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LISBOA



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Fundação para a Ciência e a Tecnologia  
MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

# Turbulent energy cascade in viscoelastic isotropic turbulence

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# Outline

- Overview of the direct numerical simulations
- Added dissipation due to polymer additives in forced HIT
- Effect of polymer additives on the turbulence energy cascade
- The inter-scale energy cascade caused by the polymers
- Summary

# DNS of statistically steady viscoelastic turbulence

## Numerical method

- Pseudo-spectral solver for velocity with 2/3<sup>rd</sup> de-aliasing
- Kurganov-Tadmor solver for conformation tensor
- Third-order Runge-Kutta in time
- FENE-P model for the polymer stress
- Alvelius (1999) forcing on first 4 waveno.

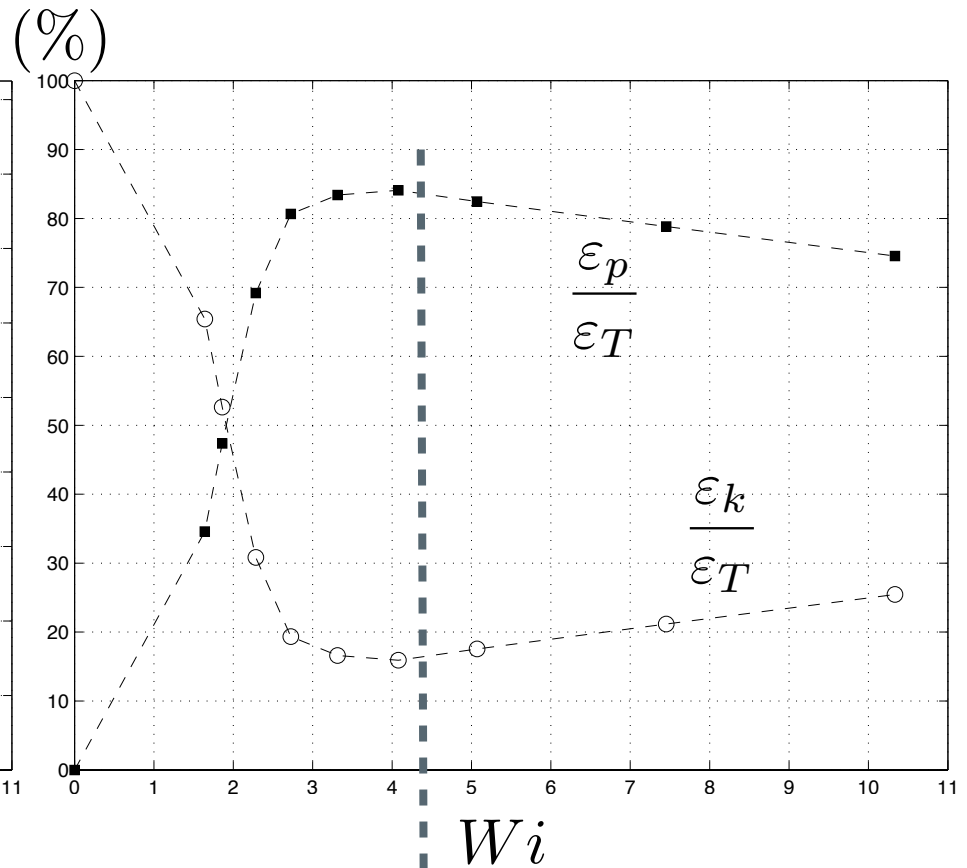
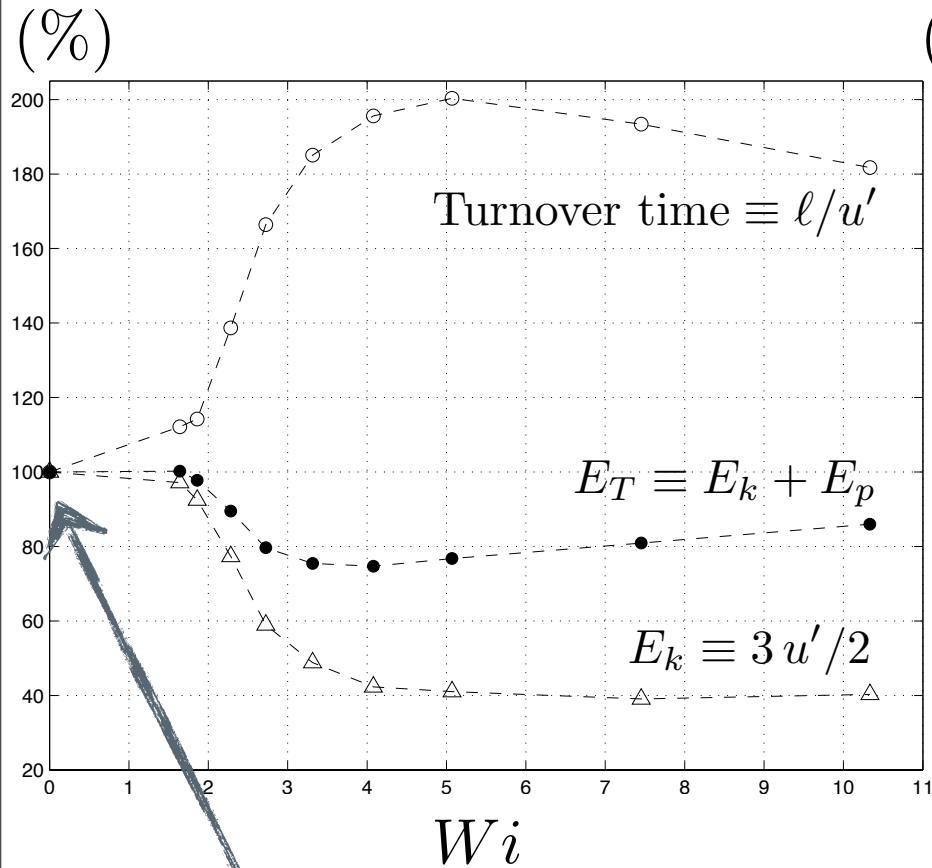
$$T_{ij}^{[p]} \equiv \frac{\mu^{[p]}}{\tau} \left( \frac{L^2 - 3}{L^2 - C_{ii}} C_{ij} - \delta_{ij} \right)$$

## DNS parameters:

- $N=192^3$  (statistically steady)
- Polymer concentration:  $\beta = 0.8$
- Relaxation time:  $\left\{ \begin{array}{l} \tau = [0.1, 0.125, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0] \\ Wi \equiv \tau/(\nu/\varepsilon^{[N]})^{1/2} = [1.6, \dots, 10.4] \\ De \equiv \tau/(\ell/u') = [0.3, \dots, 1.8] \end{array} \right.$
- Max. polymer extension:  $L = 100, \quad 3\% < \sqrt{C_{ii}}/L < 39\%$
- Reynolds number:  $50 < Re_\lambda < 70$

Added dissipation due to polymer  
additives in forced HIT

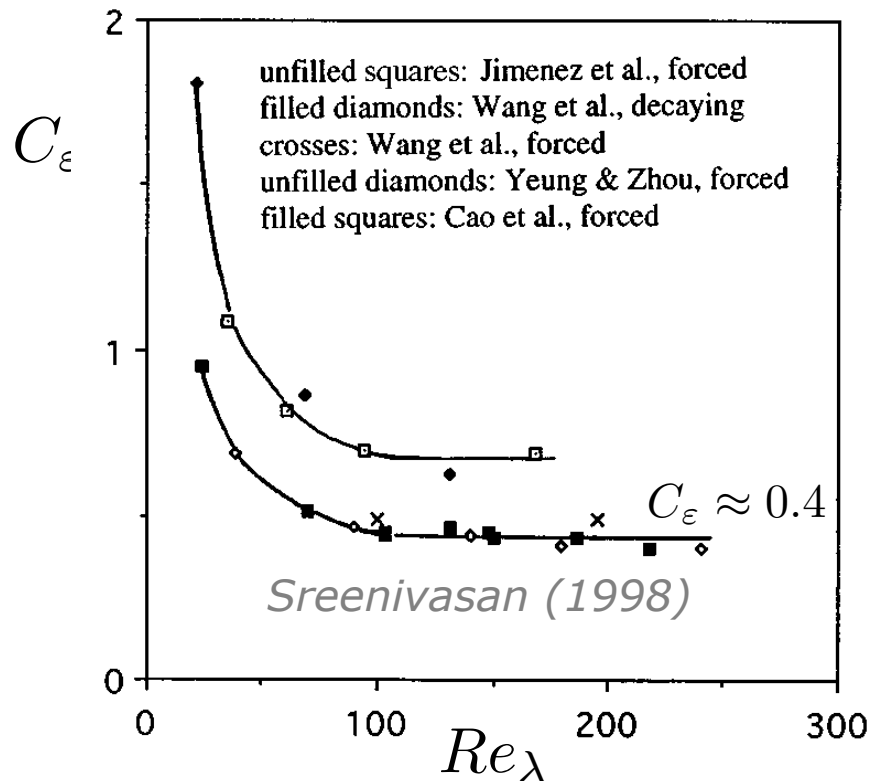
# Added dissipation due to polymer additives



# Added dissipation due to polymer additives - II

Recall: In Newtonian turbulent flows the rate of dissipation is inversely proportional to turnover time for a given turbulent kinetic energy

