MICROFLUIDIC FLOWS OF VISCOELASTIC FLUIDS

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V Brazilian Conference on Rheology

Rio de Janeiro, RJ

July 14-16, 2010

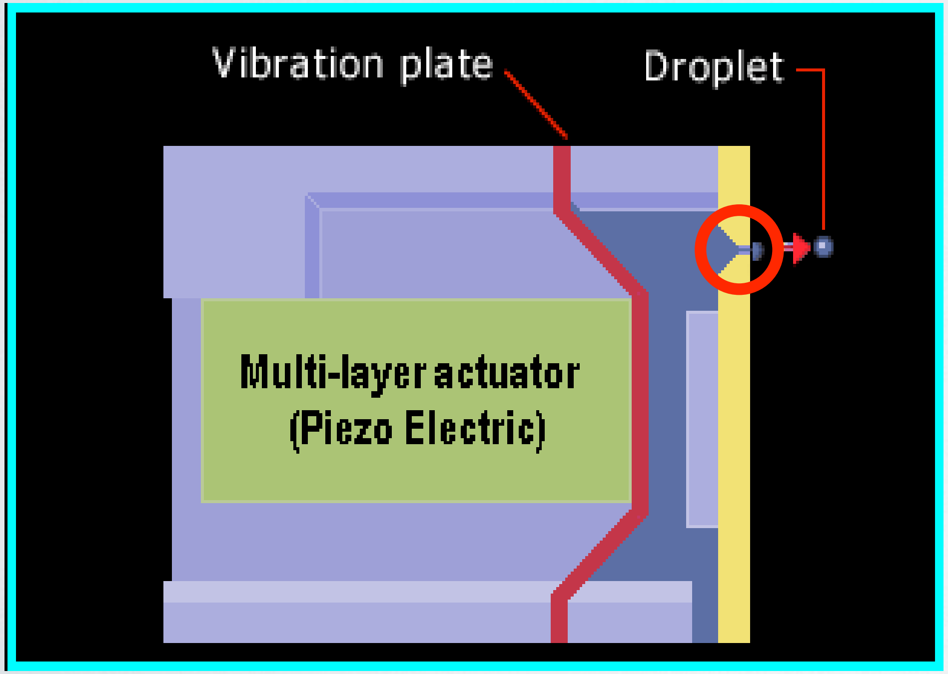
OUTLINE

- Definition, applications and our motivation and past work
- Non-dimensional numbers
- Experimental methods
- Governing equations and numerical methods
- Results
  - Hyperbolic channel: single and series (fluidic diode)
  - Flow focusing
  - Cross slot
  - Mixing-separating channel
  - 2D 4:1
- Closure

