Turbulent energy cascade in viscoelastic isotropic turbulence

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Osaka University 28 Oct 2013

Outline

- Overview of the direct numerical simulations
- Added dissipation due to polymer additives in forced HIT
- Effect of polymer additives on the turbulence energy cascade
- The inter-scale energy cascade caused by the polymers
- Summary

DNS of statistically steady viscoelastic turbulence

Numerical method

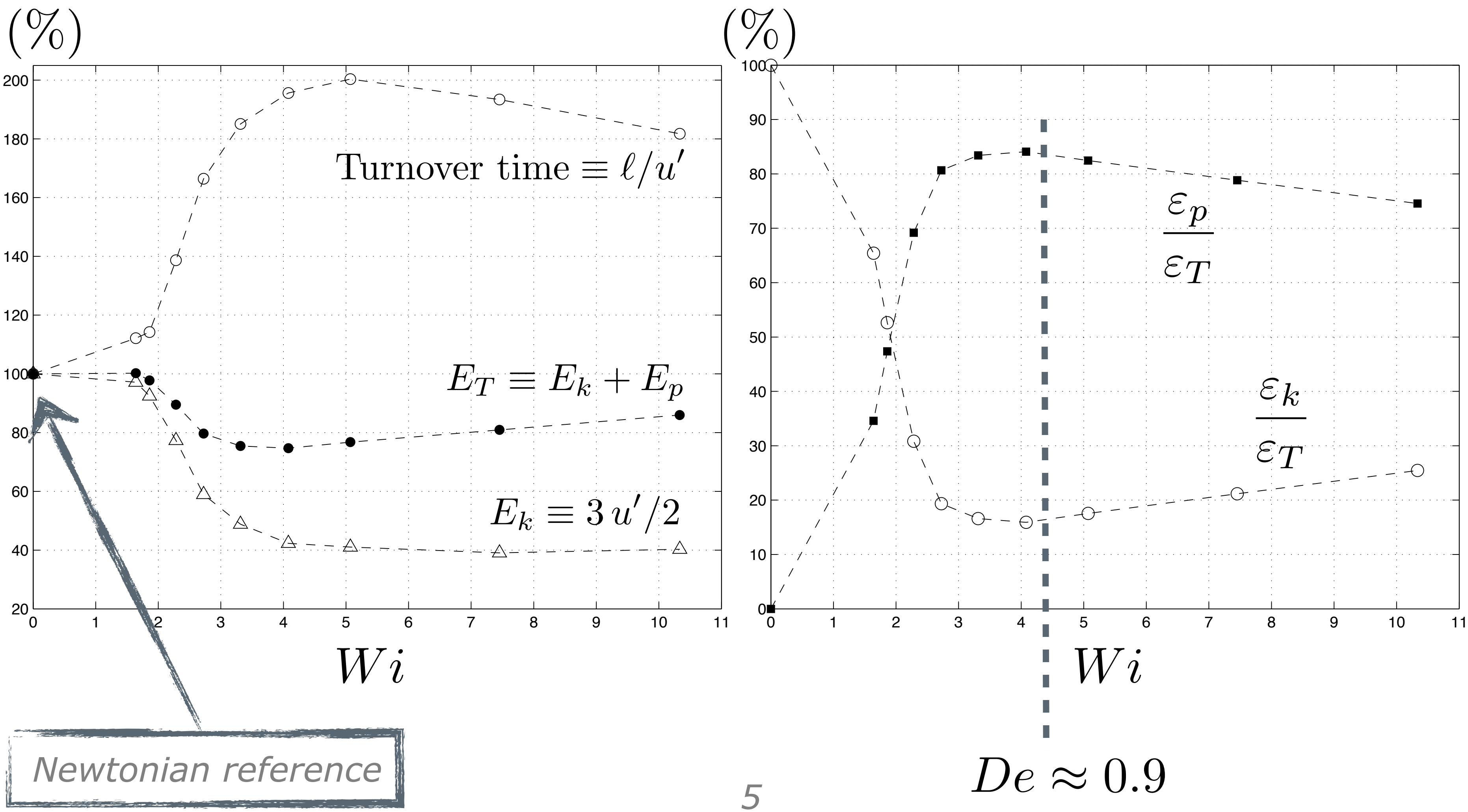
- FENE-P model for the polymer stress $$T_{ij}^{[p]} \equiv \frac{\mu^{[p]}}{\tau}\left(\frac{L^2-3}{L^2-C_{ii}}C_{ij}-\delta_{ij}\right)$$
- Pseudo-spectral solver for velocity with 2/3rd de-aliasing
- Kurganov-Tadmor solver for conformation tensor
- Third-order Runge-Kutta in time
- Alvelius (1999) forcing on first 4 waveno.

DNS parameters:

- $N=192^3$ (statistically steady)
- Solvent/total viscosity ratio: $\beta = 0.8$ $[c = 20\,\text{p.p.m.} \rightarrow 2\,\text{p.p.m.}]$
- Relaxation time: $$\begin{cases} \tau = [0.1, 0.125, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0] \\ Wi \equiv \tau/(\nu/\varepsilon^{[N]})^{1/2} = [1.6, \ldots, 10.4] \\ De \equiv \tau/(\ell/u') = [0.3, \ldots, 1.8] \end{cases}$$
- Max. polymer extension: $L = 100,$ $3\% < \sqrt{C_{ii}}/L < 39\%$
- Reynolds number: $50 < Re_\lambda < 70$

Added dissipation due to polymer additives in forced HIT

Added dissipation due to polymer additives



Added dissipation due to polymer additives

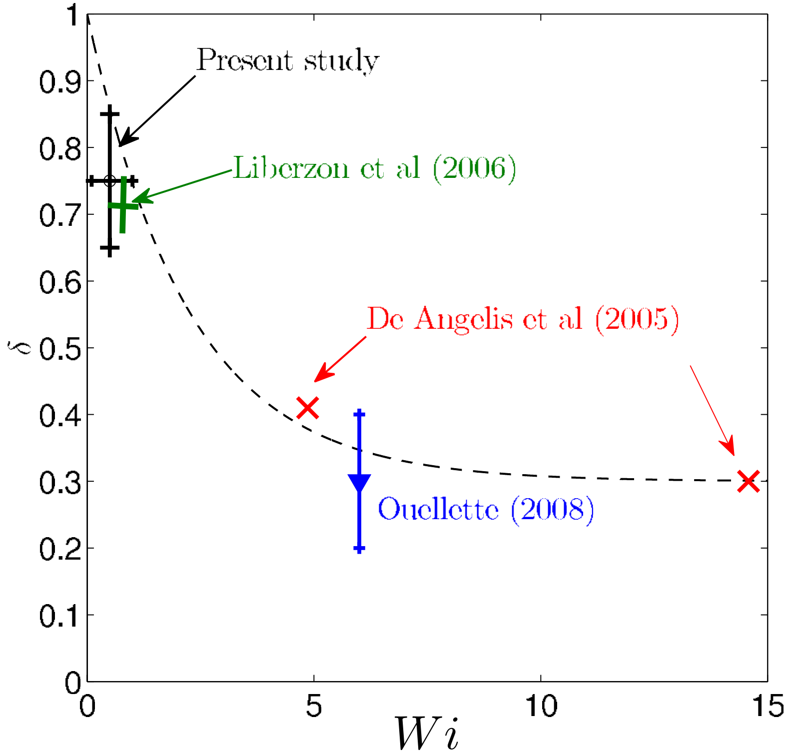
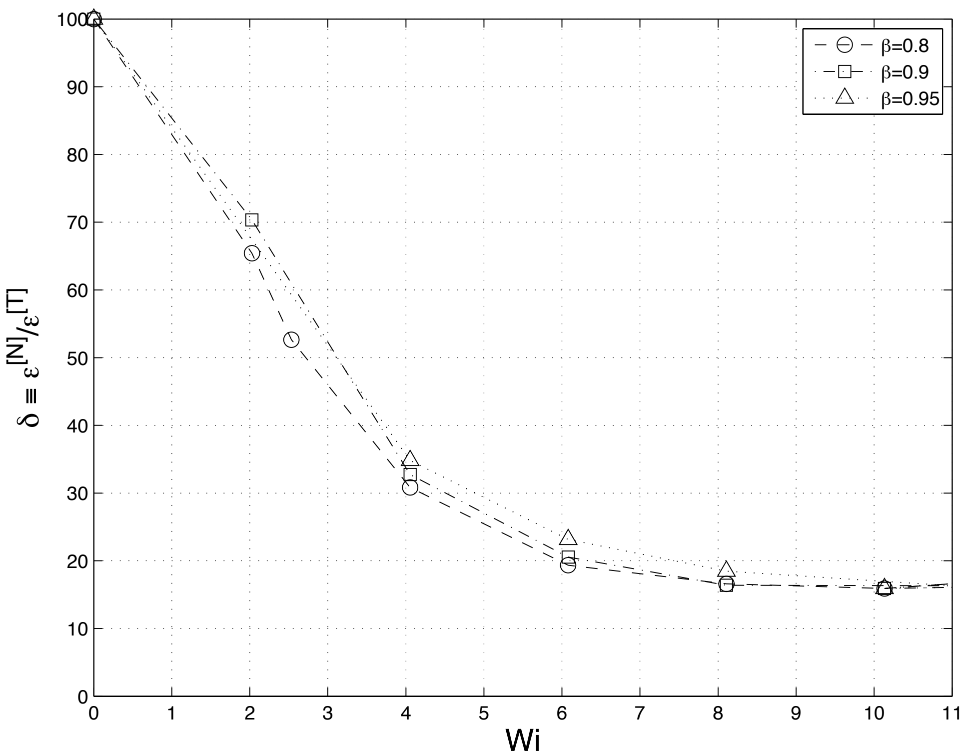


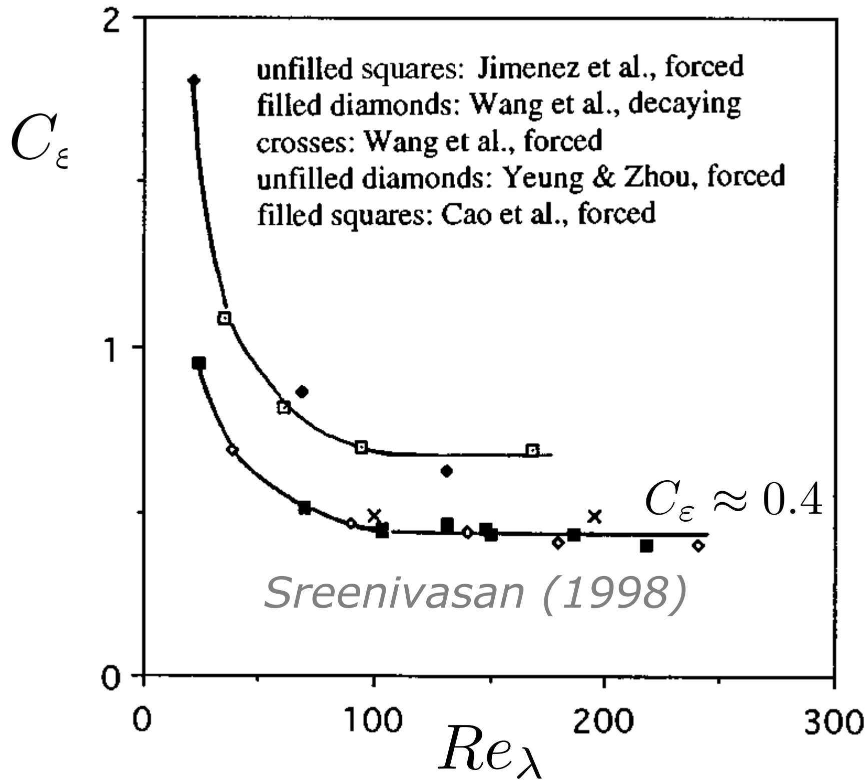
FIG. 5. (Color online) Dependence of $\delta=\varepsilon_\nu/\varepsilon_i$ on the Deborah number. × marks the numerical results (Ref. 6), a triangle marks the result of Ouellette *et al.* (Ref. 34; see also Ref. 9), + is for Ref. 18, and the circle denotes the present estimate. The error bars display the experimental uncertainty in the case of the present experiment and the variation of concentration in case of the experiment of Ref. 9, respectively.



Added dissipation due to polymer additives

Recall: In Newtonian turbulent flows the rate of dissipation is inversely proportional to turnover time for a given turbulent kinetic energy

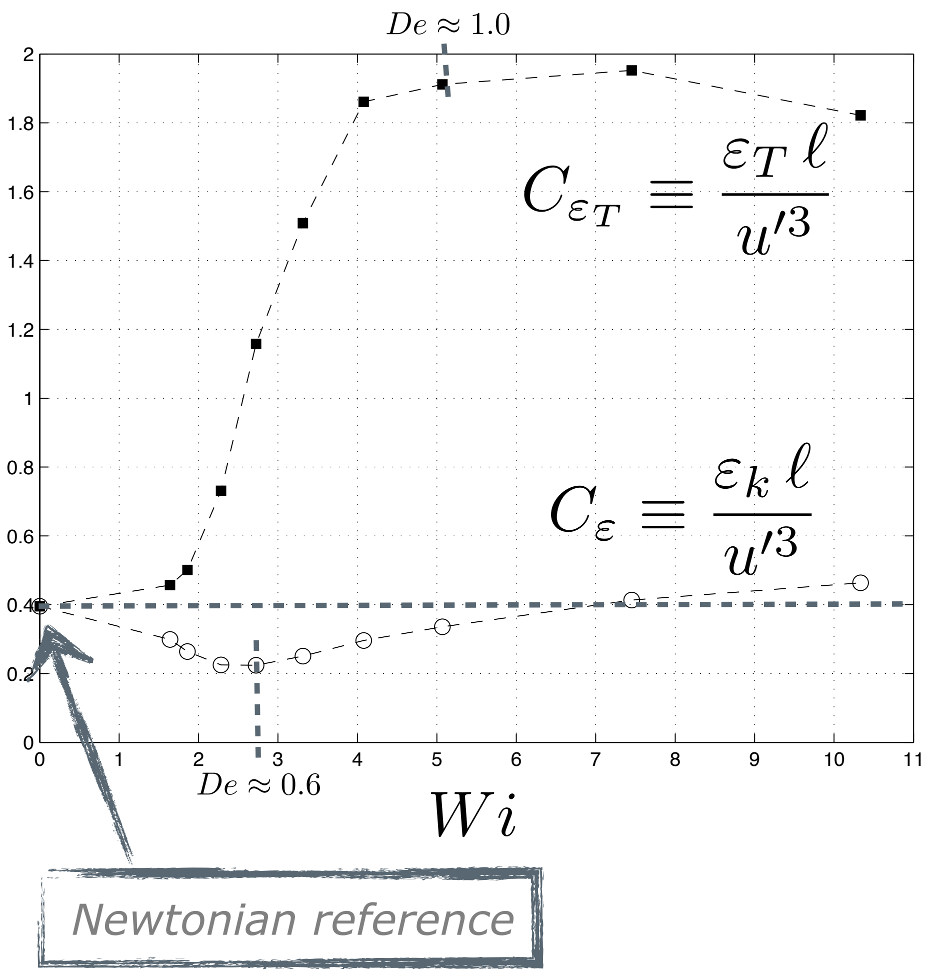
$$\varepsilon_k = C_\varepsilon u'^3/\ell \sim \frac{u'^2}{\ell/u'}$$



Previous slide:

- Decrease in kinetic energy
- Increase in turnover time
- Thus decrease in dissipation
- What about C_ε ?