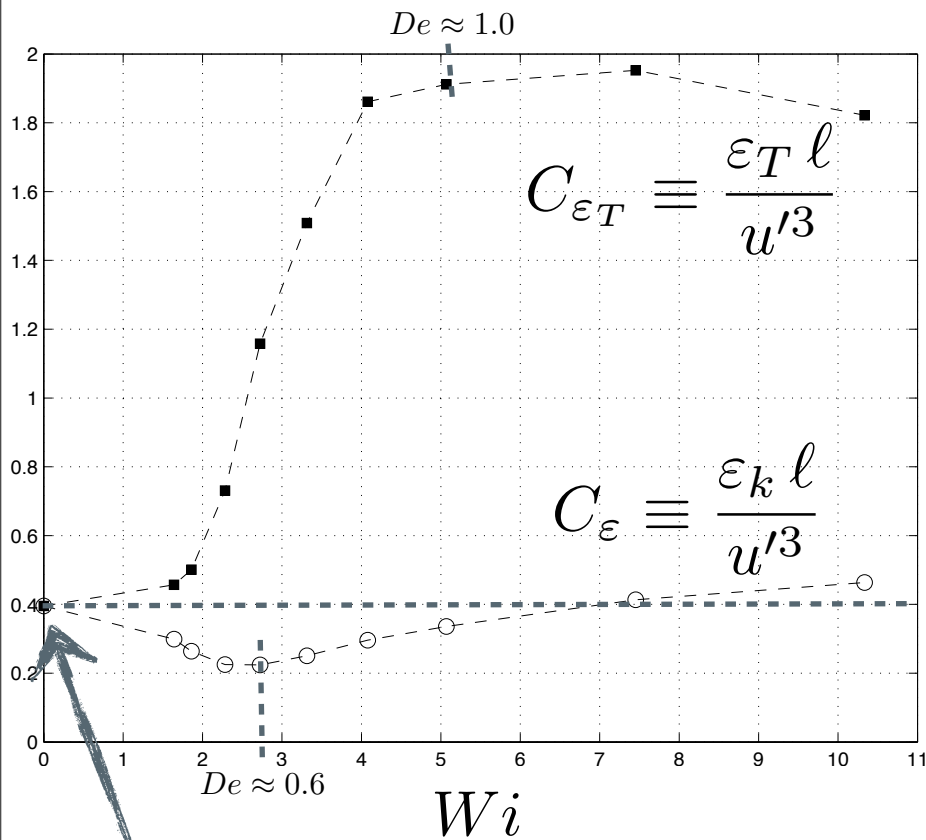
$$\varepsilon_k = C_\varepsilon u'^3 / \ell \sim \frac{u'^2}{\ell / u'}$$



Previous slide:

- Decrease in kinetic energy
- Increase in turnover time
- Thus decrease in dissipation
- What about  $C_\varepsilon$  ?

# Added dissipation due to polymer additives - III



Newtonian reference

- The polymer 'hampers' the effectiveness of turbulence in dissipating kinetic energy
- However, the polymer additives efficiently transfer kinetic to elastic energy and dissipate it
- Overall there is 'drag' increase

# Effect of polymer additives on the turbulence energy cascade



# Effect of polymer additives on the energy cascade

Lin equation for statistically steady homogeneous viscoelastic turbulence

$$f(k) = -T(k) + T^{[p]}(k) + 2\nu^{[s]}k^2 E(k)$$

External force

Non-linear transfer spectrum

Kinetic to elastic energy transfer spectrum

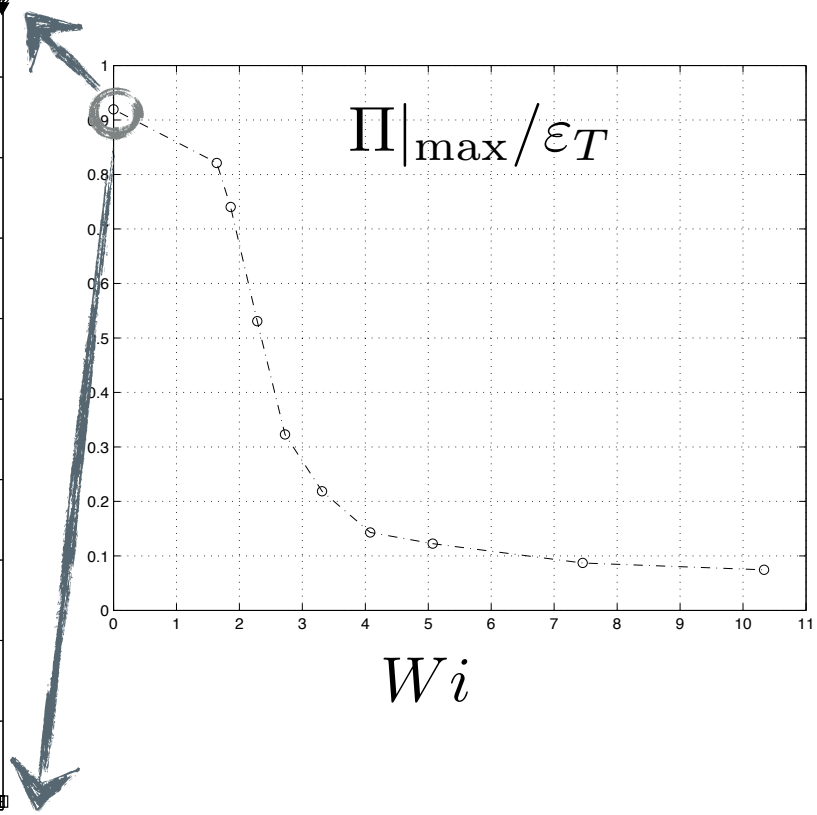
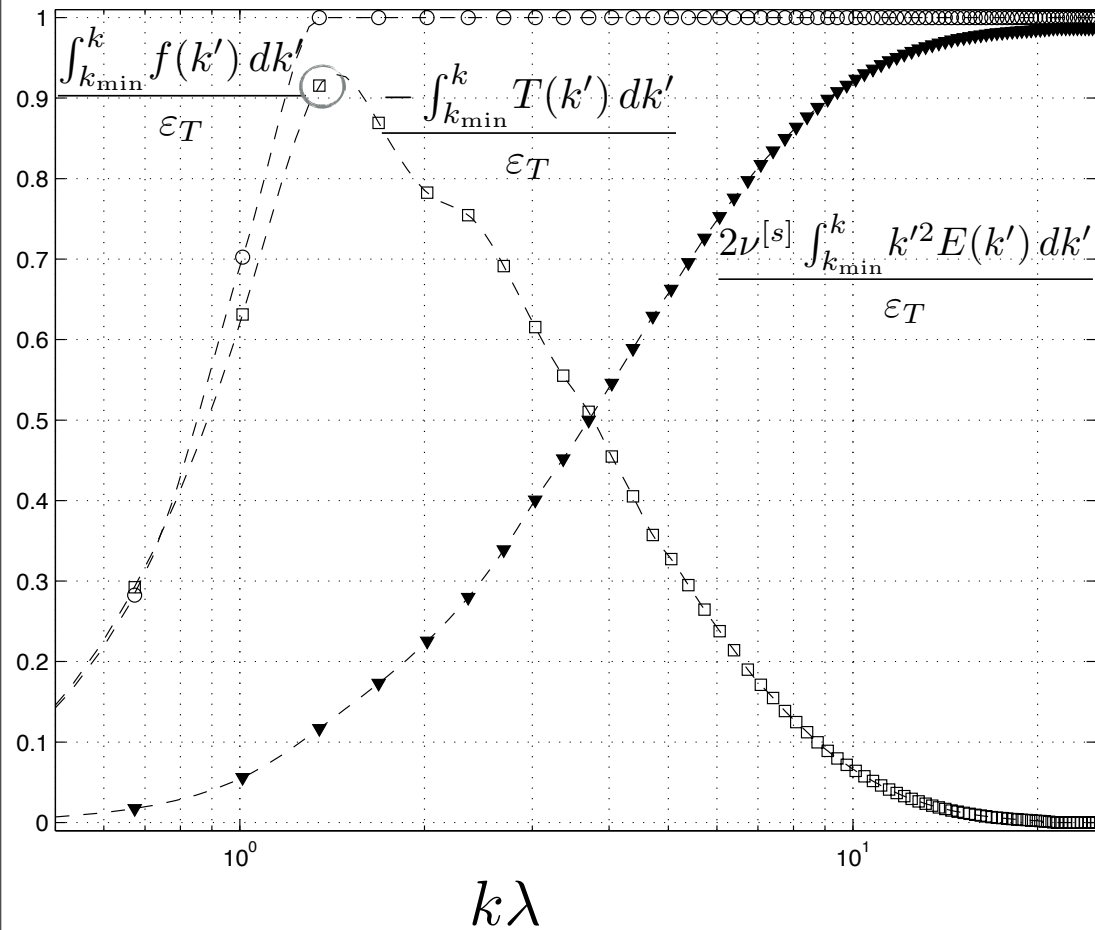
Dissipation spectrum

$$\int_{k_{\min}}^{k_{\max}} f(k) dk = P_{\text{in}} \left\{ \underbrace{\int_{k_{\min}}^{k_{\max}} T^{[p]}(k) dk}_{\varepsilon_p} + \underbrace{2\nu^{[s]} \int_{k_{\min}}^{k_{\max}} k^2 E(k) dk}_{\varepsilon_k} \right.$$

