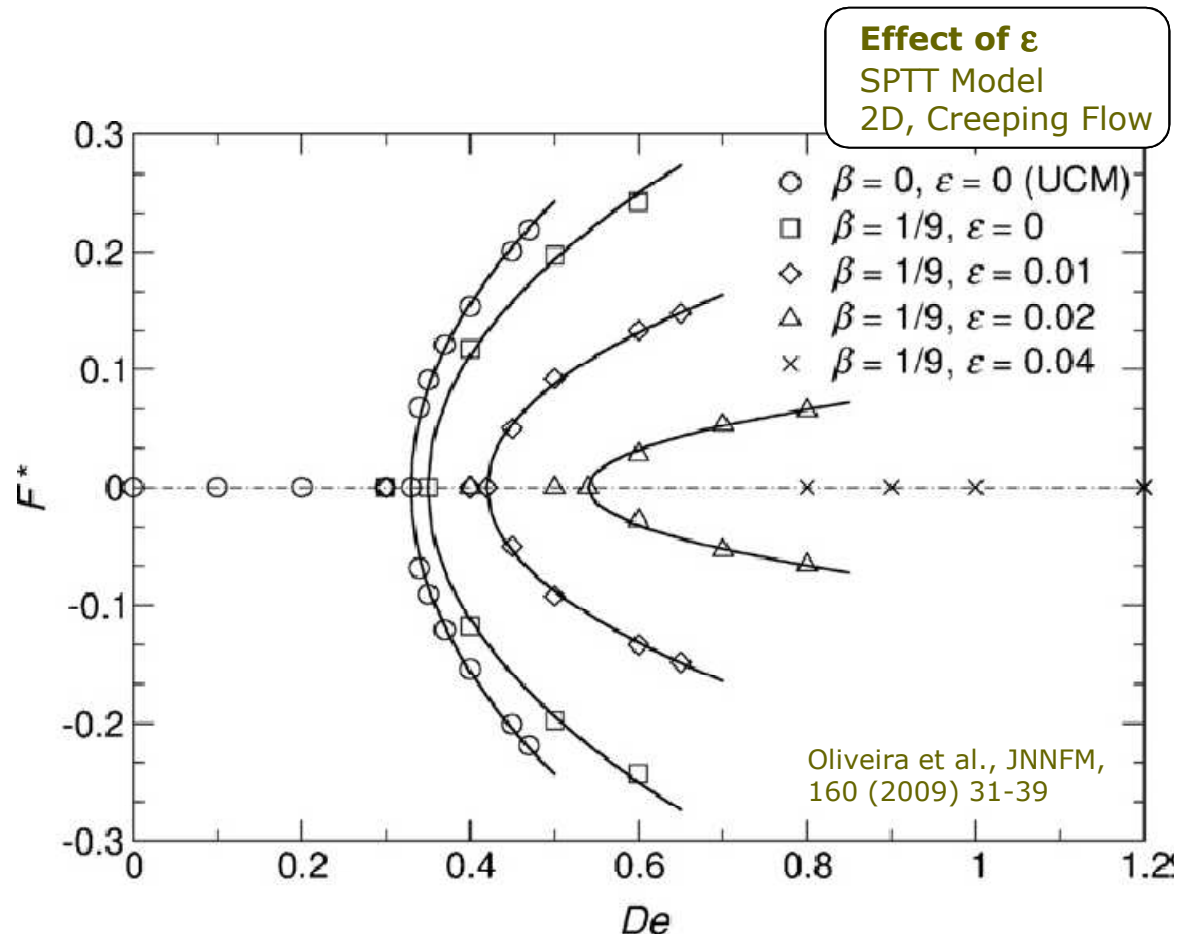
- $\beta$  has a stabilizing effect on the flow:
  - **Increase** in the critical  $De_c$
  - For  $\beta \geq 6/9$ , **no** steady asymmetry is observed.