Added dissipation due to polymer additives



- The polymer `hampers' the effectiveness of turbulence in dissipating kinetic energy
- However, the polymer additives efficiently transfer kinetic to elastic energy and dissipate it
- Overall there is <u>`drag' increase</u>

Effect of polymer additives on the turbulence energy cascade

Effect of polymer additives on the energy cascade

Lin equation for statistically steady homogeneous viscoelastic turbulence

$$f(k) = -T(k) + T^{[p]}(k) + 2\nu^{[s]} k^2 E(k)$$

External force — $f(k)$

Non-linear transfer — $-T(k)$

Kinetic to elastic energy transfer — $T^{[p]}(k)$

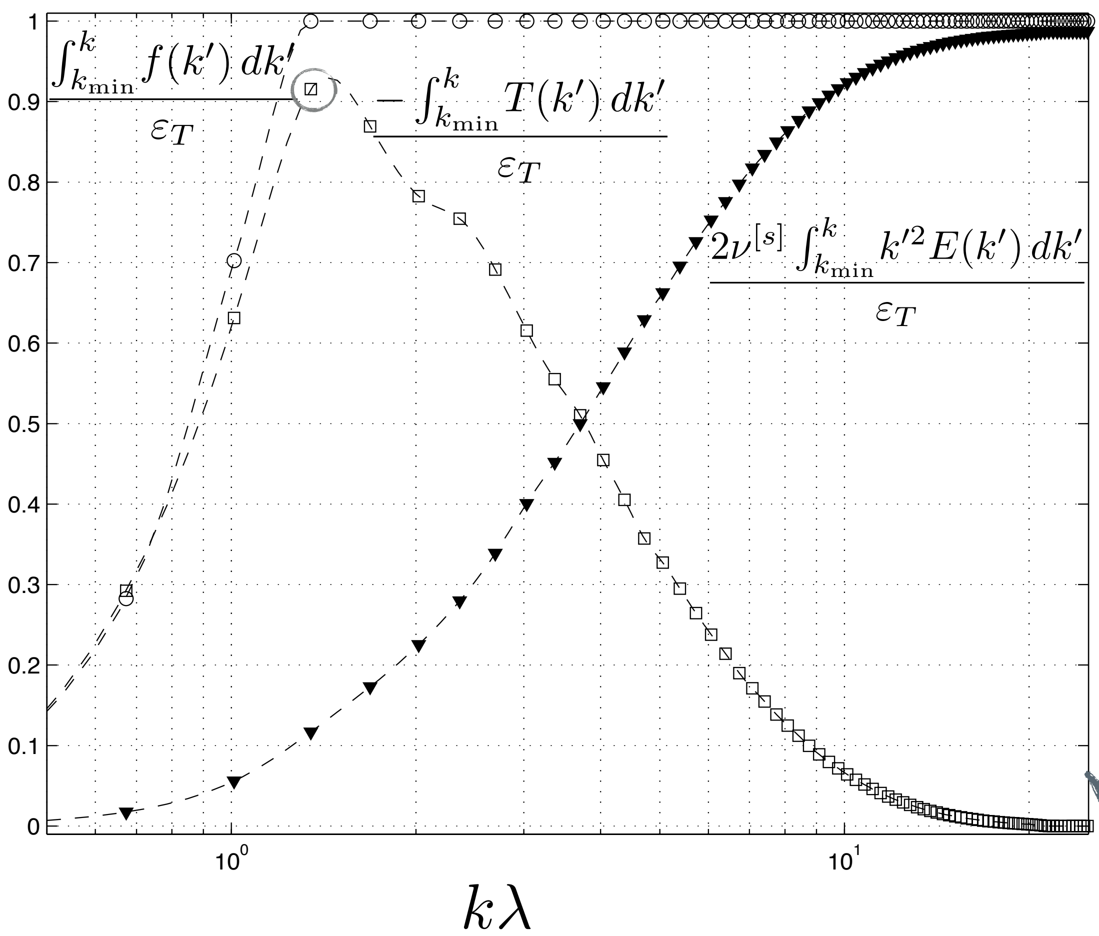
Solvent dissipation — $2\nu^{[s]} k^2 E(k)$

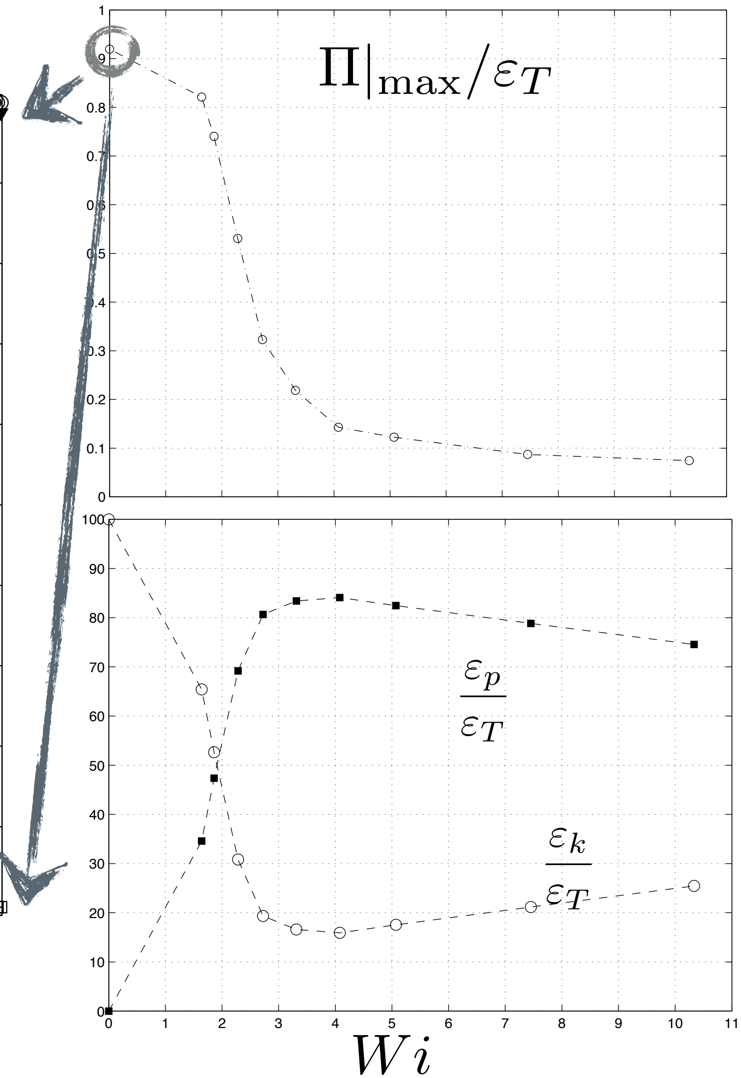
$$\int_{k_{\min}}^{k_{\max}} f(k)\, dk = P_{\text{in}} \quad \Big\{ \quad \underbrace{\int_{k_{\min}}^{k_{\max}} T^{[p]}(k)\, dk}_{\varepsilon_p} + \underbrace{2\nu^{[s]} \int_{k_{\min}}^{k_{\max}} k^2 E(k)\, dk}_{\varepsilon_k}$$

$$\int_{k_{\min}}^{k_{\max}} T(k)\, dk = 0$$

Effect of polymer additives on the energy cascade

$$f(k) = -T(k) + T^{[p]}(k) + 2\nu^{[s]} k^2 E(k)$$





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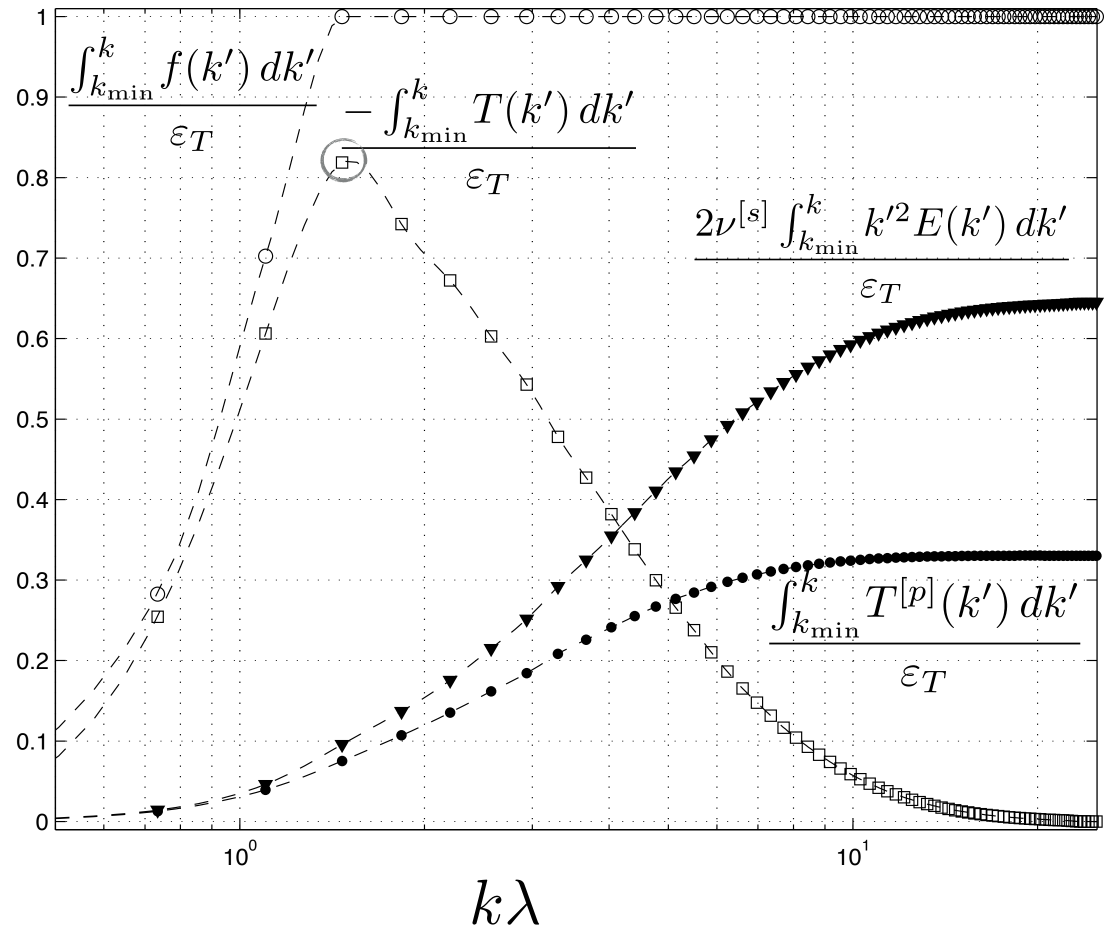