$$\int_{k_{\min}}^{k_{\max}} T(k) dk = 0$$

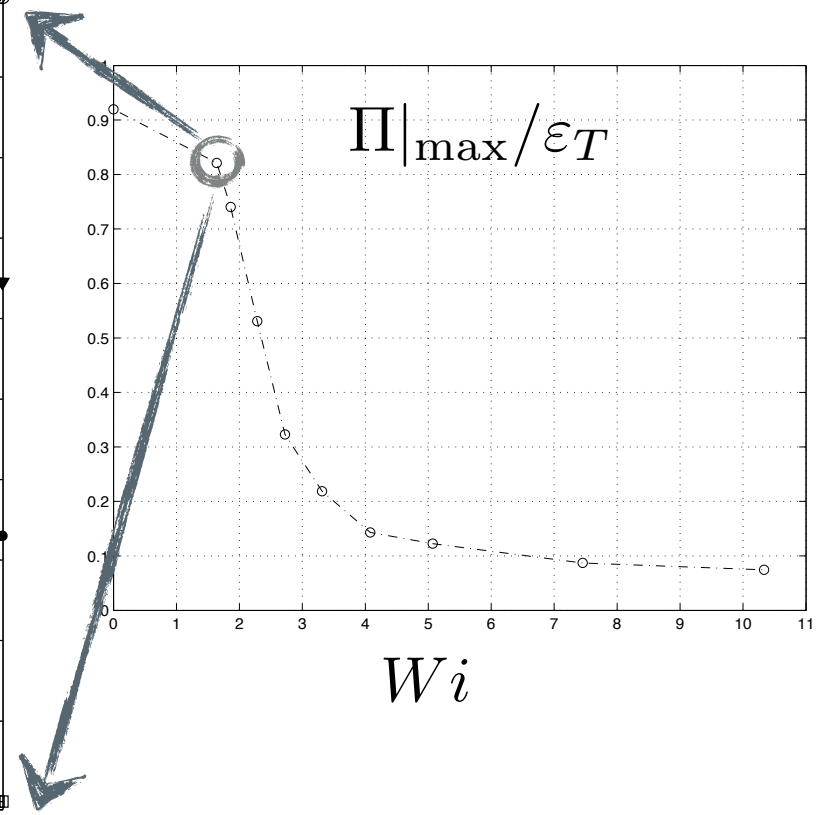
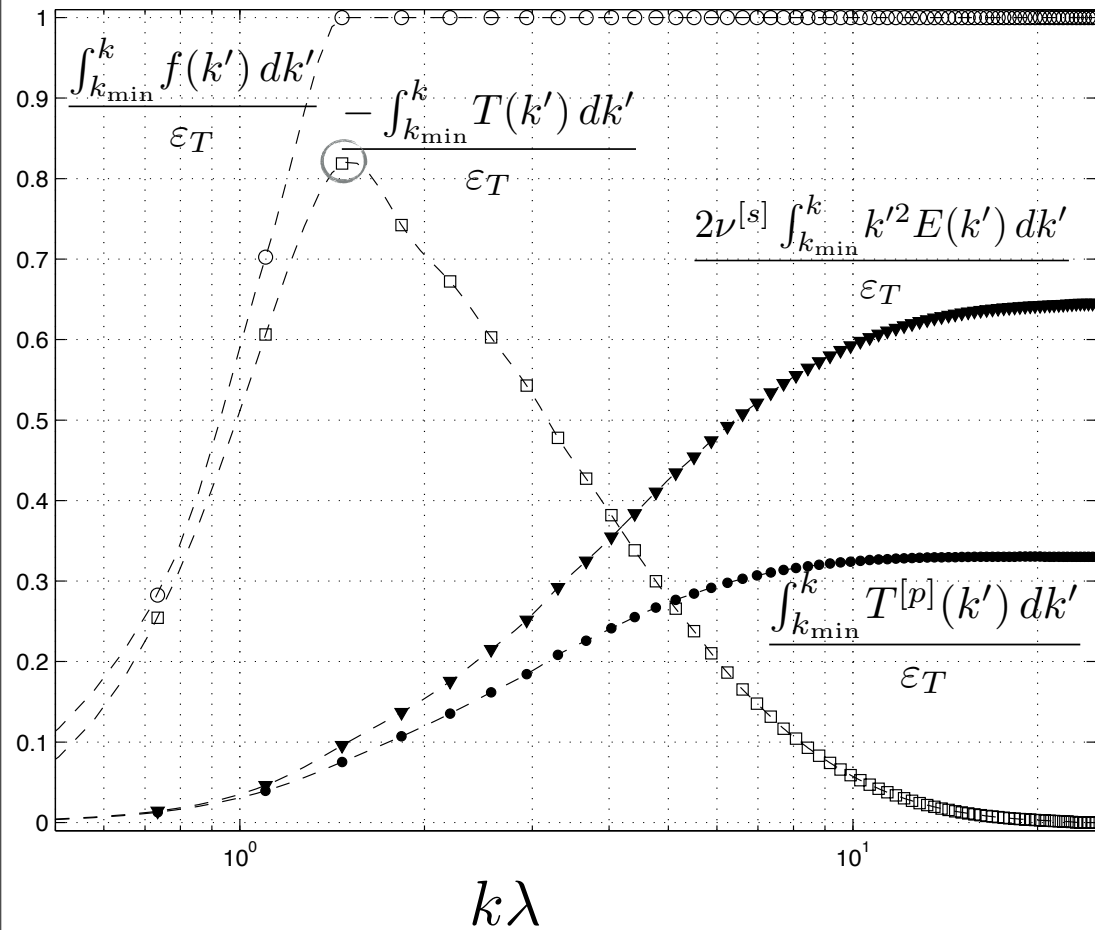
# Effect of polymer additives on the energy cascade