# Effect of $\varepsilon$ on the degree of asymmetry

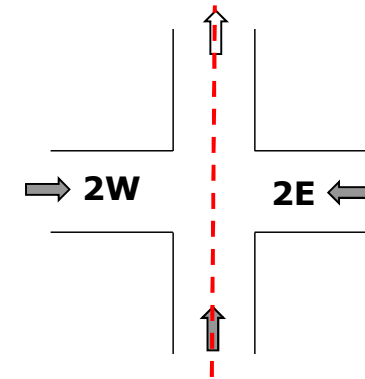
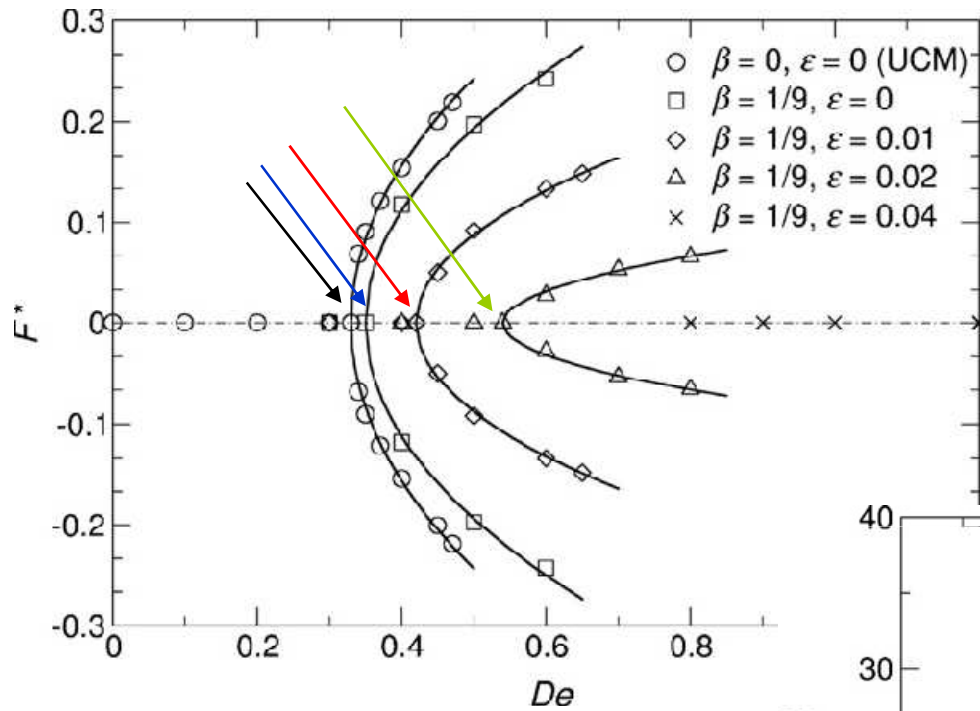


$F^*=0$  (**symmetric** flow)  
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as the flow becomes increasingly  
asymmetric.