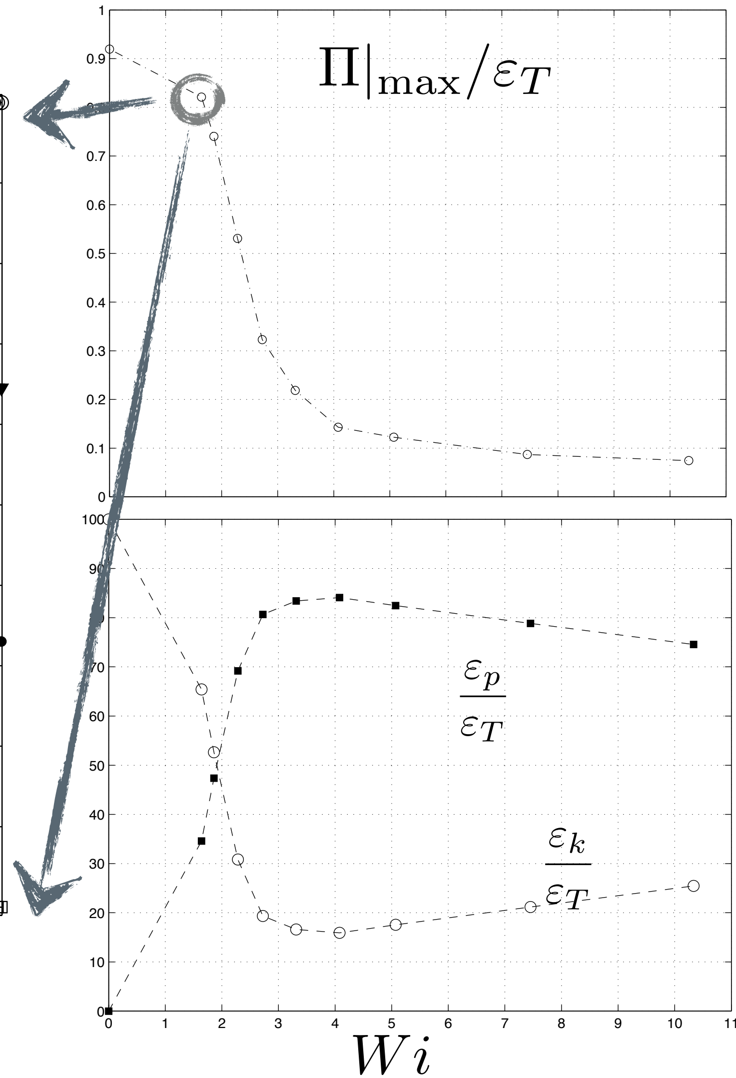

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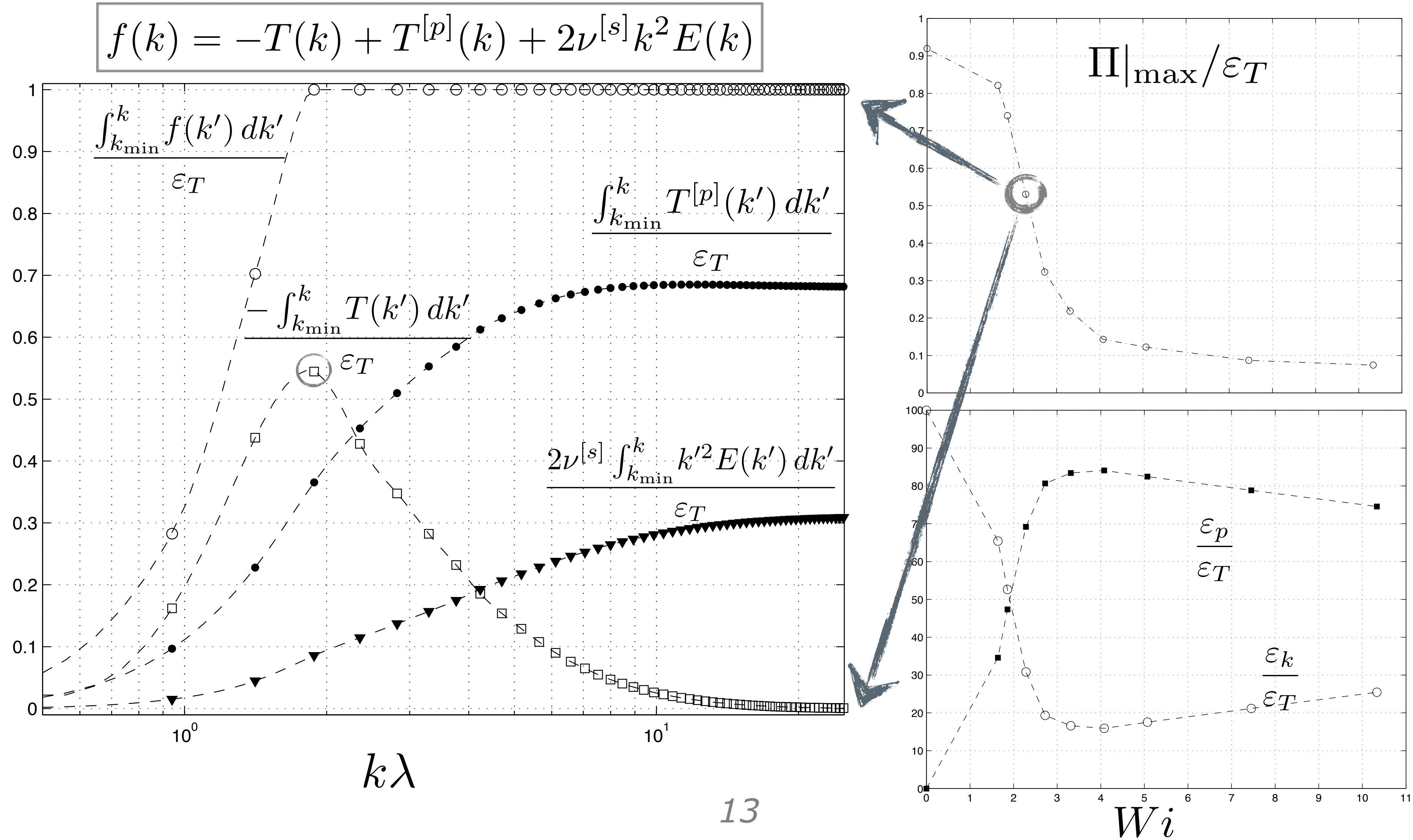

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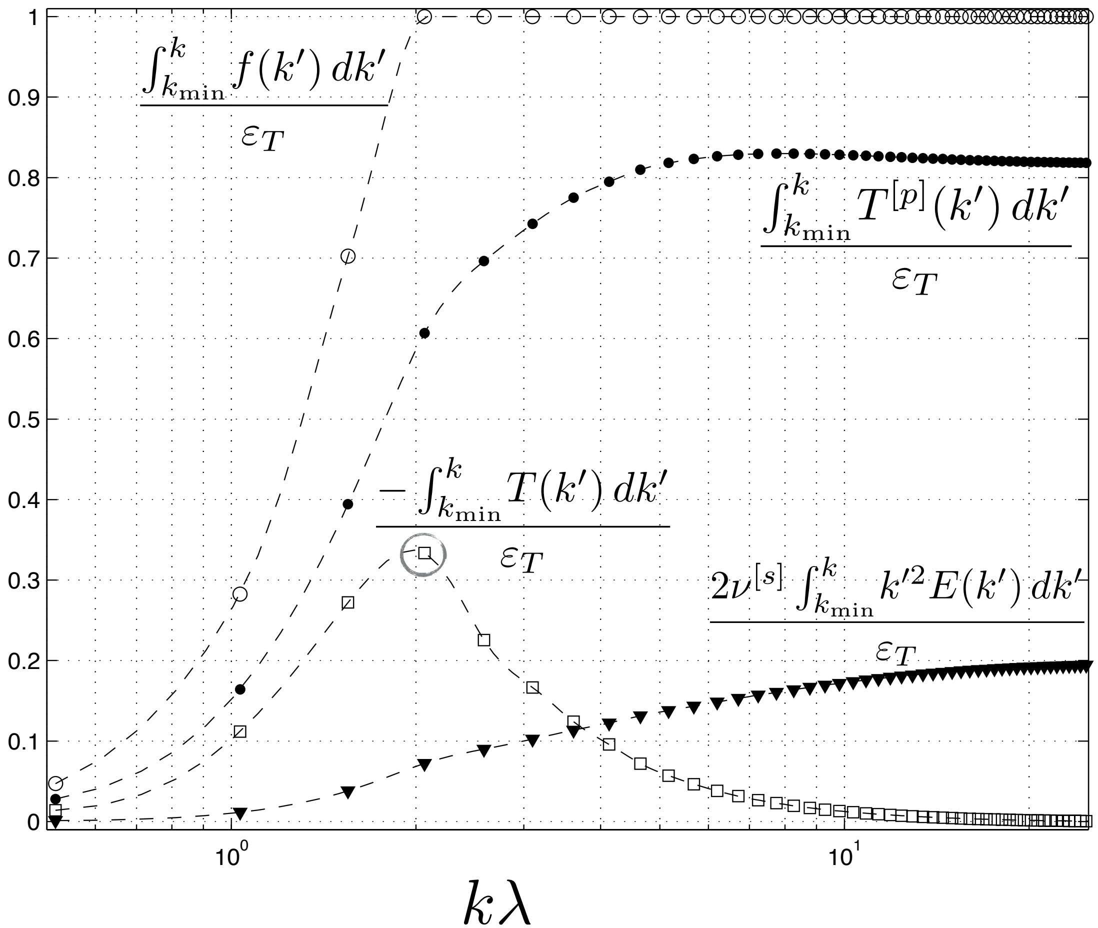

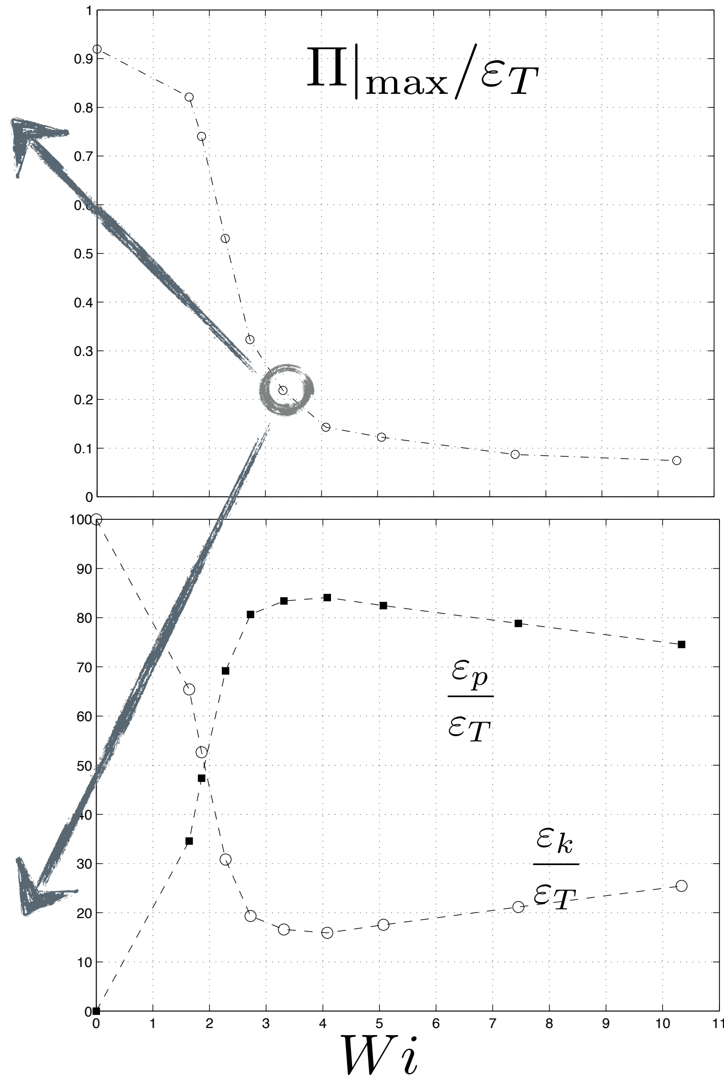

Effect of polymer additives on the energy cascade

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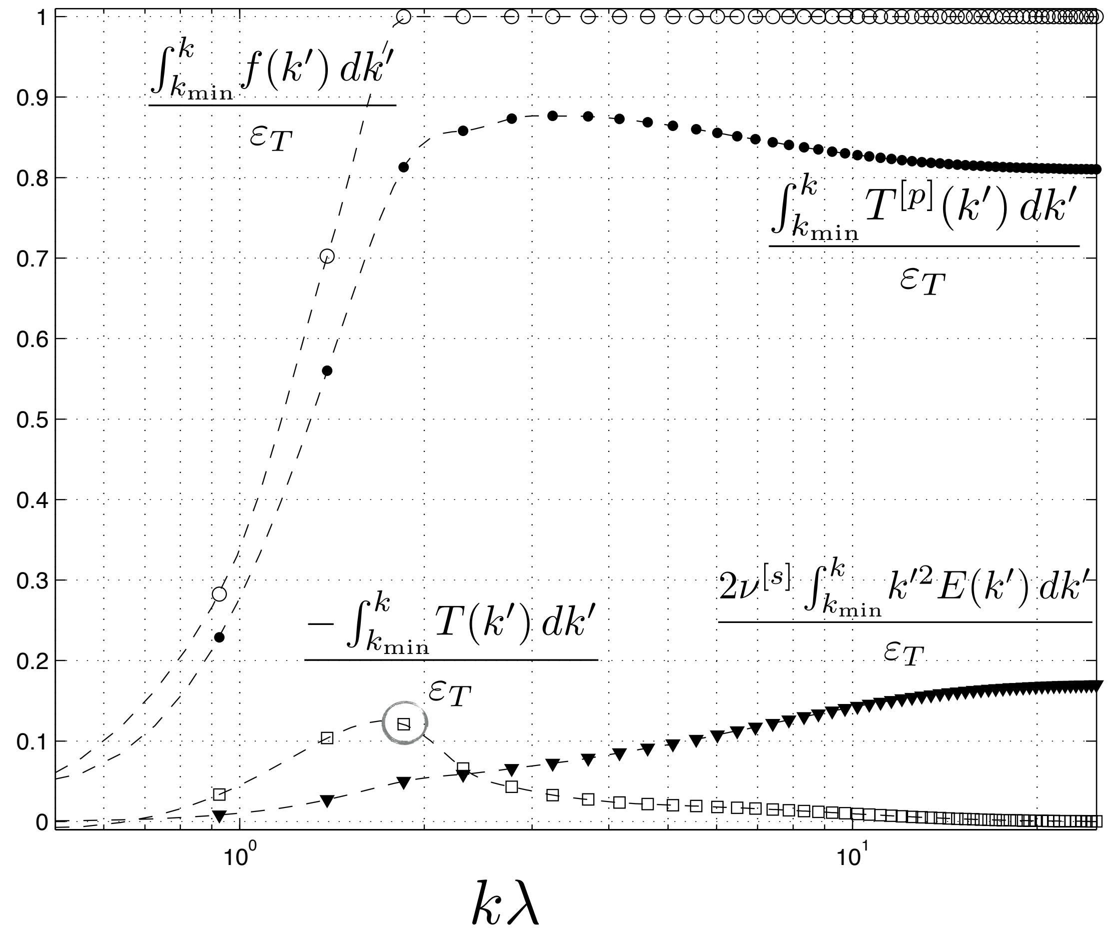

