TRANSITIONS IN SOME STAGNATION FLOWS OF VISCOELASTIC FLUIDS AT LOW REYNOLDS NUMBERS

Alexandre M. Afonso

CEFT, Departamento de Engenharia Mecânica, Faculdade de Engenharia, Universidade do Porto
Rua Dr. Roberto Frias s/n, 4200-465 Porto, Portugal, E-mail: aafonso@fe.up.pt

Daniel O. Cruz

Departamento de Engenharia Mecânica, Universidade Federal do Pará
Campus do Guamá, Belém, Pará, Brazil, E-mail: doac@ufpa.br

Manuel A. Alves

CEFT, Departamento de Engenharia Química, Faculdade de Engenharia, Universidade do Porto
Rua Dr. Roberto Frias s/n, 4200-465 Porto, Portugal, E-mail: mmalves@fe.up.pt

Fernando T. Pinho

CEFT, Departamento de Engenharia Mecânica, Faculdade de Engenharia, Universidade do Porto
Rua Dr. Roberto Frias s/n, 4200-465 Porto, Portugal, E-mail: fpinho@fe.up.pt

Flow Instabilities and Turbulence in Viscoelastic Fluids

Leiden, Holland July 19-23, 2010

OUTLINE

- Objective
- Experimental and numerical results
  - Cross slot
  - Flow focusing
- Some analytical thoughts: Stagnation + “vortex” flow
- Closure

OBJECTIVE

- **Elastic instabilities** (Re=0): enhanced mixing or upper limit in devices
- Transition from steady symmetric to steady asymmetric flow is our main interest
- When it occurs and what are the effects of solvent, inertia and extensional viscosity. Brief review in some simple flows
- Some findings about the asymmetric flow: decoupling into simpler flows
- Results: mostly numerical (some experiments) and analytical (work in

Viscoelastic instabilities in shear flows

Shaqfeh. Ann. Rev. Fluid Mech 28 (1996) 129

Taylor-Couette flow Larson et al., JFM 218 (1990) 573

Cone-plate flow McKinley et al., JNNFM 40 (1991) 201

Lid driven cavity flows Pakdel & McKinley, PRL 77 (1996) 2459

Underlying mechanism McKinley et al, JNNFM 67 (1996) 19

Pakdel & McKinley, PRL 77 (1996) 2459

$$\left(\frac{\lambda U}{\mathscr{R}}\frac{\tau_{11}}{\tau_{12}}\right) \geq M_{crit}^{2}$$

curved streamlines

Instability growth to elastic turbulence

Groisman & Steinberg, Nature 405 (2000) 53 Larson, Nature 405 (2000) 27

Microfluidics & viscoelasticity

Squires & Quake, Rev. Mod. Phys. 77 (2005) 977

NUMERICAL METHODS: SOLUTION OF THE GOVERNING EQUATIONS

- Finite-volume method (in-house code)
- Collocated block-structured mesh
- Non-orthogonal coordinates (Cartesian velocity and stress tensor)
- Diffusion: central differences (2nd order in uniform mesh)
- SIMPLEC algorithm
- Rhie-and-Chow to couple velocity and pressure
- Special scheme to couple velocity and extra stress

Oliveira et al. JNNFM, 79 (1998) 1-43.

- Advection: CUBISTA high-resolution scheme (based on QUICK, 3rd order)

Alves et al. IJNMF, 41 (2003) 47-75.

- Standard formulation and log-conformation formulation (allows higher De)

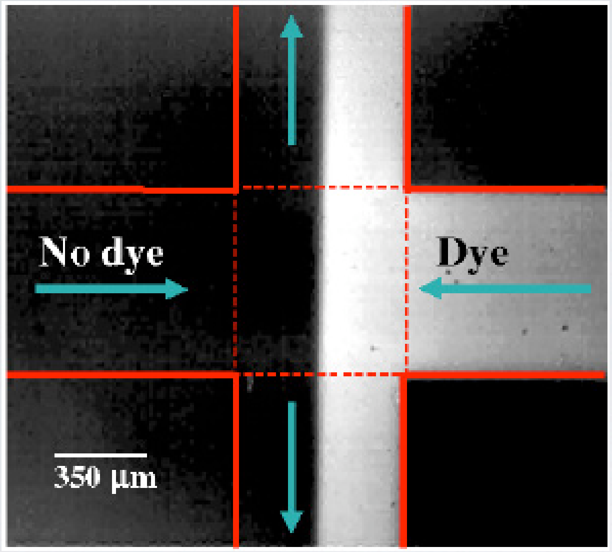
Fattal & Kupferman JNNFM, 123 (2004) 281-285.

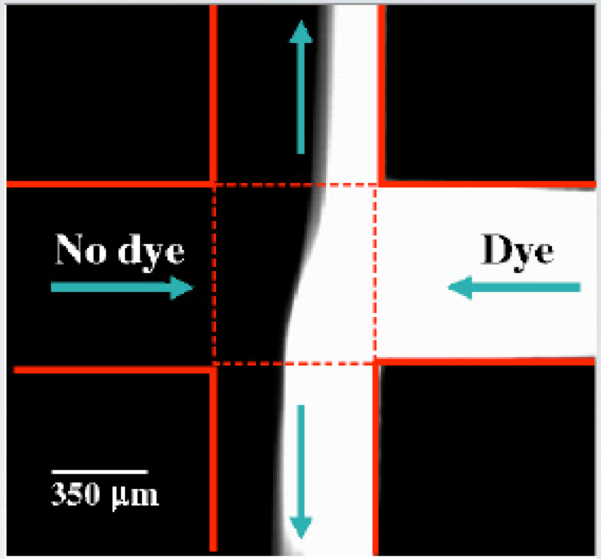
More details for FVM: Afonso et al. JNNFM 157 (2009) 55-65

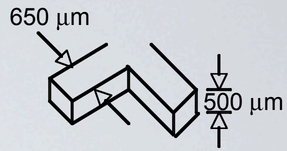
CROSS SLOT

2D CROSS SLOT

Arratia et al., PRL 96 (2006) 144502



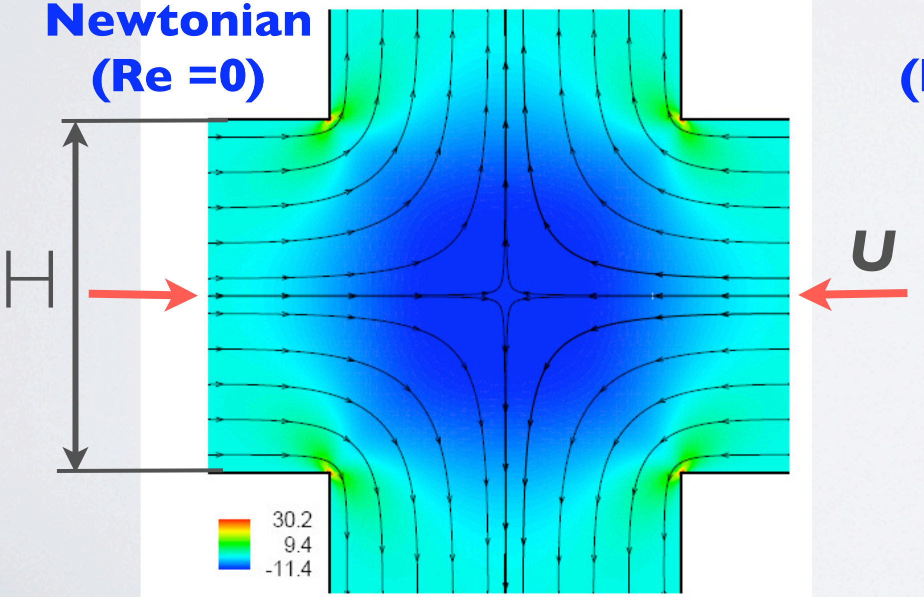




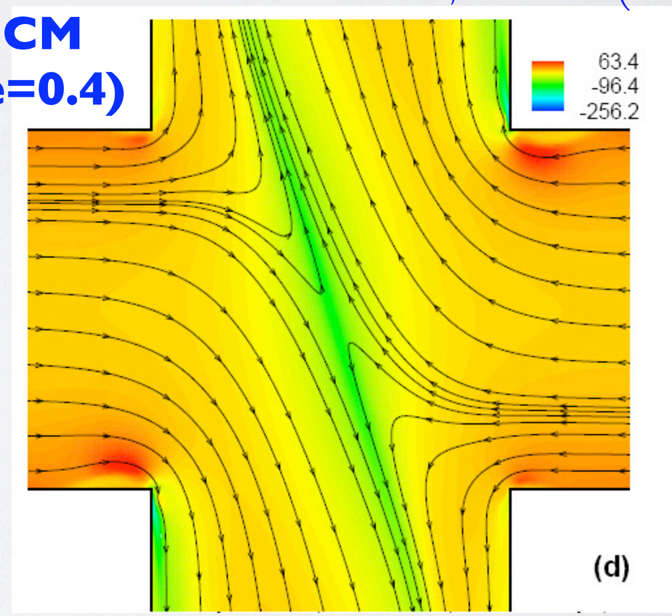
Newtonian (Re < 10^{-2}) **PAA Boger fluid (Re < 10^{-2}, De=4.5)**

Poole et al., PRL 99 (2007) 164503

Newtonian (Re =0)



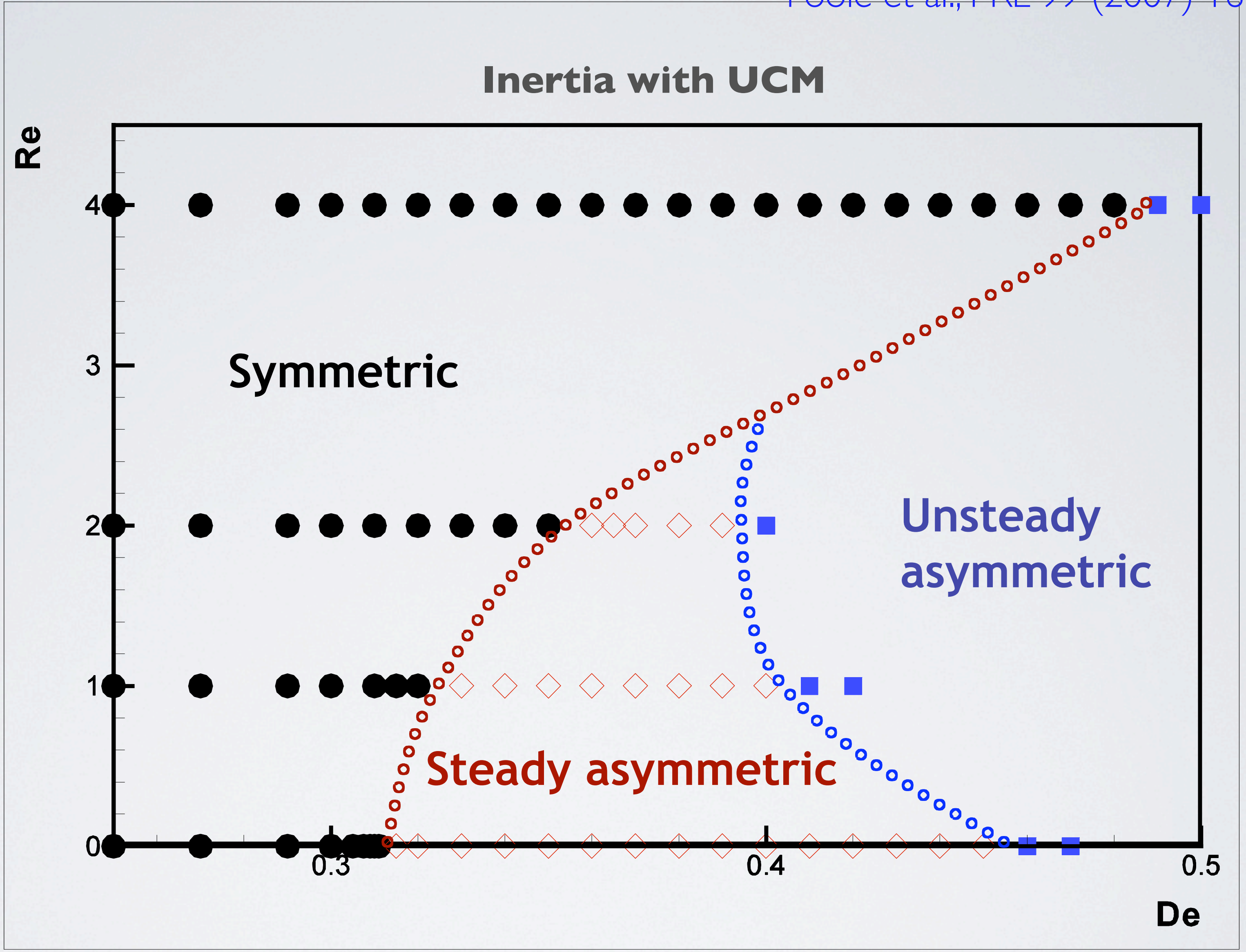
UCM (De=0.4)



2D CROSS SLOT WITH UCM: EFFECT OF INERTIA

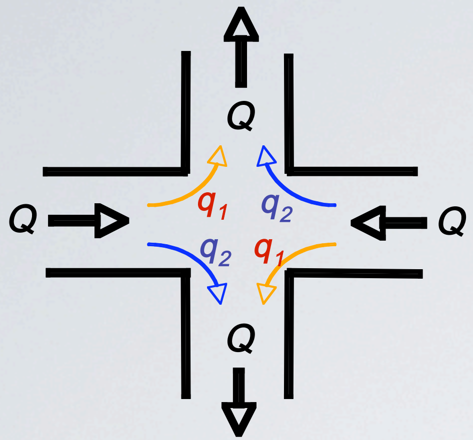
Poole et al., PRL 99 (2007) 164503

$$De = \frac{\lambda U}{H}$$



Inertia decreases degree of asymmetry and stabilizes the flow

2D CROSS SLOT: OLDROYD-B — EFFECT OF SOLVENT — CREEPING FLOW



Oldroyd-B

Re=0

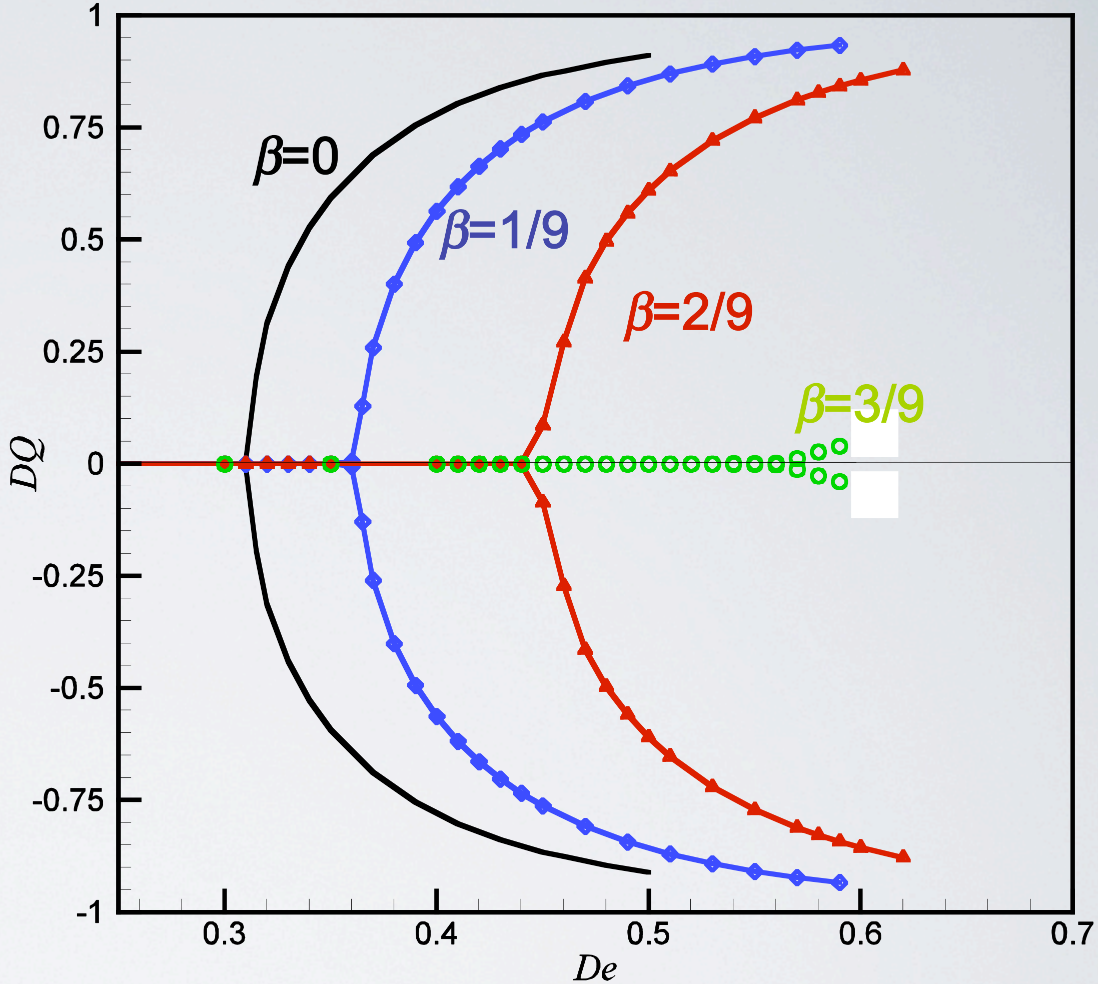
$$DQ = \frac{q_2 - q_1}{q_2 + q_1} = \frac{q_2 - q_1}{Q}$$

$DQ = 0 \rightarrow$ symmetric

$DQ = \pm 1 \rightarrow$ completely asymmetric

$$\beta = \frac{\eta_s}{\eta_s + \eta_p}$$

Poole et al., SoR 2007



Increasing the solvent viscosity

Increases De_{CR}

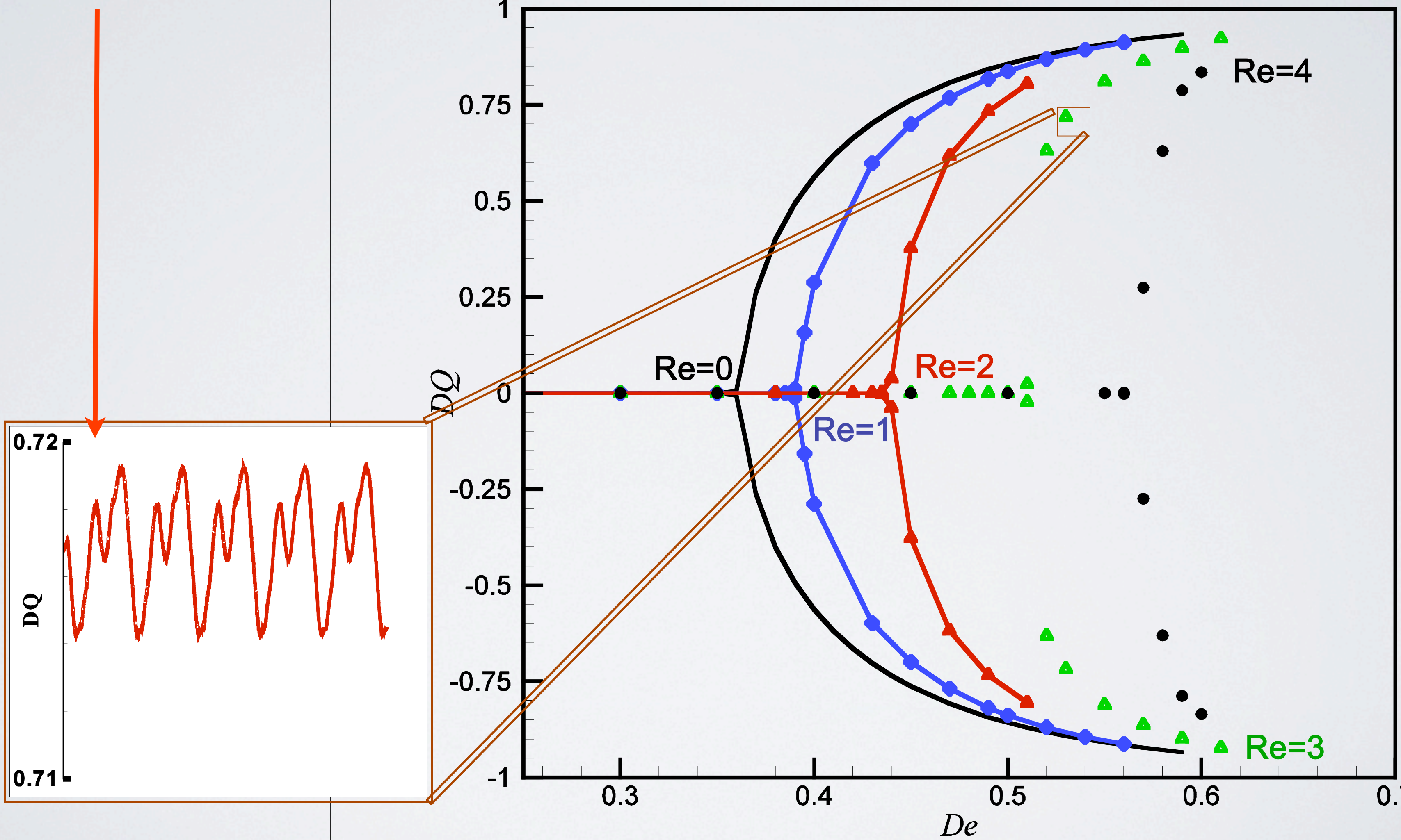
For β>3/9 flow becomes asymmetric unsteady (as in flow focusing)