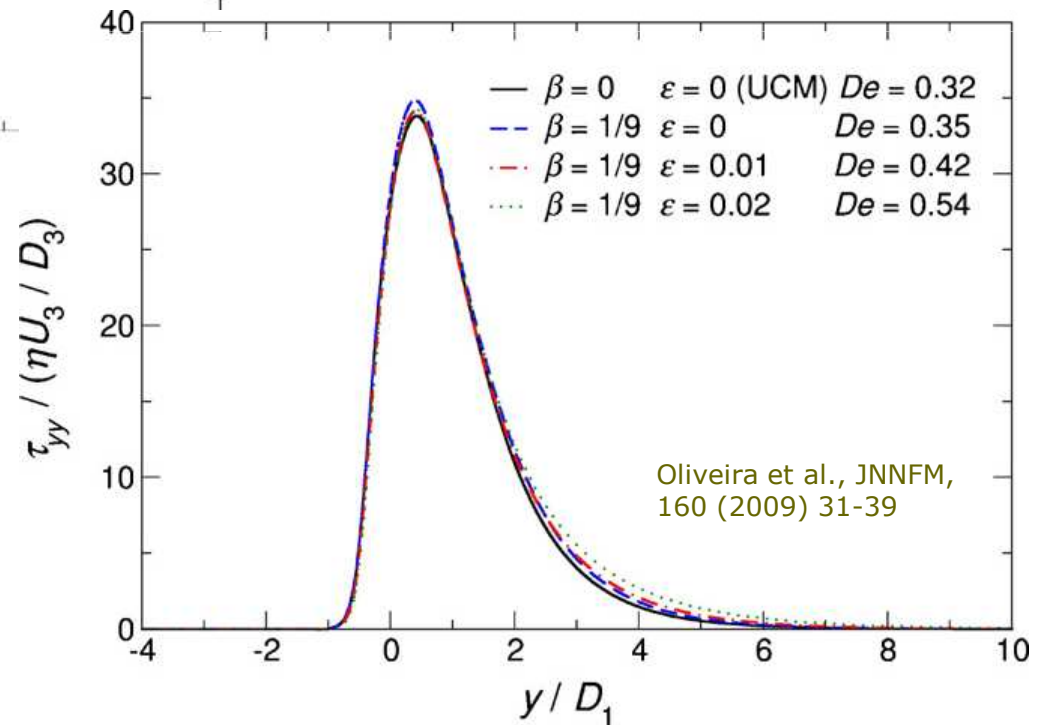


- **Increase** in the critical  $De_c$
- Decrease in the degree of asymmetry ( $\varepsilon < 0.04$ )
- For  $\varepsilon \geq 0.04$ , the steady asymmetry no longer observed. The flow transitions directly from steady symmetric to unsteady.

# Axial Normal Stress Profiles



**Axial normal stress profiles along the centerline ( $x = 0$ )**



- Similar levels of normal stresses achieved near critical conditions.
- Extensional properties decisive for the onset of flow asymmetry.

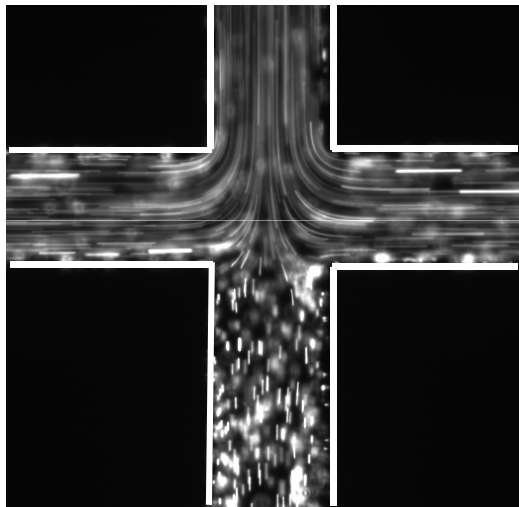
Oliveira et al., JNNFM, 160 (2009) 31-39

# Viscoelastic Fluid: PAA125+NaCl

$Q_1 = 0.01$  ml/h

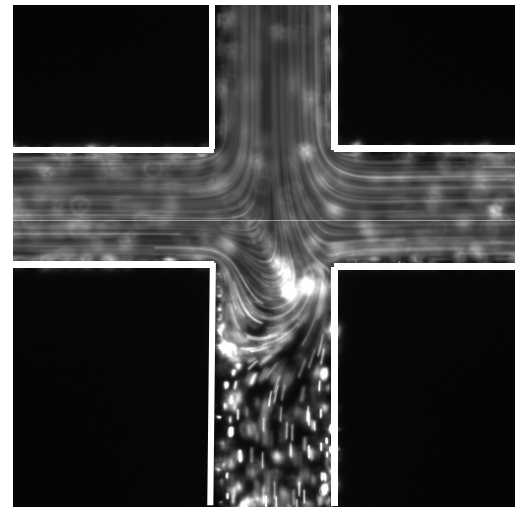
increasing  $Q_2$

Viscoelastic



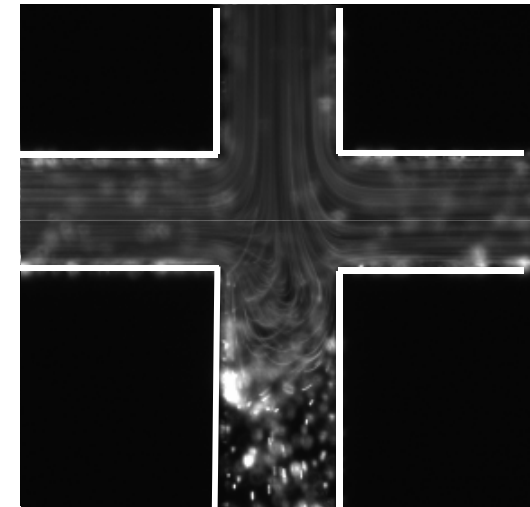
$Q_2 = 0.05$  ml/h,  $VR = 5$   
 $Re = 0.23$ ,  $De = 0.38$