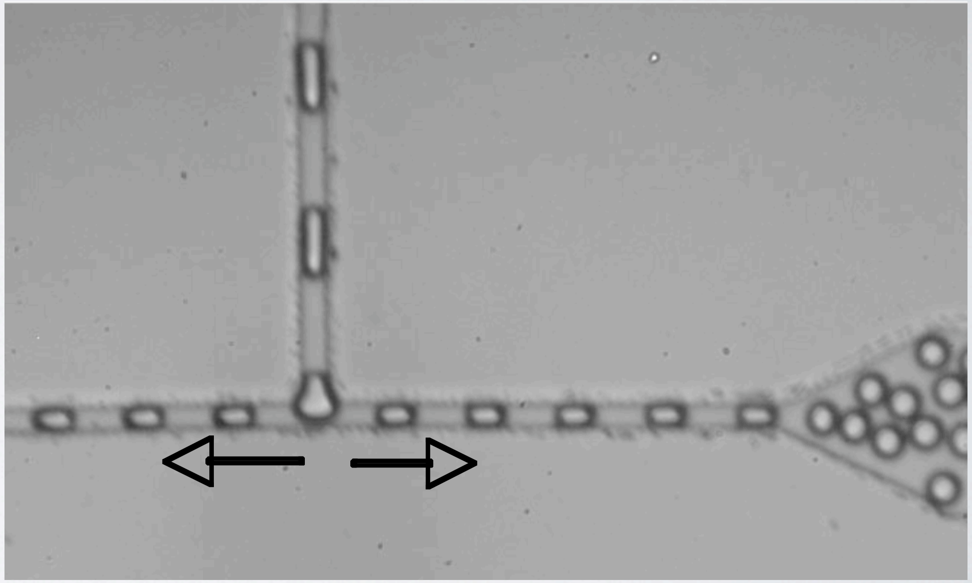
DEFINITION AND APPLICATIONS

- Fluid mechanics at the micro-scale: 100 nm - 500 µm
  nanofluidics: 10 nm - 1 µm
- Handles nano- & picolitre of fluid, miniaturization, coupling w/ electronics
- Applications: inkjet printing, analytical chemistry, micro-rheology, biology, DNA separation and sequencing, medicine, control systems, heat dissipation of micro-electronics, fuel cells, energy & display technology

Inkjet printing, spray drying, precise reactant delivery



Drop fission



Link et al. PRL 92 (2004) 54503

RELEVANCE & MOTIVATION

- Motivation: **(1) mixing with complex fluids; interaction with fluid elasticity; (2) limit of operation of micro-rheometers**

→ **Viscoelastic flow instabilities**

Mixing at very low Re → **Absence of turbulence**

Absence of chaotic advection

Other non-linear effects may help mixing: elasticity

$$Re = \frac{\rho U L}{\eta} = \frac{UL}{\nu}$$

$$De = \frac{t_{fluid}}{t_{flow}} = \frac{\lambda}{L/U} = \frac{\lambda U}{L}$$

Liquids

Small Γ, *Pe* is large

$$Pe = \frac{UL}{\Gamma} = Re.Pr = Re.Sc$$

Short transit times
Poor mixing

RELEVANT PAST WORK

Viscoelastic instabilities in shear flows

Shaqfeh. Ann. Rev. Fluid Mech 28 (1996) 129

Taylor-Couette flow Larson et al., JFM 218 (1990) 573

Cone-plate flow McKinley et al., JNNFM 40 (1991) 201

Lid driven cavity flows Pakdel & McKinley, PRL 77 (1996) 2459

Underlying mechanism McKinley et al, JNNFM 67 (1996) 19

Pakdel & McKinley, PRL 77 (1996) 2459

$$\left(\frac{\lambda U}{\mathscr{R}}\frac{\tau_{11}}{\tau_{12}}\right)\geq M_{crit}^{2}$$

on curved streamlines

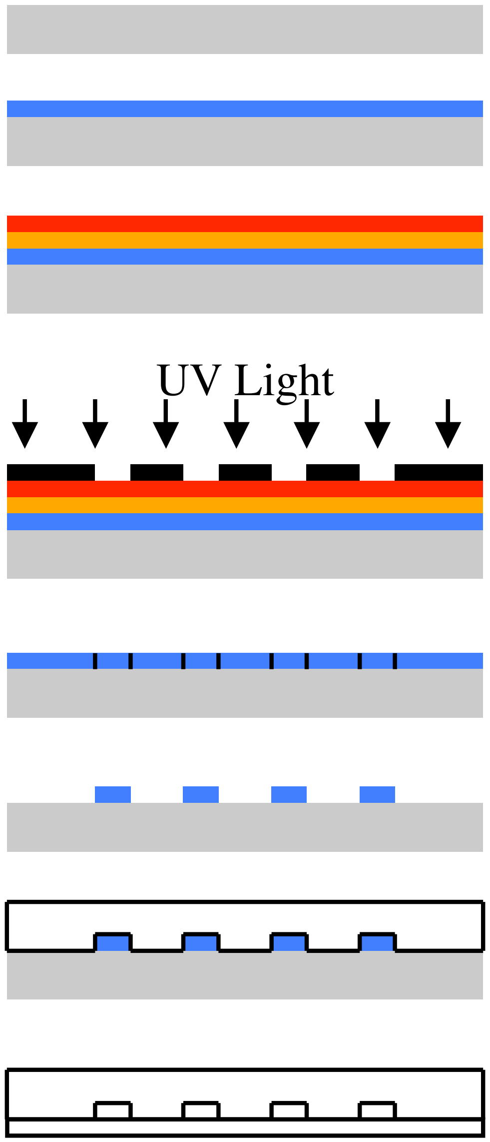
Instability growth to elastic turbulence