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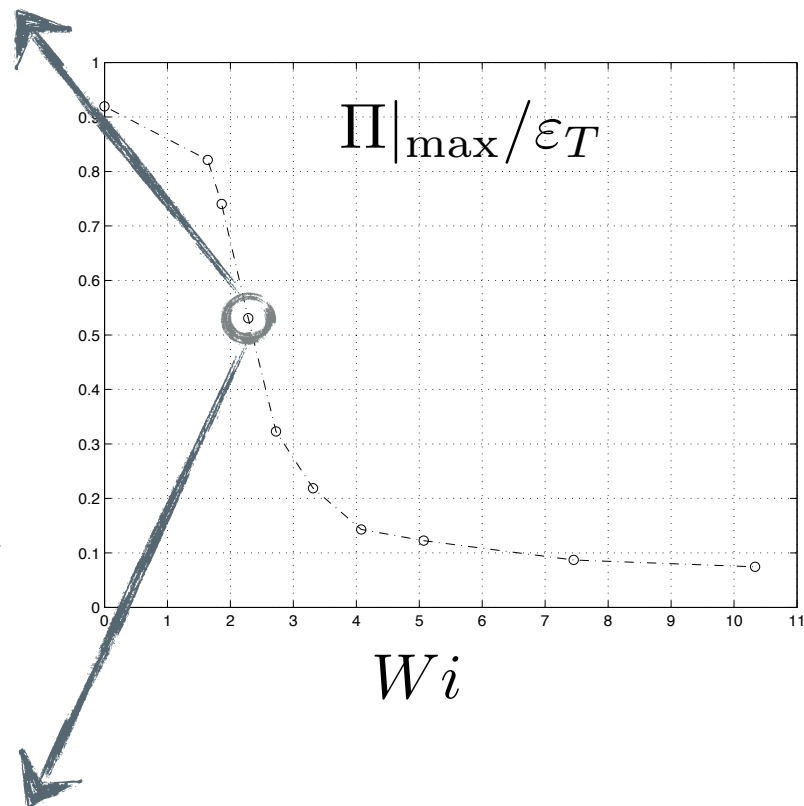
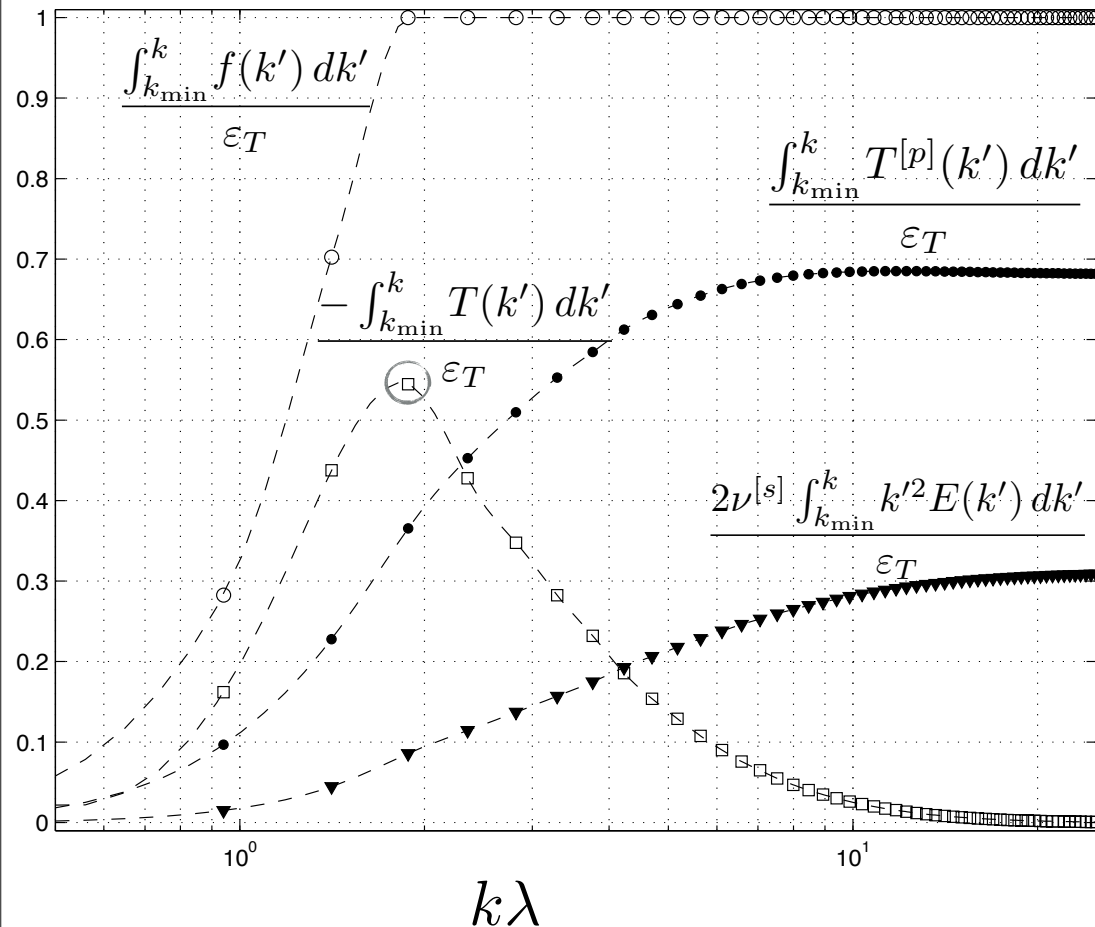
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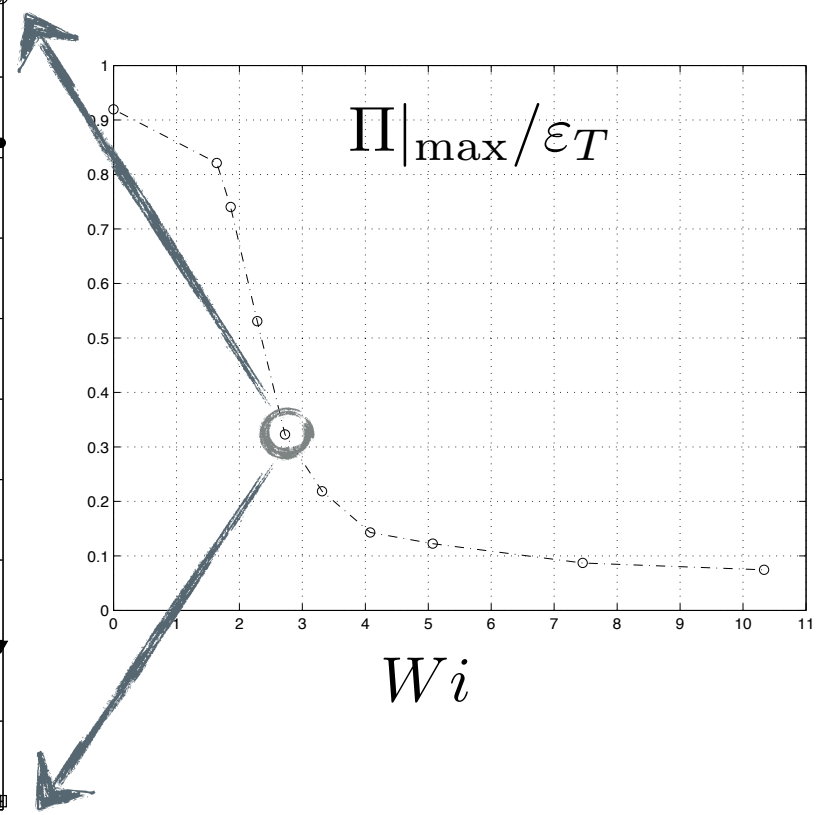
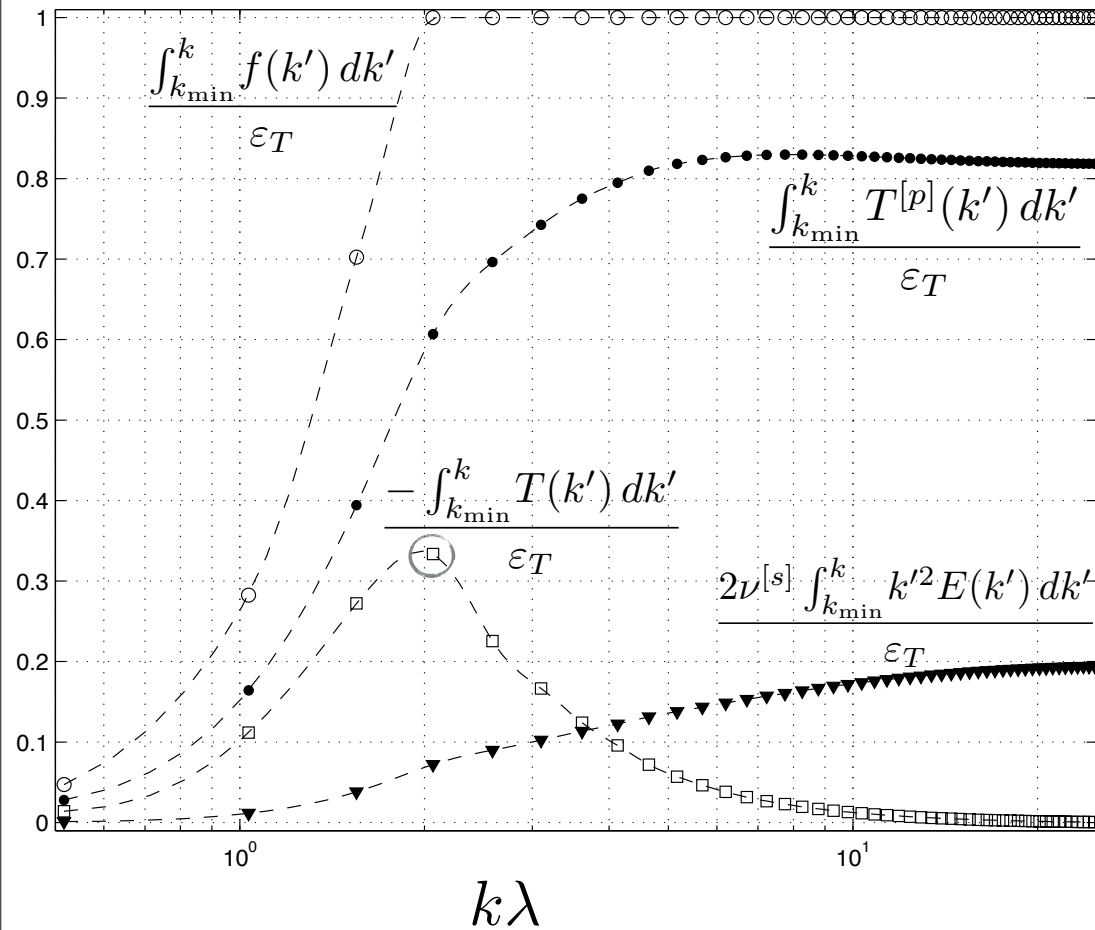
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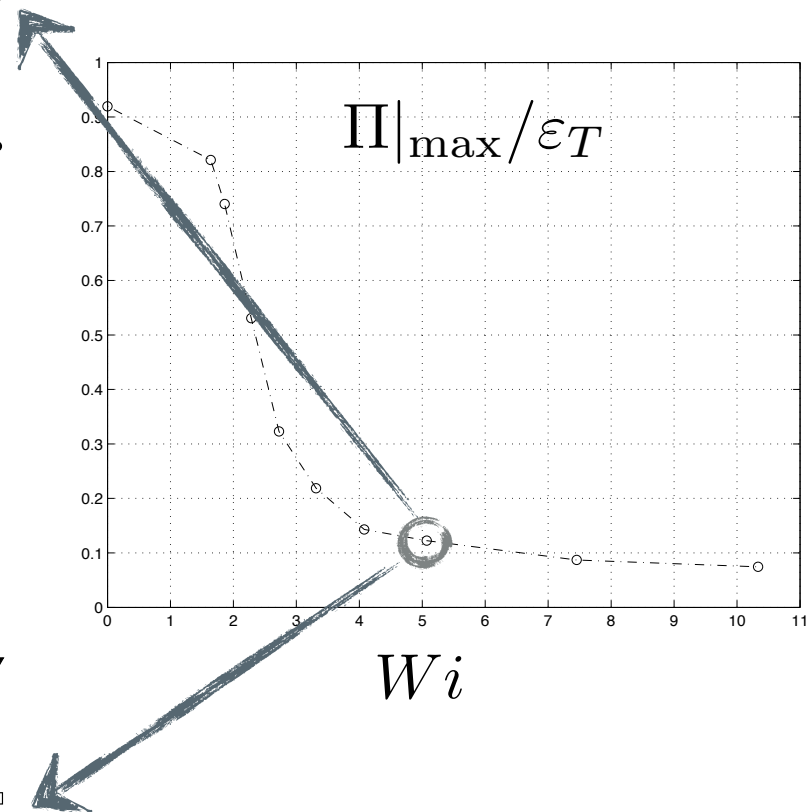