Symmetric



$Q_2 = 0.2$  ml/h,  $VR = 20$   
 $Re = 0.87$ ,  $De = 1.41$

Steady Asymmetric



$Q_2 = 0.5$  ml/h,  $VR = 50$   
 $Re = 2.15$ ,  $De = 3.479$

Unsteady 3D

# Viscoelastic Fluid: PAA125+NaCl

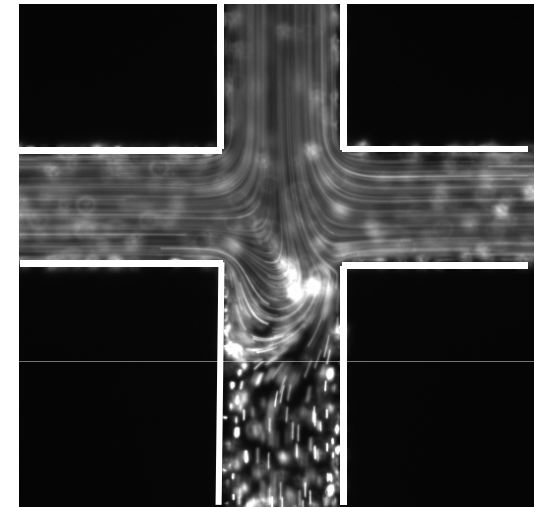
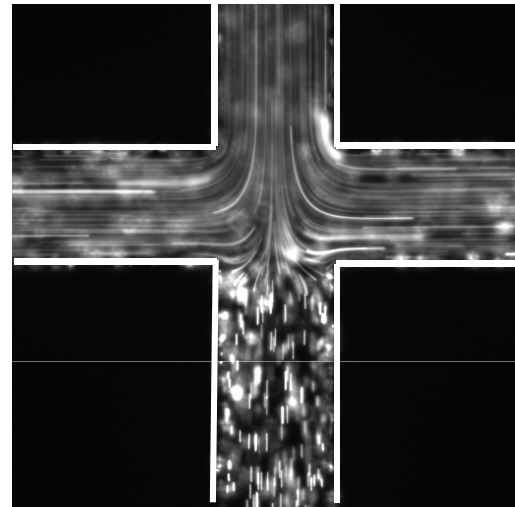
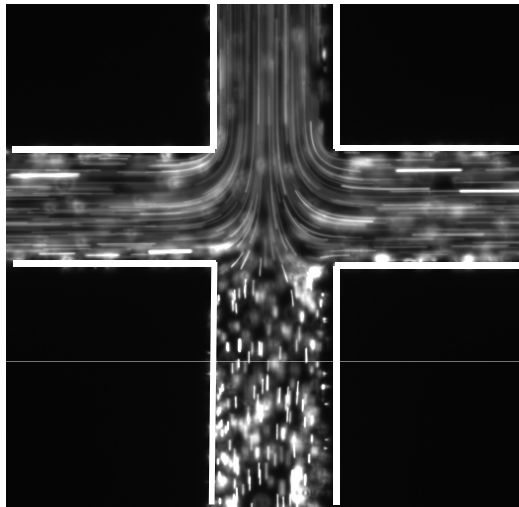
$Q_1 = 0.01$  ml/h