Poole et al., SoR 2007

Increasing Re

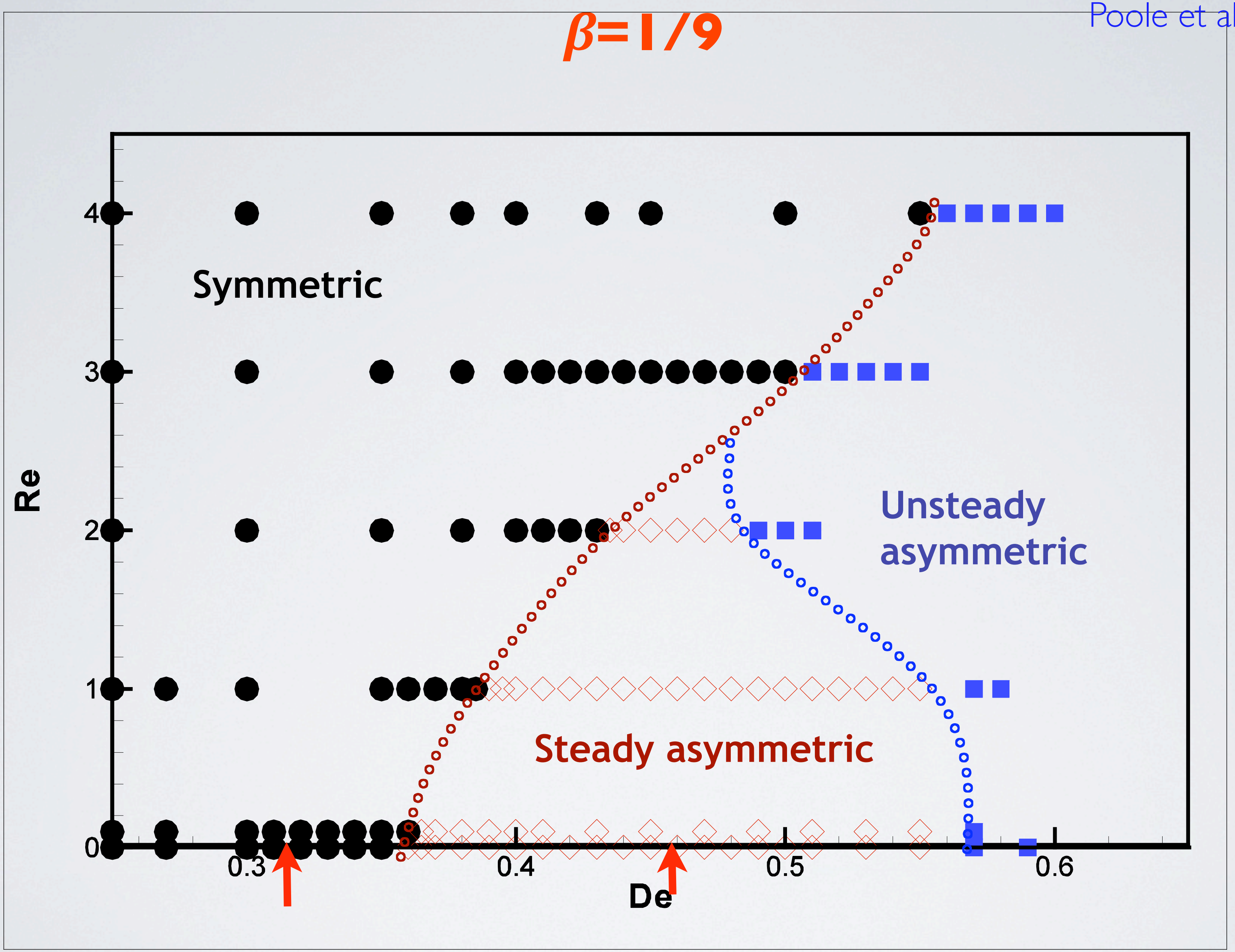
β=1/9

- Increases De_{CR}
- Decreases degree of asymmetry
- For Re > 2 unsteady asymmetric flow



2D CROSS SLOT: OLDROYD-B — STABILITY MAP

Poole et al., SoR 2007



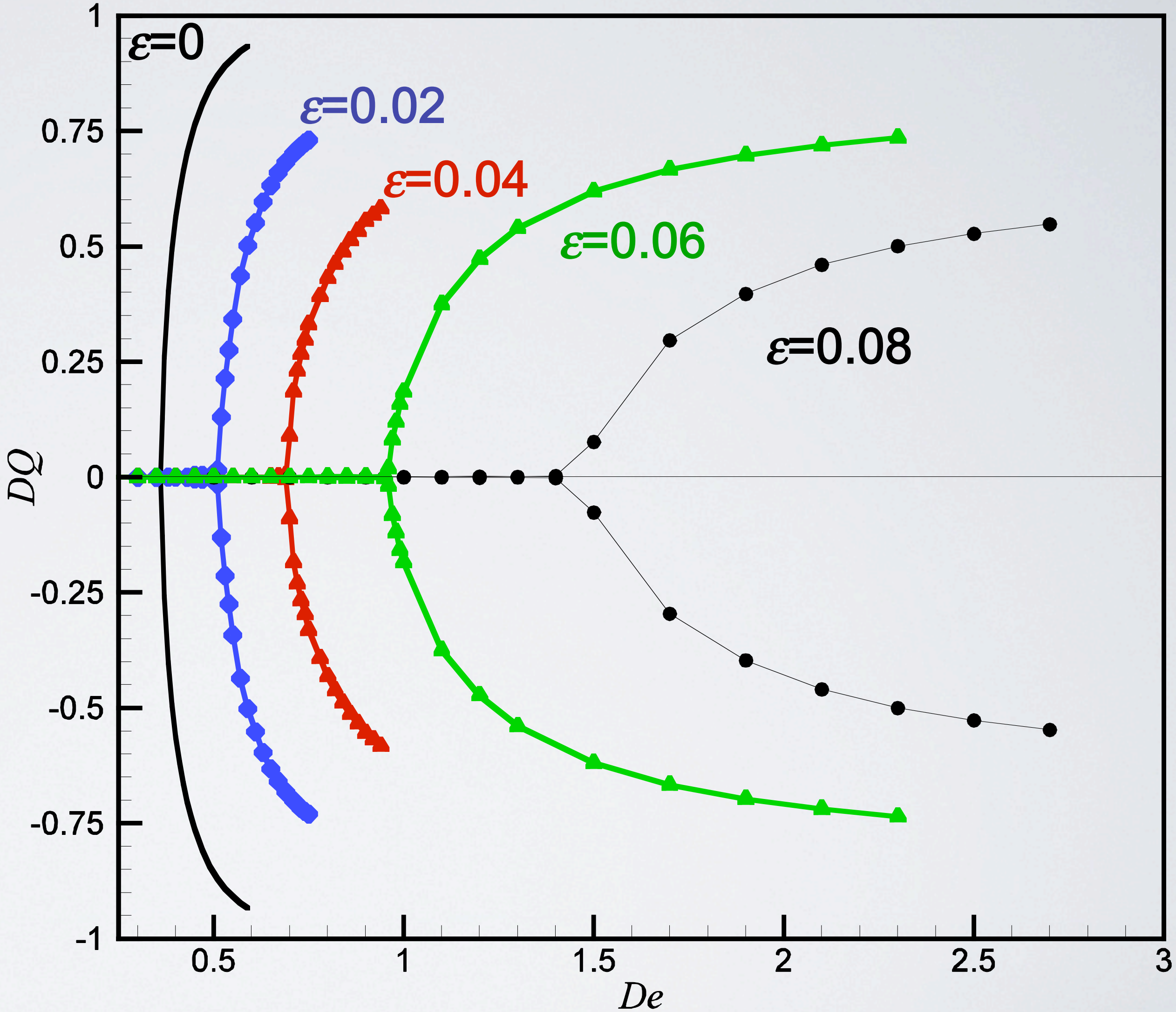
2D CROSS SLOT: SPTT — EFFECT OF EPSILON

Poole et al., SoR 2007

β=1/9

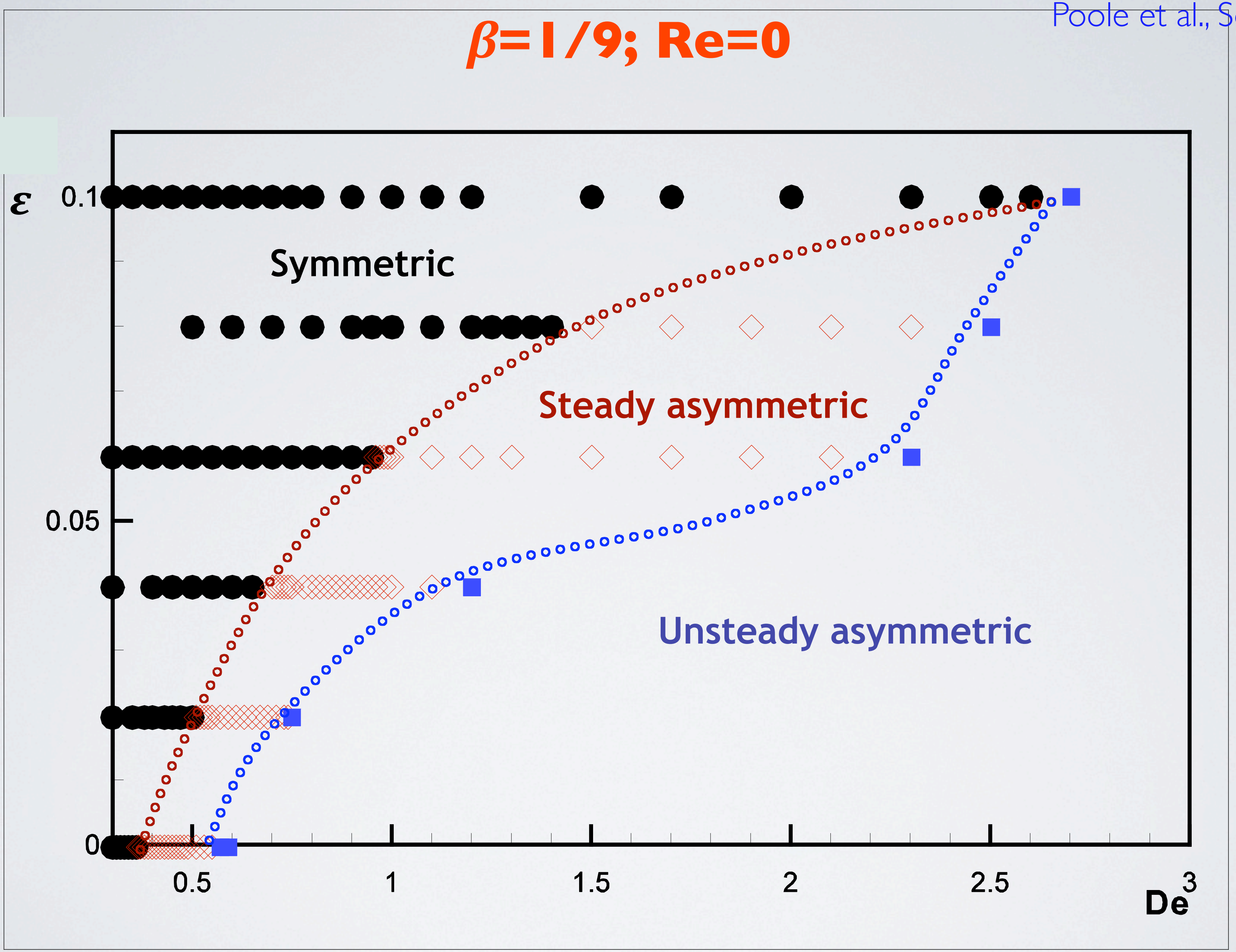
Increasing ε

- Increases De_{CR}
- Decreases degree of asymmetry (ε<0.04)
- Increases degree of asymmetry and extension in De (ε>0.04)
- Asymmetric stable flow disappears for ε>0.08



2D CROSS SLOT: SPTT — STABILITY MAP

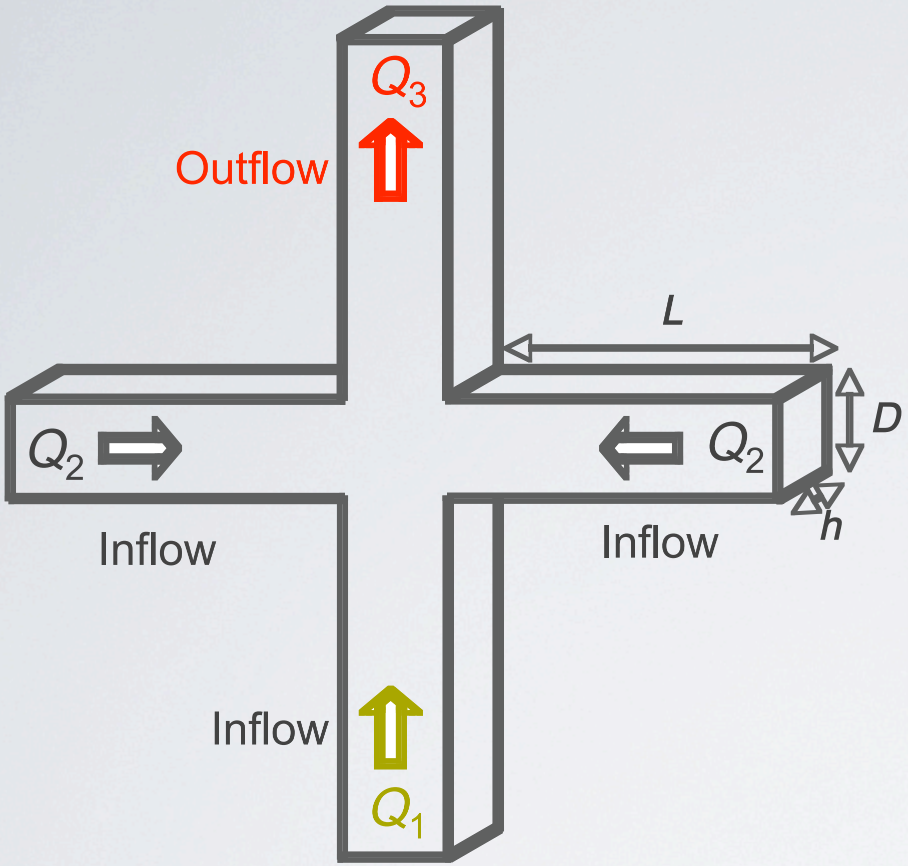
Poole et al., SoR 2007



FLOW FOCUSING
(extensional flow "without" shear)

FLOW FOCUSING

Oliveira et al. JNNFM 160 (2009) 31-39



Cross-slot with 3 inlets and 1 outlet

Operational Variables

$$Q_1, Q_2$$

$$Q_3 = 2 \times Q_2 + Q_1$$

Dimensionless Variables

$$FR = \frac{Q_2}{Q_1}$$

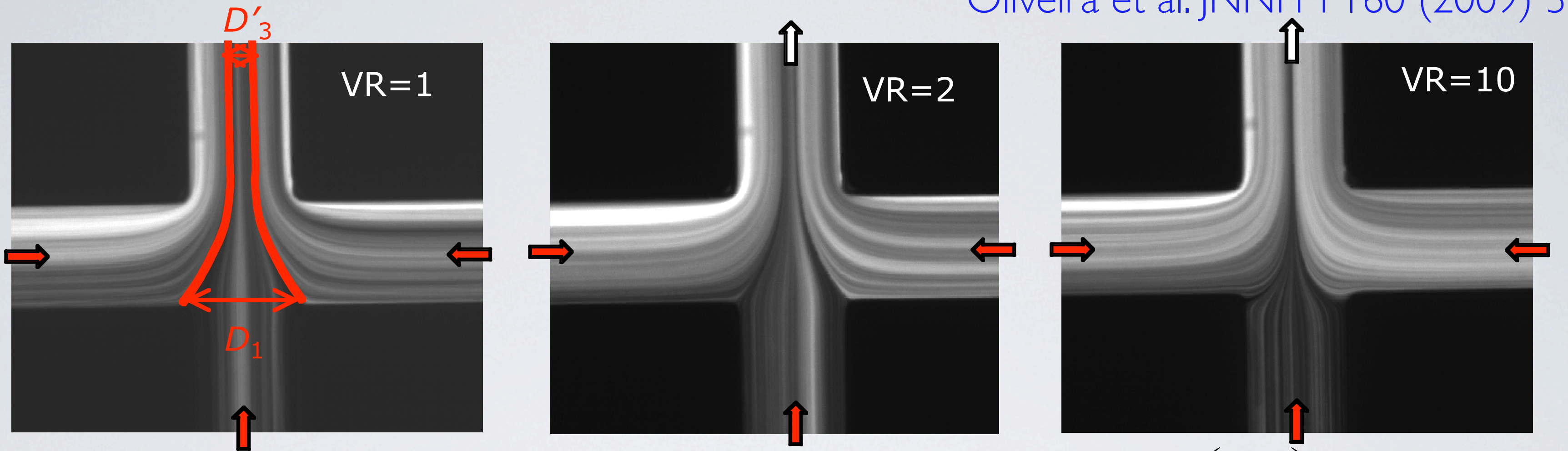
$$VR = \frac{U_2}{U_1} \quad (= FR)$$

$$\left. \begin{array}{l} \mathrm{Re} = \dfrac{\rho U_2 D}{\eta_0} \\ De = \dfrac{\lambda U_2}{D} \end{array} \right\} El = \frac{De}{\mathrm{Re}}$$

All dimensions kept constant in experiments and calculations

FLOW FOCUSING: NEWTONIAN

Oliveira et al. JNNFM 160 (2009) 31-39

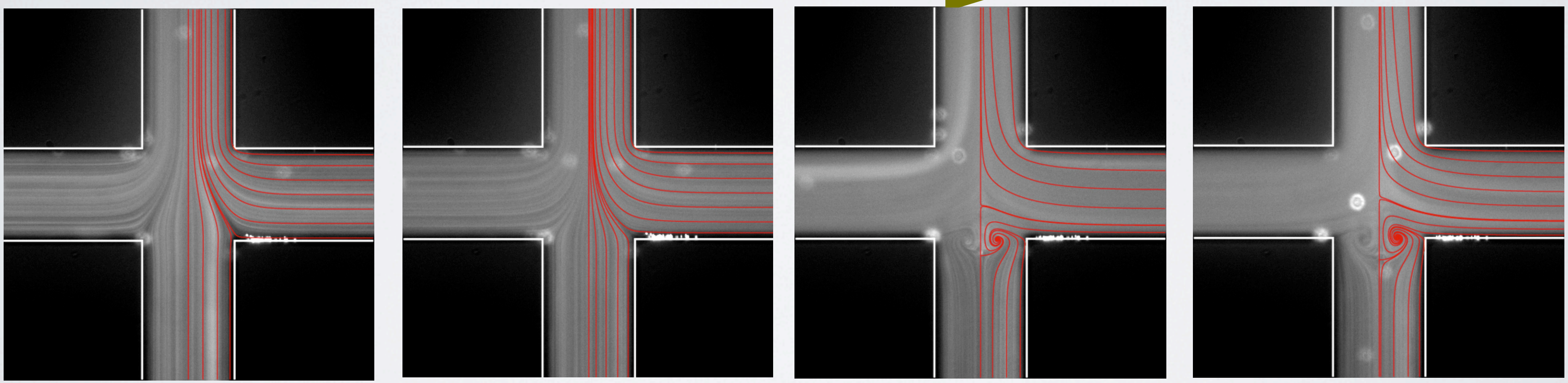


Separation streamlines: nearly hyperbolic shape

$$\varepsilon_H = \ln\left(\frac{D_1}{D_3}\right) = \ln\left[\frac{3}{2}\left(1+2VR\right)\right]$$

$Q_1 = 0.01$ ml/h

increasing Q_2



$Q_2 = 0.3$ ml/h	$Q_2 = 0.9$ ml/h	$Q_2 = 15$ ml/h	$Q_2 = 18$ ml/h
$VR = 1$, $Re_3 = 2.8$	$VR = 3$, $Re_3 = 6.5$	$VR = 50$, $Re_3 = 94.2$	$VR = 60$, $Re_3 = 112.8$