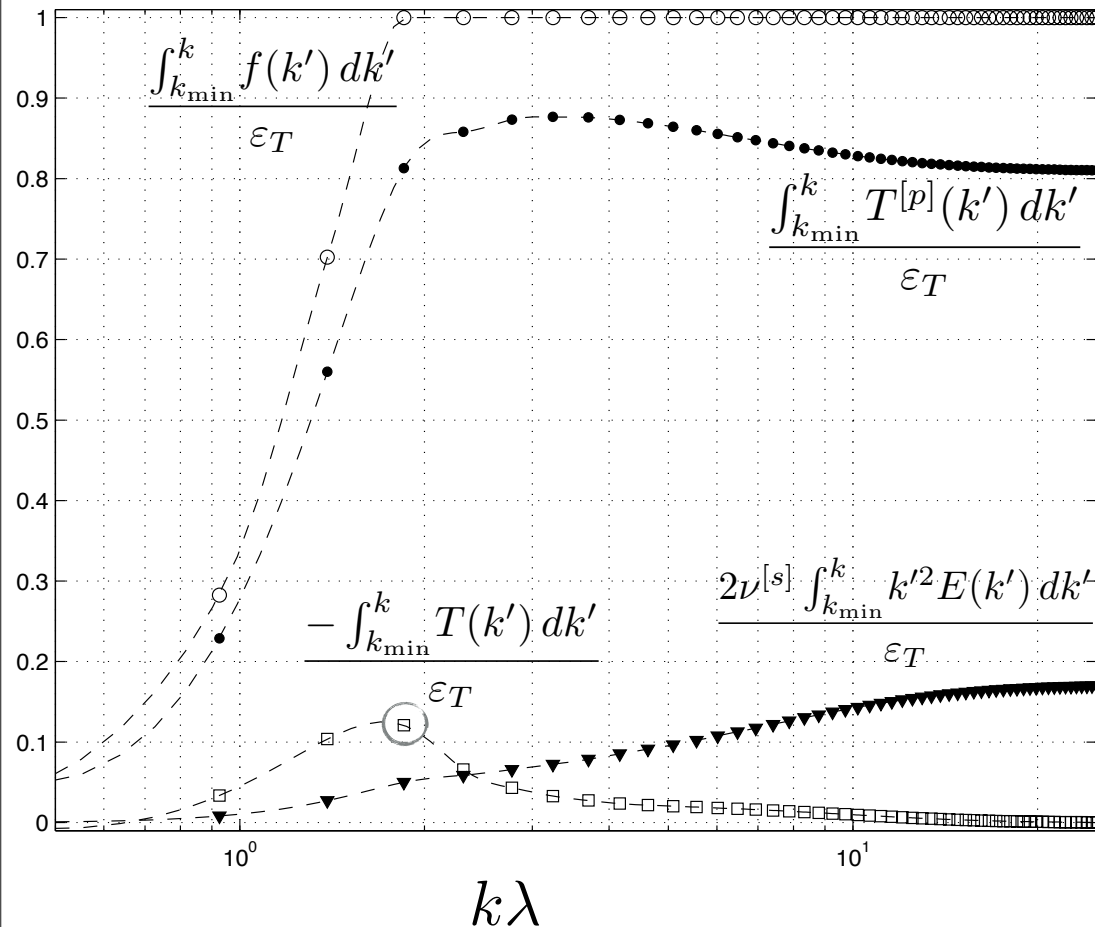
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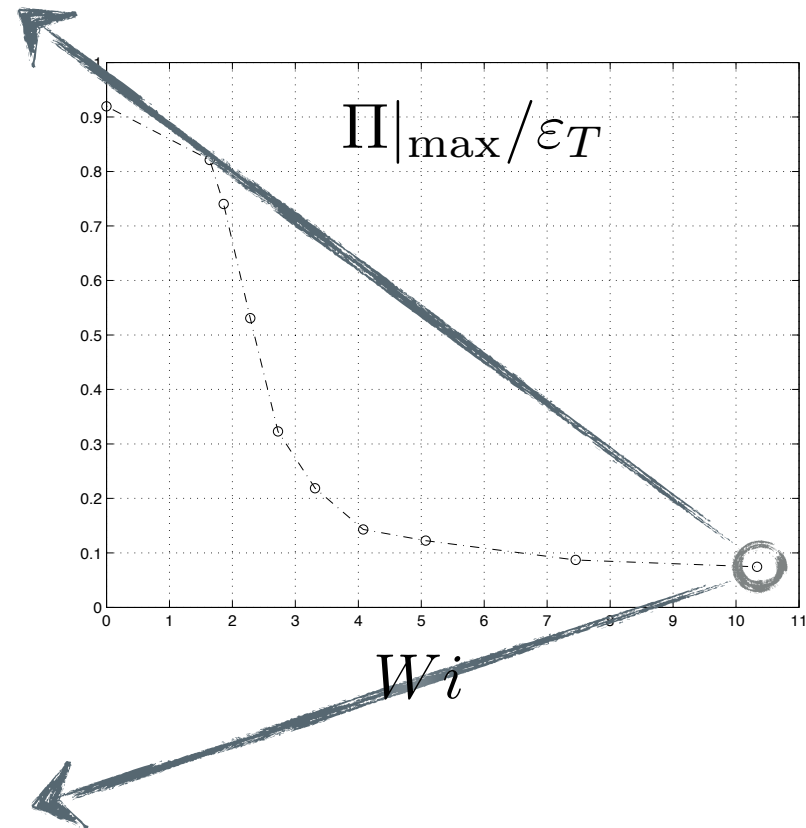
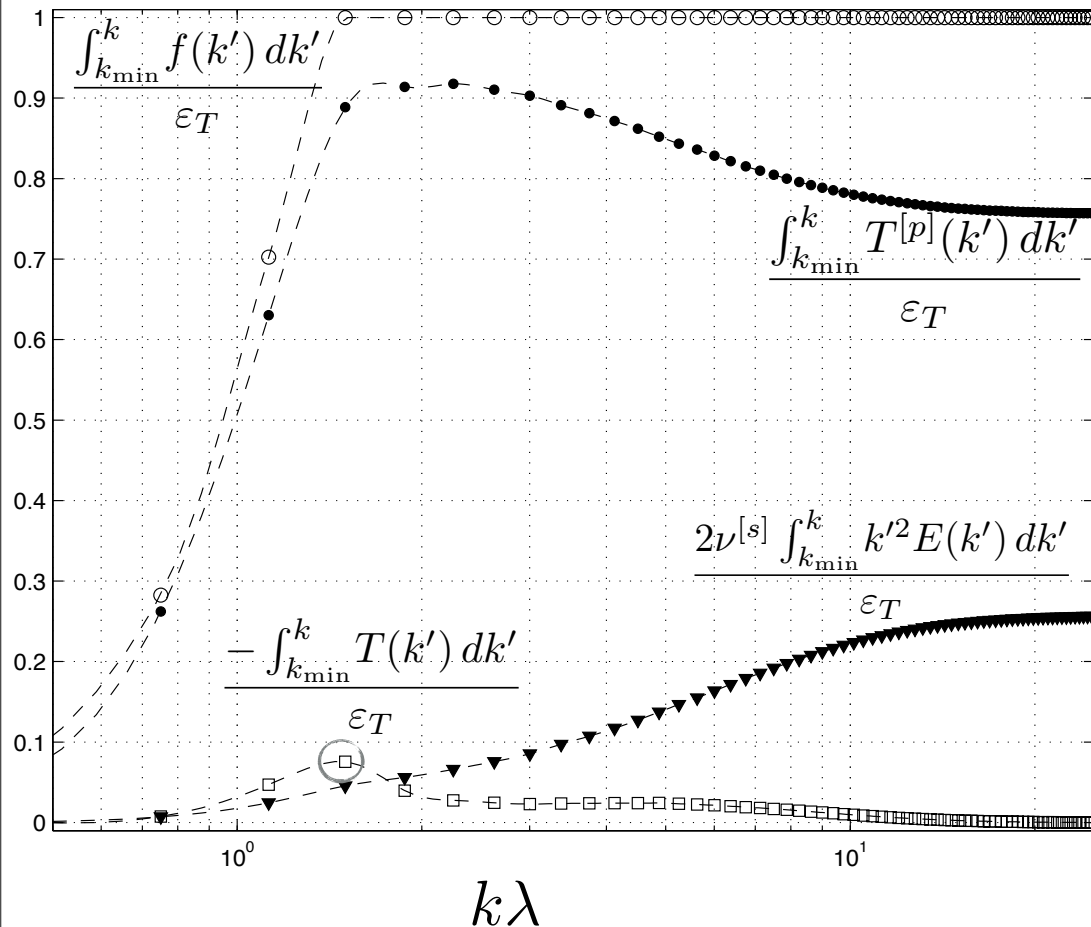
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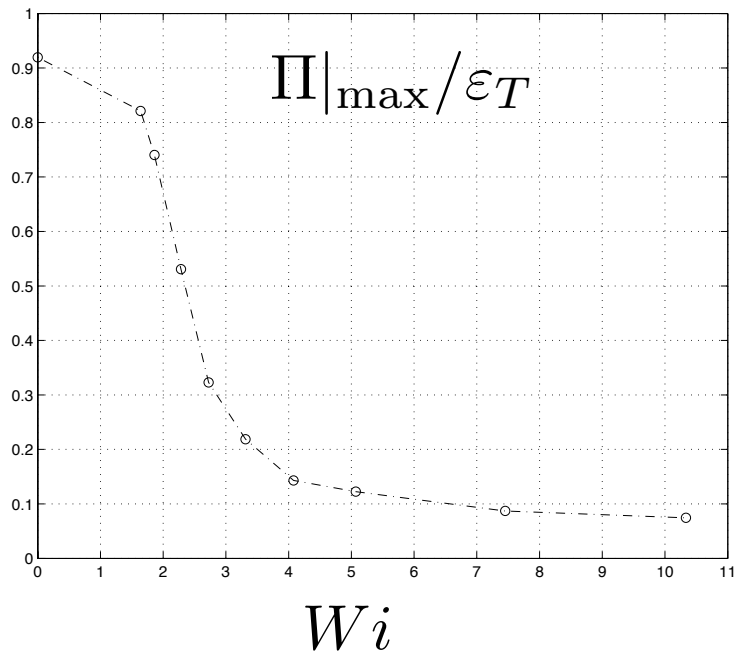
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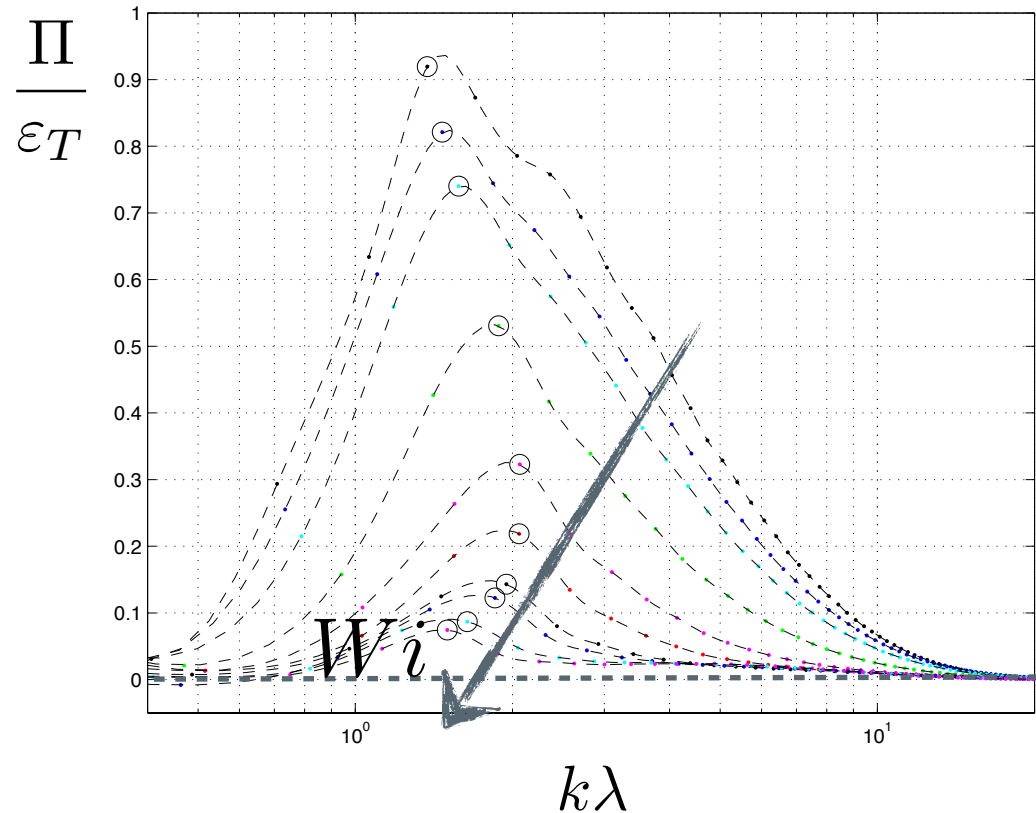