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 $Re = 0.23$ ,  $De = 0.38$

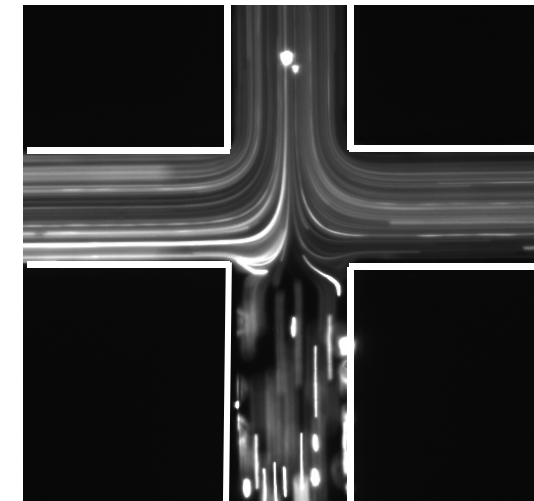
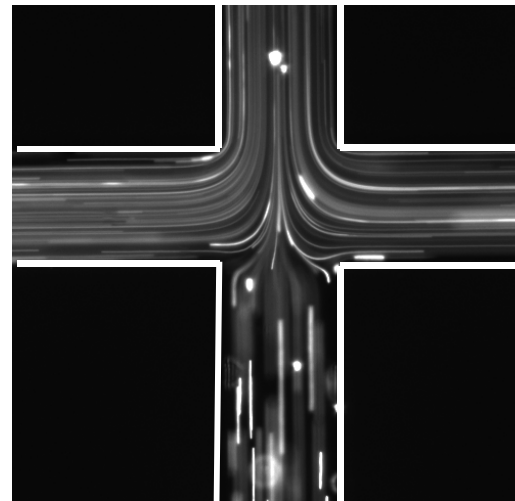
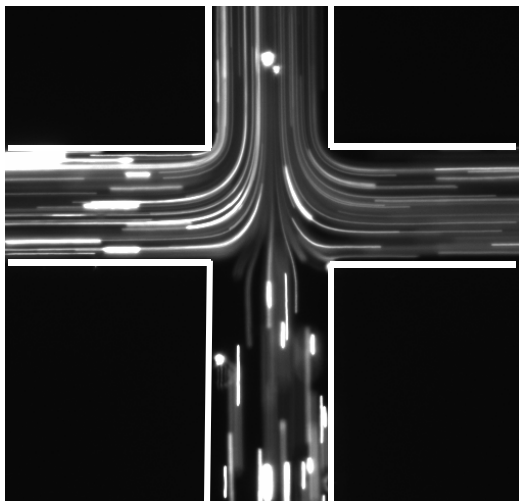
$Q_2 = 0.1$  ml/h,  $VR = 10$   
 $Re = 0.45$ ,  $De = 0.723$