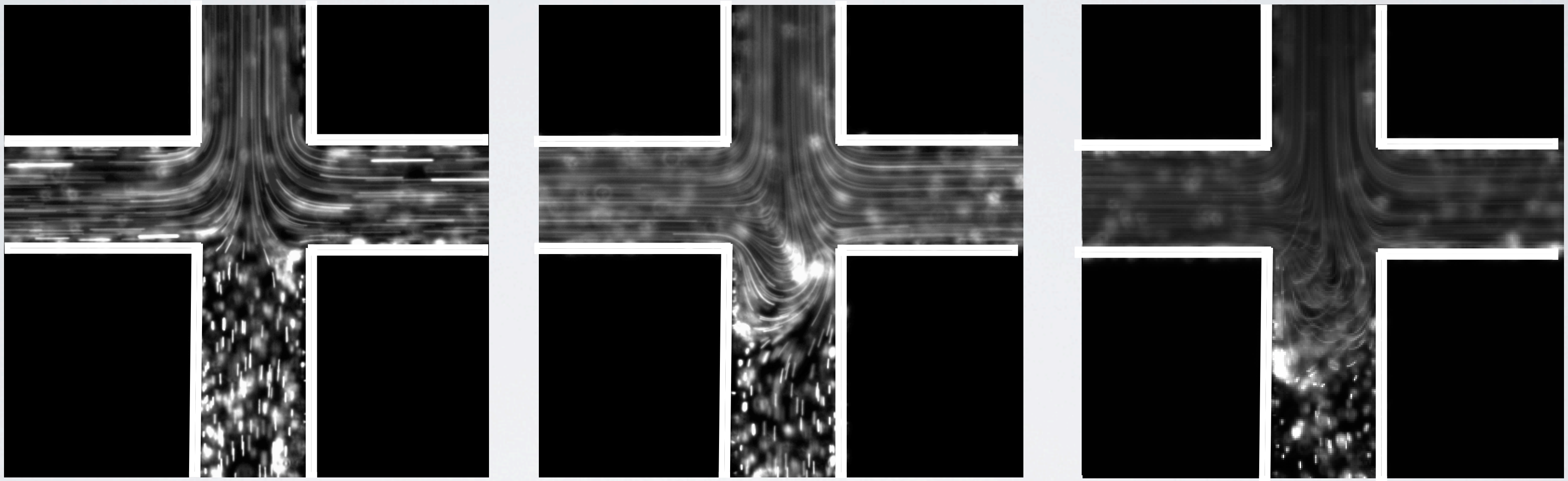
FLOW FOCUSING: PAA125

Oliveira et al. JNNFM 160 (2009) 31-39

$$Q_1 = 0.01 \text{ ml/h}$$

increasing Q_2 →

Viscoelastic



$Q2$ = 0.05 ml/h, VR = 5
Re = 0.23, De = 0.38

Symmetric

$Q2$ = 0.2 ml/h, VR = 20
Re = 0.87, De = 1.41

Steady Asymmetric

$Q2$ = 0.5 ml/h, VR = 50
Re = 2.15, De = 3.479

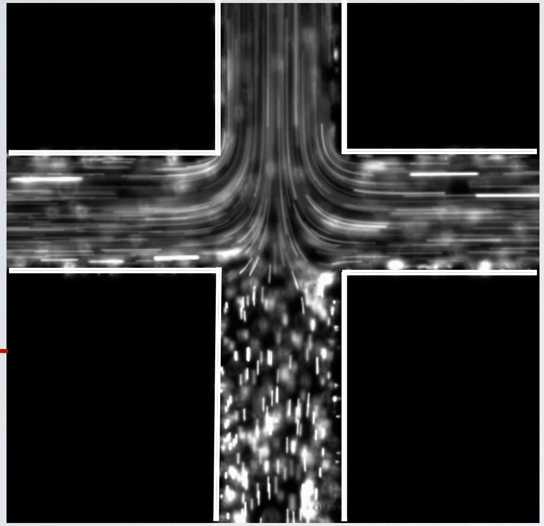
Unsteady 3D

FLOW FOCUSING: PAA125

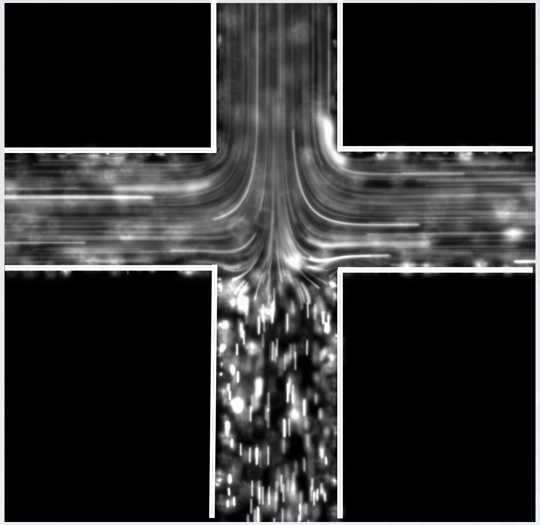
$Q_1 = 0.01$ ml/h

Oliveira et al. JNNFM 160 (2009) 31-39

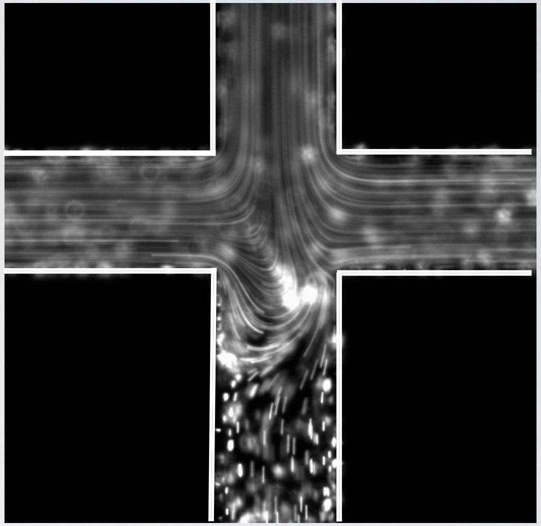
Viscoelastic Experimental



$Q2$ = 0.05 ml/h, VR = 5
Re = 0.23, De = 0.38

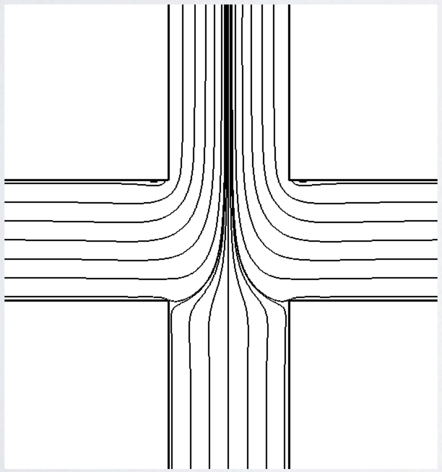


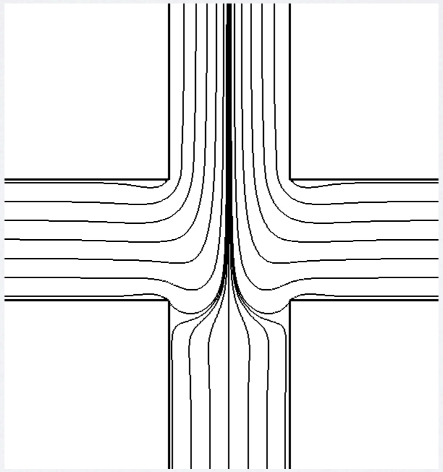
$Q2$ = 0.1 ml/h, VR = 10
Re = 0.45, De = 0.723

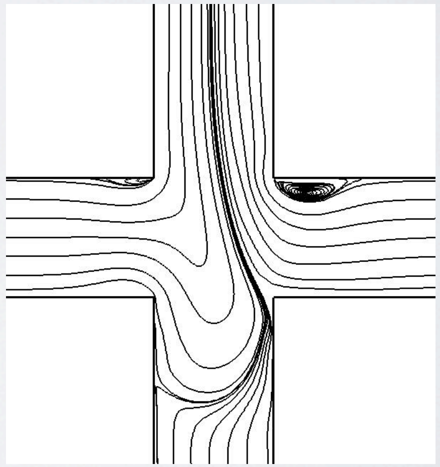


$Q2$ = 0.2 ml/h, VR = 20
Re = 0.87, De = 1.41

UCM 2D Calculations

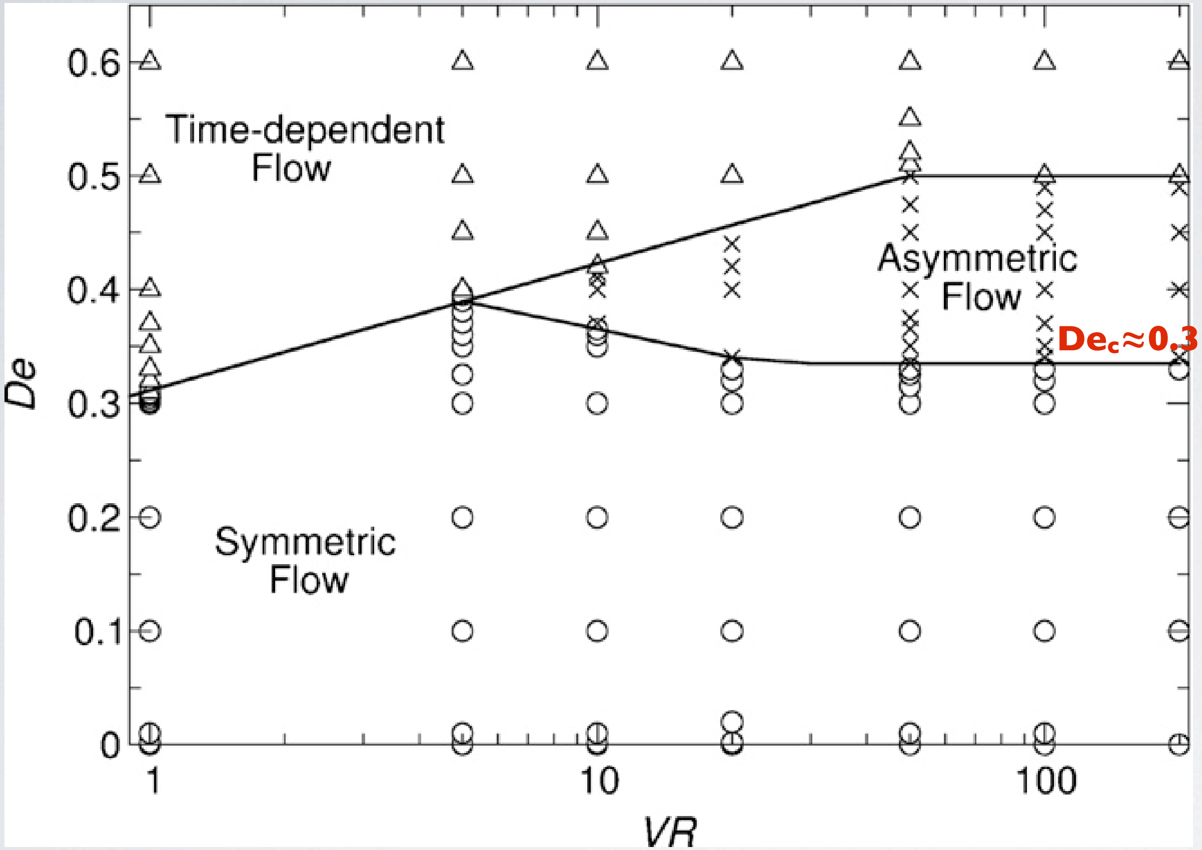




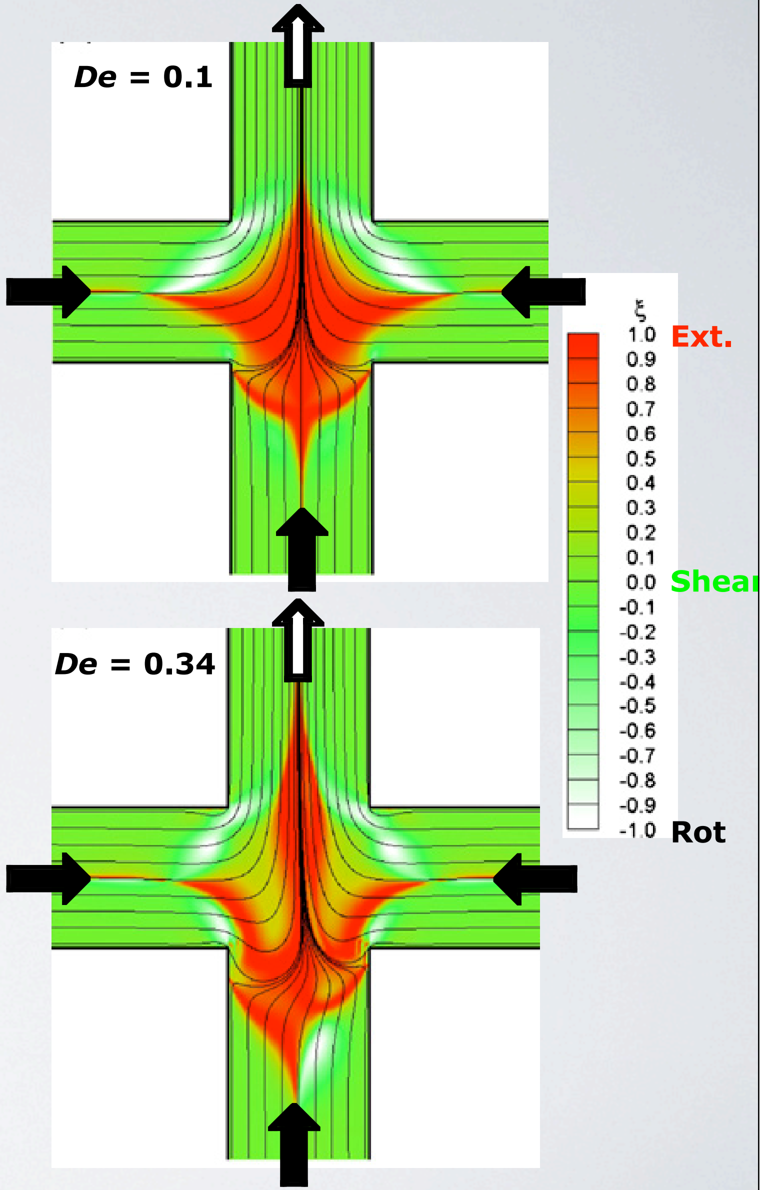


FLOW FOCUSING: VISCOELASTIC

Oliveira et al. JNNFM 160 (2009) 31-39



De$_c$≈0.33



$$\xi = \frac{1-R}{1+R}$$

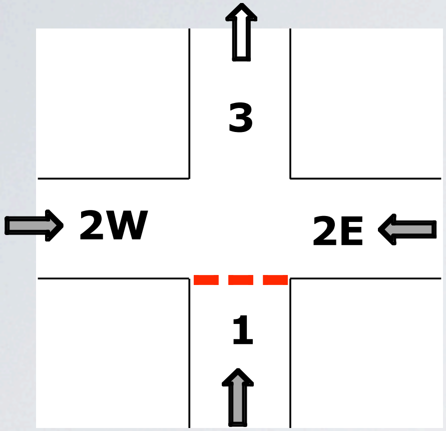
$$R = \frac{tr\tilde{\mathbf{W}}^2}{tr\mathbf{D}^2}$$

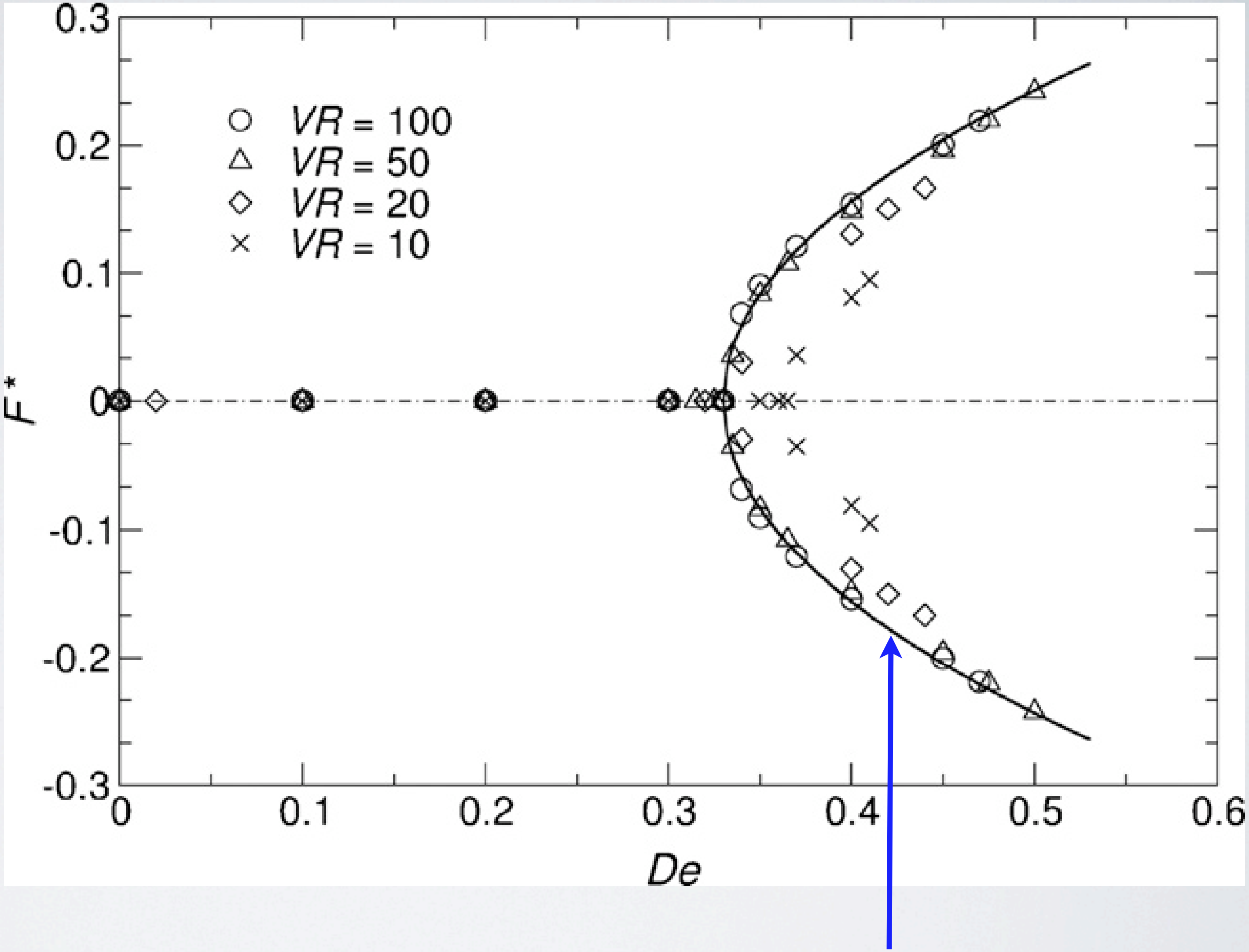
Astarita, JNNFM 6 (1979) 69
Thompson et al., JNNFM 86 (1999) 375
Mompean et al., JNNFM 111 (2003) 151

FLOW FOCUSING: EFFECT OF VR

Oliveira et al. JNNFM 160 (2009) 31-39

$$F^* = \frac{F_W - F_E}{F_3}$$





$$F^* = 0.59\sqrt{De - 0.33}$$

Bistable flow

High VR:

- constant De_c
- evolution independent of VR
- supercritical pitchfork bifurcation