Groissman & Steinberg, Nature 405 (2000) 53 Larson, Nature 405 (2000) 27

Microfluidics & viscoelasticity

Squires & Quake, Rev. Mod. Phys. 77 (2005) 977

EXPERIMENTAL METHODS: MICROFABRICATION BY SOFT LITHOGRAPHY



Xia & Whitesides, (1998)
Ann. Rev. Mat. Sci. 28,153-184

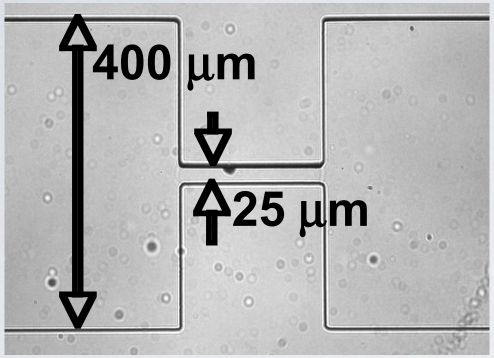
1. **Silicon Wafer**
2. Spin coat photoresist **SU-8** and prebake
3. Spin coat **barrier coat (CEM-BC7.5)** and **contrast enhancer (CEM 388SS)** (vertical walls).
4. **Chrome Mask** over coated wafer
5. **UV Exposure** – **cross-link SU-8**
6. Wash **barrier coat** and **contrast enhancer**
7. Post-bake and develop **SU-8**
8. Pour **PDMS** over substrate and cure (80°C, 25 mins)
9. Peel off substrate
10. Treat surfaces with air plasma, seal with glass slide