$Q_2 = 0.2$  ml/h,  $VR = 20$   
 $Re = 0.87$ ,  $De = 1.41$

Viscoelastic



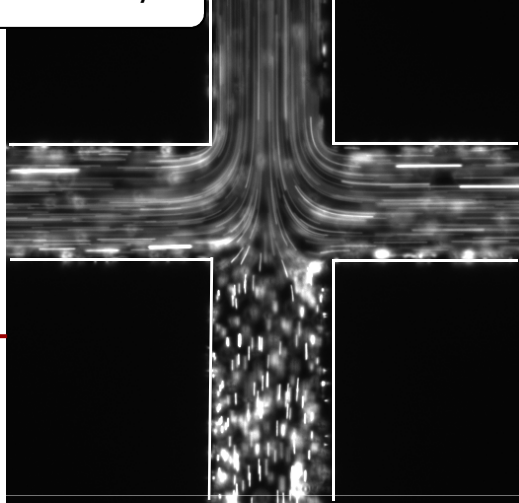
Newtonian



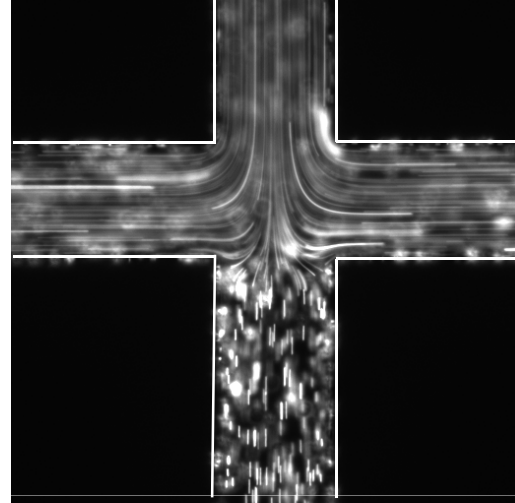
# Viscoelastic Fluid: PAA125+NaCl

$Q_1 = 0.01$  ml/h

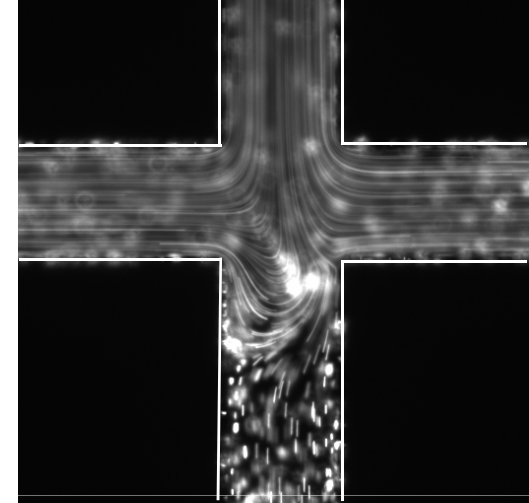
Viscoelastic  
Experimental



$Q_2 = 0.05$  ml/h,  $VR = 5$   
 $Re = 0.23$ ,  $De = 0.38$

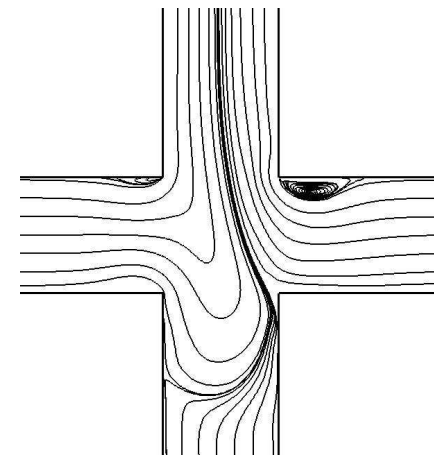
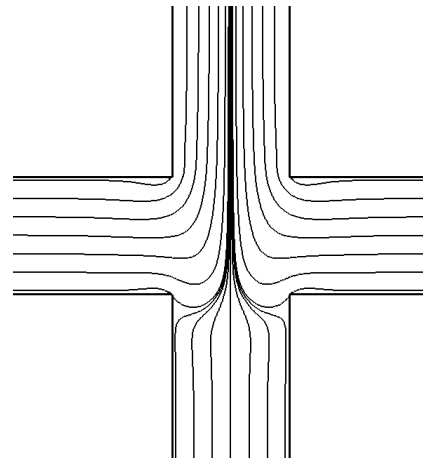
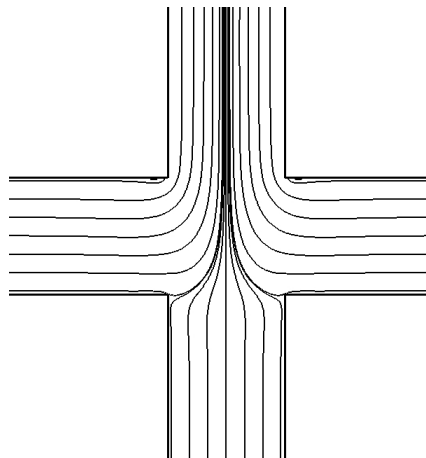


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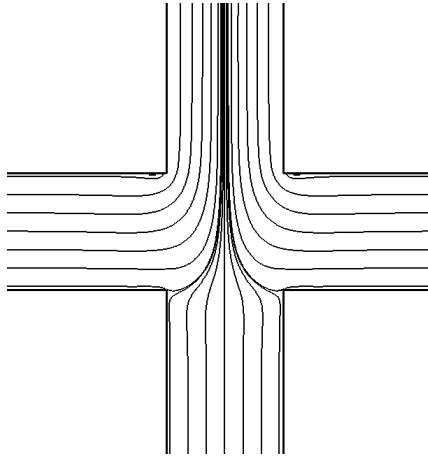
UCM  
2D Calculations