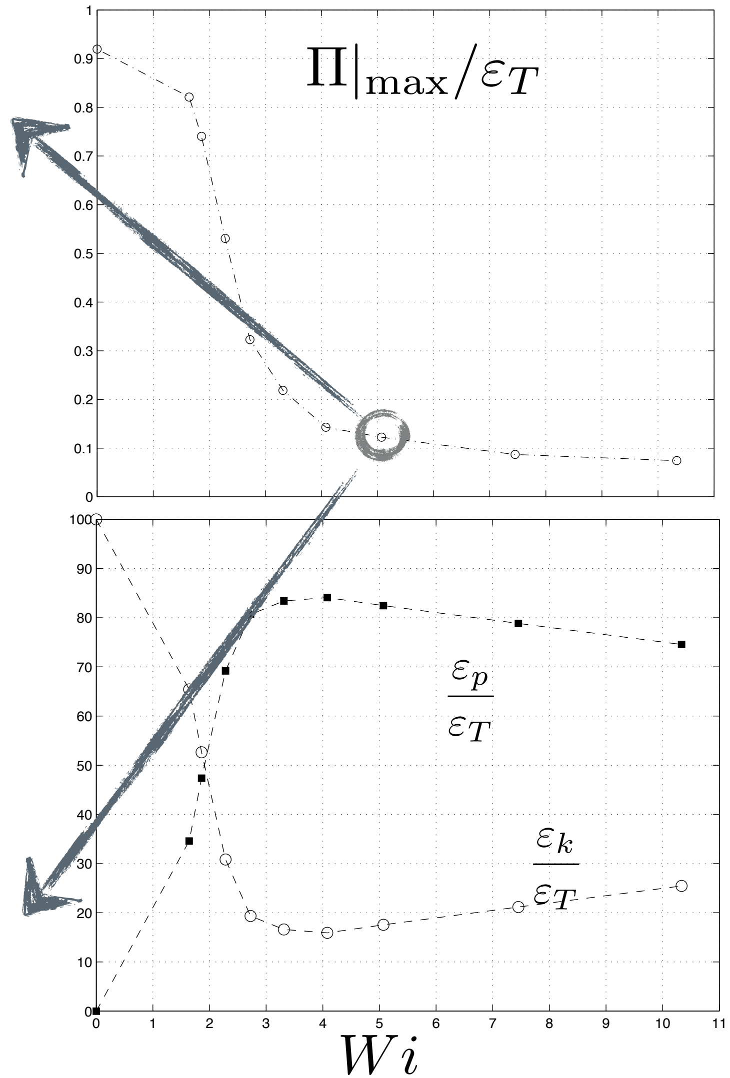

Effect of polymer additives on the energy cascade

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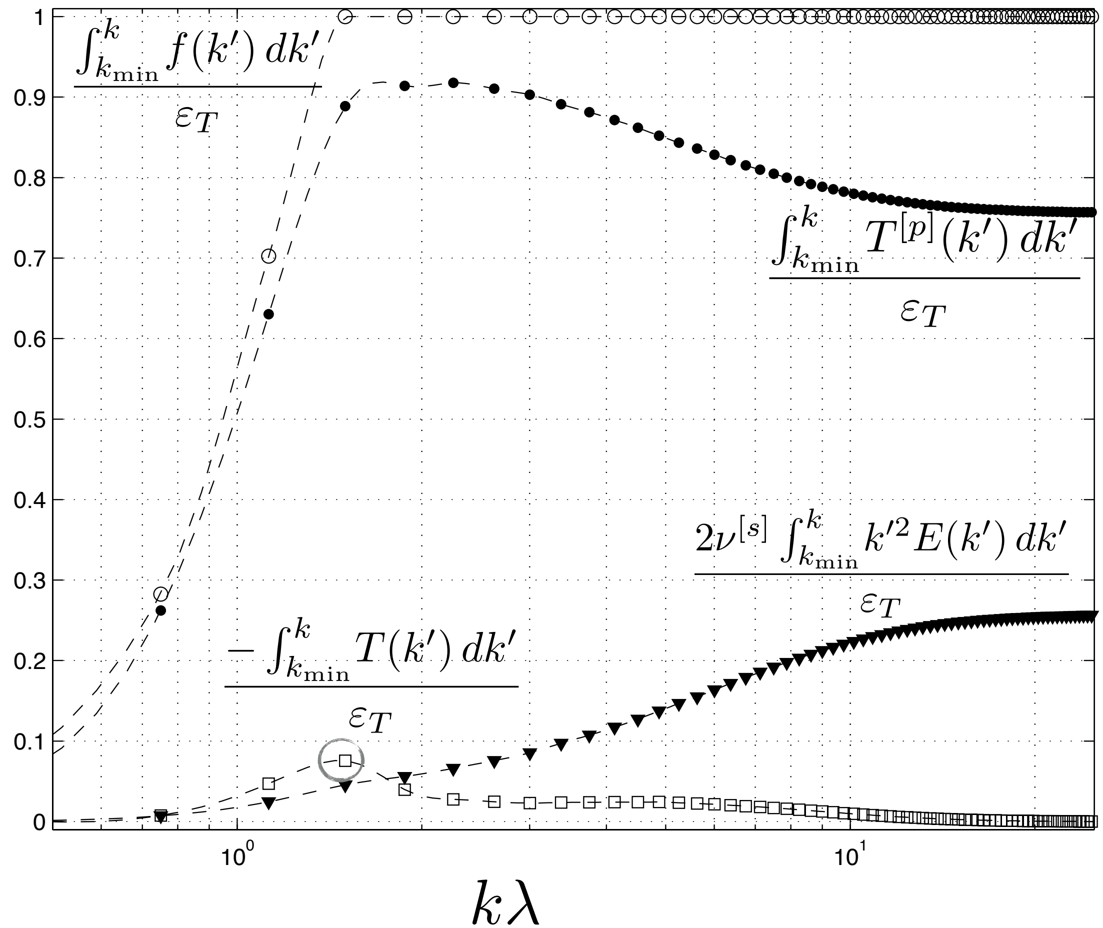
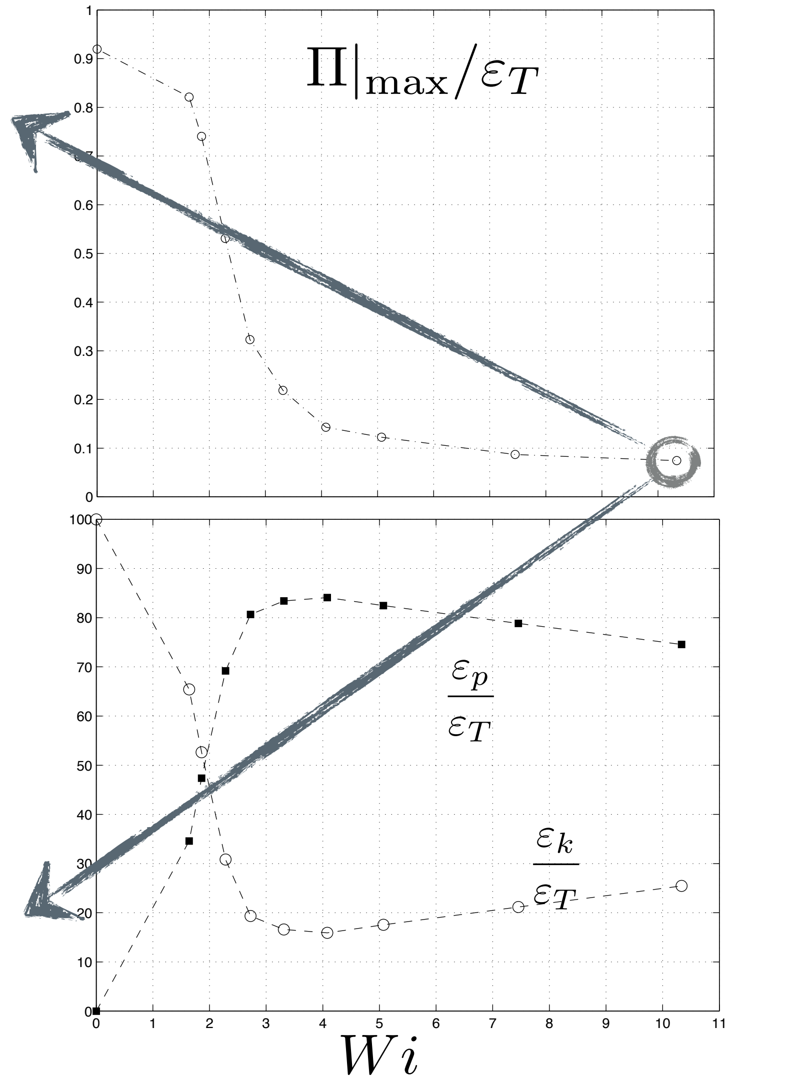
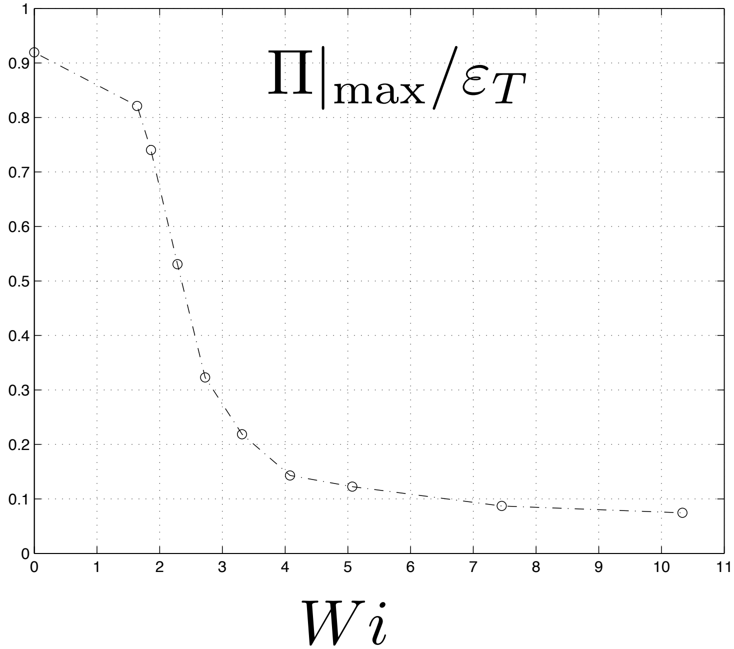


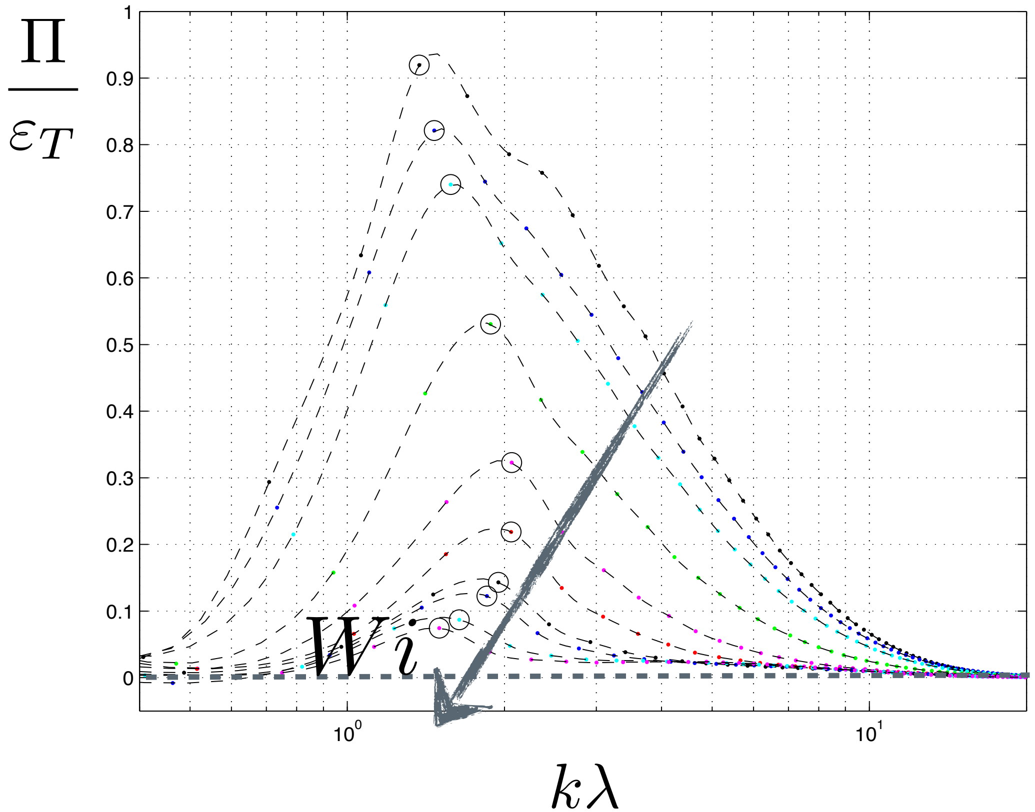


Effect of polymer additives on the energy cascade

The non-linear transfer of energy decreases with increasing `Wi'

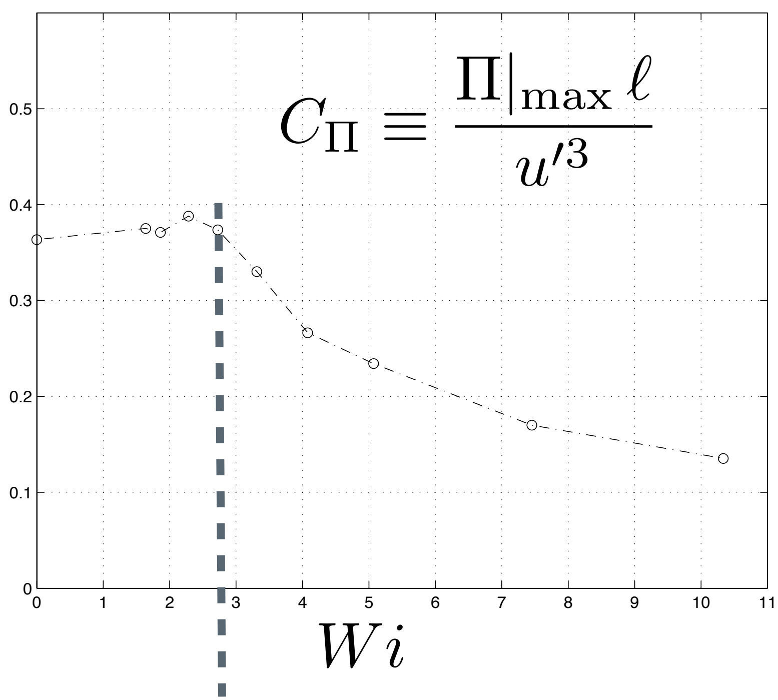
$$\Pi \equiv -\int_{k_{\min}}^{k} T(k')\,dk'$$