Needs access to fairly clean environment

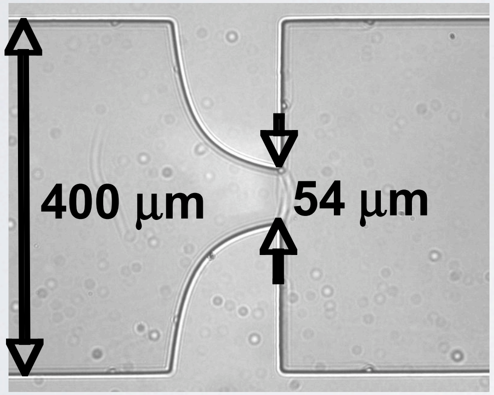
MICROGEOMETRIES

Planar hyperbolic contraction- sudden expansion

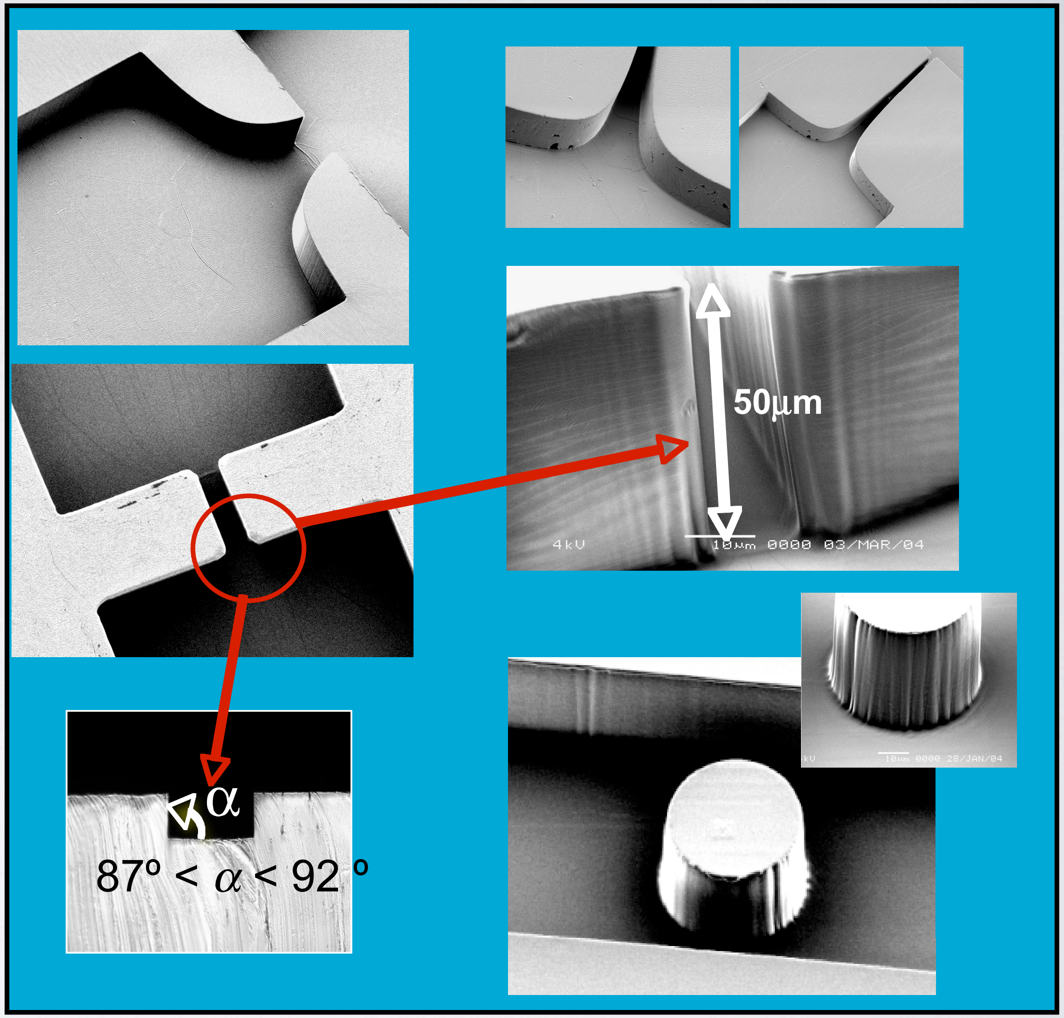
Abrupt contraction-expansion
(CR=ER=16)



Hyperbolic contraction ($\varepsilon_H = 2$)



SEM Images



Accuracy of dimensions to within 5%

Near vertical walls:

tapering angle 87° < α < 92°

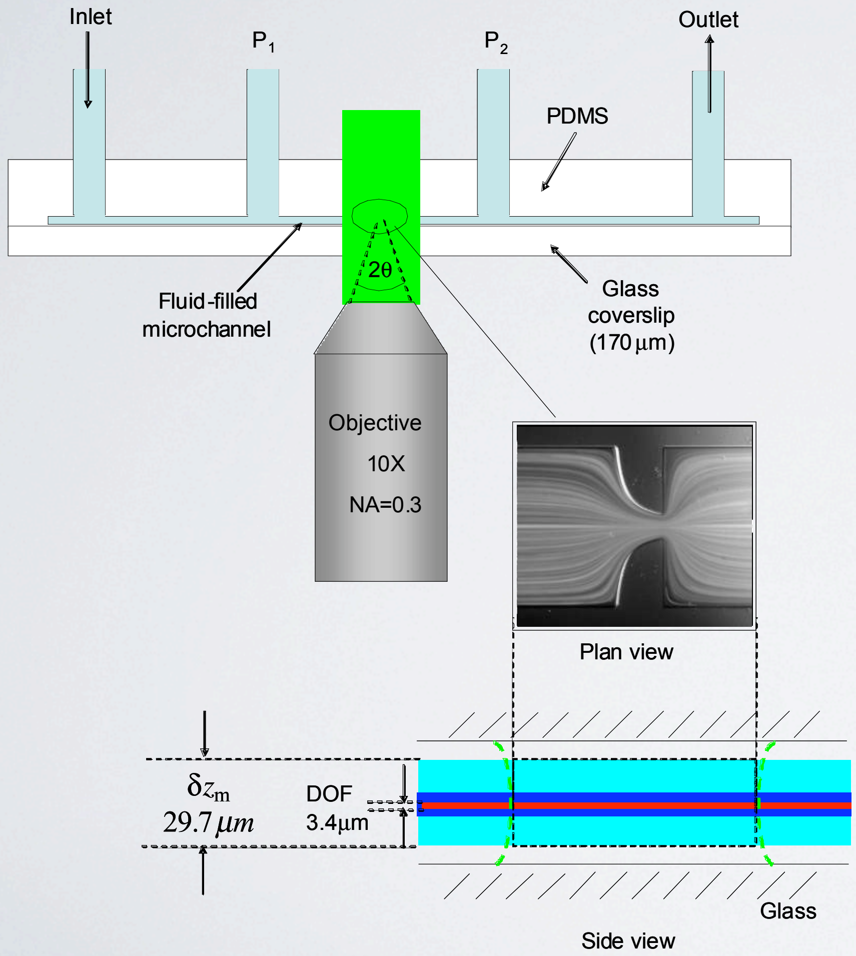
EXPERIMENTAL METHODS: FLOW VISUALIZATION & MICRO-PIV

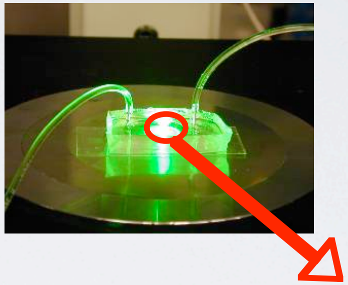
Streakline imaging

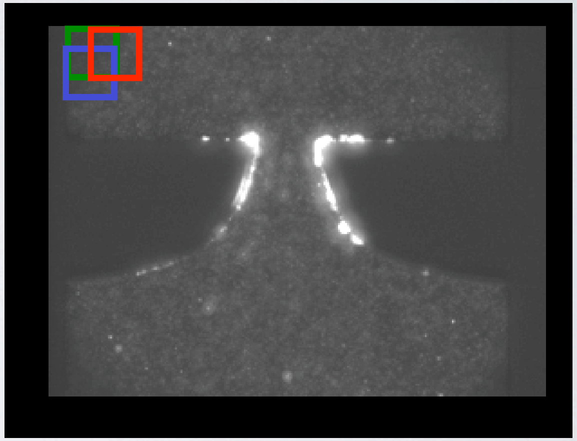
1 μm fluorescent particles
Mercury lamp
Long exposure
10X lens (NA=0.3, measurement depth= 30 μm

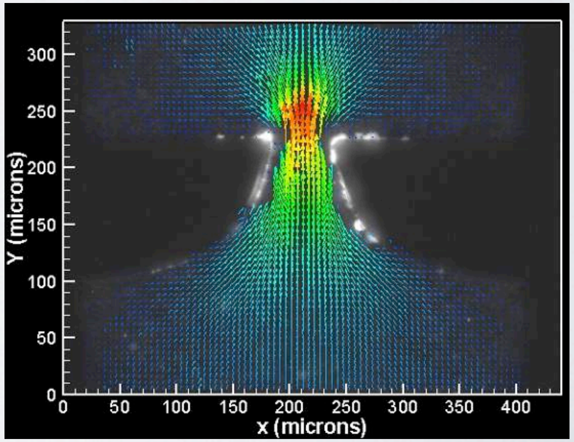
μPIV

500 nm fluorescent particles
Double-pulsed laser, Volume illumination
Double-frame camera
20X lens (NA=0.5, measurement depth= 12 μm
32x32 pixel interrogation, 50& overlap