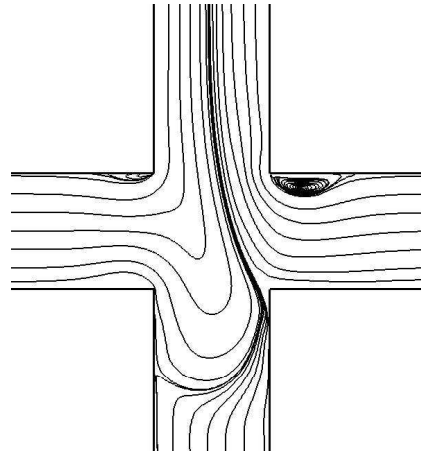
# Viscoelastic Model

$Q_1 = 0.01 \text{ ml/h}$

**UCM**  
2D Calculations



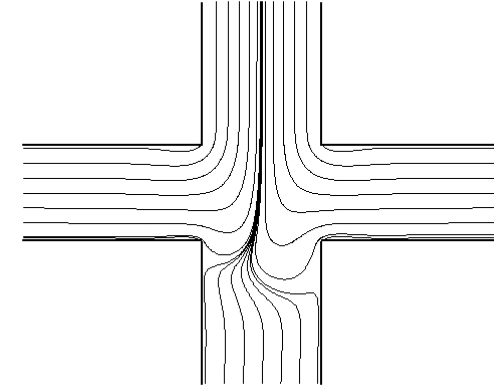
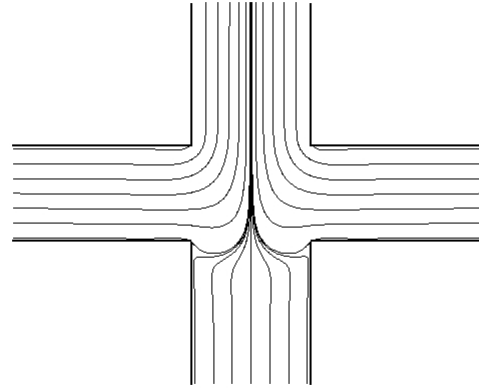
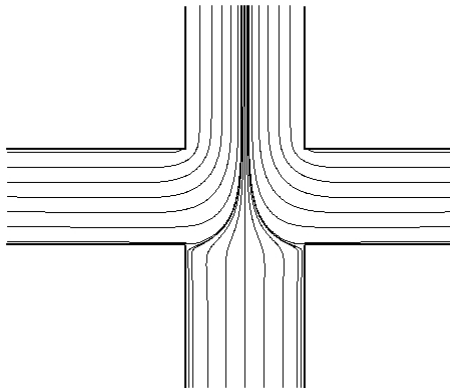
$Q_2 = 0.05 \text{ ml/h}, VR = 5$   
 $Re = 0.23, De = 0.38$