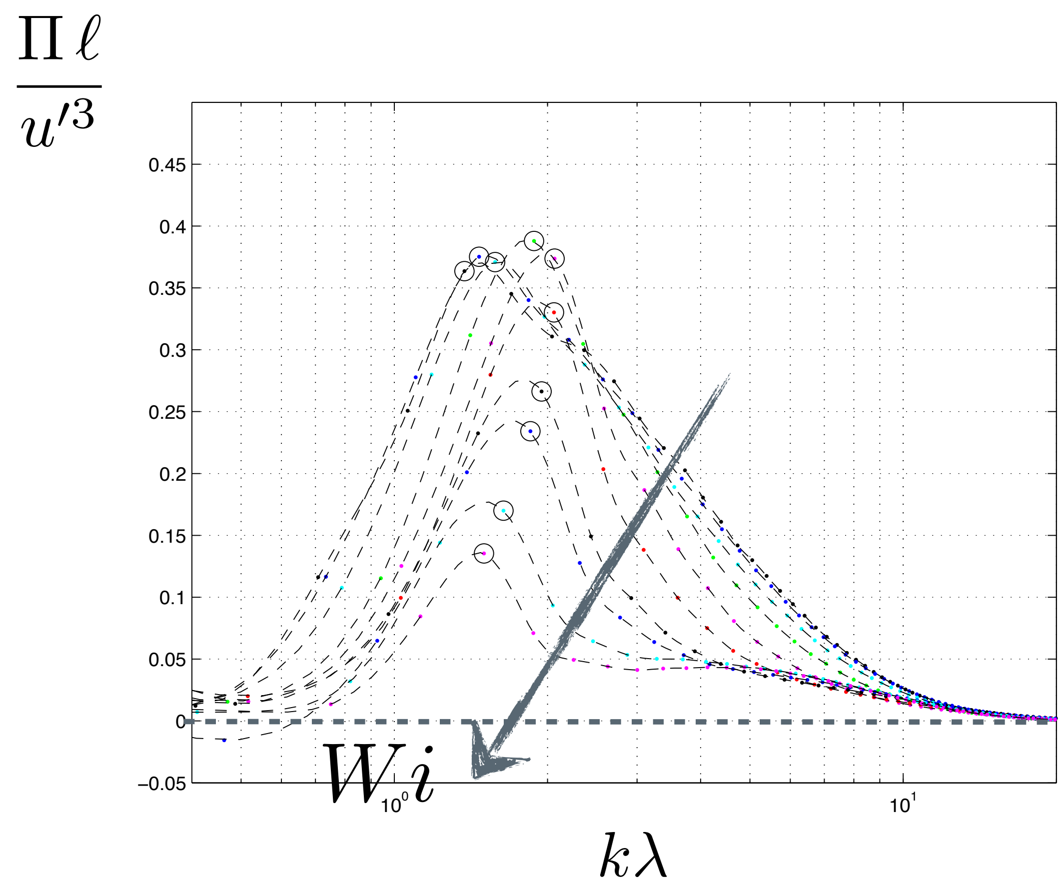


Effect of polymer additives on the energy cascade

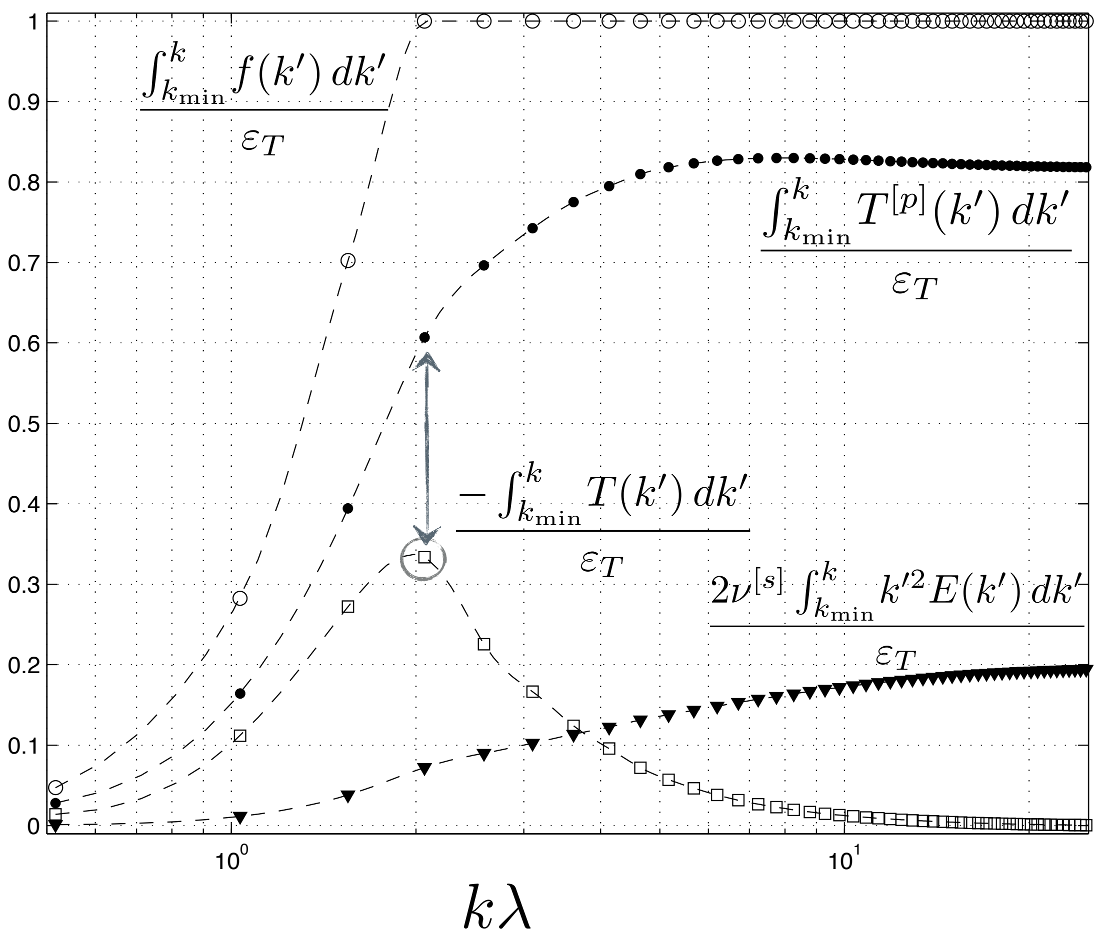
However ...



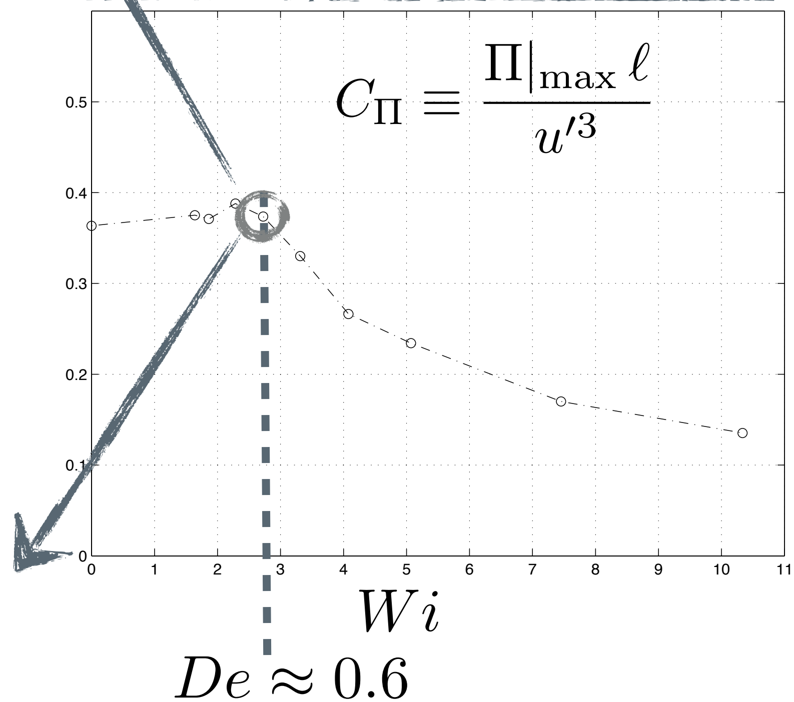
$De \approx 0.6$



Effect of polymer additives on the energy cascade



Kinetic -> Polymer transfer
overwhelms
Nonlinear energy transfer
by factor of **2**
but $C_\Pi \simeq$ **0.4**



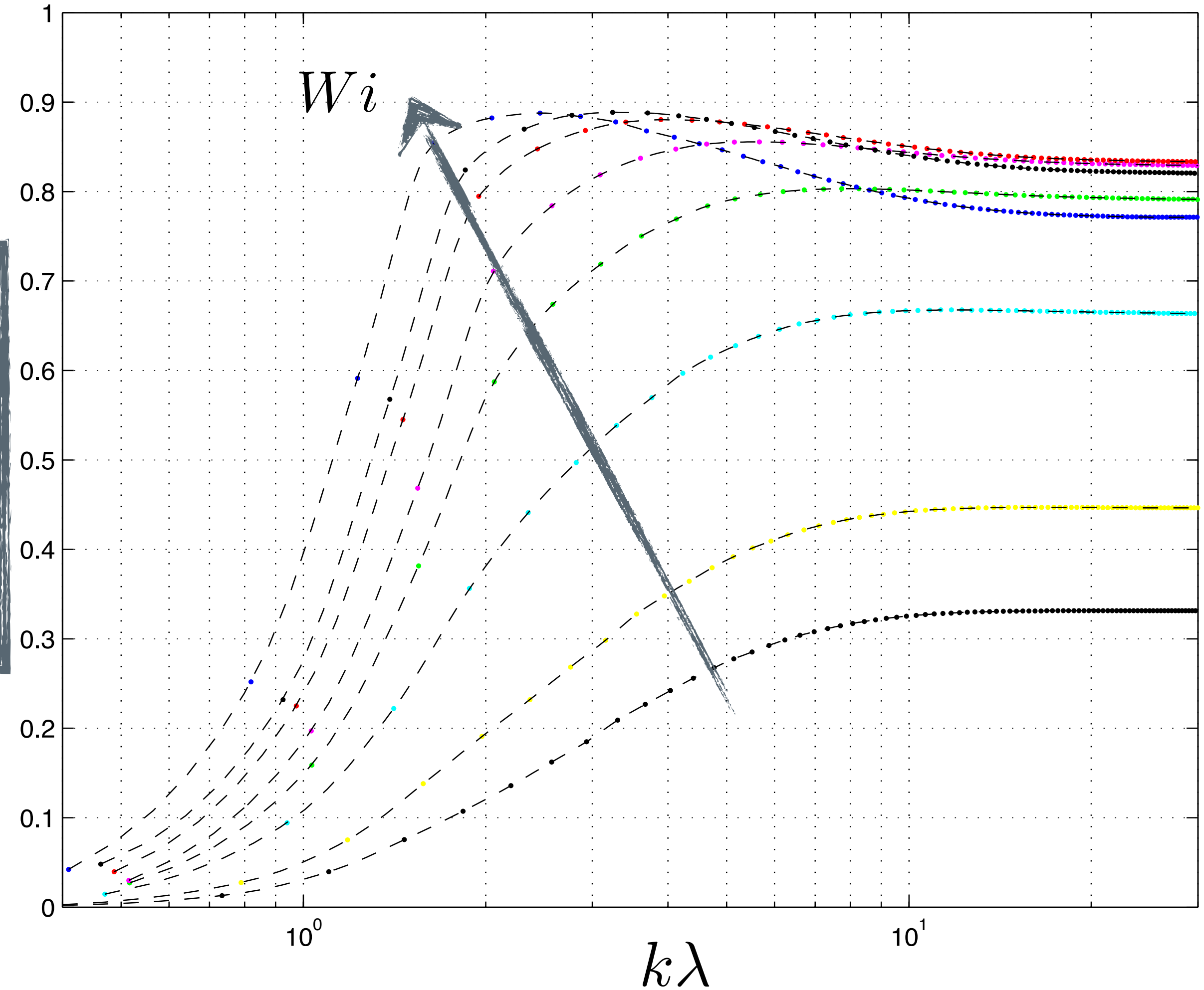
The inter-scale energy cascade caused by the polymers

The inter-scale energy transfer caused by the polymers

$$\frac{\int_{k_{\min}}^{k} T^{[p]}(k')\,dk'}{\varepsilon_T}$$

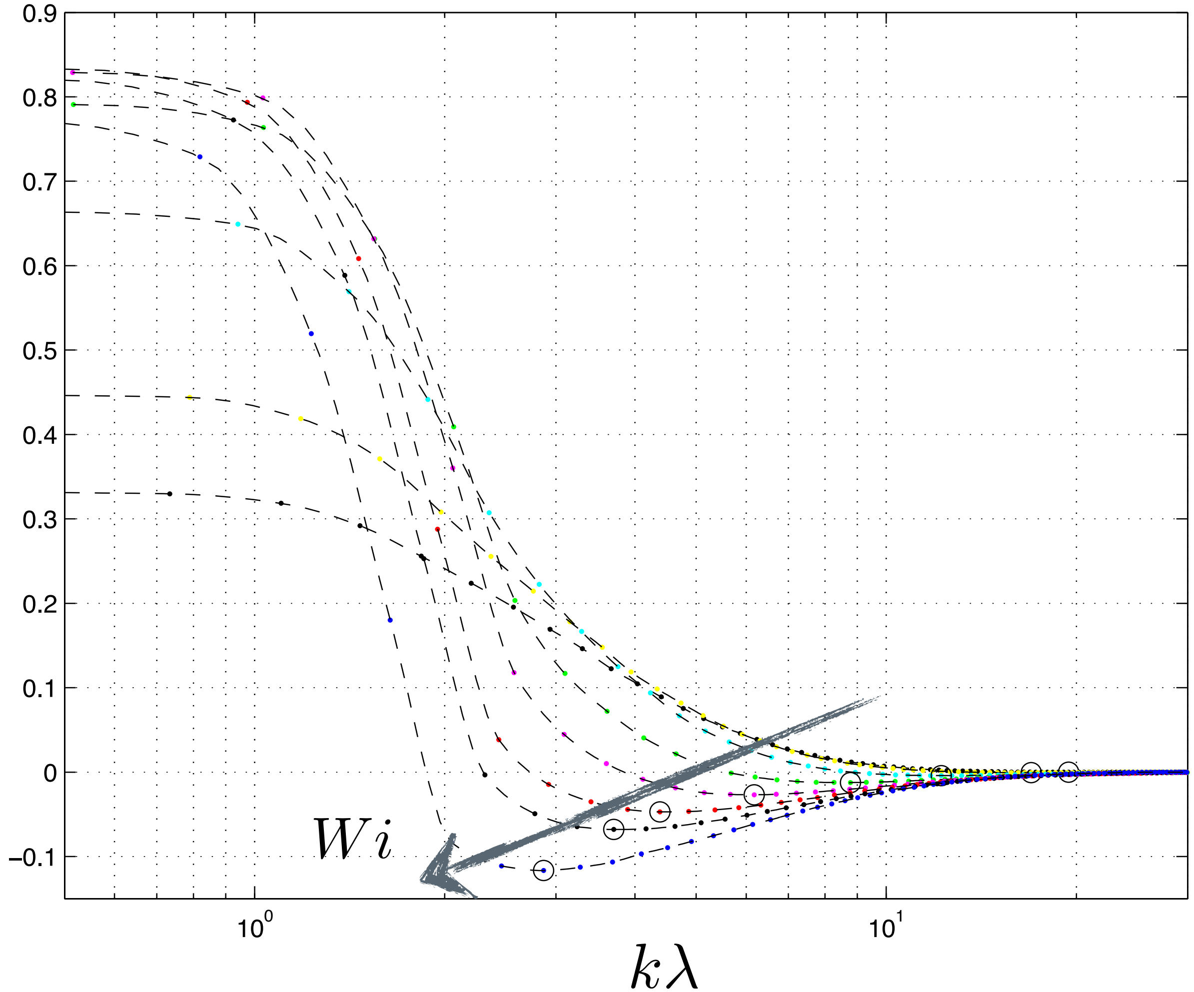
Beyond Wi ~ 3

Energy is returned back at high wavenumbers



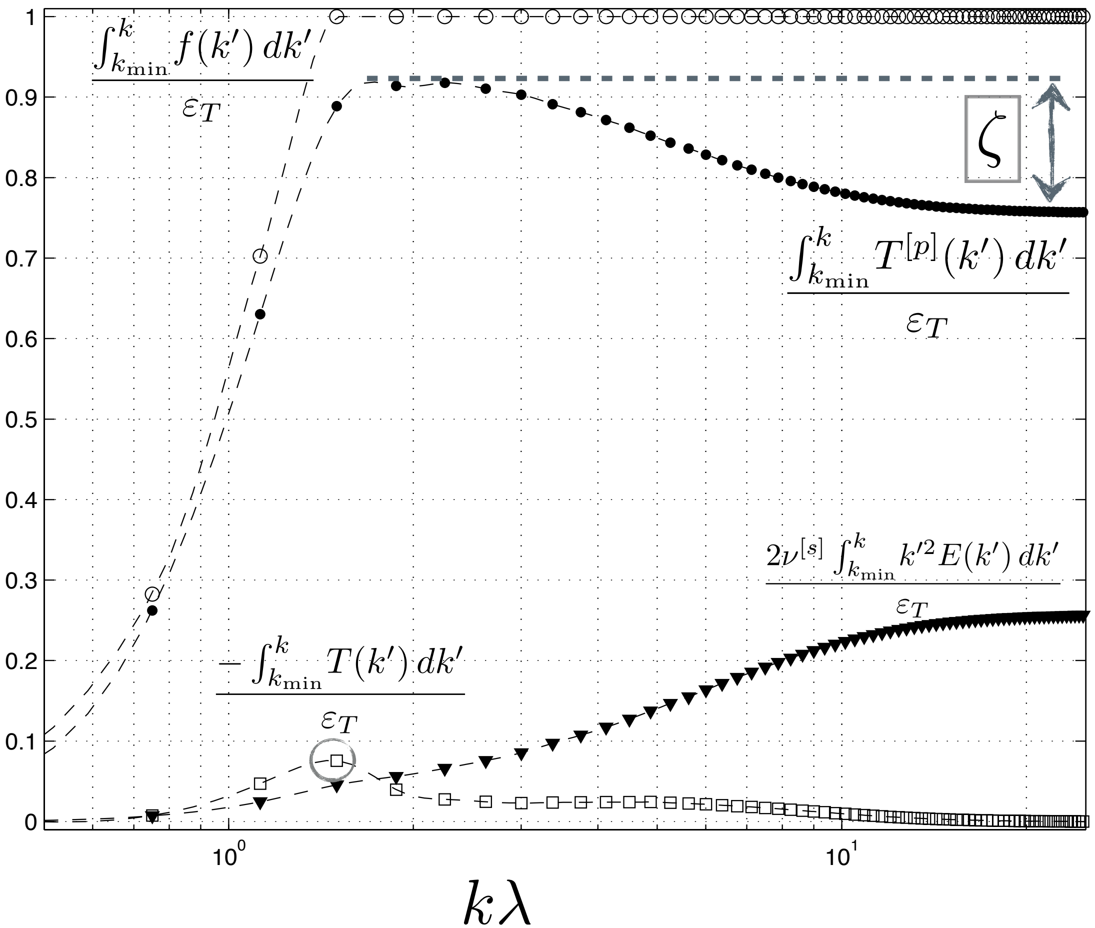
The inter-scale energy transfer caused by the polymers