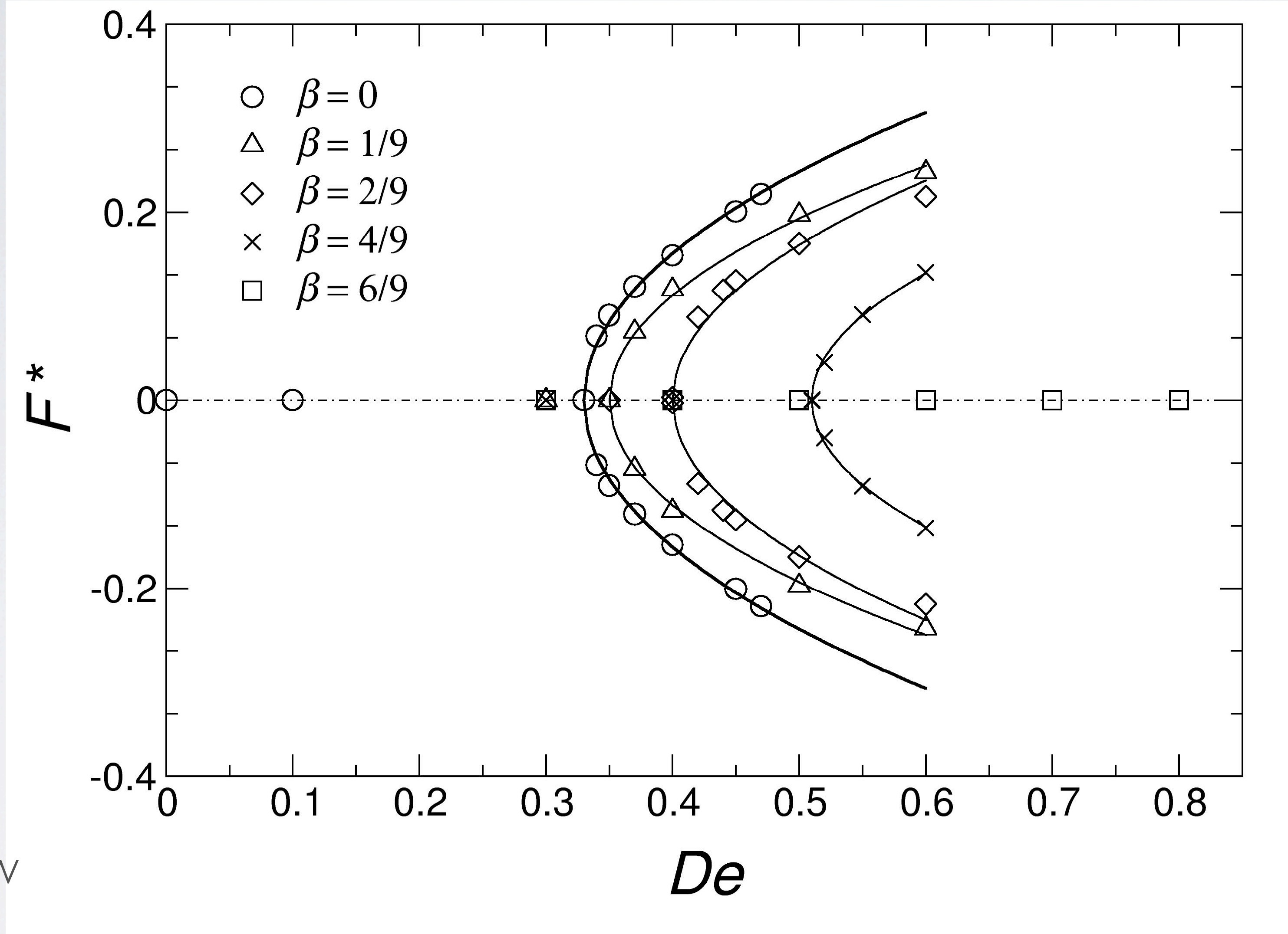
FLOW FOCUSING: EFFECT OF β

Oliveira et al. JNNFM 160 (2009) 31-39

$$\beta = \frac{\eta_s}{\eta_s + \eta_p}$$

↓

Oldroyd-B



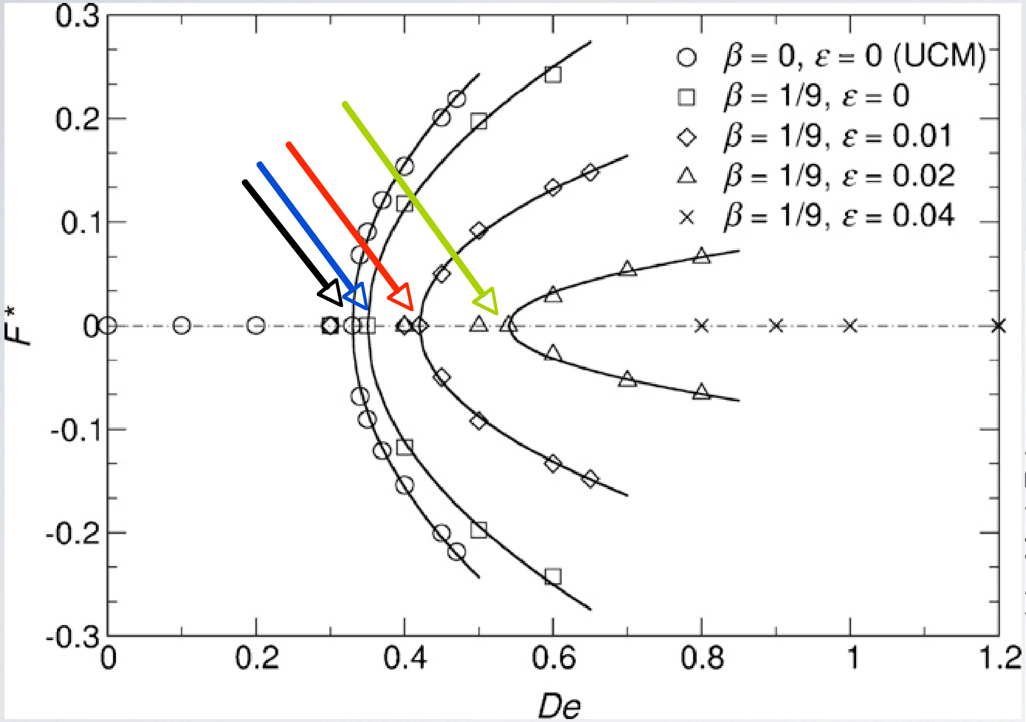
β stabilizes the flow

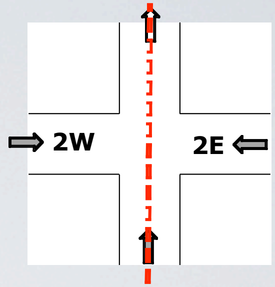
increases De_c

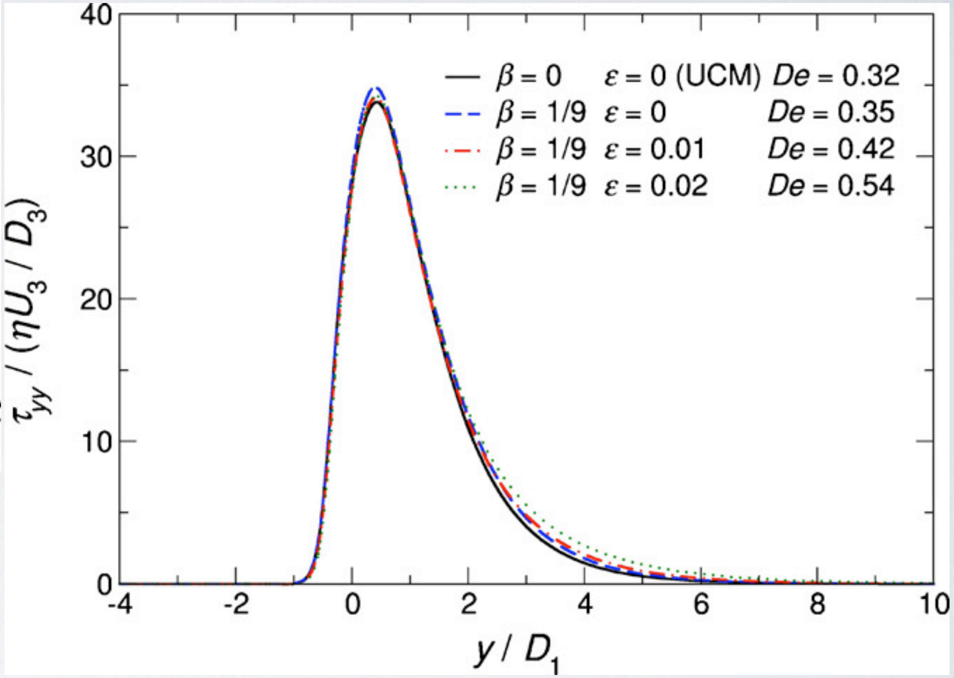
$\beta \geq 6/9$, no steady asymmetry

FLOW FOCUSING: EFFECT OF ε

sPTT





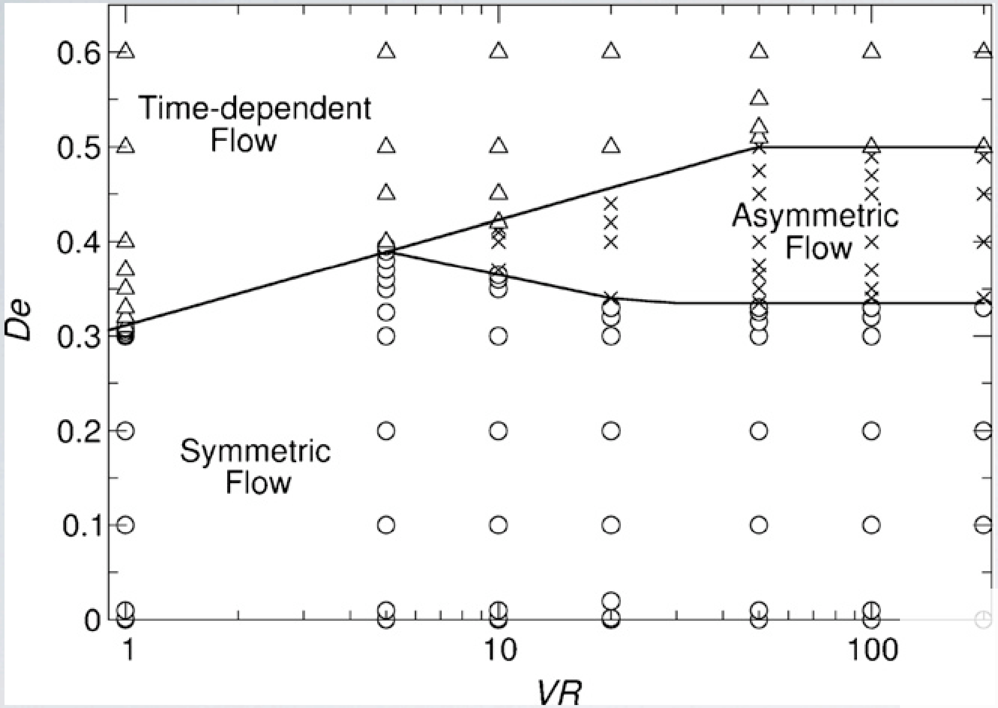


- ε stabilizes the flow
  - increases De_c
  - decreases degree of asymmetry
  - $\varepsilon \geq 0.04$ steady asymmetry disappears
  - **(Transition directly to unsteady flow)**

- Similar levels of normal stresses achieved near critical conditions
- Extensional properties decisive for onset of flow asymmetry

FLOW FOCUSING: NUMERICAL VERSUS EXPERIMENTS (PAA 125)

Oliveira et al. JNNFM 160 (2009) 31-39

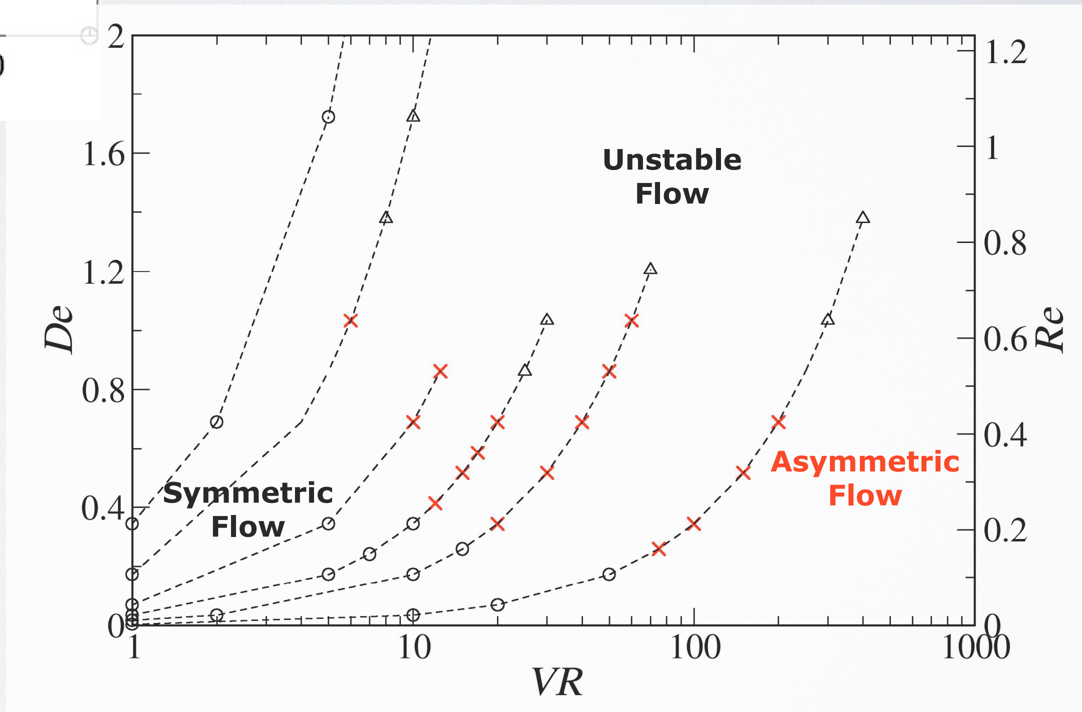


Numerical

UCM, 2D, Re=0

Experimental

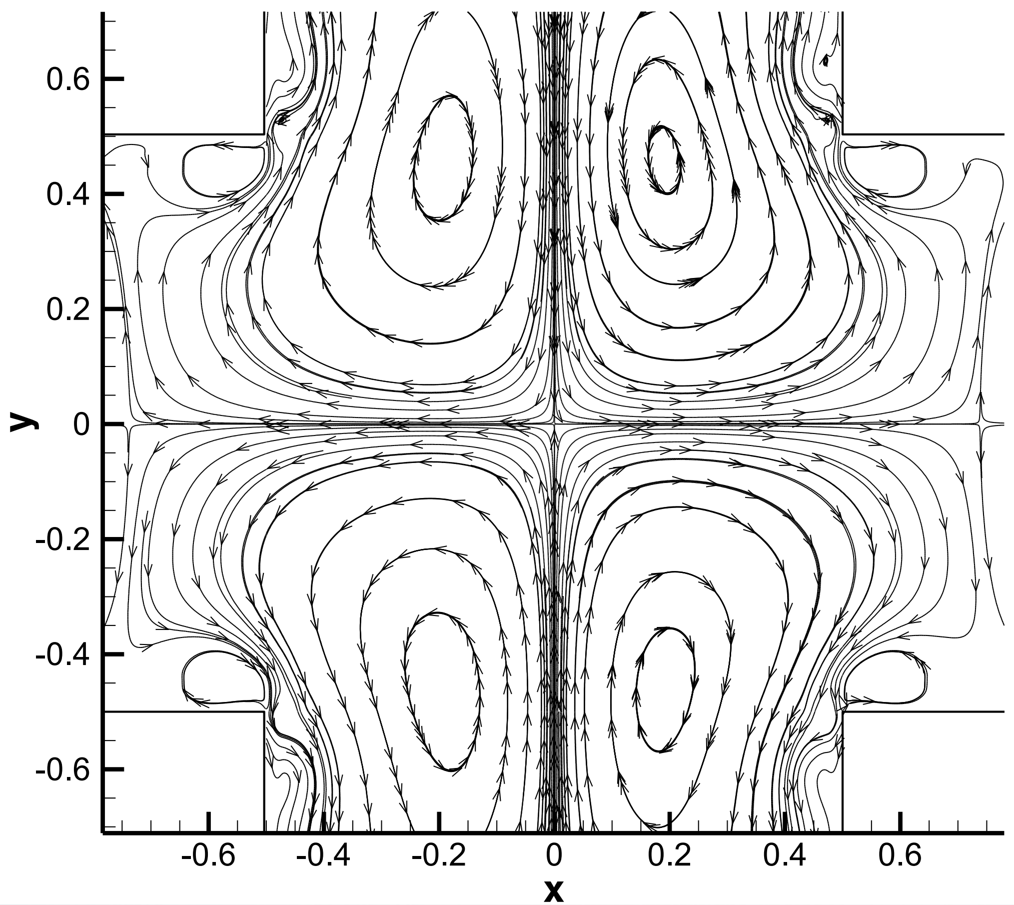
PAA 125 + NaCl

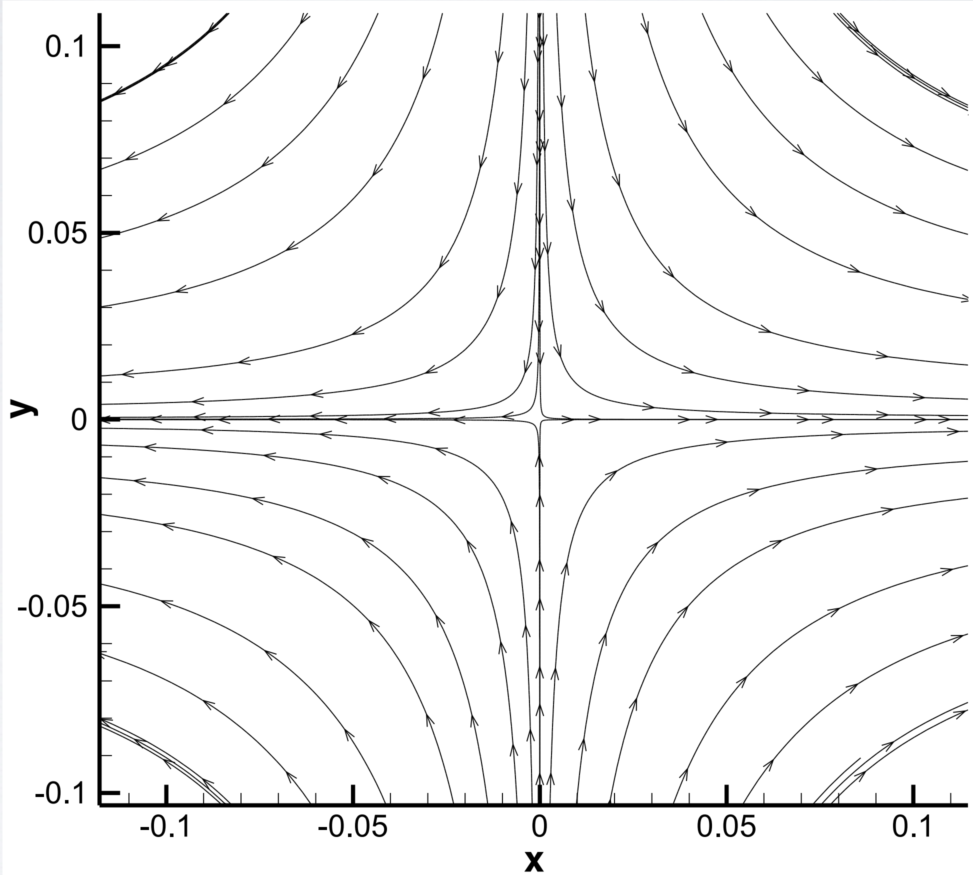


STAGNATION FLOW
Symmetry & asymmetry
Some observations from numerics on cross flow