$Q_2 = 0.2 \text{ ml/h}, VR = 20$   
 $Re = 0.87, De = 1.41$

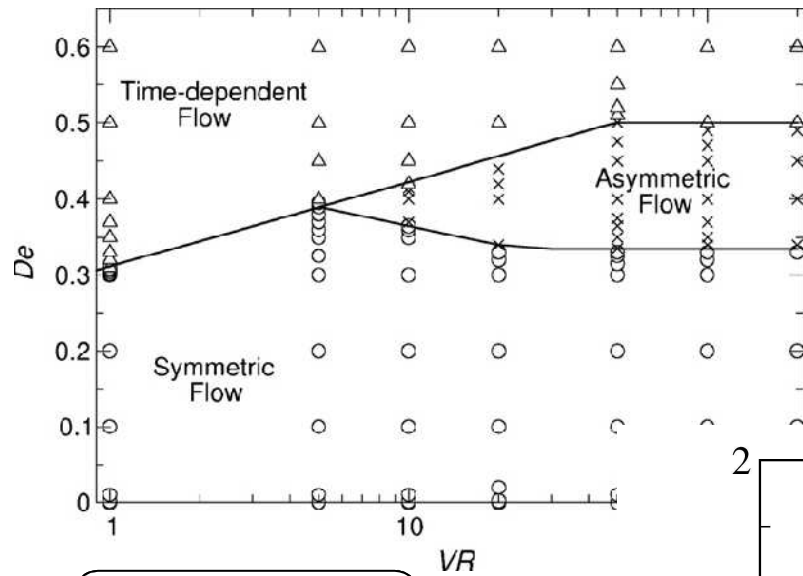
Unsteady 3D

**Oldroyd-B**  
2D Calculations



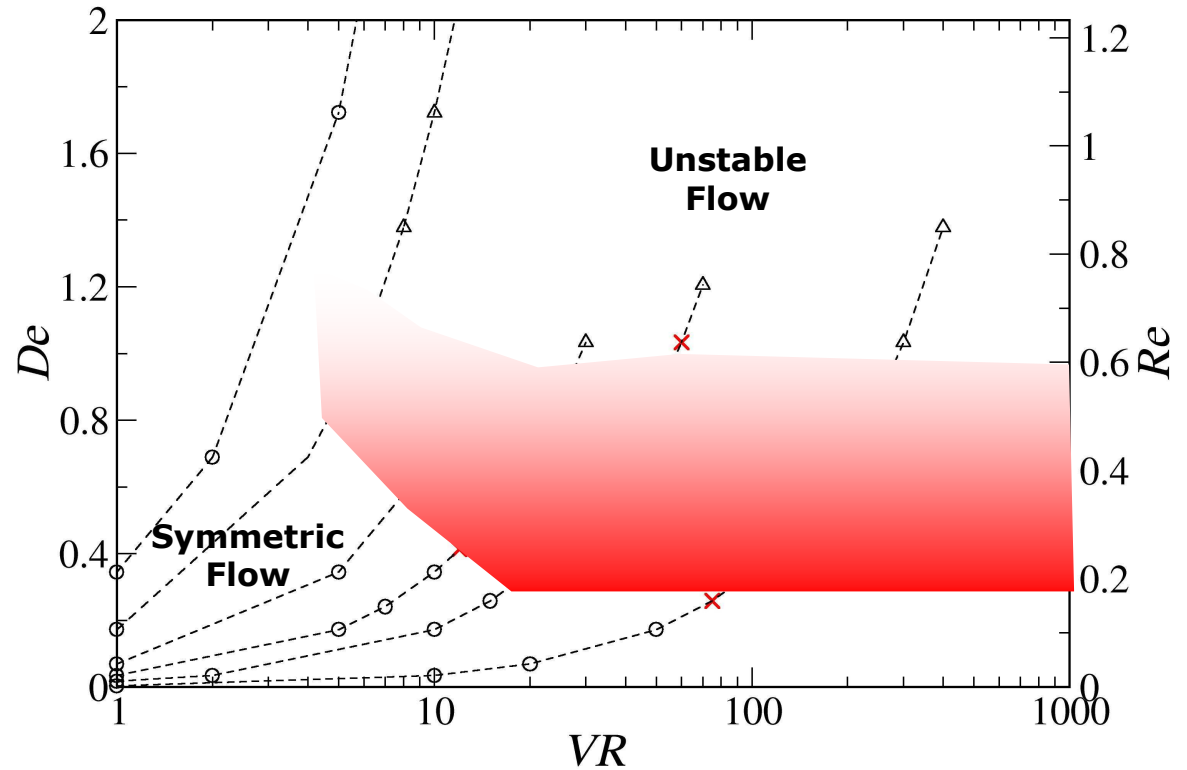
$Q_2 = 0.35 \text{ ml/h}, VR = 35$   
 $Re = 0.87, De = 1.41$

# Experimental Flow Map



**Numeric**  
UCM Model  
2D, Creeping Flow

**Experimental**  
PAA 125 + NaCl



# Summary

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- Elastically-driven asymmetries at the microscale.
- **Symmetric-breaking bifurcation** at high  $De$  and  $VR$ .
- **Unsteady 3D** flow above a critical  $De$ : useful for mixing purposes.
- Numerical 2D calculations reproduce qualitatively the experimental results.

## **Ongoing work:**

- Experimental:  $\mu$ PIV
- Numerical: 3D simulations



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