$$\frac{\int_k^{k_{\max}} T^{[p]}(k')\,dk'}{\varepsilon_T}$$

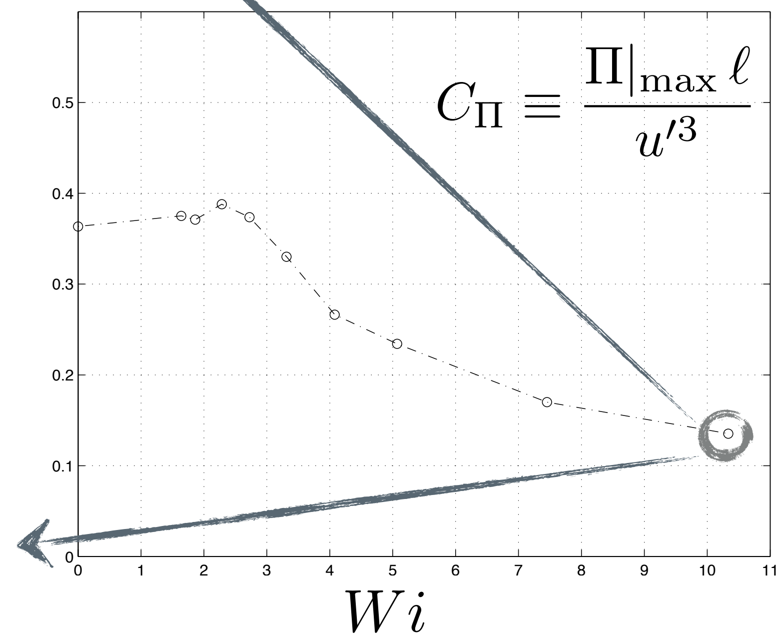


Circles: maximum energy returned to turbulence from polymers (up to 10% of total energy dissipated)

The inter-scale energy transfer caused by the polymers



ζ - *represents Polymer -> Kinetic energy transfer* at high waveno.

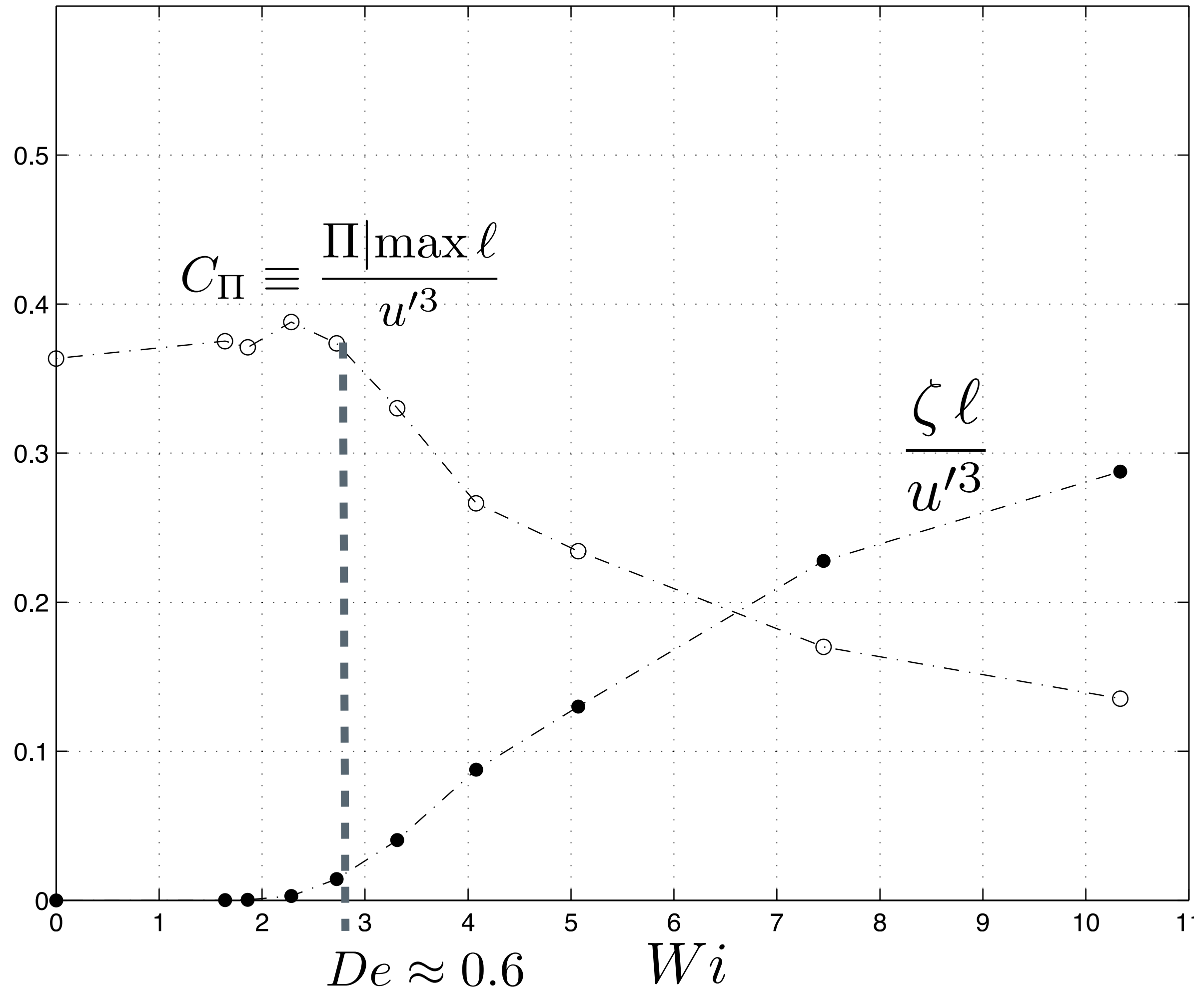


The inter-scale energy transfer caused by the polymers

The tampering of the energy cascade

seems related to

small-scale polymer energy feedback



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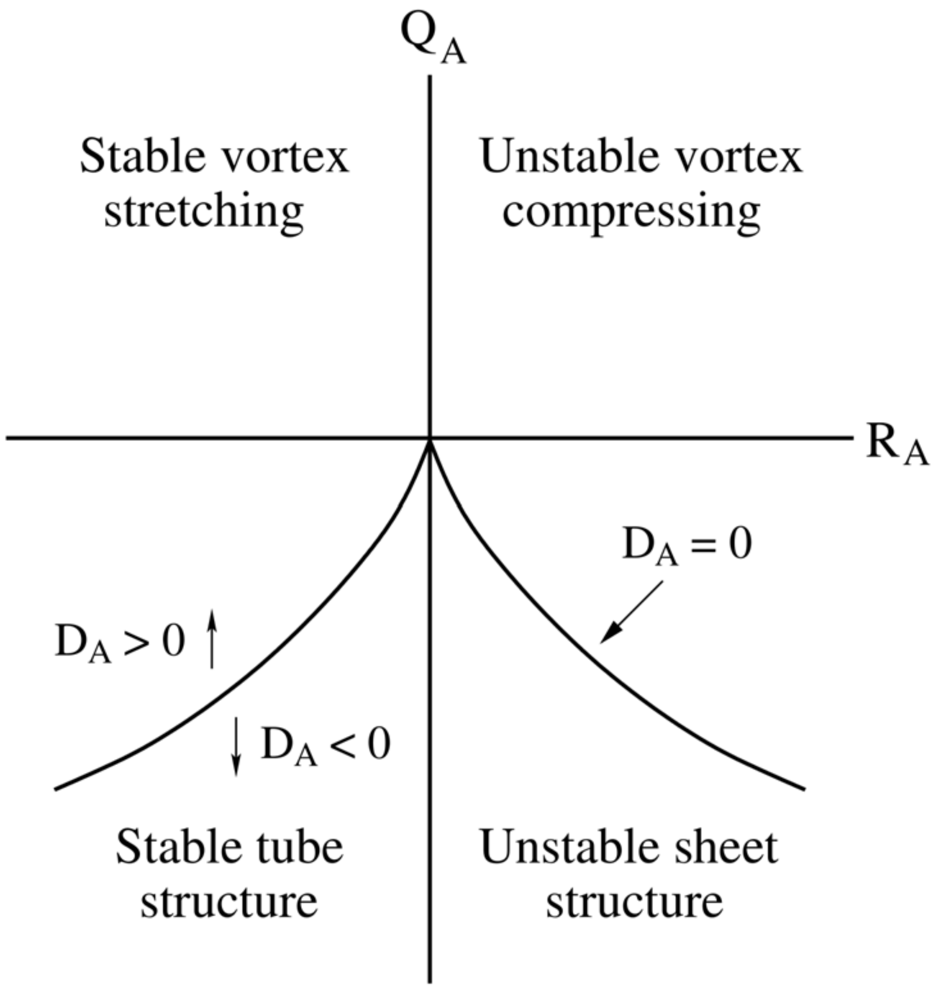
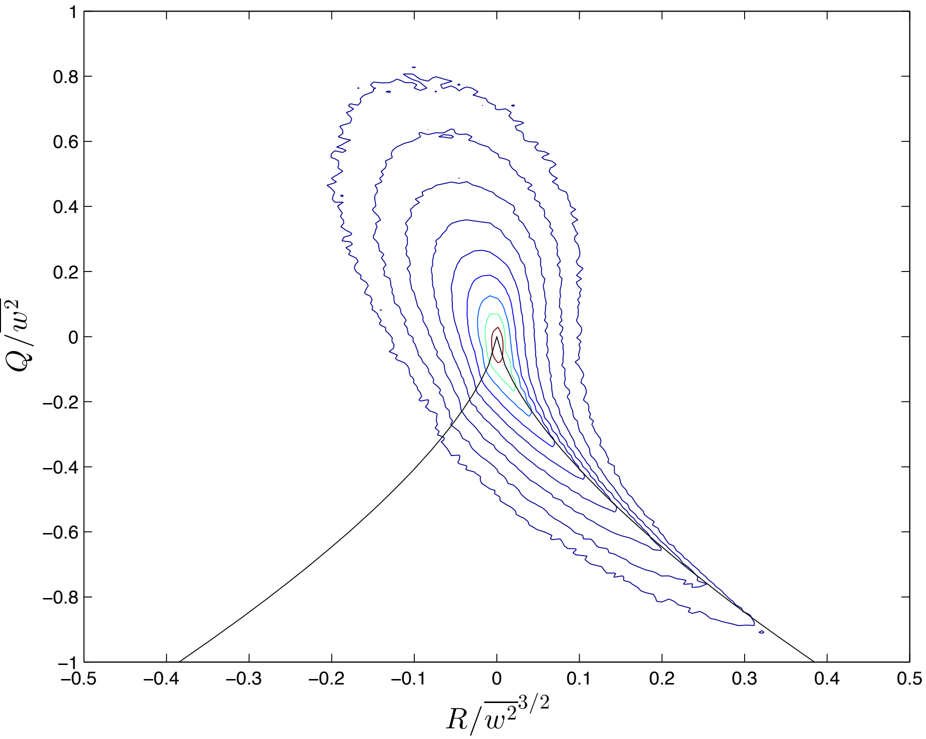
How do the geometrical statistics look like?