DIFFERENCE OF TWO SYMMETRIC FLOWS FAR FROM TRANSITION

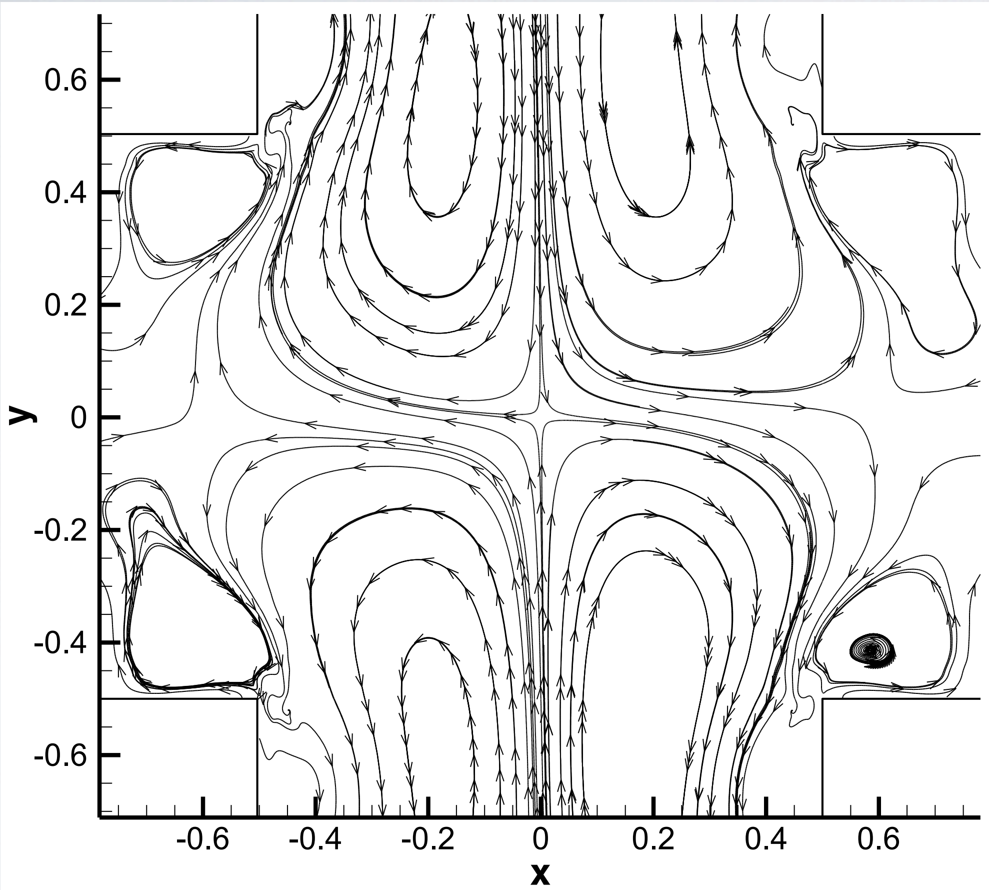
$$De = 0.20 - De = 0.19$$

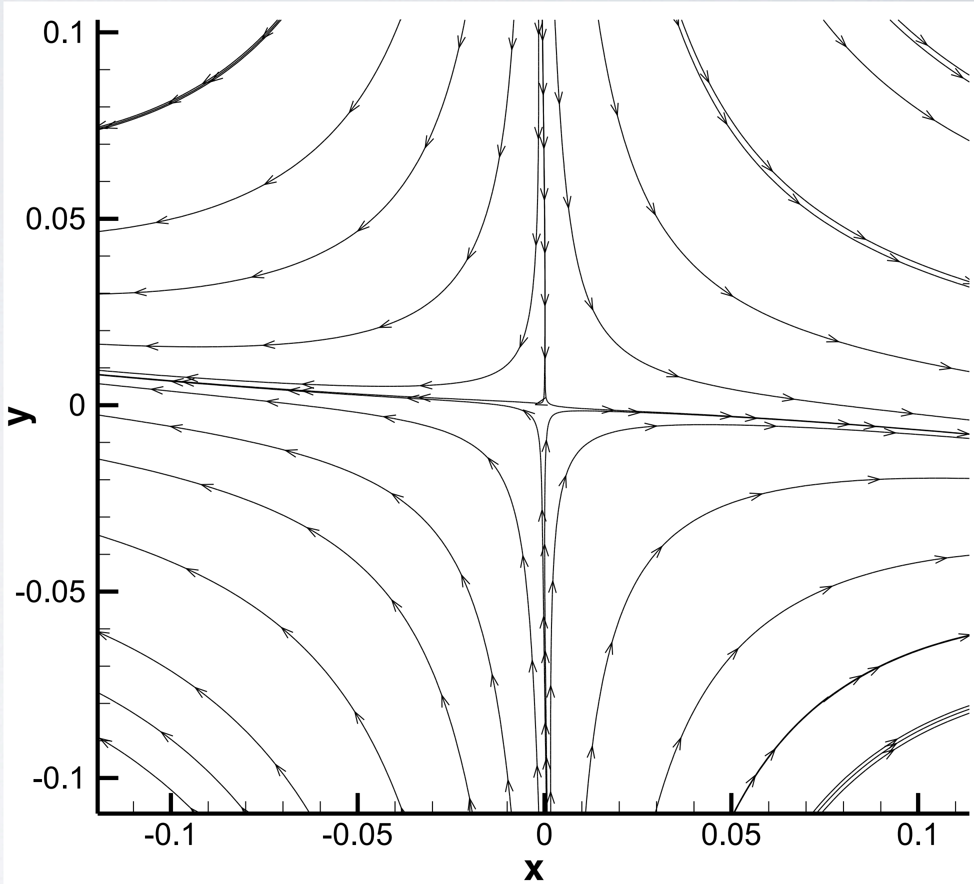




DIFFERENCE OF TWO SYMMETRIC FLOWS CLOSE TO TRANSITION

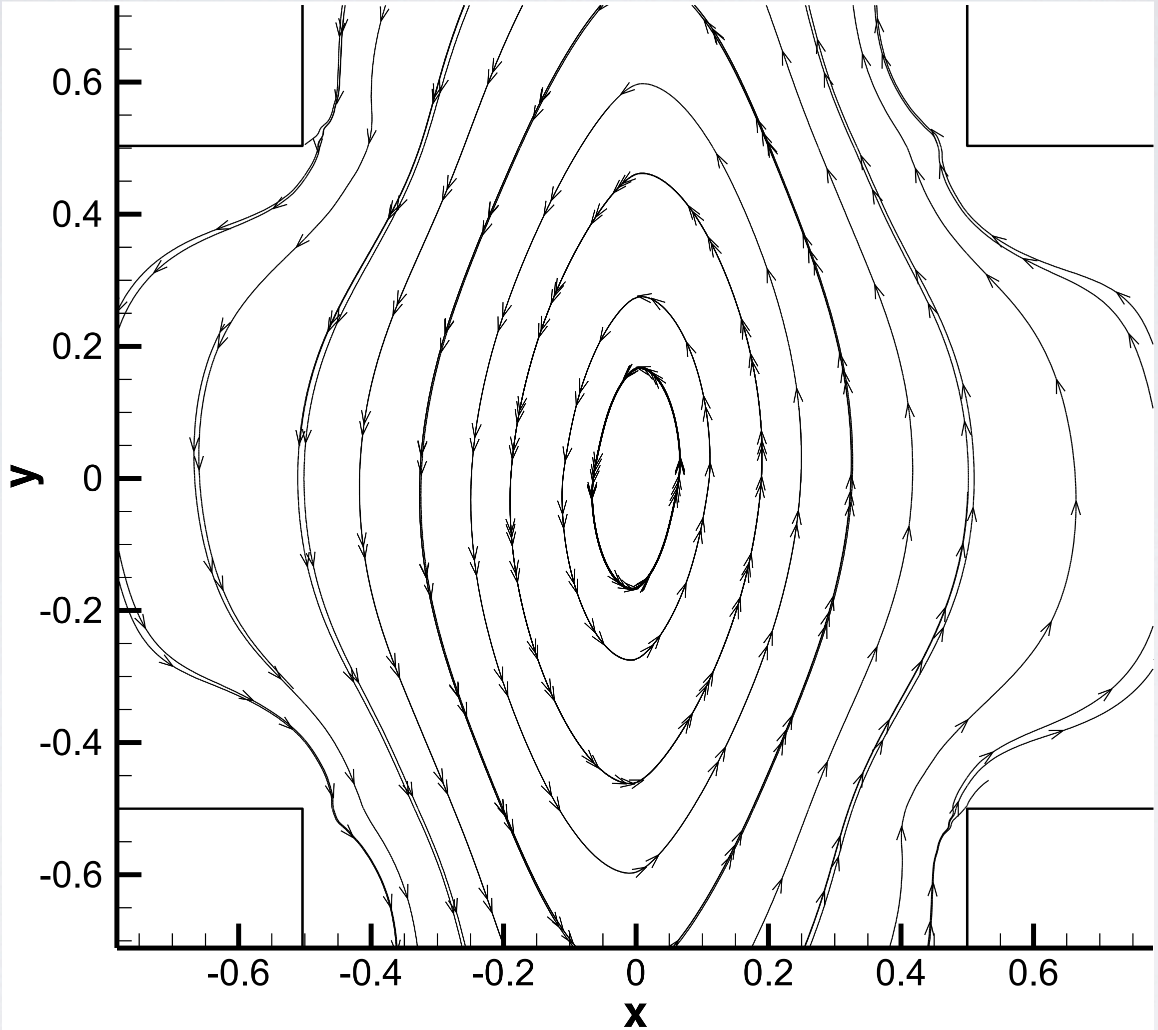
$$De = 0.308 - De = 0.307$$





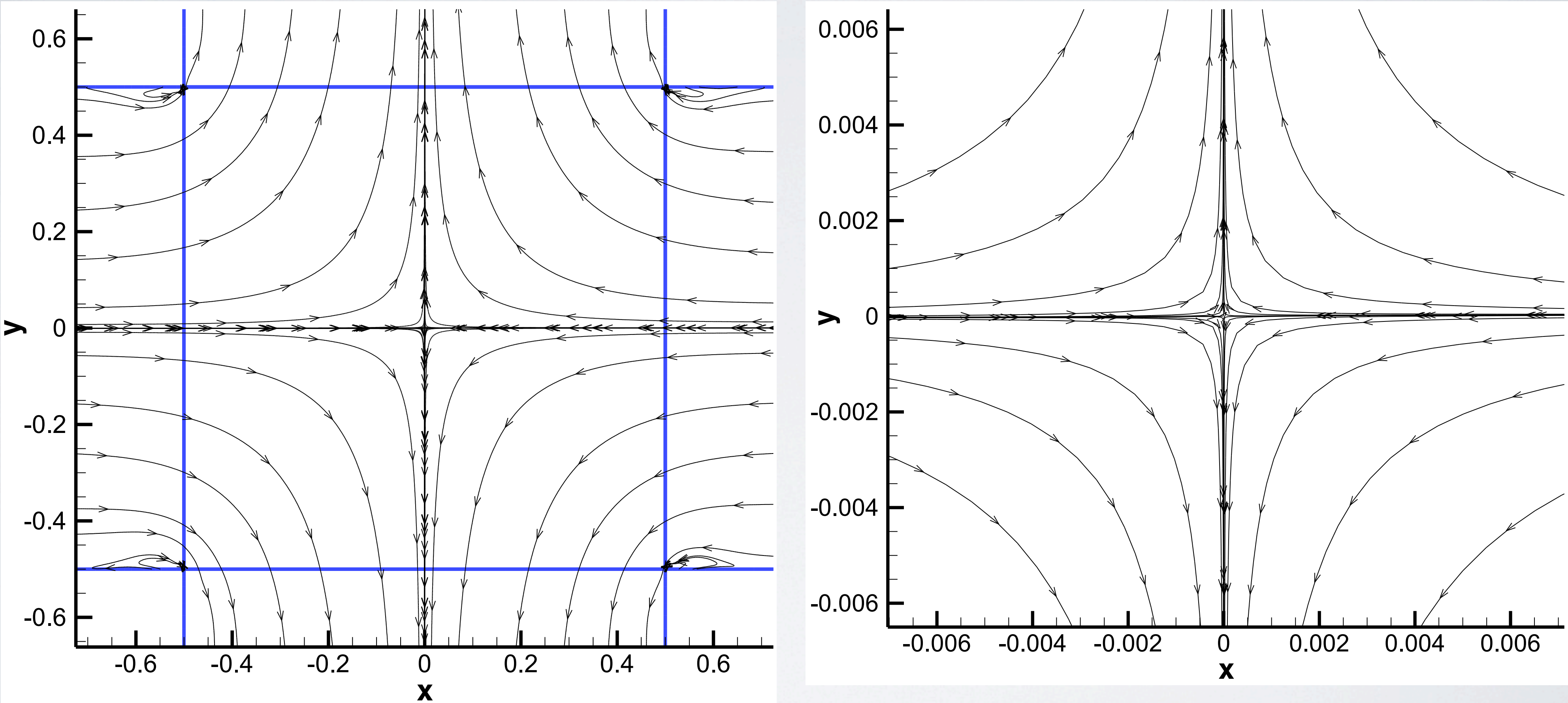
CRITICAL FLOW - SYMMETRIC FLOW

$$De = 0.310 - De = 0.309$$



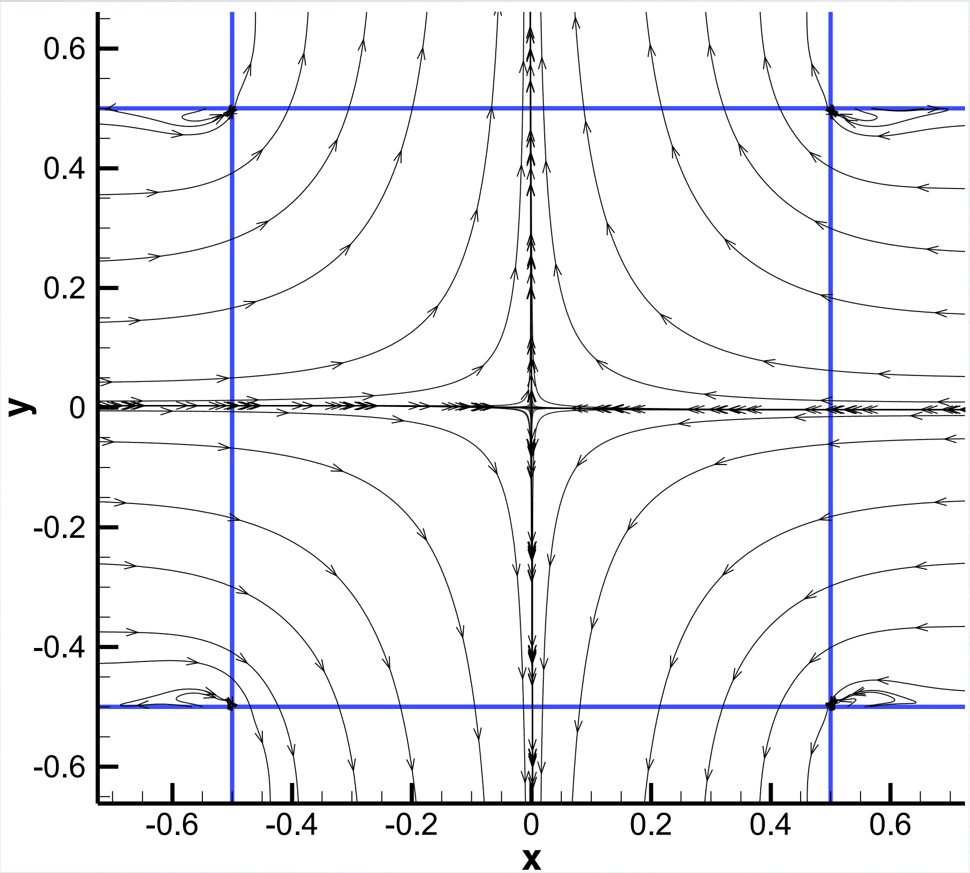
STREAMLINES FOR SYMMETRIC FLOW

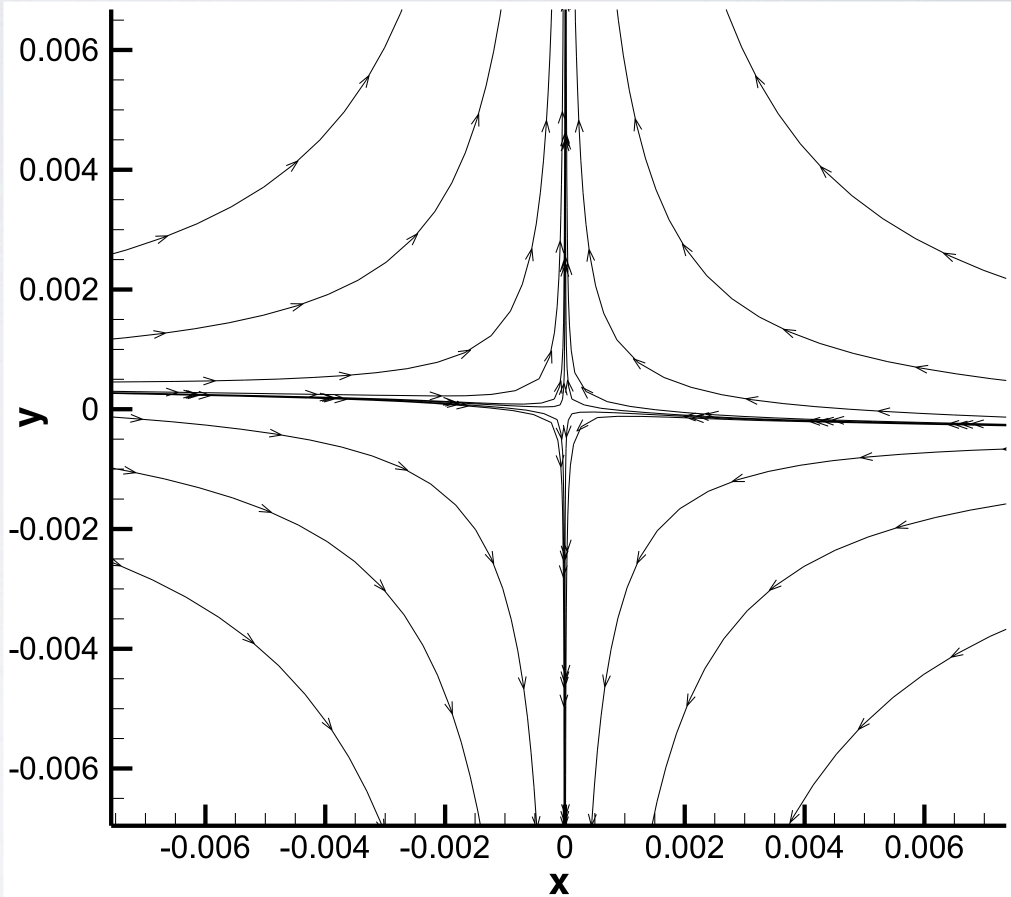
$$De = 0.309$$



STREAMLINES FOR CRITICAL FLOW

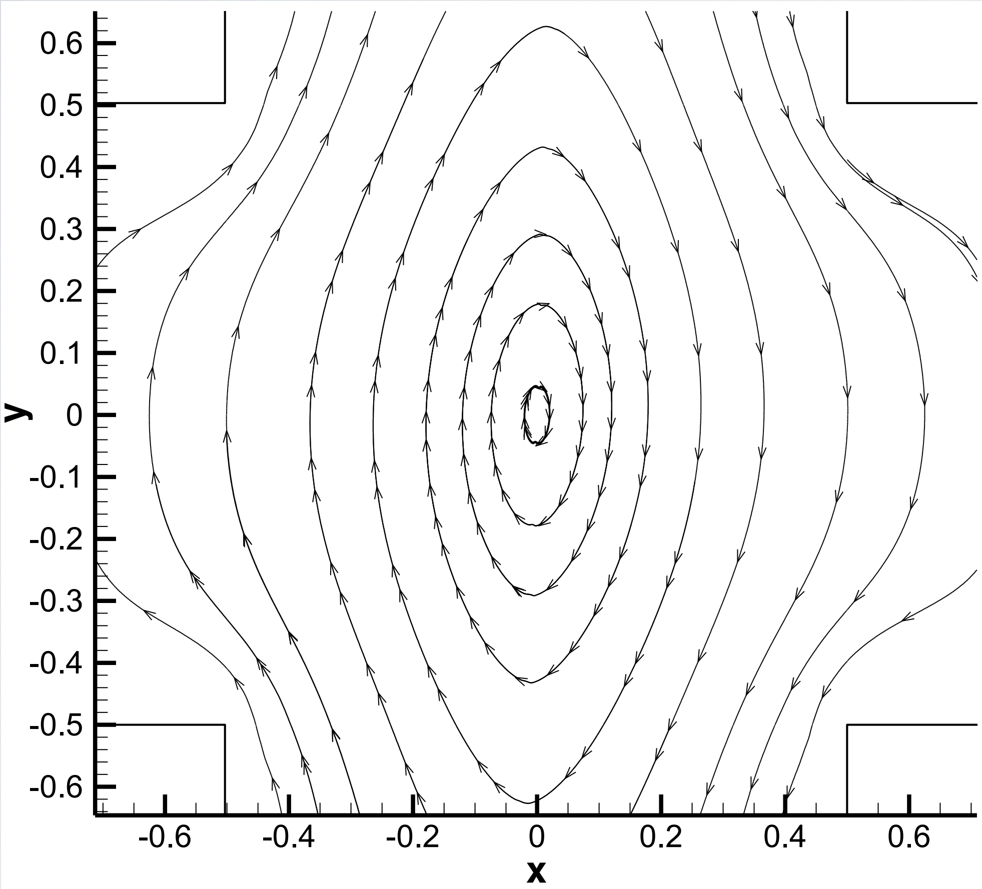
$$De = 0.310$$



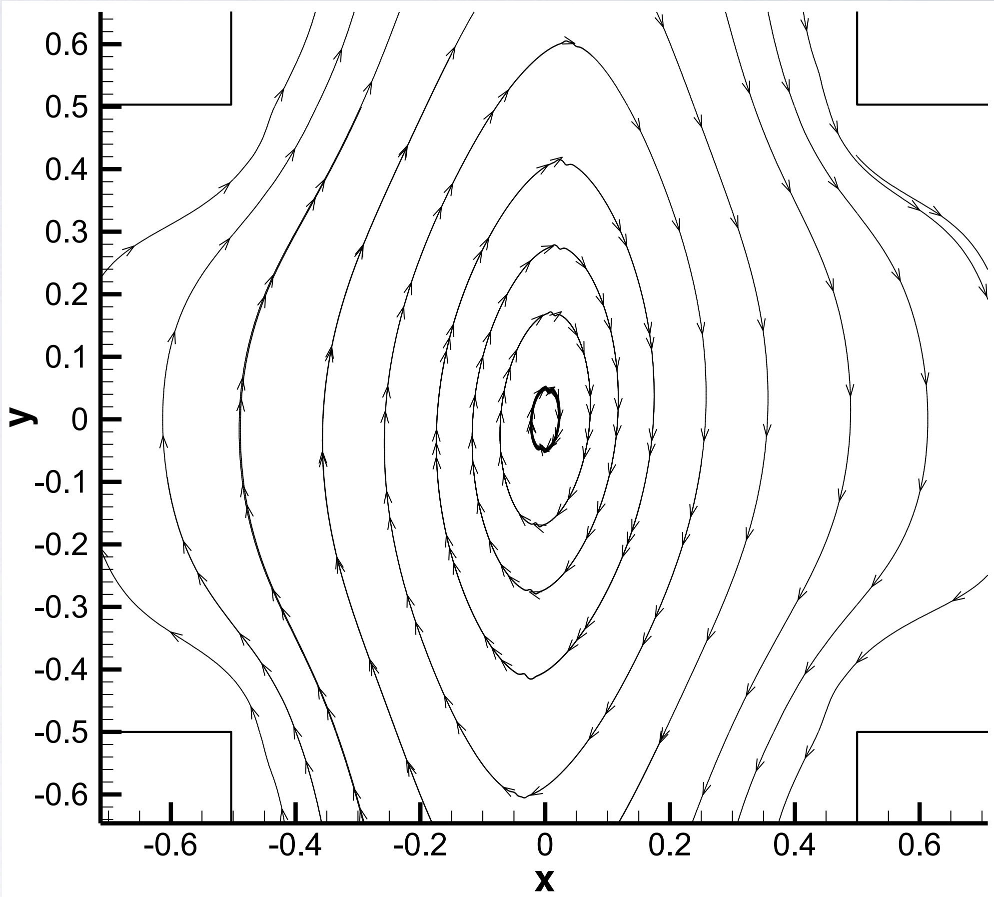


SMALL DIFFERENCE BETWEEN TWO ASYMMETRIC FLOWS

$De = 0.312 - De = 0.311$



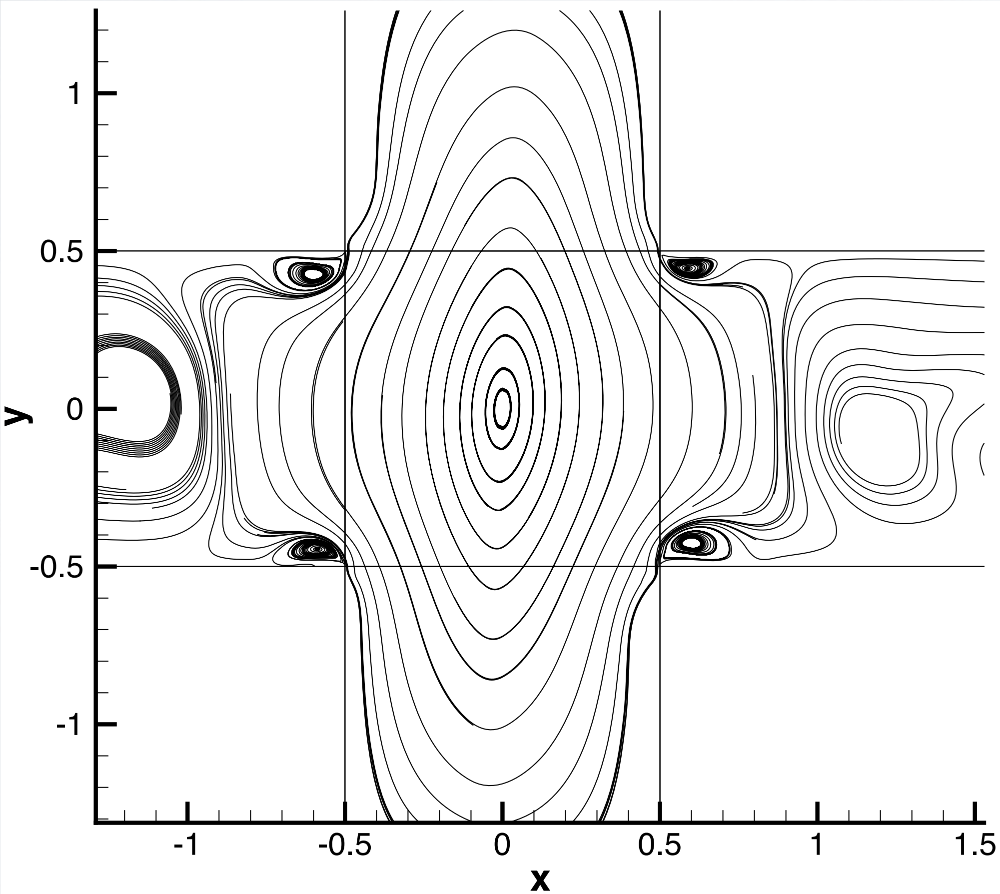
$De = 0.315 - De = 0.314$



LARGER DIFFERENCES

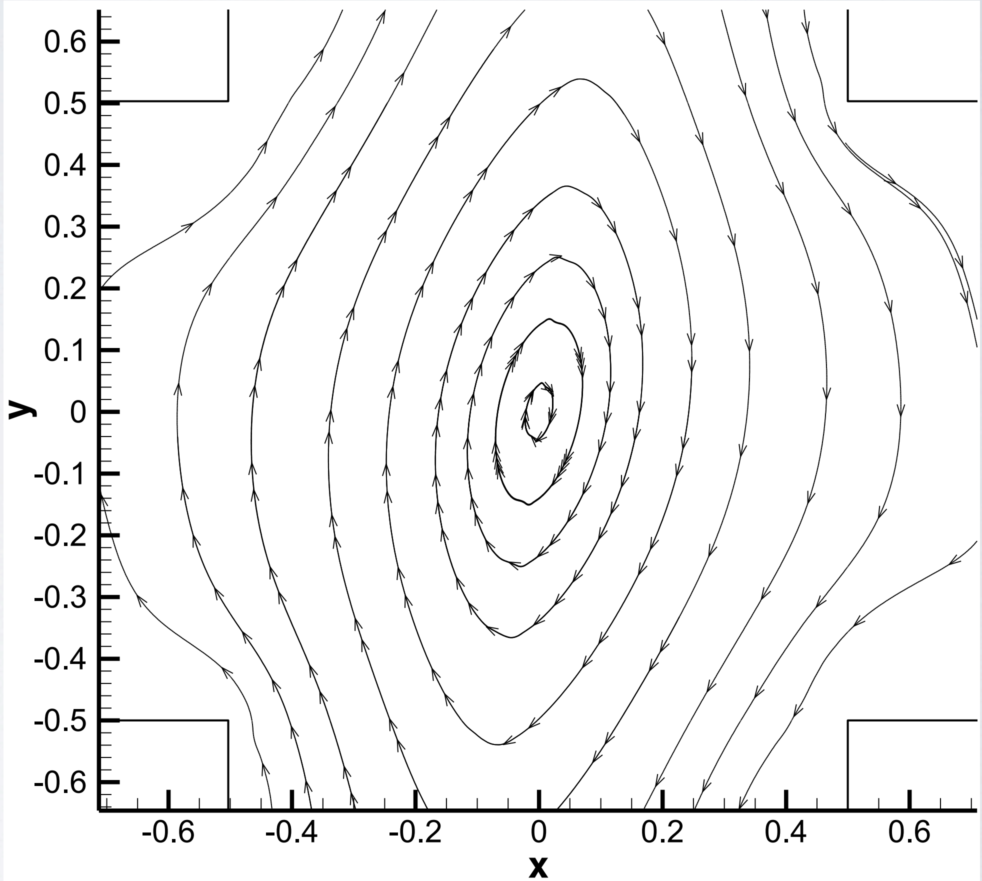
Asymmetric- critical

$$De = 0.32 - De = 0.31$$



Asymmetric- asymmetric

$$De = 0.34 - De = 0.32$$



STAGNATION + VORTEX FLOW
An analytical solution

PROBLEM FORMULATION: UCM

Stagnation flow

$$u_{sta} = ax$$

$$v_{sta} = -ay$$

"Vortex" flow

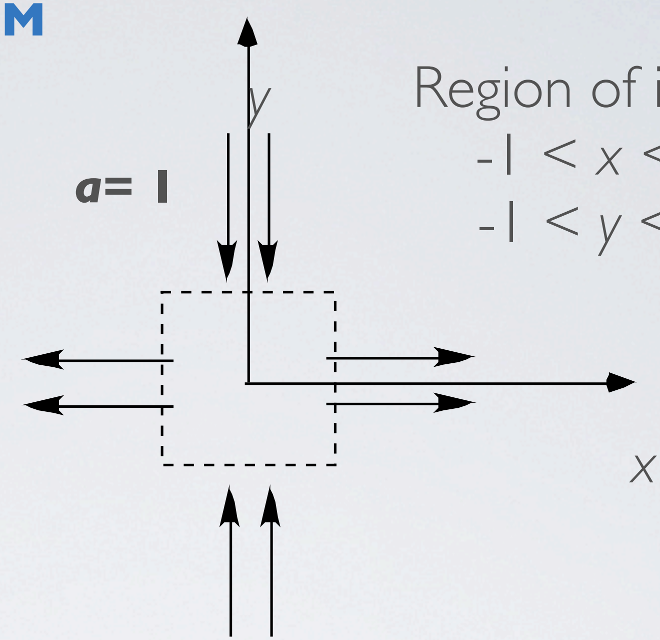
$$u_{vor} = b_u y$$

$$v_{vor} = b_v x$$

Stagnation + "vortex" flow

$$u = ax + b_u y$$

$$v = -ay + b_v x$$



Region of interest

$-1 < x < +1$

$-1 < y < +1$

Numerical: *a*= -1

$$\tau_{xx} + De\left[u\frac{\partial \tau_{xx}}{\partial x} + v\frac{\partial \tau_{xx}}{\partial y} - 2\left(\tau_{xx}\frac{\partial u}{\partial x} + \tau_{xy}\frac{\partial v}{\partial x}\right)\right] = 2\frac{\partial u}{\partial x}$$

$$\tau_{xy} + De\left[u\frac{\partial \tau_{xy}}{\partial x} + v\frac{\partial \tau_{xy}}{\partial y} - \left(\tau_{xx}\frac{\partial u}{\partial y} + \tau_{yy}\frac{\partial v}{\partial x}\right)\right] = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\tau_{yy} + De\left[u\frac{\partial \tau_{yy}}{\partial x} + v\frac{\partial \tau_{yy}}{\partial y} - 2\left(\tau_{xy}\frac{\partial u}{\partial y} + \tau_{yy}\frac{\partial v}{\partial y}\right)\right] = 2\frac{\partial v}{\partial y}$$