# Effect of polymer additives on the energy cascade

The non-linear transfer of energy decreases with increasing 'Wi'



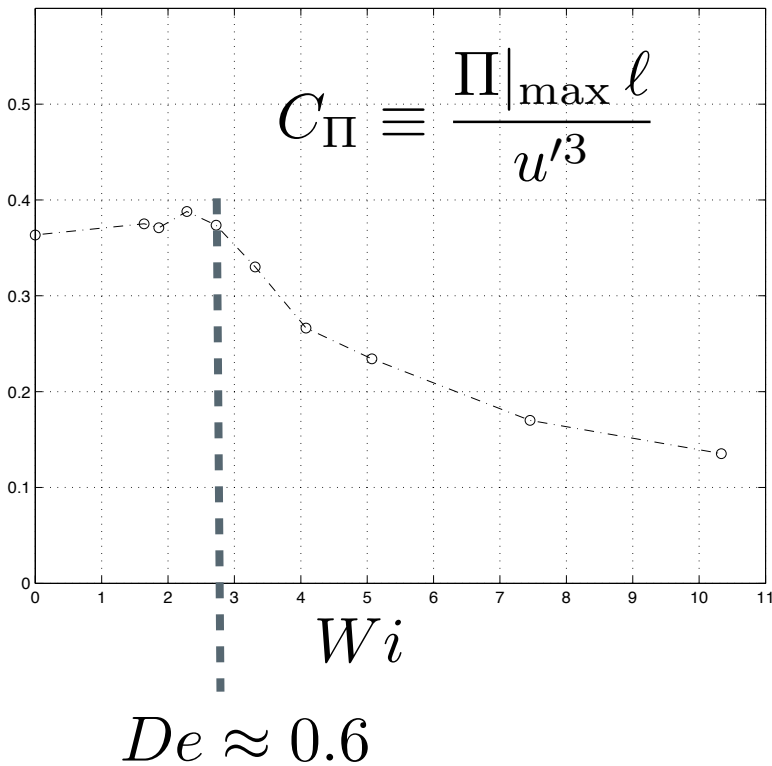
$$\Pi \equiv - \int_{k_{\min}}^k T(k') dk'$$