Q - R : Invariants of velocity derivative tensor

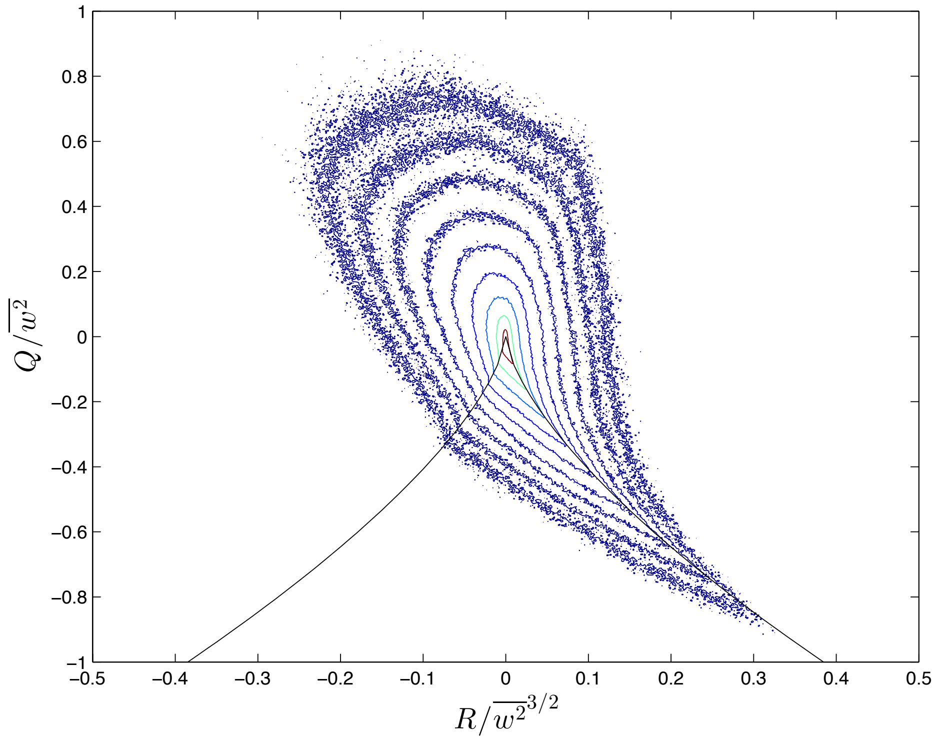


Q - R : Invariants of velocity derivative tensor

Newtonian



$Wi = 3$



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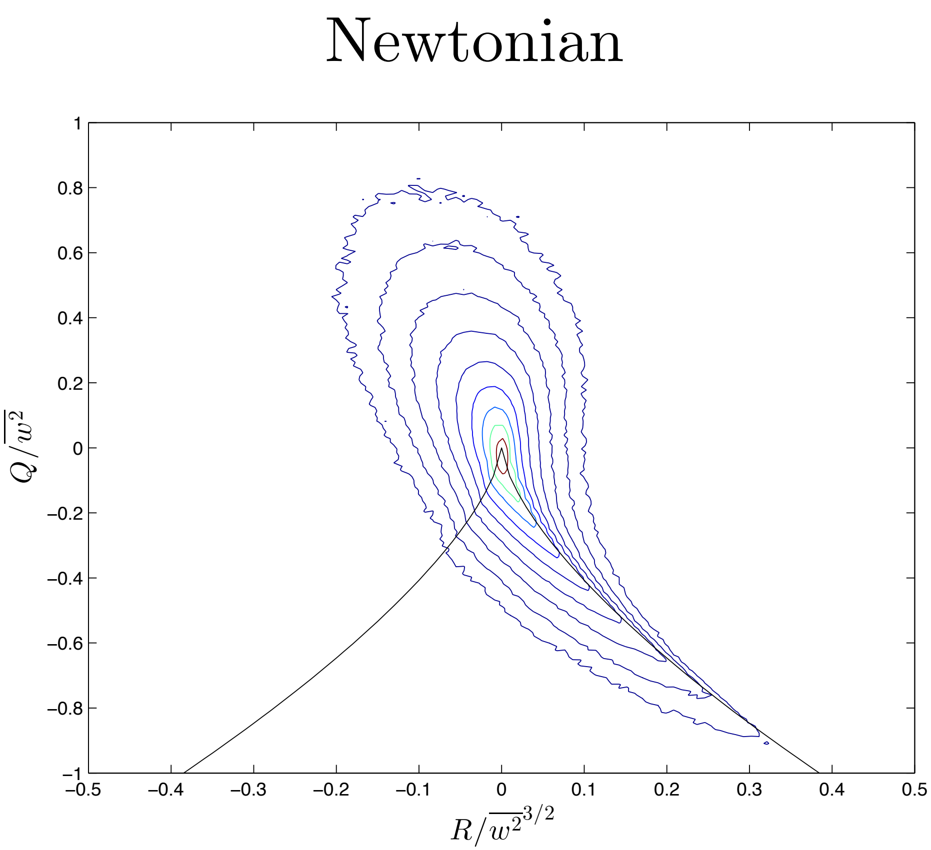
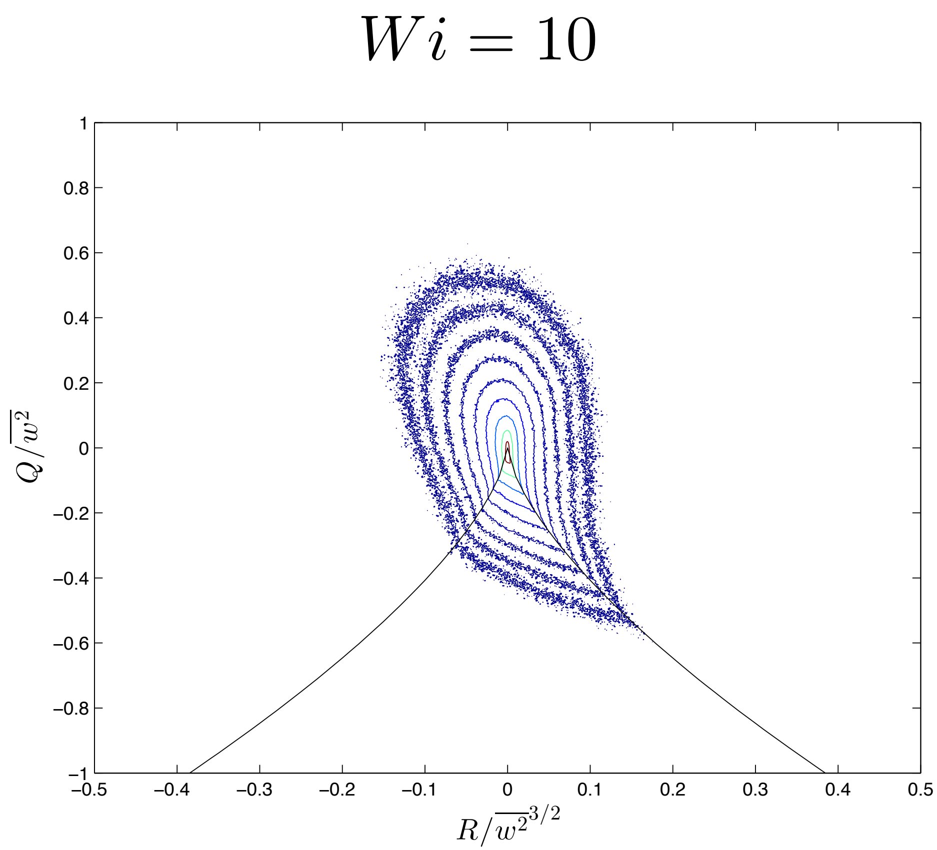
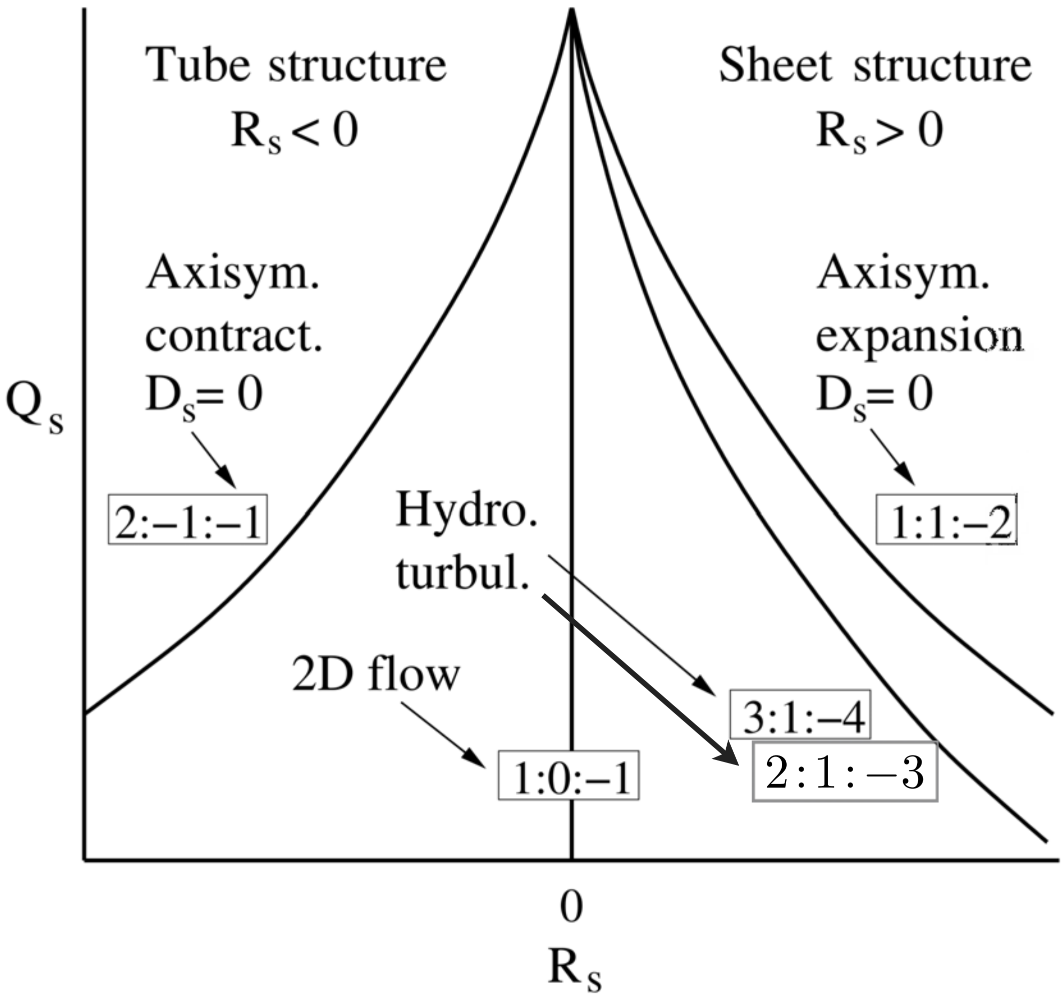




Qs- Rs : Invariants of rate of strain tensor

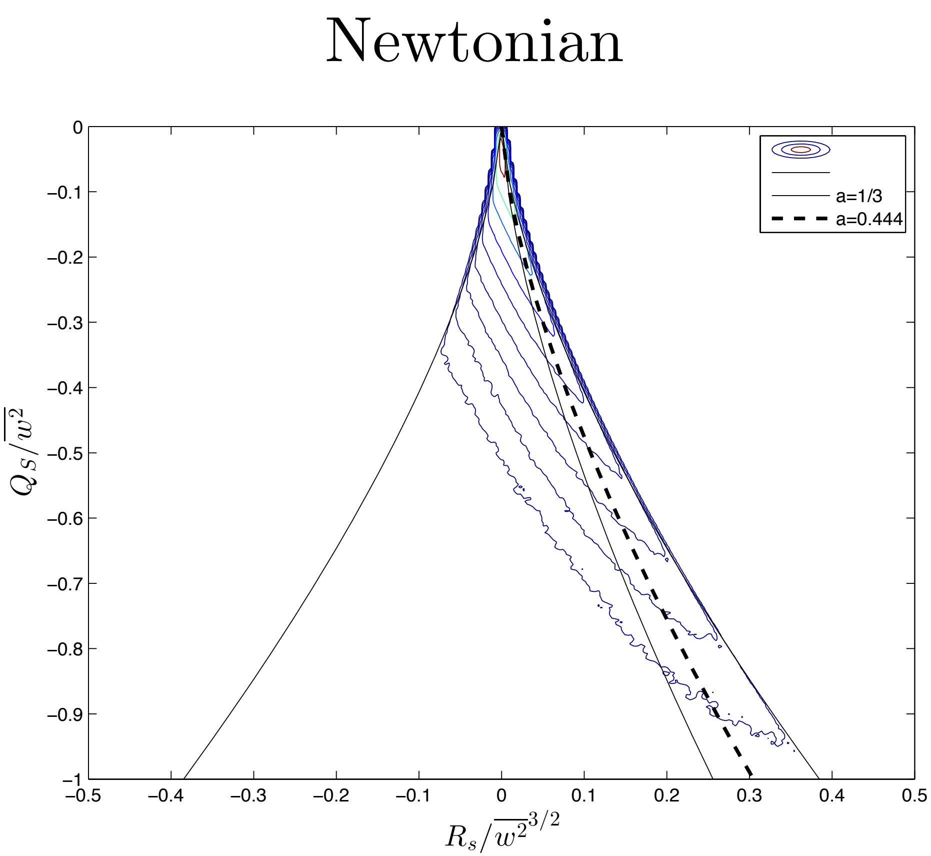


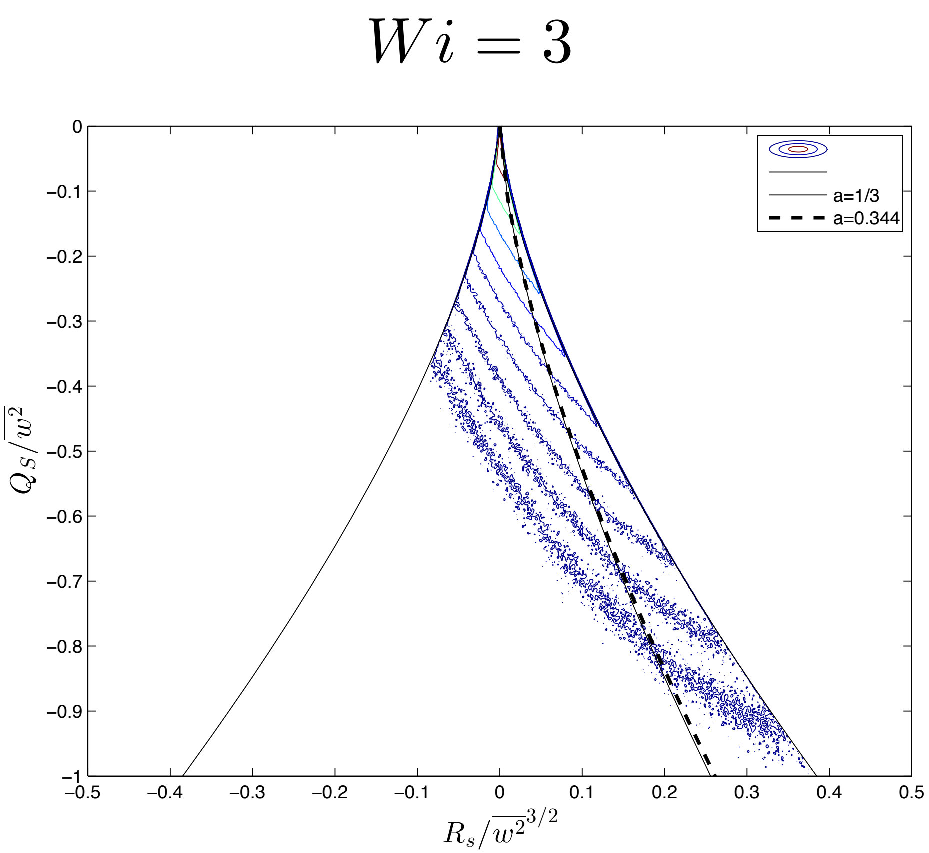
$$\lambda_1 : \lambda_2 : \lambda_3$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 0$$