GENERAL SOLUTION & CONSTANT SOLUTION

$$\tau_{xx} + De\left[\left(b_u y + ax\right)\frac{\partial \tau_{xx}}{\partial x} + \left(b_v x - ay\right)\frac{\partial \tau_{xx}}{\partial y} - 2\left(a\tau_{xx} + b_v \tau_{xy}\right)\right] = 2a$$

$$\tau_{xy} + De\left[\left(b_v x - ay\right)\frac{\partial \tau_{xy}}{\partial x} + \left(b_u y + ax\right)\frac{\partial \tau_{xy}}{\partial y} - \left(b_u \tau_{xx} + b_v \tau_{yy}\right)\right] = b_u + b_v$$

$$\tau_{ij} = \left(\tau_{ij}\right)\Big|_{const} + \left(\tau_{ij}\right)\Big|_{\text{homogeneous}}$$

$$\tau_{yy} + De\left[\left(b_v x - ay\right)\frac{\partial \tau_{yy}}{\partial x} + \left(b_u y + ax\right)\frac{\partial \tau_{yy}}{\partial y} - 2\left(b_u \tau_{xy} - a\tau_{yy}\right)\right] = -2a$$

$$\frac{\partial \tau_{ij}}{\partial x_k} = 0 \Rightarrow \quad \tau_{xx} = -\frac{2\left[a + 2a^2 De + b_v\left(b_u + b_v\right)De\right]}{-1 + 4\left(a^2 + b_u b_v\right)De^2}$$

$$\tau_{xy} = -\frac{b_u + b_v + 2a\left(b_u - b_v\right)De}{-1 + 4\left(a^2 + b_u b_v\right)De^2} \qquad \tau_{yy} = \frac{2\left[a - 2a^2 De - b_u\left(b_u + b_v\right)De\right]}{-1 + 4\left(a^2 + b_u b_v\right)De^2}$$

This solution absorbs the constants on the rhs of constitutive equation

HOMOGENEOUS SOLUTION (I)

$$\tau_{xx} + De\left[\left(b_u y + ax\right)\frac{\partial \tau_{xx}}{\partial x} + \left(b_v x - ay\right)\frac{\partial \tau_{xx}}{\partial y} - 2\left(a\tau_{xx} + b_v \tau_{xy}\right)\right] = 0$$

$$\tau_{xy} + De\left[\left(b_v x - ay\right)\frac{\partial \tau_{xy}}{\partial x} + \left(b_u y + ax\right)\frac{\partial \tau_{xy}}{\partial y} - \left(b_u \tau_{xx} + b_v \tau_{yy}\right)\right] = 0$$

$$\tau_{yy} + De\left[\left(b_v x - ay\right)\frac{\partial \tau_{yy}}{\partial x} + \left(b_u y + ax\right)\frac{\partial \tau_{yy}}{\partial y} - 2\left(b_u \tau_{xy} - a\tau_{yy}\right)\right] = 0$$

Solution hypothesis (I): $\tau_{ij}\left(x,y\right) = \tau_{ij}\left(\phi\right)$ with $\phi = kx + Ty$

$$m\phi De\sqrt{a^2 + b_u b_v}\,\frac{d\tau_{xx}}{d\phi} = \left(-1 + 2aDe\right)\tau_{xx} + 2b_v De\tau_{xy}$$

$$m\phi De\sqrt{a^2 + b_u b_v}\,\frac{d\tau_{xy}}{d\phi} = b_u De\tau_{xx} + b_v De\tau_{yy}$$

$$m\phi De\sqrt{a^2 + b_u b_v}\,\frac{d\tau_{yy}}{d\phi} = -\left(1 + 2aDe\right)\tau_{yy} + 2b_u De\tau_{xy}$$

$$k = \frac{Tb_v}{-a \pm \sqrt{a^2 + b_u b_v}}$$

$m = \pm 1$

Solution hypothesis (2): $\tau_{ij}(\phi) = \alpha_{ij}\phi^q$

as in stagnation flow [1,2]

[1] Renardy JNNFM 138 (2006) 204-205
[2] Becherer, Morozov, van Saarloos JNNFM 153 (2008) 183-190

$$\left[\left(-1+2aDe-mqDe\sqrt{a^2+b_ub_v}\right)\alpha_{xx}+2b_vDe\alpha_{xy}\right]\phi^q=0$$

$$\left[b_uDe\alpha_{xx}+b_vDe\alpha_{yy}-\left(1+mqDe\sqrt{a^2+b_ub_v}\right)\alpha_{xy}\right]\phi^q=0$$

$$\left[2b_uDe\alpha_{xy}-\left(1+2aDe+mqDe\sqrt{a^2+b_ub_v}\right)\alpha_{yy}\right]\phi^q=0$$

$$\alpha_{xx}=\frac{2b_vDe\alpha_{xy}}{-1+2aDe-mqDe\sqrt{a^2+b_ub_v}} \qquad \alpha_{yy}=\frac{2b_uDe\alpha_{xy}}{-1+2aDe+mqDe\sqrt{a^2+b_ub_v}}$$

HOMOGENEOUS SOLUTION (3)

Back-substituting, three possible values of q and three possible stress fields

(1)

$$q = \frac{2}{m} - \frac{1}{mDe\sqrt{a^2 + b_u b_v}}$$

$$\alpha_{xx} = \frac{b_v \alpha_{xy}}{-a + \sqrt{a^2 + b_u b_v}}$$

$$\alpha_{yy} = \frac{b_u \alpha_{xy}}{a + \sqrt{a^2 + b_u b_v}}$$

(2)

$$q = -\frac{1}{mDe\sqrt{a^2 + b_u b_v}}$$

$$\alpha_{xx} = \frac{-b_v \alpha_{xy}}{a}$$

$$\alpha_{yy} = \frac{b_u \alpha_{xy}}{a}$$

(3)

$$q = -\frac{2}{m} - \frac{1}{mDe\sqrt{a^2 + b_u b_v}}$$

$$\alpha_{xx} = \frac{-b_v \alpha_{xy}}{a + \sqrt{a^2 + b_u b_v}}$$

$$\alpha_{yy} = \frac{b_u \alpha_{xy}}{a - \sqrt{a^2 + b_u b_v}}$$

Homogeneous solution is sum of all

Momentum not yet enforced

No boundary conditions imposed

MOMENTUM EQUATION (I)

$$\frac{\partial}{\partial y}\left(-\frac{\partial p}{\partial x}+\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}\right)-\frac{\partial}{\partial x}\left(-\frac{\partial p}{\partial y}+\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}\right)=0$$

Case I

$m=-1$ avoids singularity at x=0, y=0 when De <<1

~~$b_u = -\frac{a^2}{b_v}$~~ ⟶ singularities at all *De*

Stagnation + "vortex" flow

$$u = ax + b_u y$$

$$v = -ay + b_v x$$

$$b_u = b_v$$

$$b_u = \frac{1-9a^2De^2}{9b_vDe^2}$$

$$b_u = \frac{1-4a^2De^2}{4b_vDe^2}$$

possible forms to obey simultaneously momentum & UCM

Note change of signs at *De*= 1/(3a) and 1/(2a)

MOMENTUM EQUATION (2)

Case 2

$$b_u = -\frac{a^2}{b_v}$$ → singularities

$$b_u = \frac{1-a^2 De}{b_v De^2}$$ → ok, but needs to be compatible with case (1) (restrict values of *a* and *De*)

$$b_u = -2ia - b_v$$

$$b_u = 2ia - b_v$$

neglected at this stage

We will consider no contributions from case 2 to the solution (k=0 and $\alpha_{xy}=0$)

Case 3

After substitution of stresses all terms in equation are multiplied by (1+*m*). Since m=-1, momentum is automatically satisfied

STRESS FIELD

$$\tau_{xx} = \frac{b_v \alpha_{xy1}}{-a+\sqrt{a^2+b_u b_v}} \phi^{\frac{2}{m}-\frac{1}{mDe\sqrt{a^2+b_u b_v}}} - \frac{b_v \alpha_{xy3}}{a+\sqrt{a^2+b_u b_v}} \phi^{-\frac{2}{m}-\frac{1}{mDe\sqrt{a^2+b_u b_v}}} - \frac{2\left(a+2Dea^2+b_u b_v De+b_v^2 De\right)}{4a^2De^2-1+4b_u b_v De^2}$$

$$\tau_{xy} = \alpha_{xy1}\phi^{\frac{2}{m}-\frac{1}{mDe\sqrt{a^2+b_u b_v}}} + \alpha_{xy3}\phi^{-\frac{2}{m}-\frac{1}{mDe\sqrt{a^2+b_u b_v}}} - \frac{b_u+b_v+2aDe\left(b_u-b_v\right)}{4a^2De^2-1+4b_u b_v De^2}$$

$$\tau_{yy} = \frac{b_u \alpha_{xy1}}{a+\sqrt{a^2+b_u b_v}} \phi^{\frac{2}{m}-\frac{1}{mDe\sqrt{a^2+b_u b_v}}} + \frac{b_u \alpha_{xy3}}{a-\sqrt{a^2+b_u b_v}} \phi^{-\frac{2}{m}-\frac{1}{mDe\sqrt{a^2+b_u b_v}}} - \frac{2\left(-a+2Dea^2+b_u b_v De+b_u^2 De\right)}{4a^2De^2-1+4b_u b_v De^2}$$

with $\alpha_{xy1} = \alpha_{xy1}\left(a,b_u,b_v\right), \alpha_{xy3} = \alpha_{xy3}\left(a,b_u,b_v\right)$ such as

$$\alpha_{xy1} = \frac{\alpha_1\left(-a+\sqrt{a^2+b_u b_v}\right)}{b_v} \qquad \alpha_{xy3} = \frac{\alpha_3\left(a+\sqrt{a^2+b_u b_v}\right)}{b_v}$$

$a=1, b_u=0, b_v=0 \Rightarrow$ Becherer et al. JNNFM 153 (2008) 183

STREAMLINES AND STRESSES (I)

Stream function $\psi = axy + b_u \dfrac{y^2}{2} - b_v \dfrac{x^2}{2}$

Stream function of vortex $\psi_1 = \psi_{total} - \psi_{stagnation} = b_u \dfrac{y^2}{2} - b_v \dfrac{x^2}{2}$

$$b_u = b_v$$

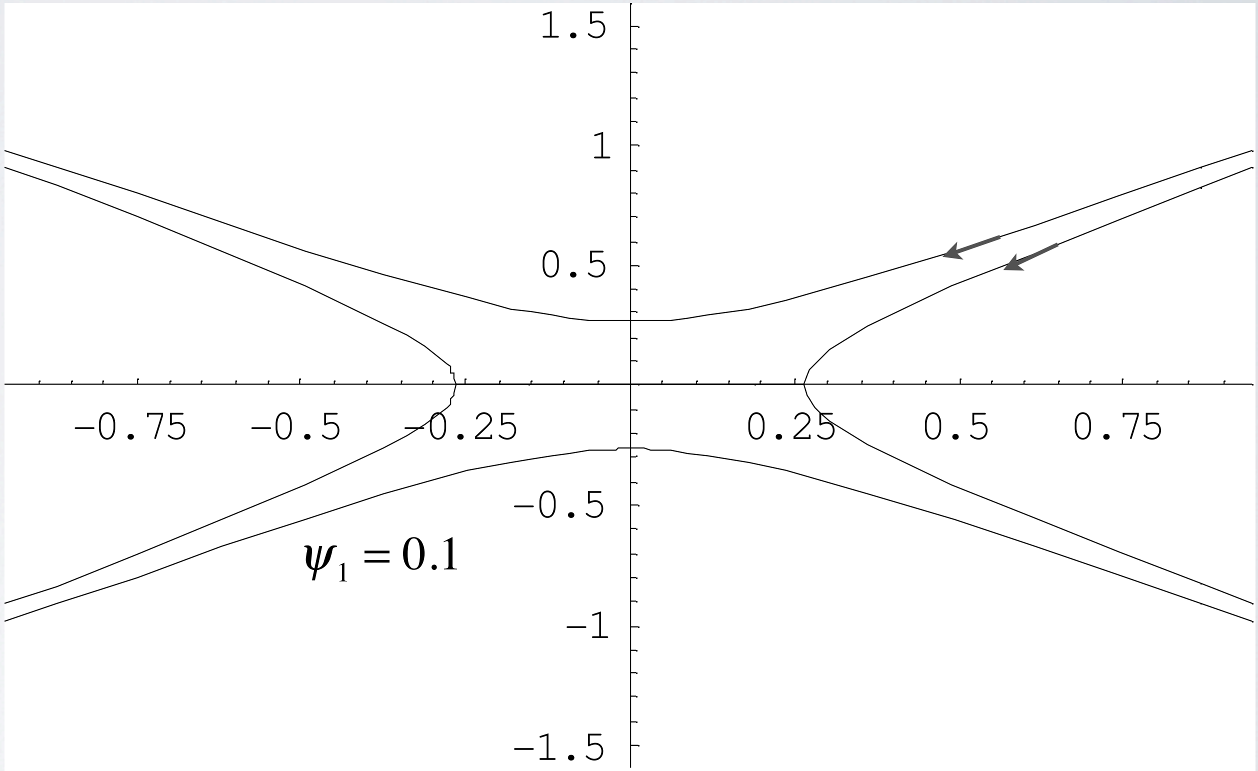
$$b_u = \frac{1 - 9a^2 De}{9 b_v De^2}$$

$$b_u = \frac{1 - 4a^2 De}{4 b_v De^2}$$

(1) $De < \dfrac{1}{\sqrt{9a^2}}$

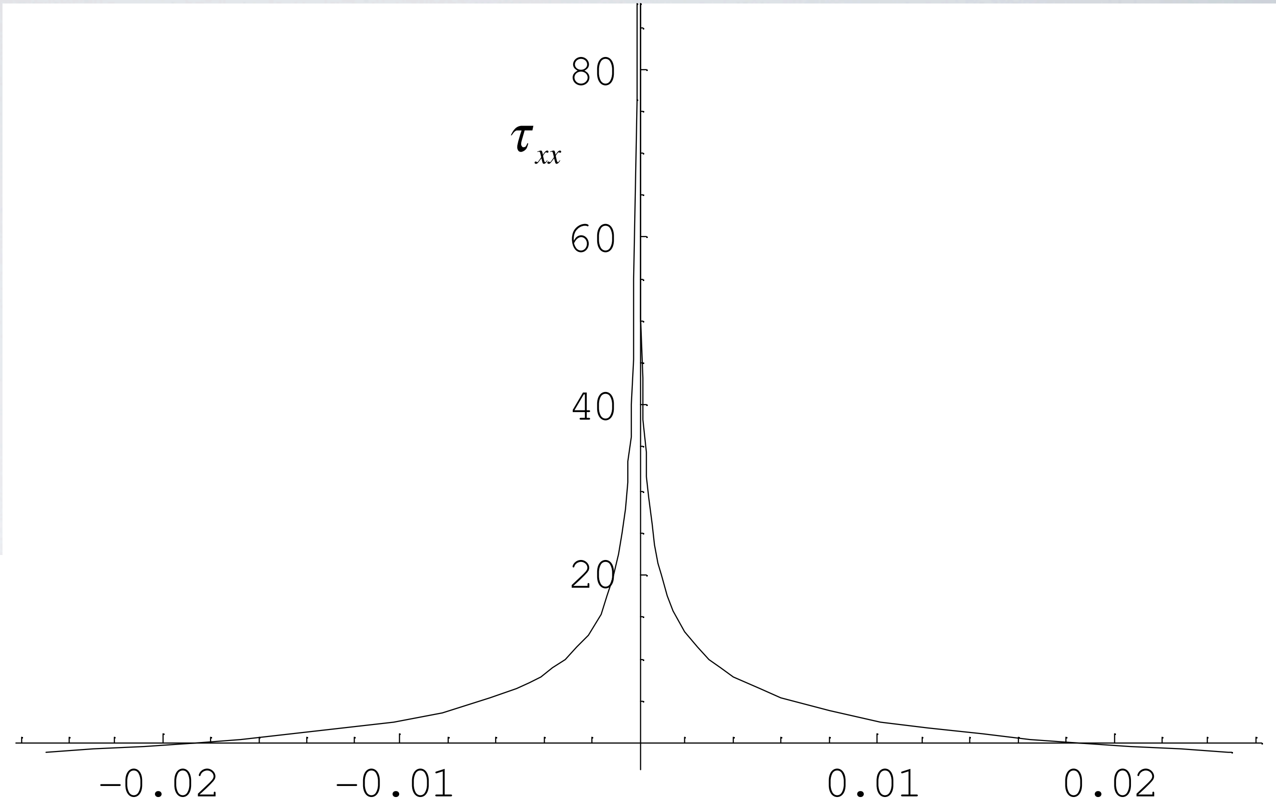
$b_u = b_v = 2.85; a = -1;$

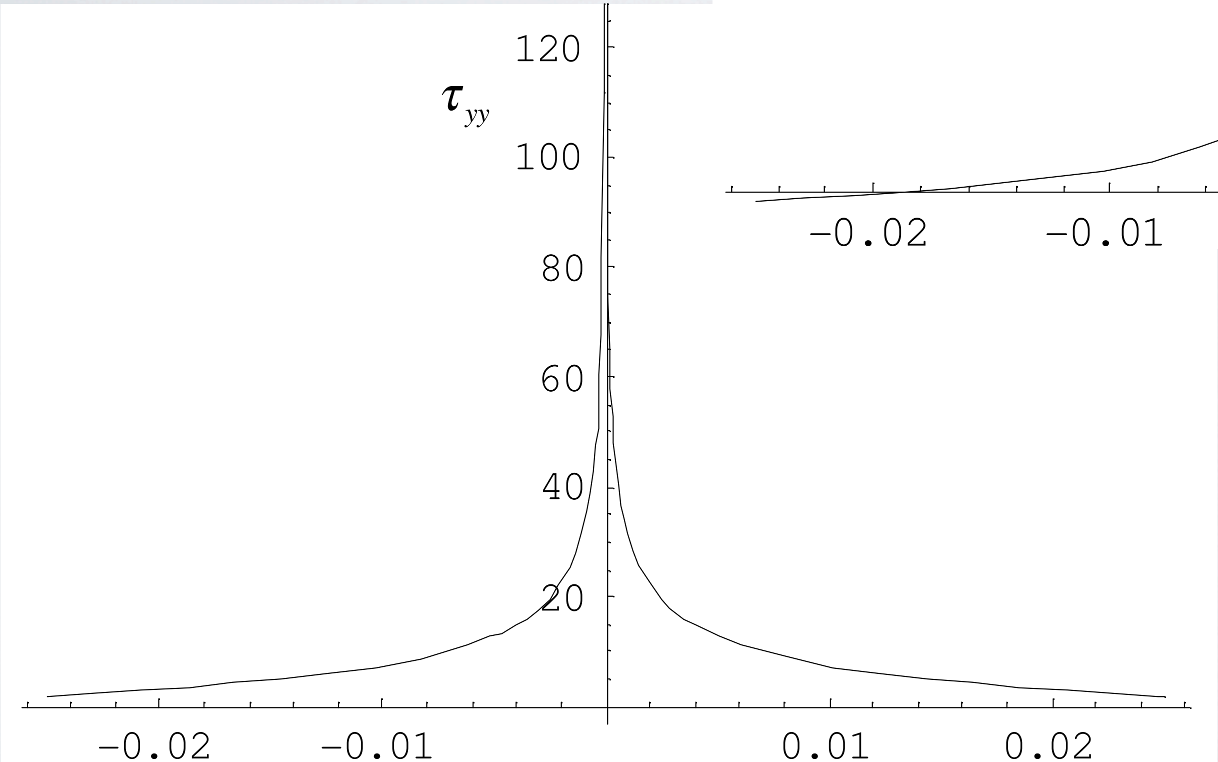
$De = 0.2$



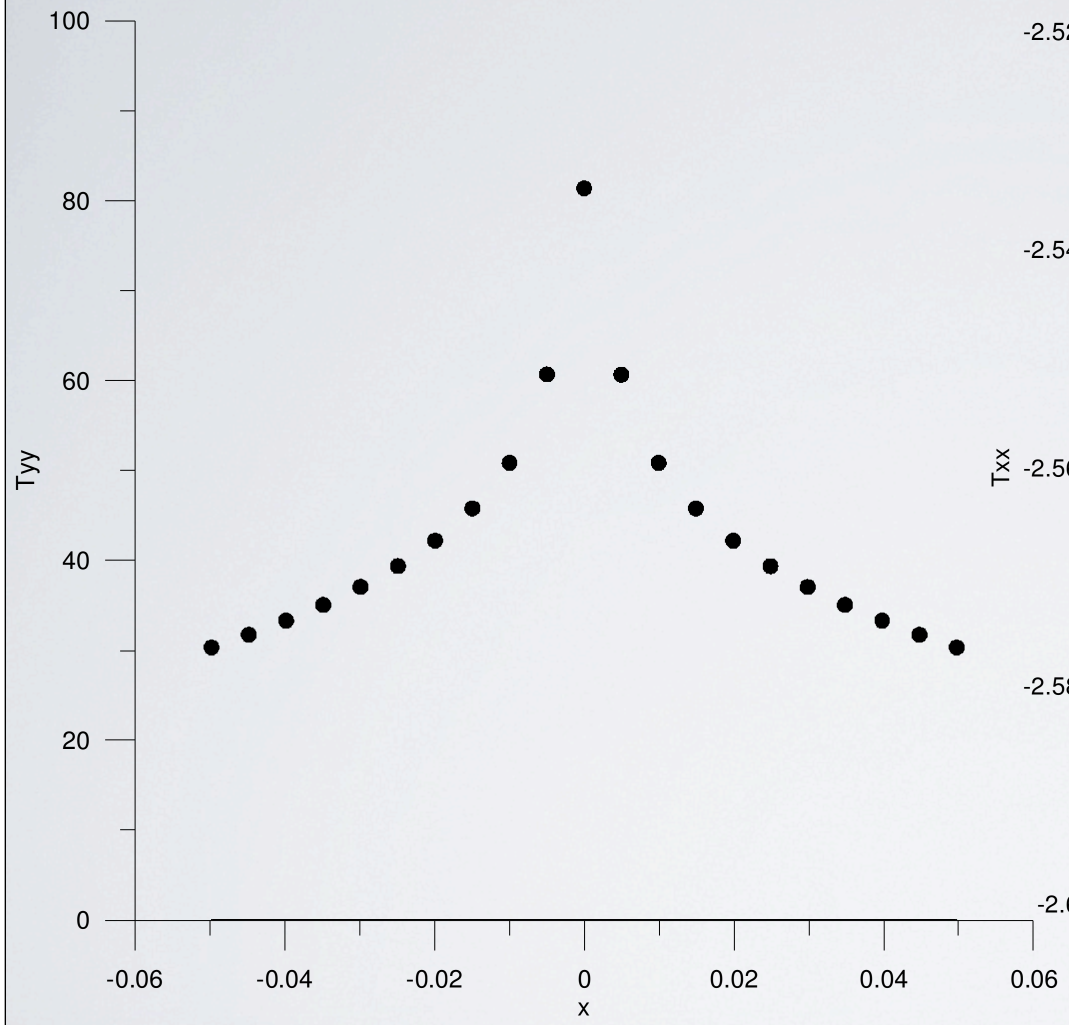
STREAMLINES AND STRESSES (2)