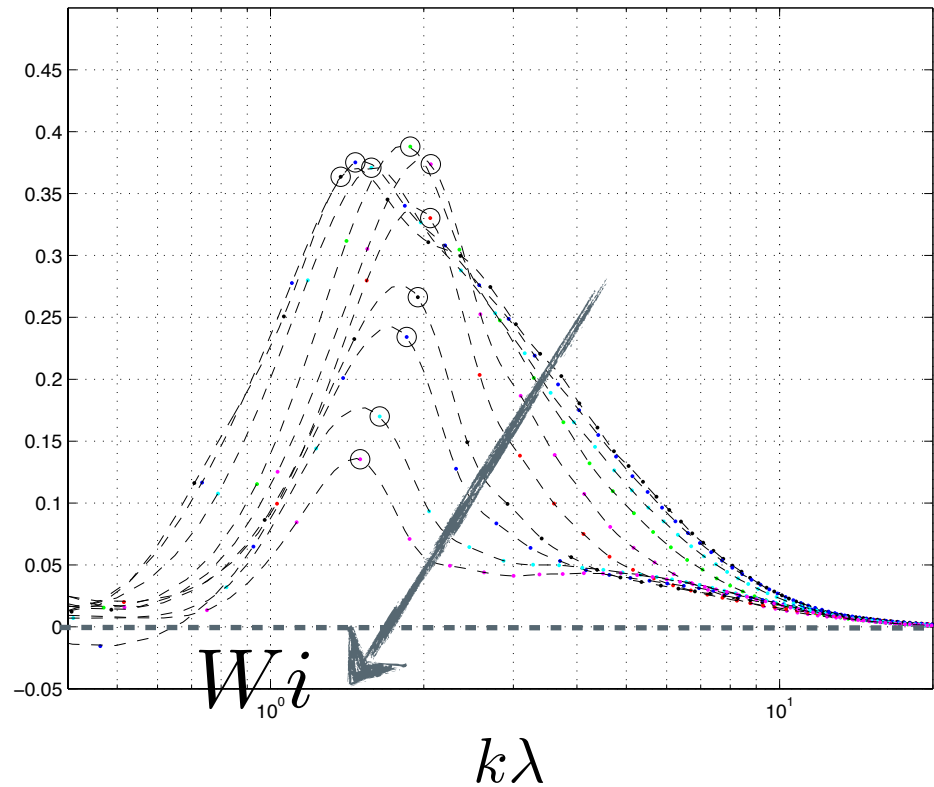


# Effect of polymer additives on the energy cascade

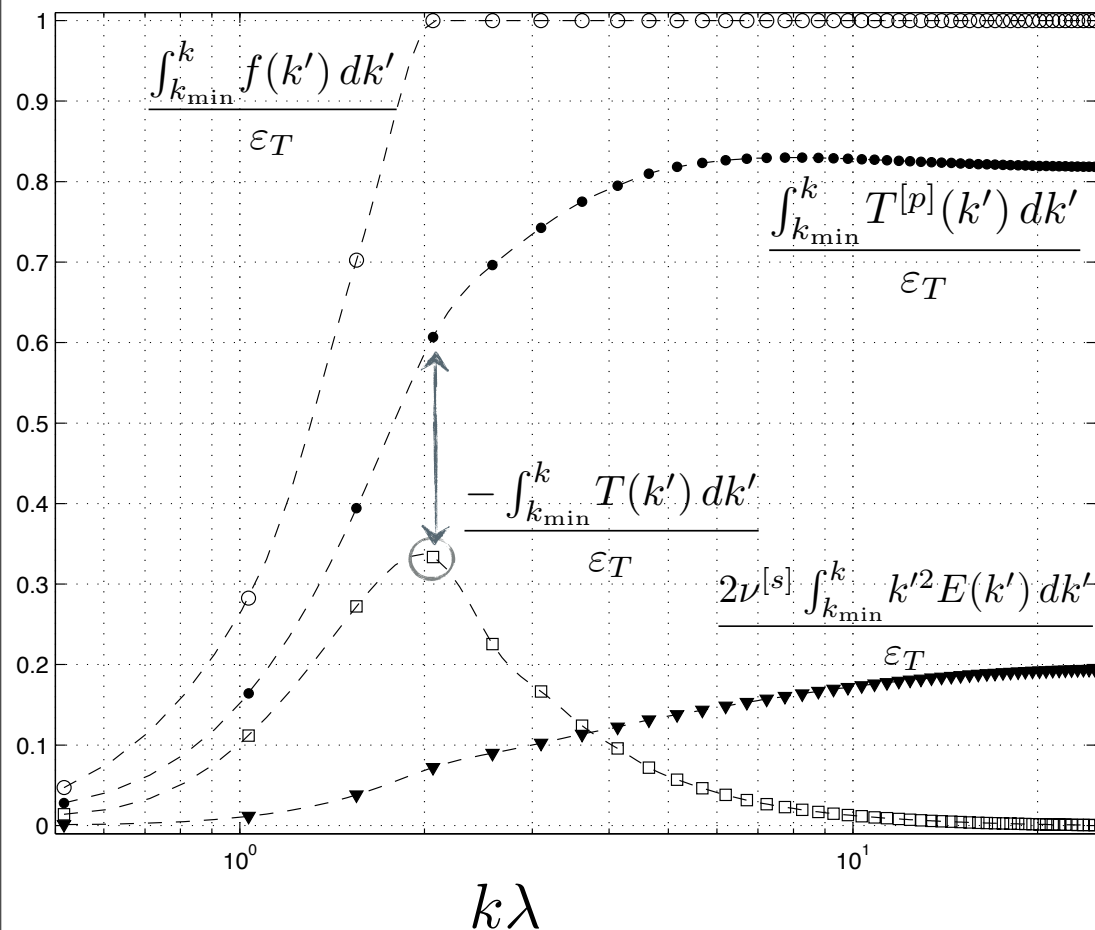
However ...



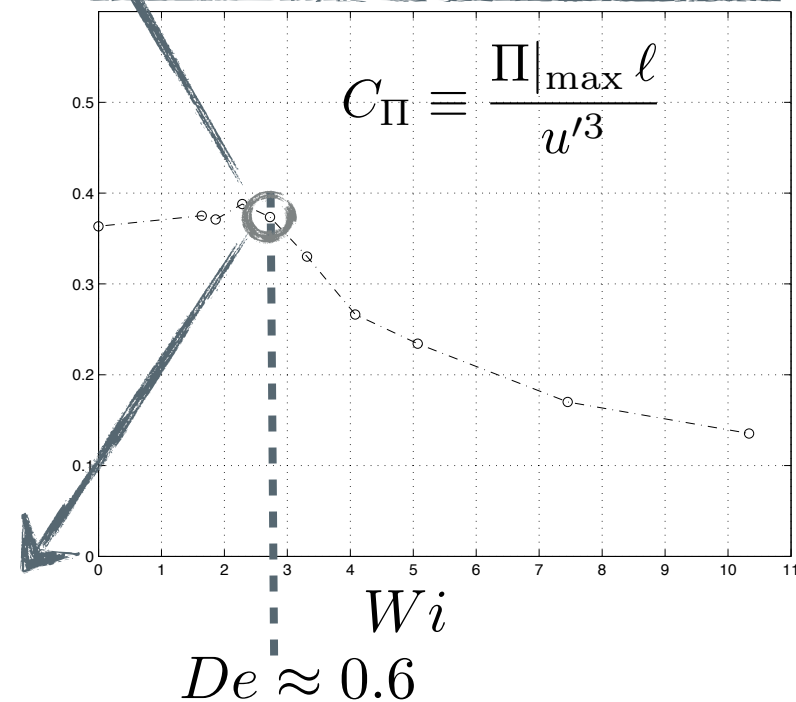
$$\frac{\Pi \ell}{u'^3}$$



# Effect of polymer additives on the energy cascade

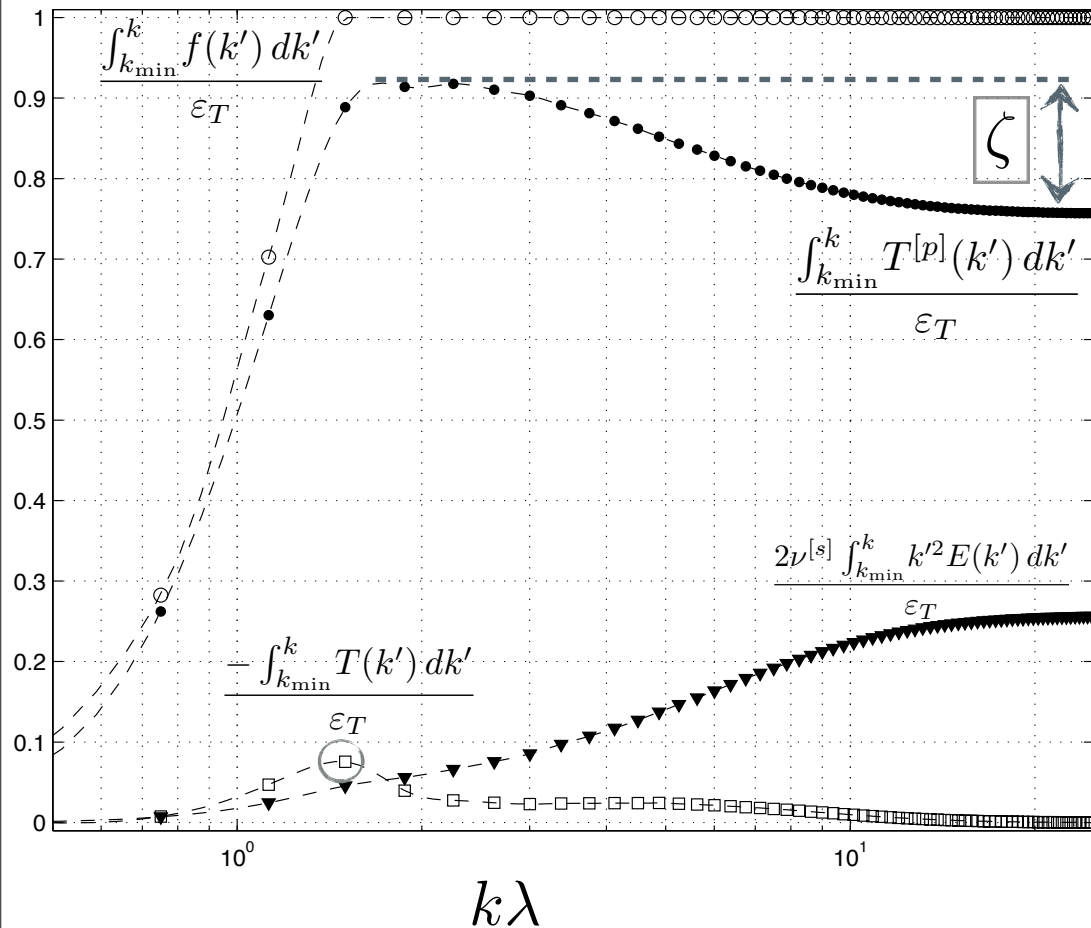


*Kinetic -> Polymer transfer overwhelms  
Nonlinear energy transfer  
 by factor of 2  
 but  $C_{\Pi} \approx 0.4$*