$$a = \lambda_1 / \lambda_2$$

Qs- Rs : Invariants of rate of strain tensor





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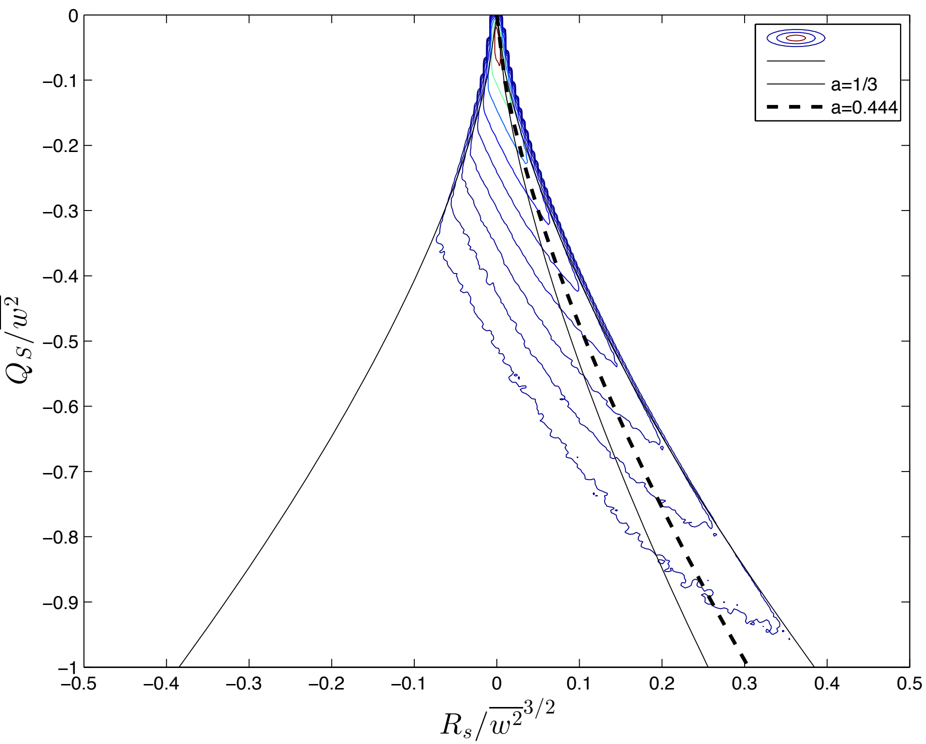
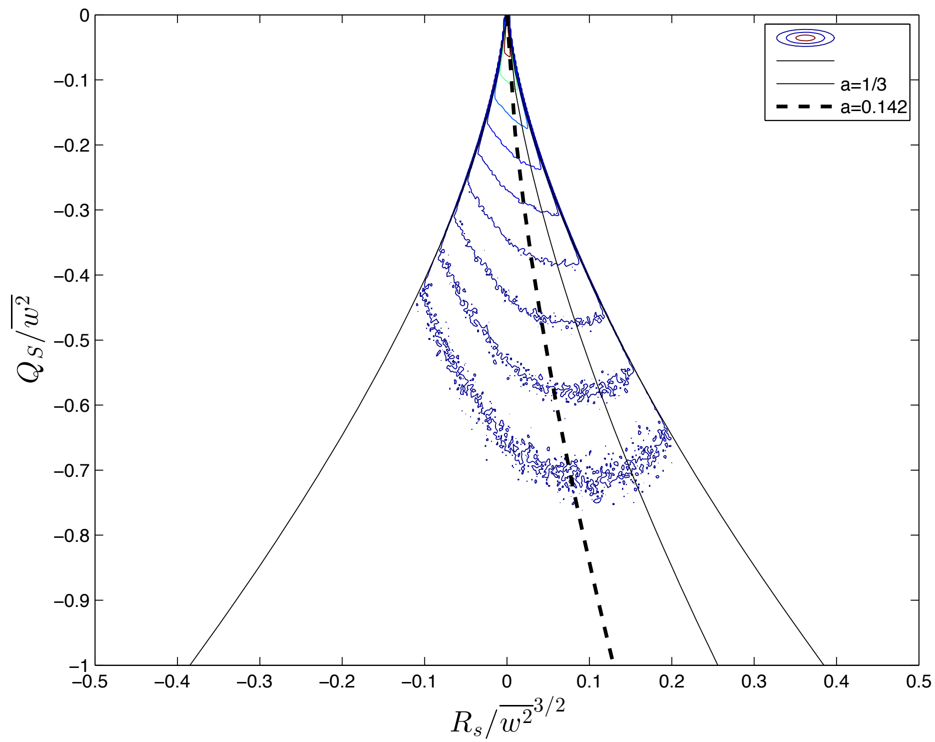
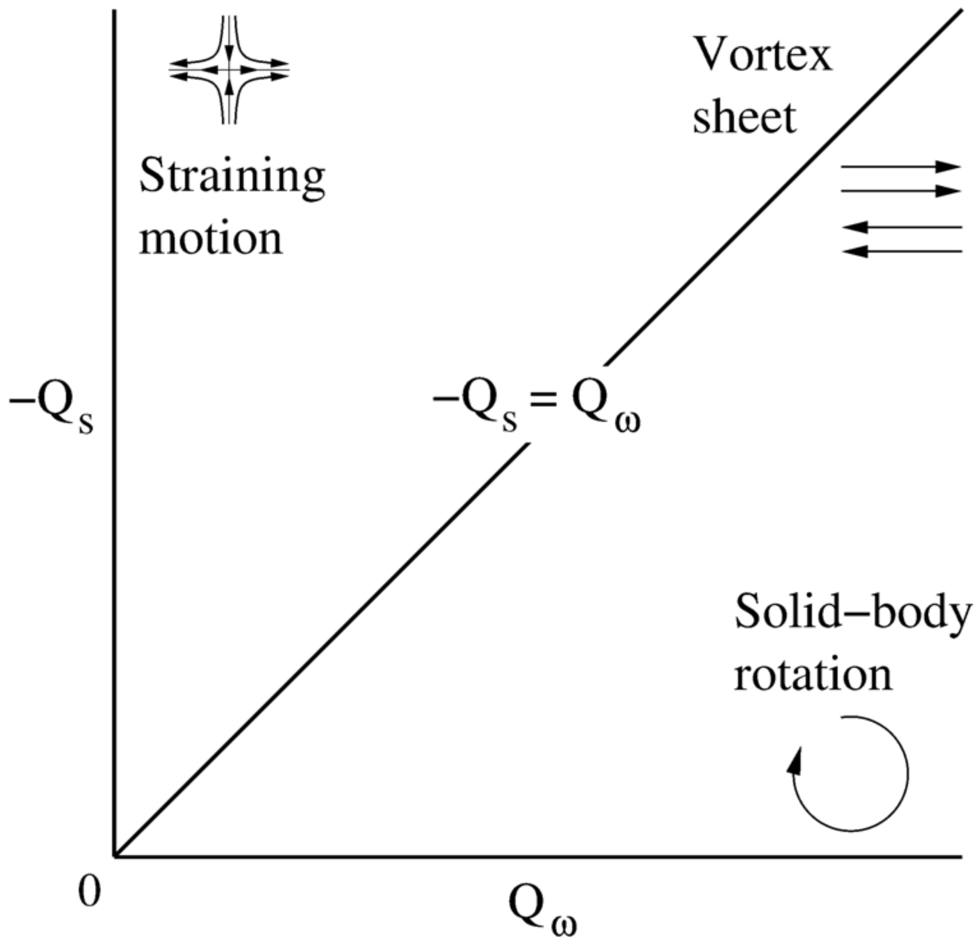


$Wi = 10$



Qs- Qw invariants



Qs- Qw invariants

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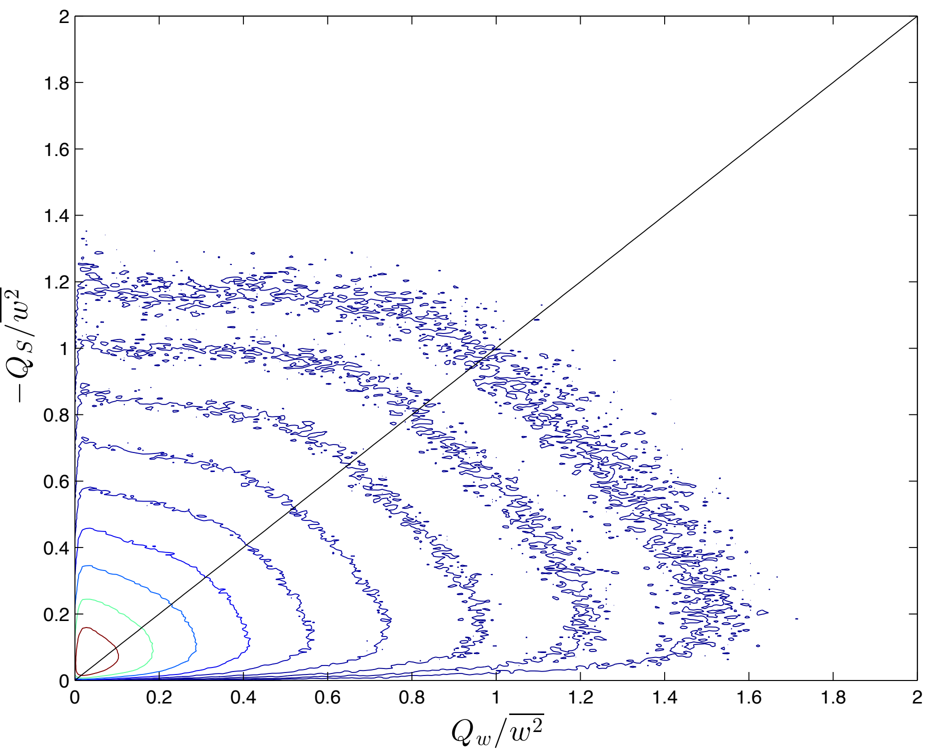

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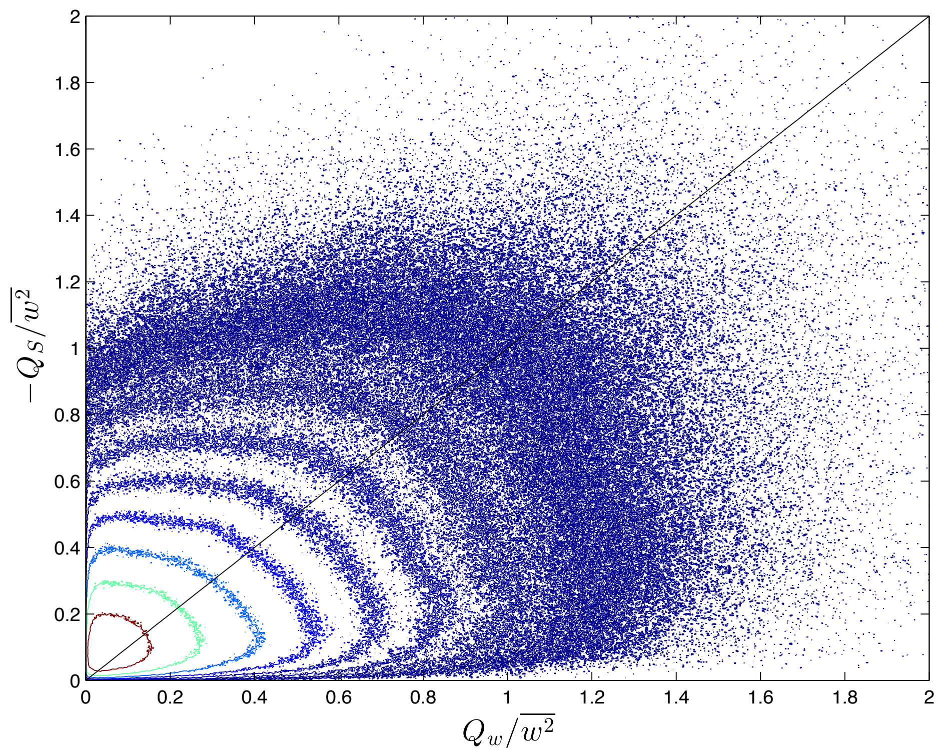
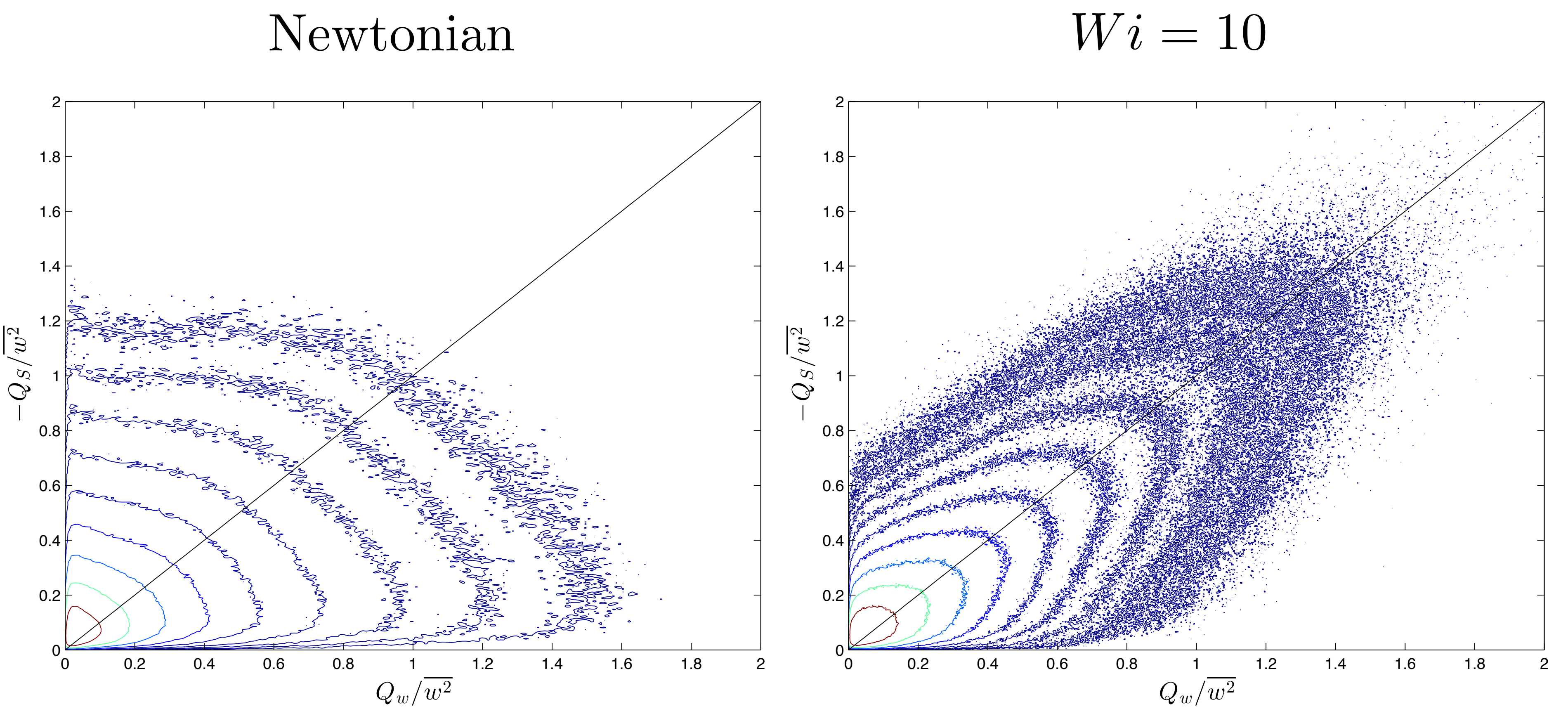


Qs- Qw invariants



Summary

Summary

- Polymers offer an additional energy dissipation mechanism causing \`drag increase'
- Little change on energy cascade flux, w.r.t eddy turnover time even when polymers dissipate 80% of the total power input but no high waveno. energy feedback from polymers
- For higher Wi, polymers remove more energy at large scales than they are able to dissipate and feedback the deficit at small scales
- Changes in cascade flux relative to turnover time seems to be closely related to high wavenumber energy feedback from polymers
- Massive changes in geometrical statistics for large Wi