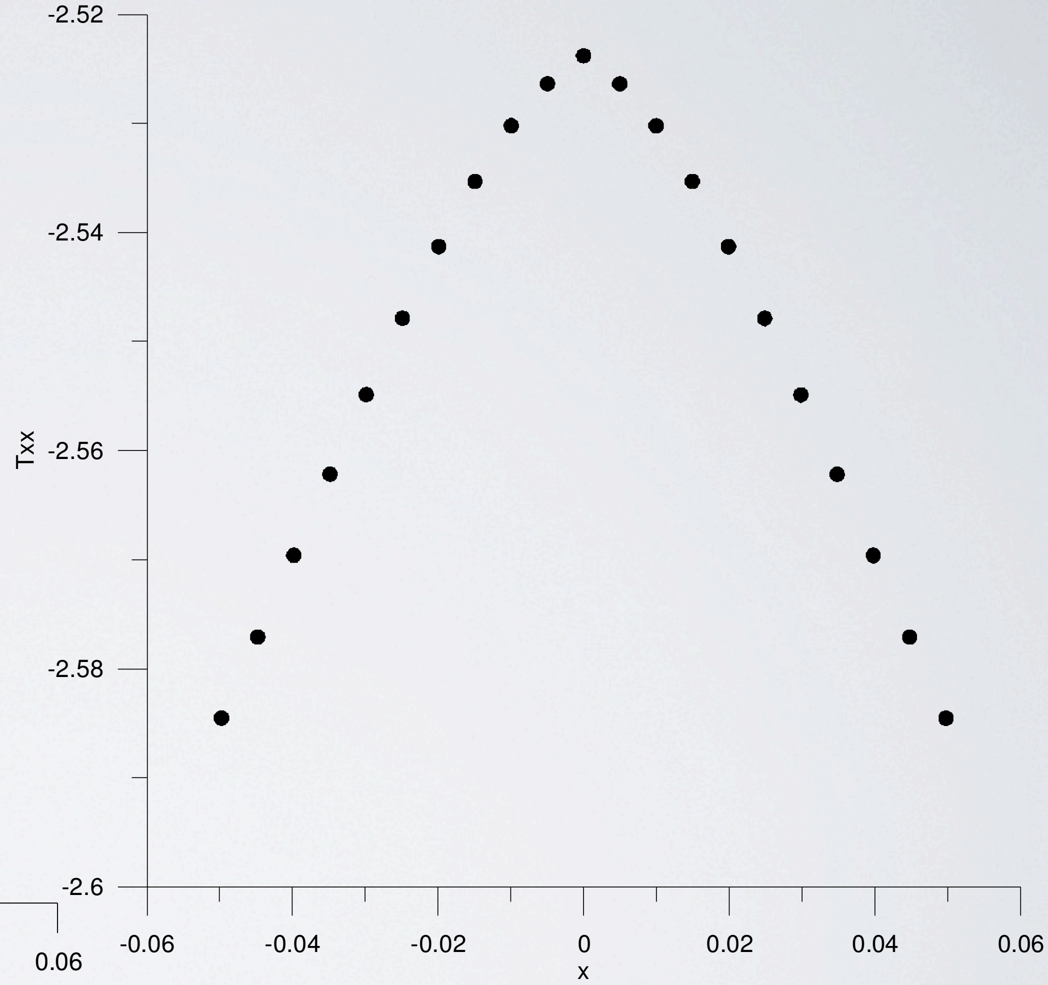
$$y = 0.000025$$





STRESSES FROM NUMERICAL RESULTS: DE=0.2 (I)

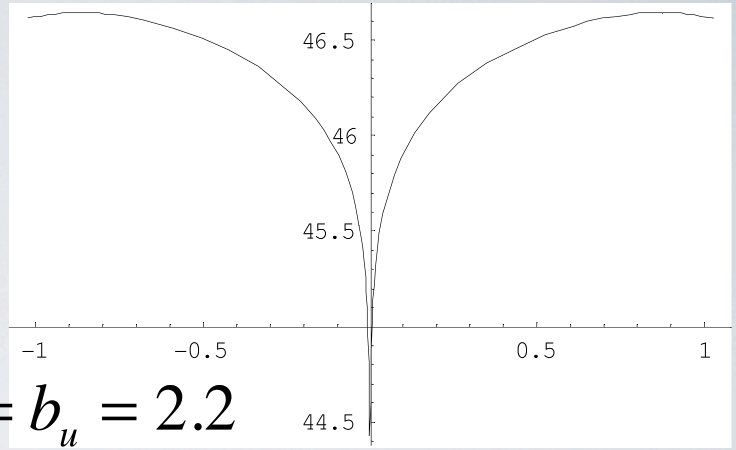




STRESSES (2)

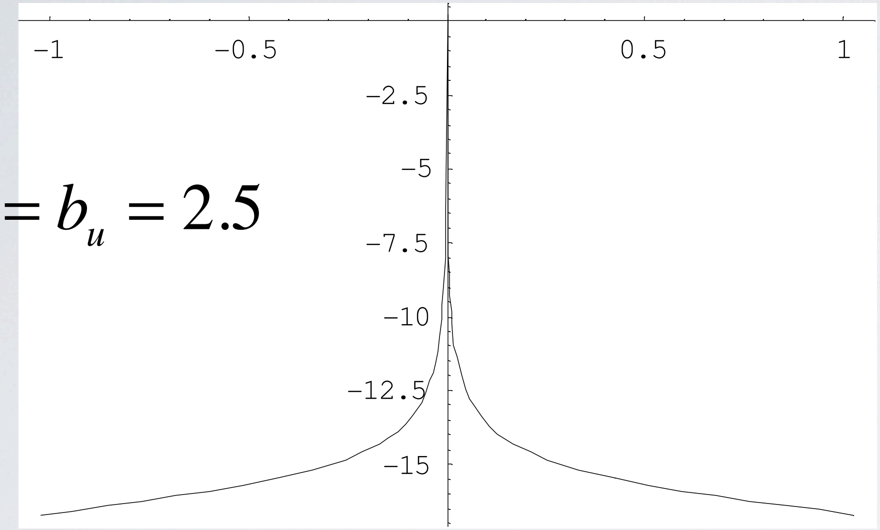
(1a) $De < \frac{1}{\sqrt{9a^2}}$ $\quad De = 0.2; a = -1$

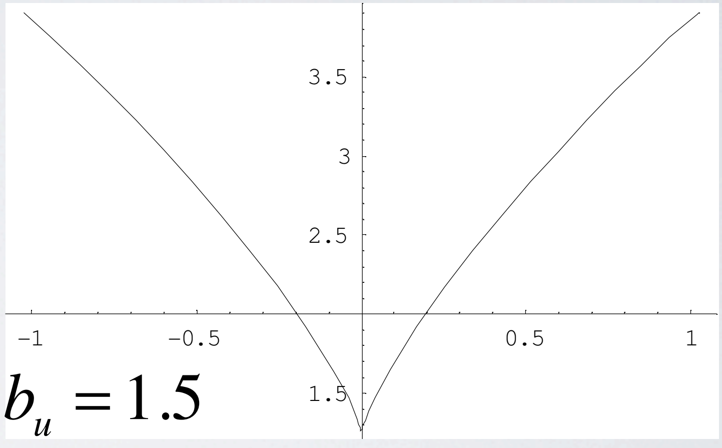
τ_{xx}



$b_v = b_u = 2.2$

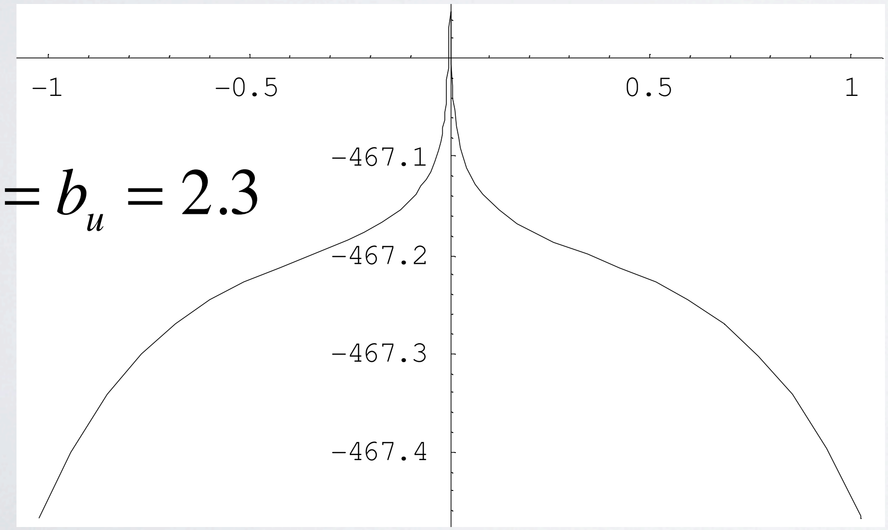
$b_v = b_u = 2.5$

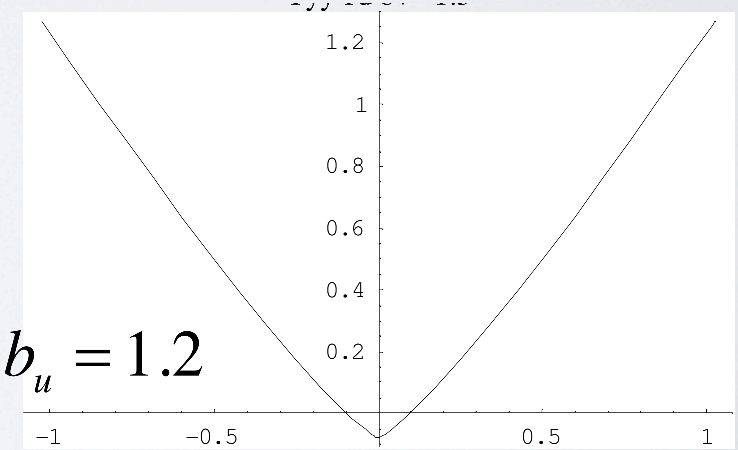




$b_v = b_u = 1.5$

$b_v = b_u = 2.3$





$b_v = b_u = 1.2$

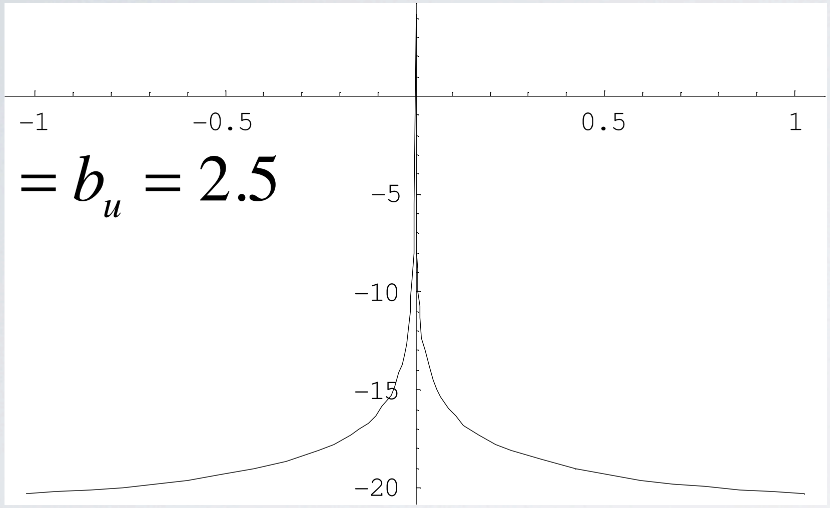
Txx 1e bv =1.5

STRESSES (3)

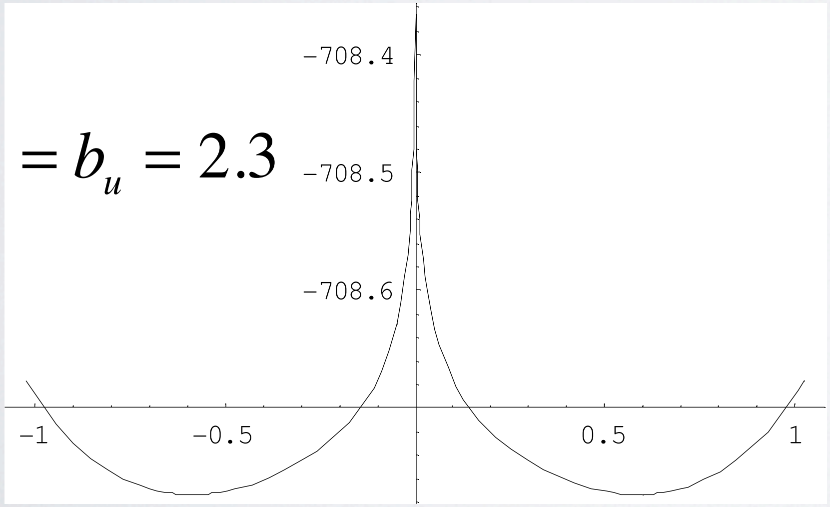
(1) $De < \dfrac{1}{\sqrt{9a^2}}$ $\quad De = 0.2; a = -1$

τ_{yy}

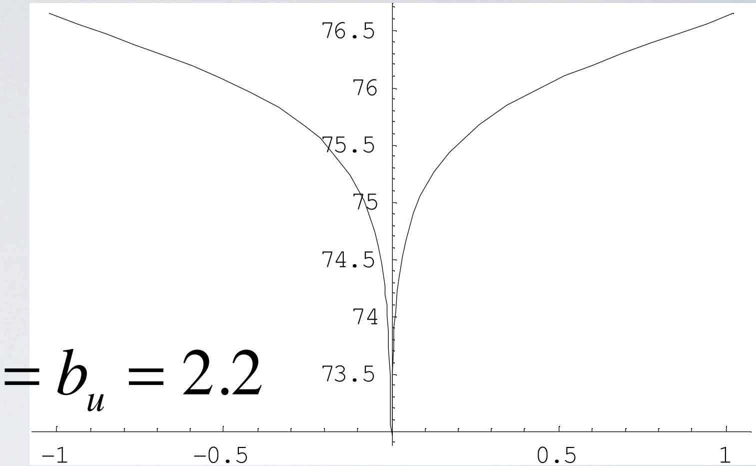
$b_v = b_u = 2.5$



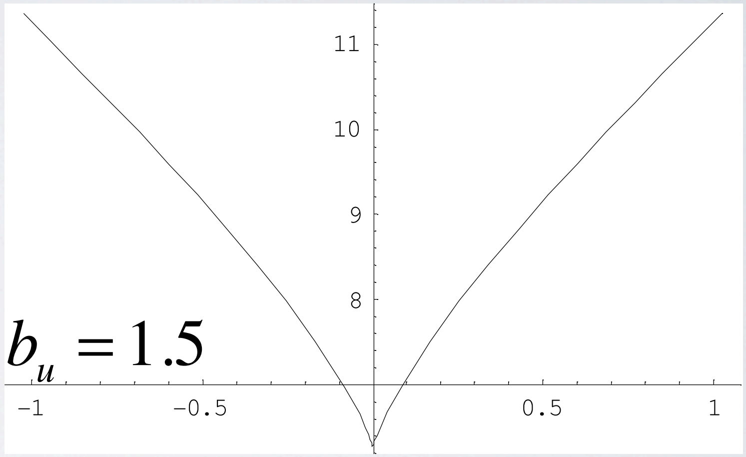
$b_v = b_u = 2.3$



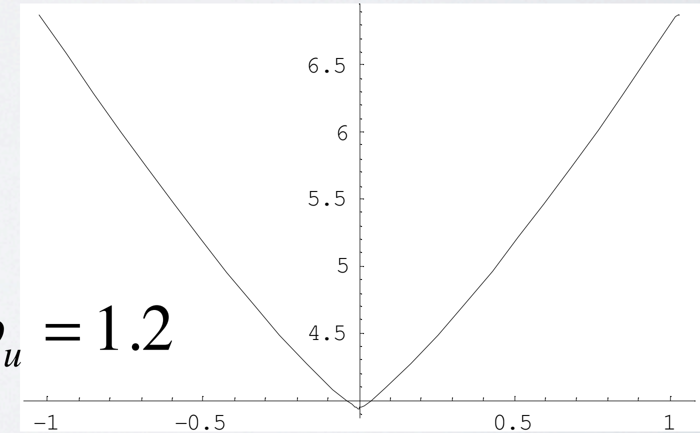
$b_v = b_u = 2.2$



$b_v = b_u = 1.5$



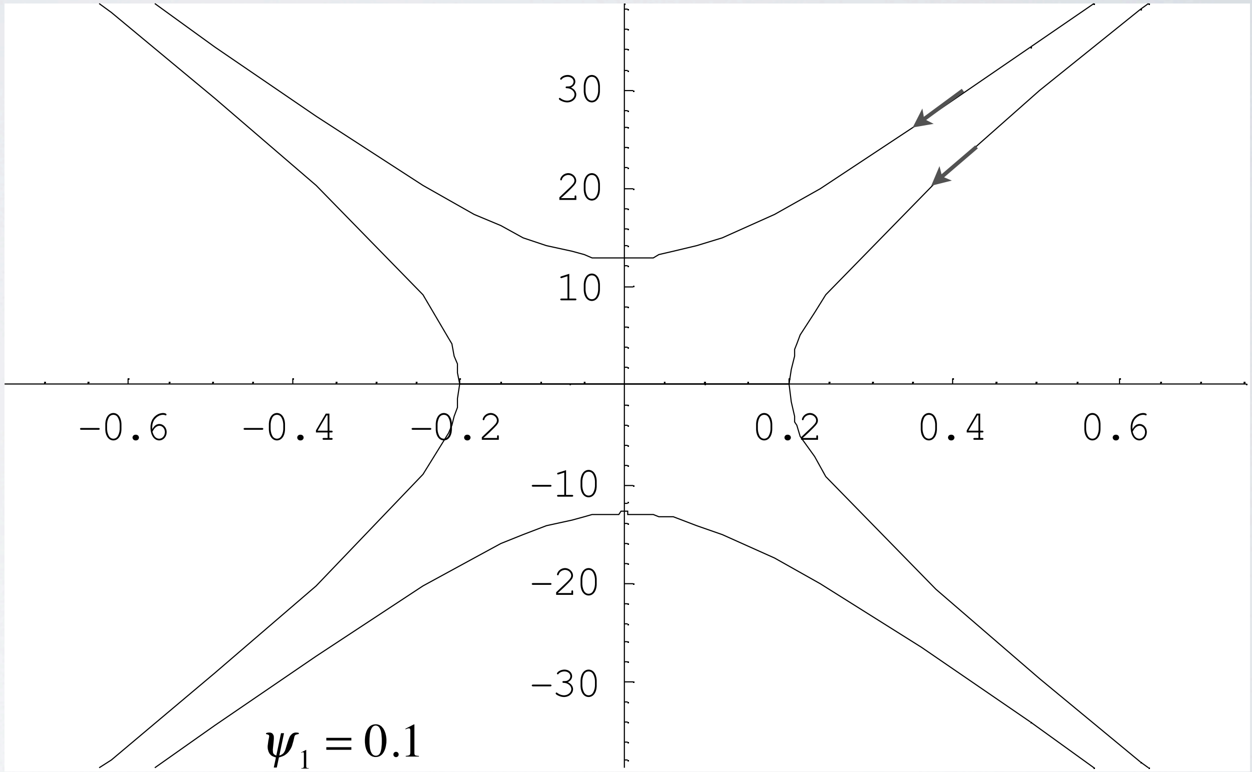
$b_v = b_u = 1.2$



STREAMLINES AND STRESSES (3)

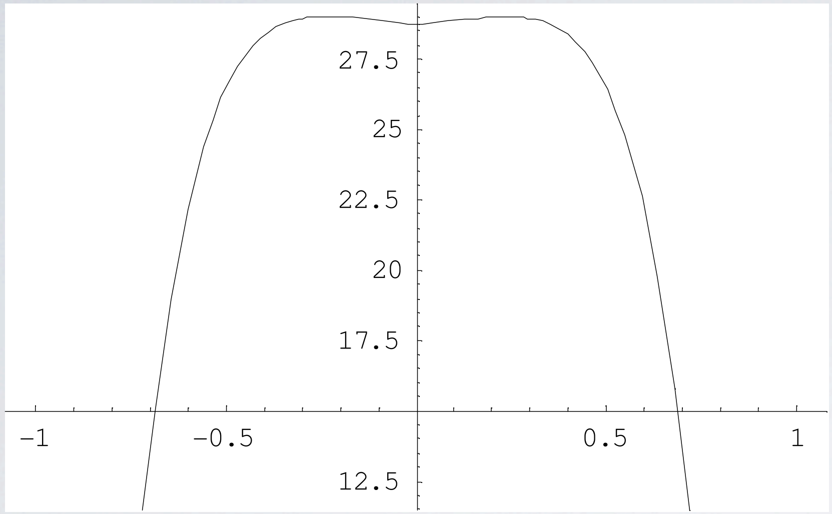
(2) $De < \frac{1}{\sqrt{9a^2}}$

$$b_u = \frac{1-9a^2De^2}{9b_vDe^2}; a=-1; b_v=5; De=\frac{1}{3\sqrt{a^2}}-0.001$$