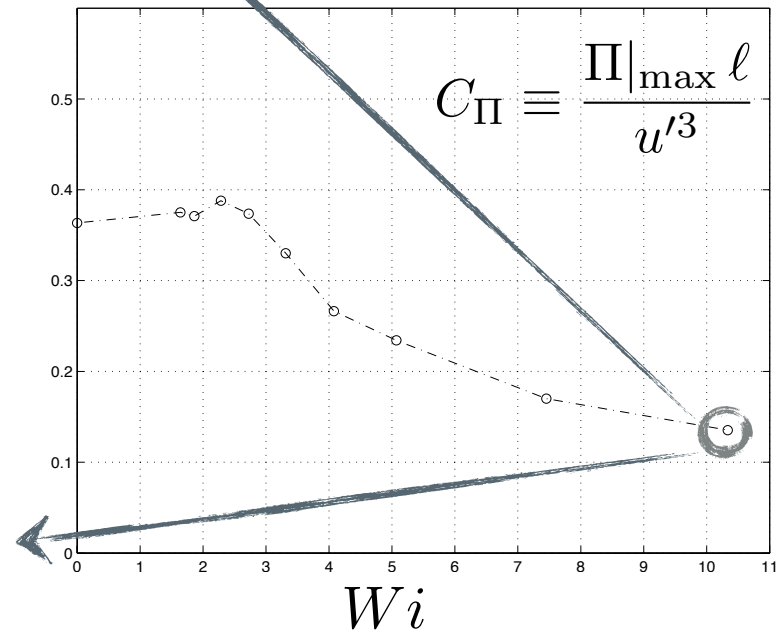


# The inter-scale energy cascade caused by the polymers

# Effect of polymer additives on the energy cascade

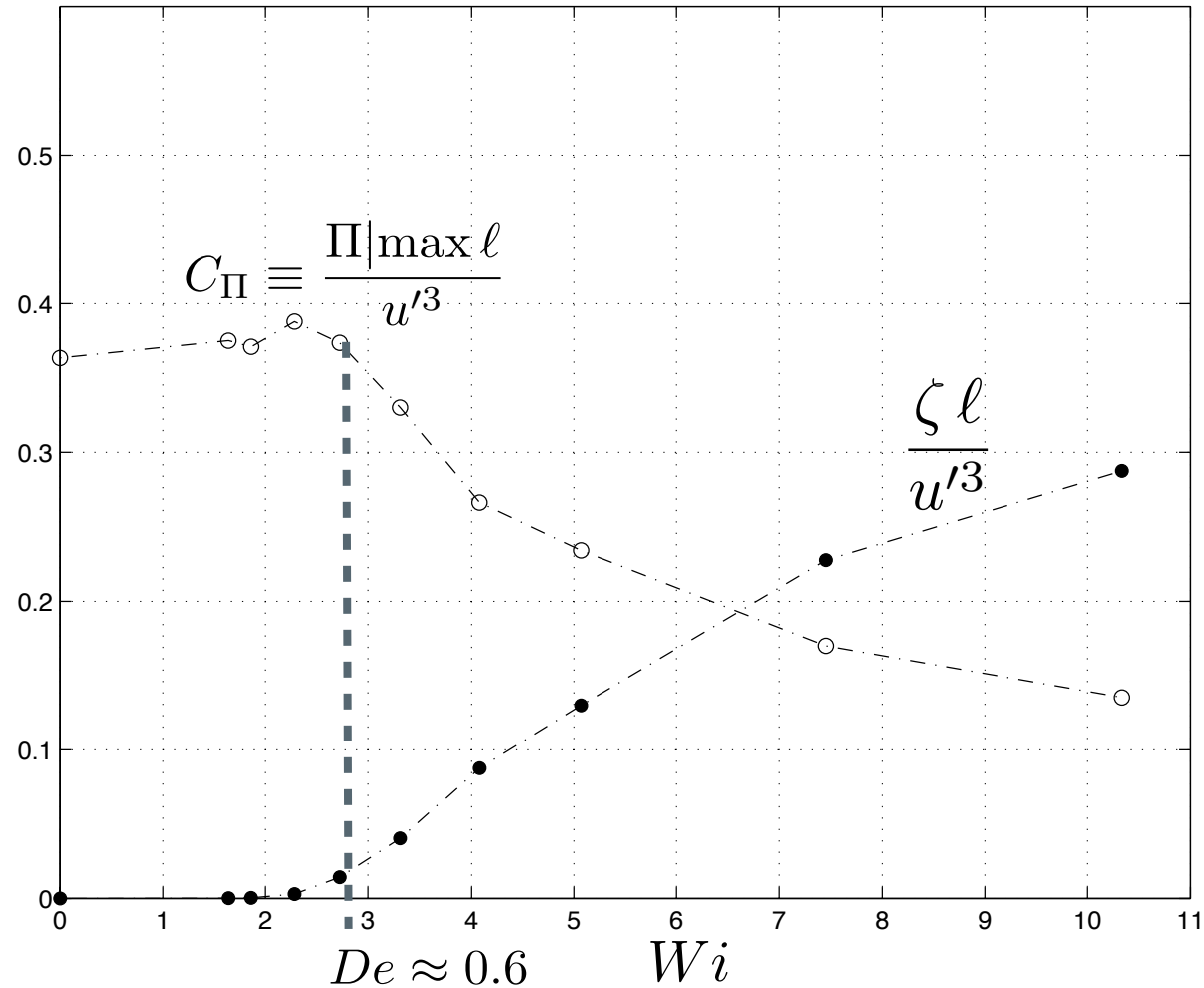


$\zeta$  - represents Polymer  $\rightarrow$  Kinetic energy transfer at high waveno.

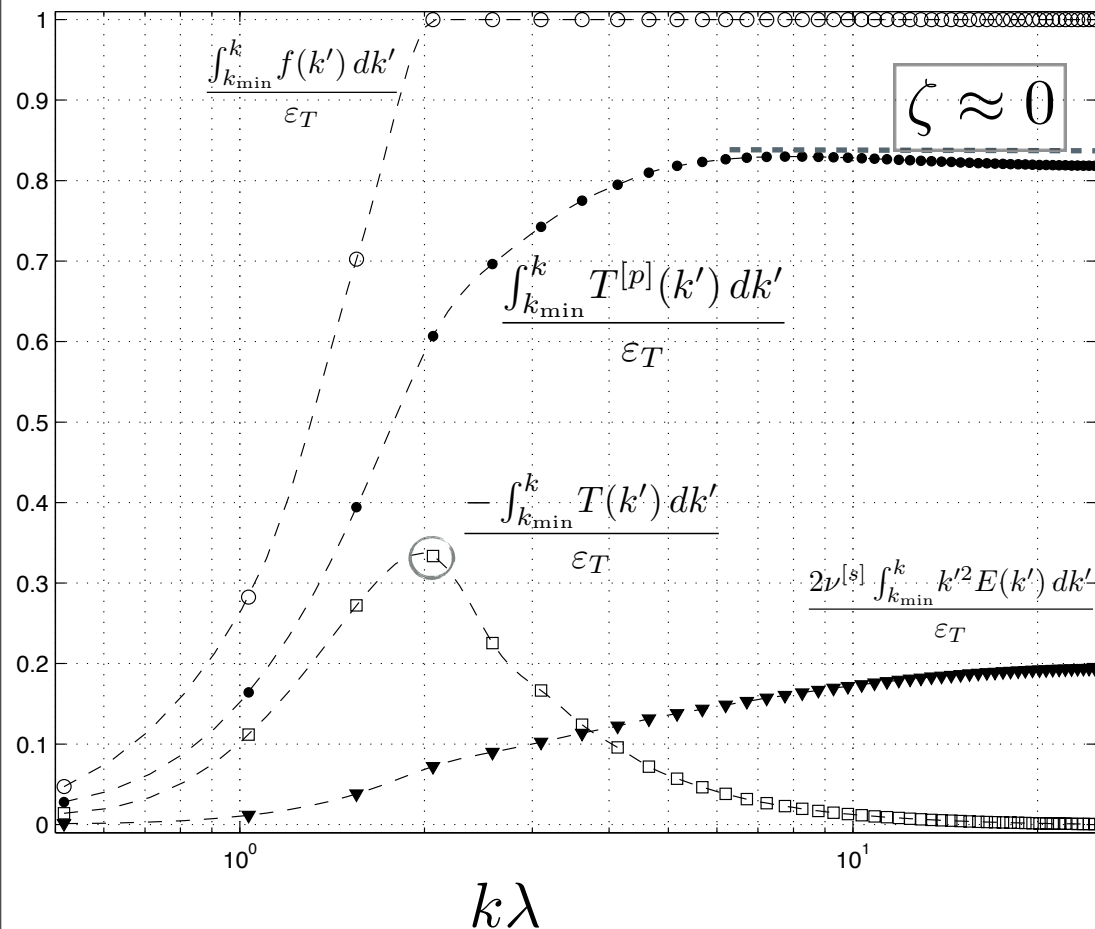


# The inter-scale energy transfer caused by the polymers

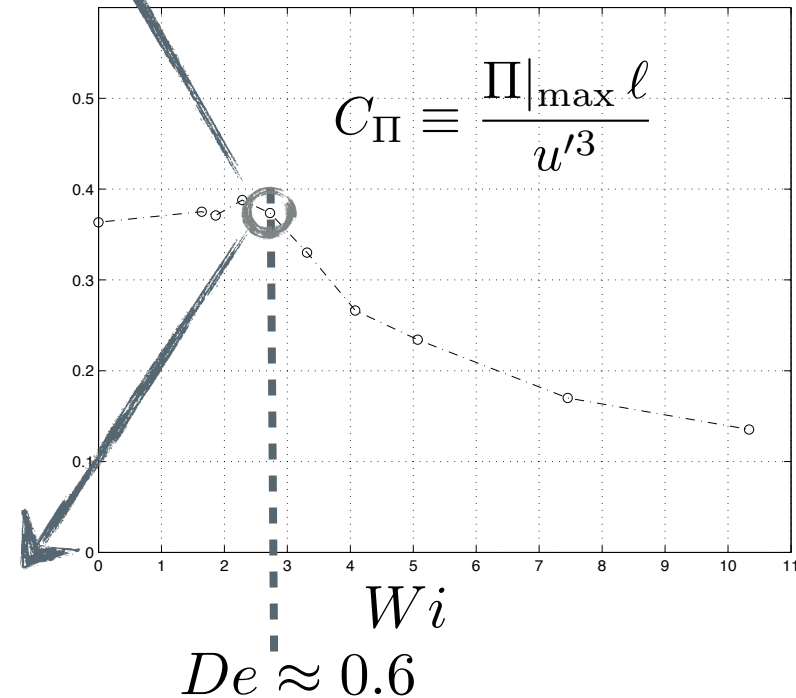
*The tampering of the energy cascade seems related to small-scale polymer energy feedback*



# Effect of polymer additives on the energy cascade



**Negligible**  
 Polymer → Kinetic transfer  
**Negligible effect on**  
 Nonlinear energy cascade



# Summary

# Summary

- Polymers offer an additional energy dissipation mechanism causing 'drag increase'
- Little change on energy cascade flux, w.r.t eddy turnover time even when polymers dissipate 80% of the total power input but no high waveno. energy feedback from polymers
- For higher  $Wi$ , polymers remove more energy at large scales than they dissipate and feedback the deficit at small scales
- Changes in cascade flux relative to turnover time seems to be closely related to high waveno. energy feedback from polymers