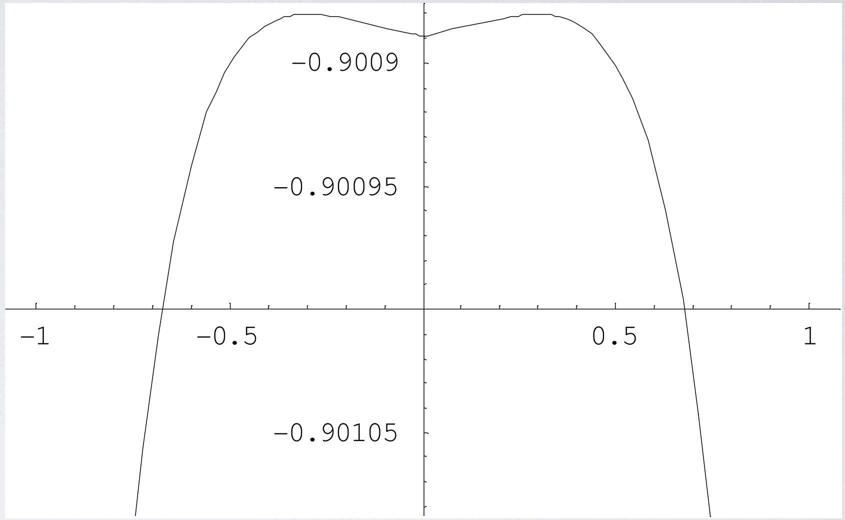


STREAMLINES AND STRESSES (4)

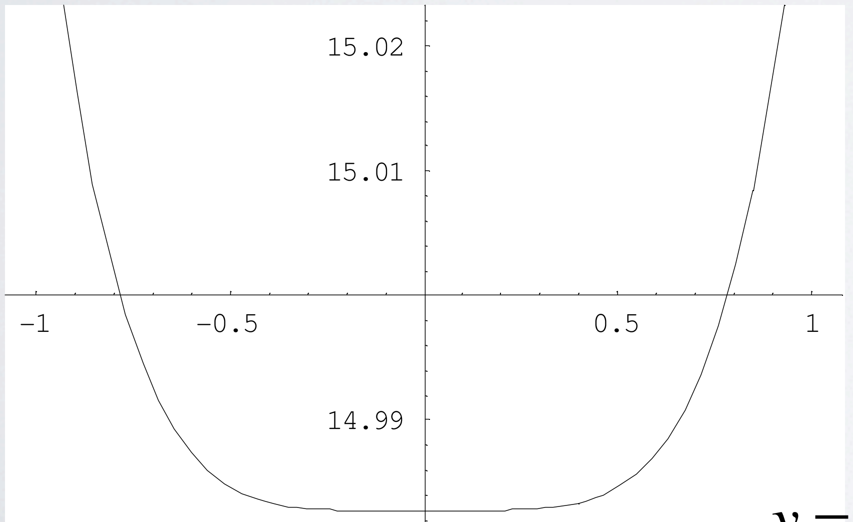
$b_v = 5$ τ_{xx}



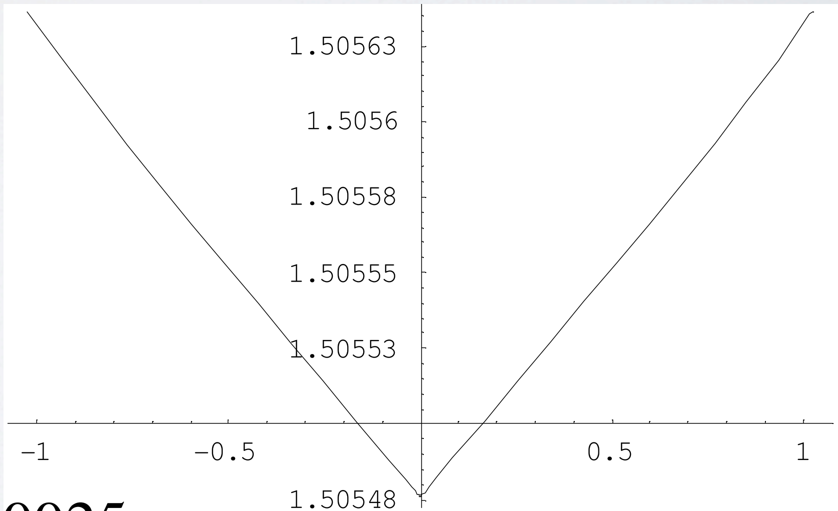
τ_{xx} $b_v = 0.5$



τ_{yy}



τ_{yy}



$y = 0.000025$

STREAMLINES AND STRESSES (5)

(3) $De > \frac{1}{\sqrt{9a^2}}$

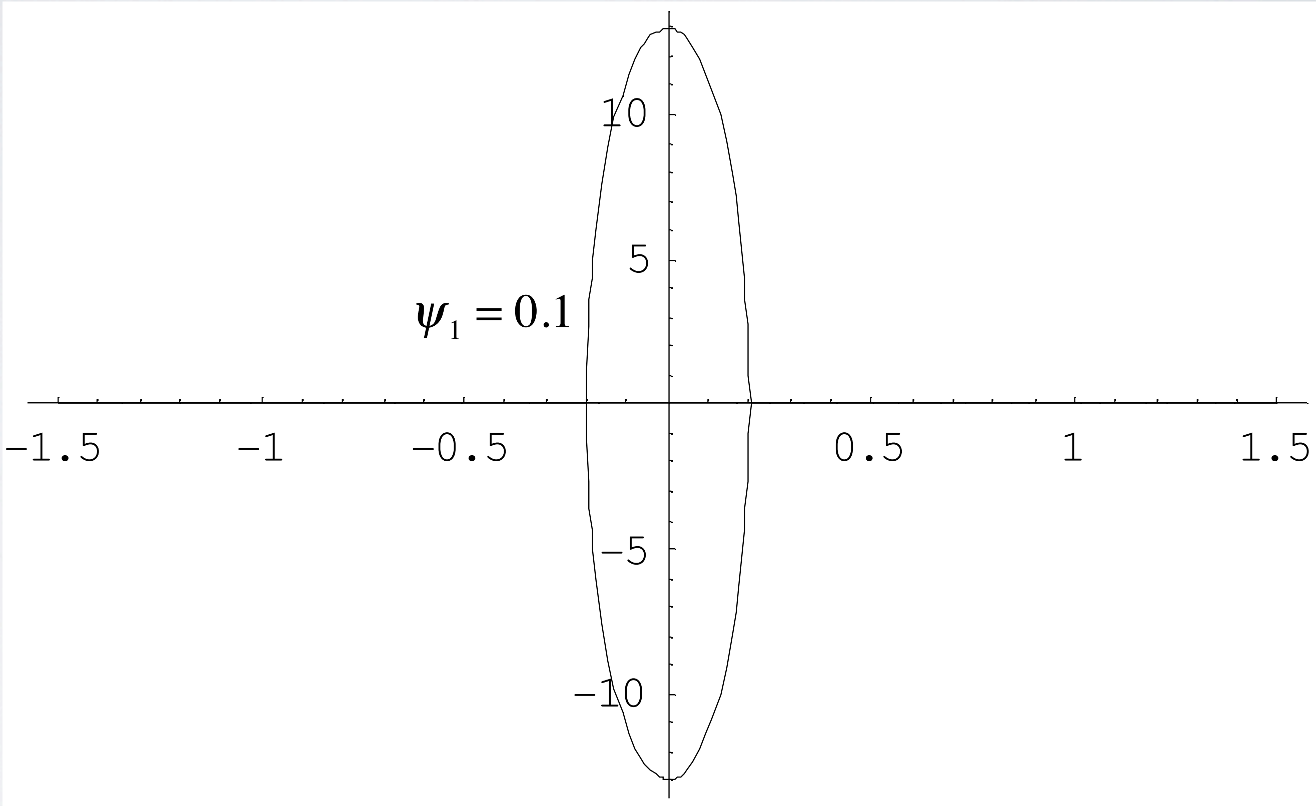
$$b_u = \frac{1-9a^2De^2}{9b_v De^2}; a=-1; b_v=5; De=\frac{1}{3\sqrt{a^2}}+0.001$$

b_u, b_v

Opposite signs

↓

Vortex



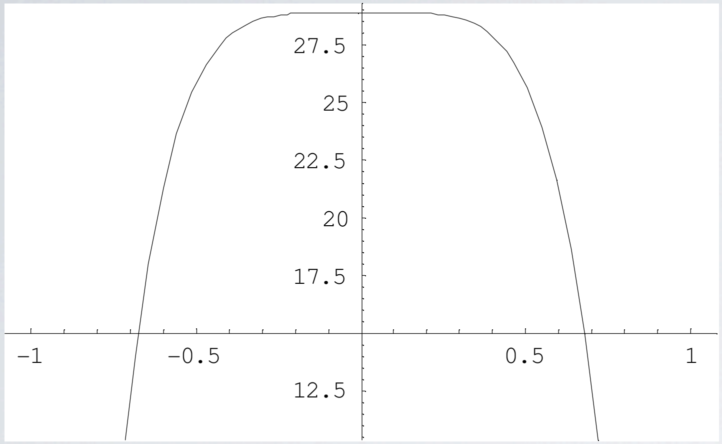
$De < De_c$ **Vortex enclosing stagnation point is not possible.**

$De > De_c$ **Vortex enclosing stagnation point is possible.**

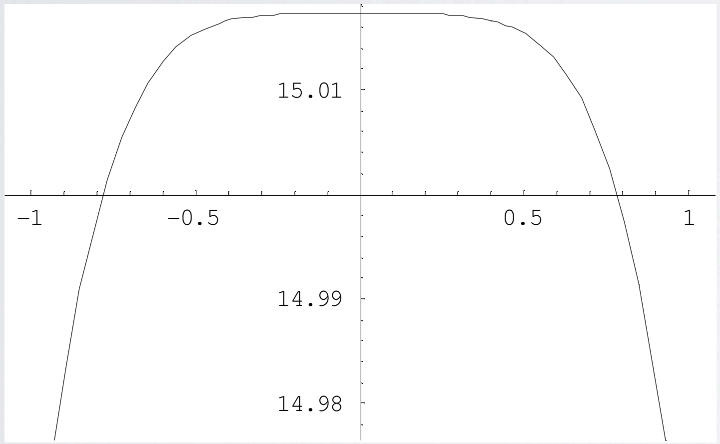
STREAMLINES AND STRESSES (6)

$b_v = 5$

τ_{xx}

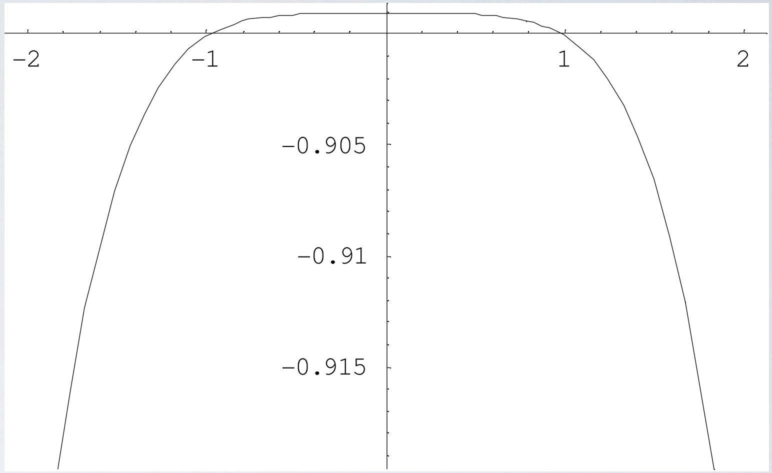


τ_{yy}

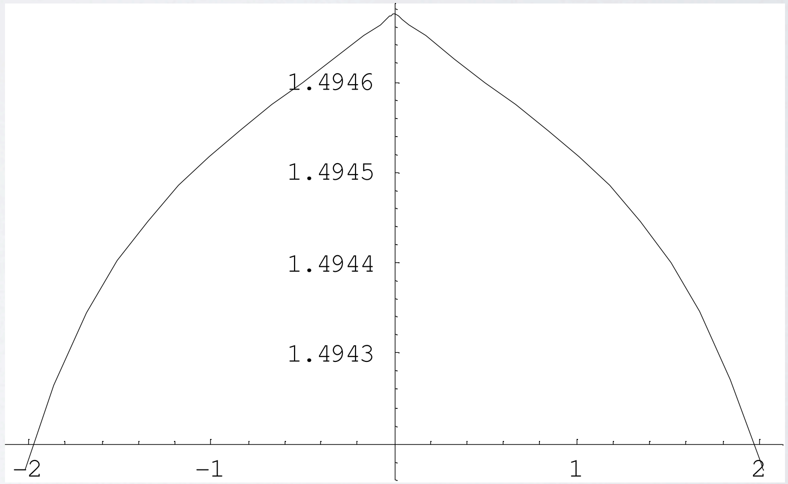


τ_{xx}

$b_v = 0.5$



τ_{yy}

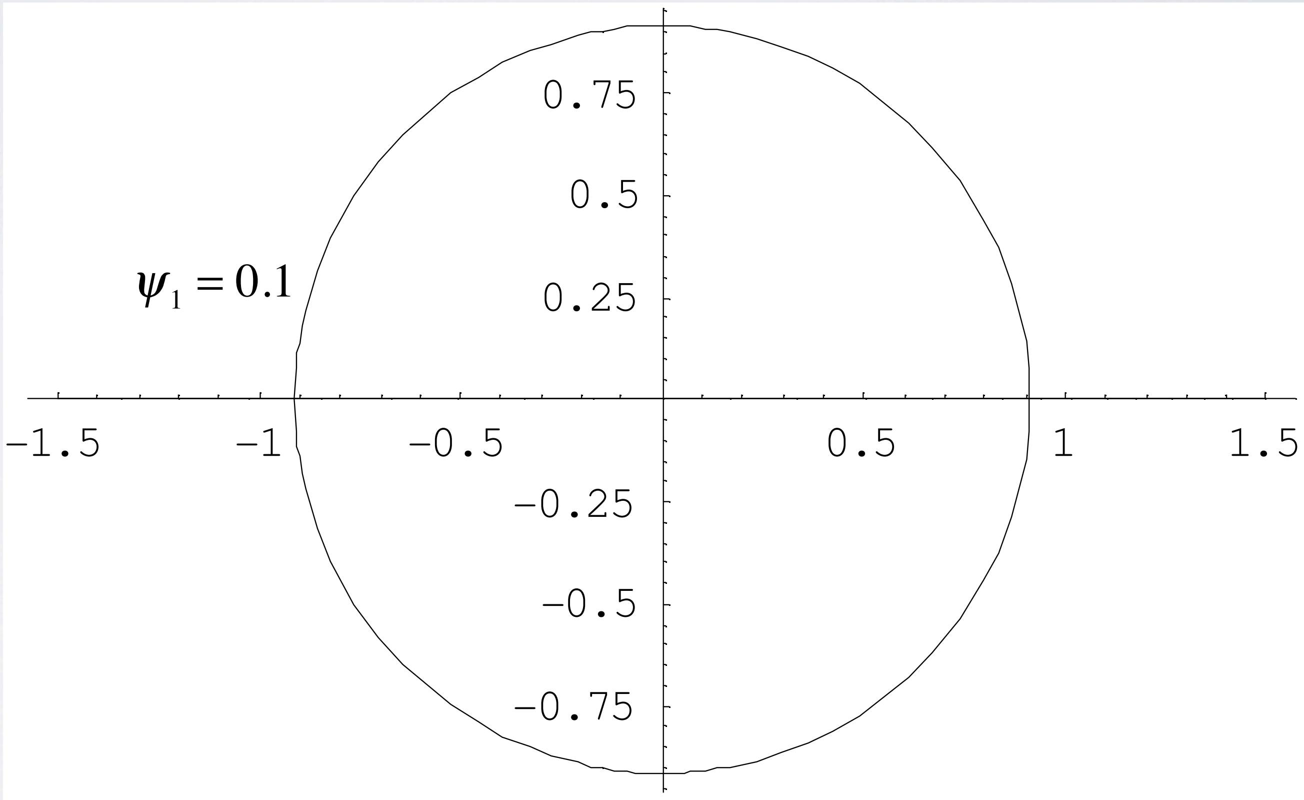


$y = 0.000025$

STREAMLINES AND STRESSES (7)

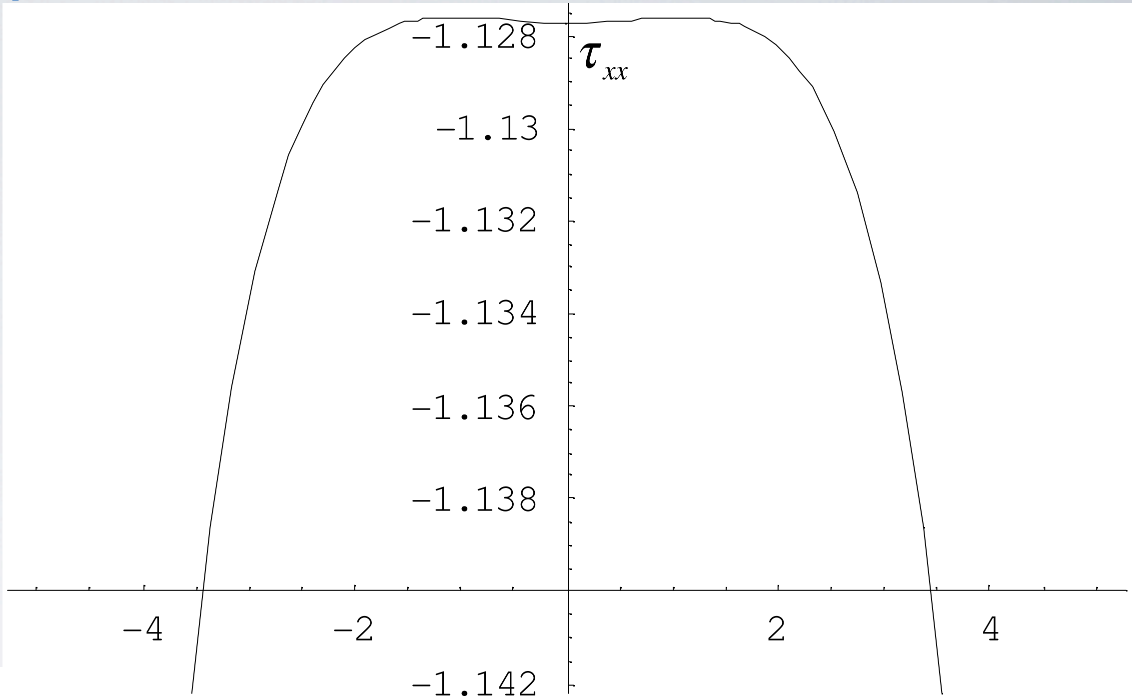
(4) $De > \frac{1}{\sqrt{9a^2}}$ - Circular vortex

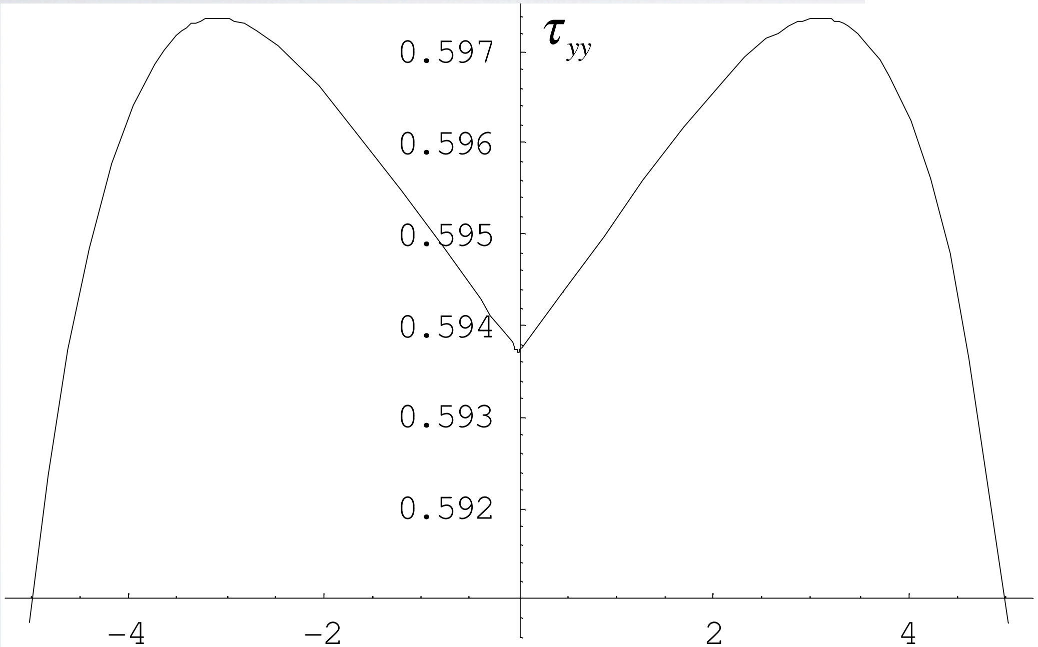
$$b_u = \frac{1 - 9a^2 De^2}{9 b_v De^2}; a = -1; b_v = 0.23999; De = \frac{1}{3\sqrt{a^2}} + 0.01$$



STREAMLINES AND STRESSES (8)

$$y = 0.000025$$





CLOSURE

- Steady symmetric to steady asymmetric is a purely elastic instability. Inertia and solvent delays and eliminates this transition.
- This transition exists with bounded extensional viscosity, but is weakened with $\boldsymbol{\varepsilon}$
- Steady asymmetric flow is a combination of a planar stagnation and a vortex
- Analytical solution obtained enforcing UCM constitutive equation and momentum. It shows closed vortex cannot exist below $De<1/(3a)$
- Behavior of the solution currently under investigation: need to impose BC
- Need for stability analysis on the analytical solution.

ACKNOWLEDGEMENTS

- Fundação para a Ciência e a Tecnologia & Feder: PTDC/EQU-FTT/71800/2006, PTDC/EQU-FTT/70727/2006, PTDC/EME-MFE/70186/2006
- CNpQ 200120/2009-3