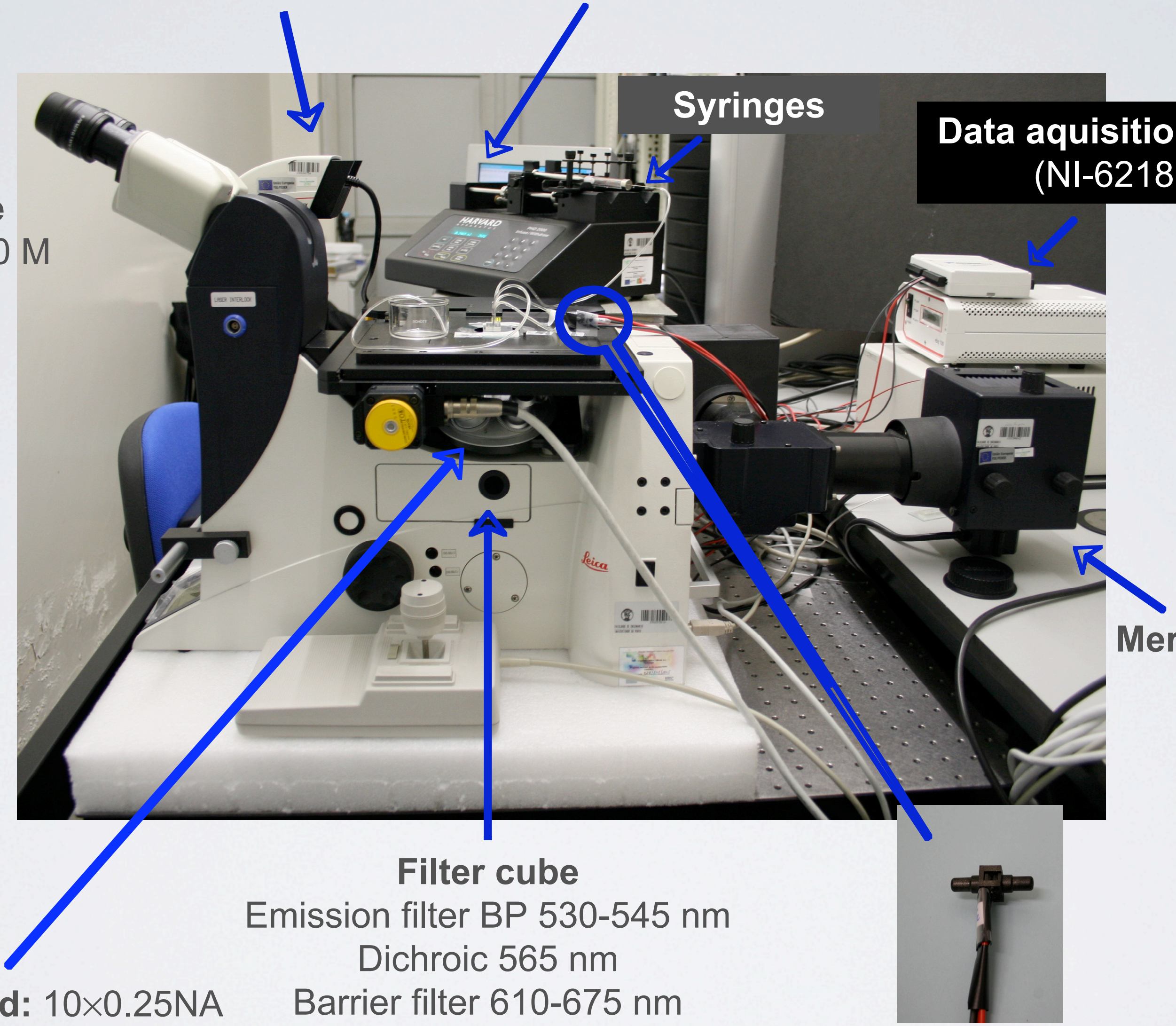








EXPERIMENTAL METHODS: FLOW VISUALIZATION (2)



Digital camera (Leica DFC 350 FX)

Syringe pump (Harvard apparatus PHD 2000)

Microscope
Leica DMI5000 M

Syringes

Data aquisition card (NI-6218)

Mercury lamp

Filter cube
Emission filter BP 530-545 nm
Dichroic 565 nm
Barrier filter 610-675 nm

Objectives used: 10×0.25NA
5 ×0.12NA

Differencial pressure sensor (Honeywell 26PC series)

EXPERIMENTAL METHODS: RHEOLOGY - DATA FOR FLOW FOCUSING

Newtonian fluid: water

Viscoelastic fluid: PAA 125 ppm +1% NaCl
PAA (M_w= 18 x 10^6 g/mol)

Shear, Physica MCR 301
(cone-plate, d= 75 mm, 1°)

T= 20°C

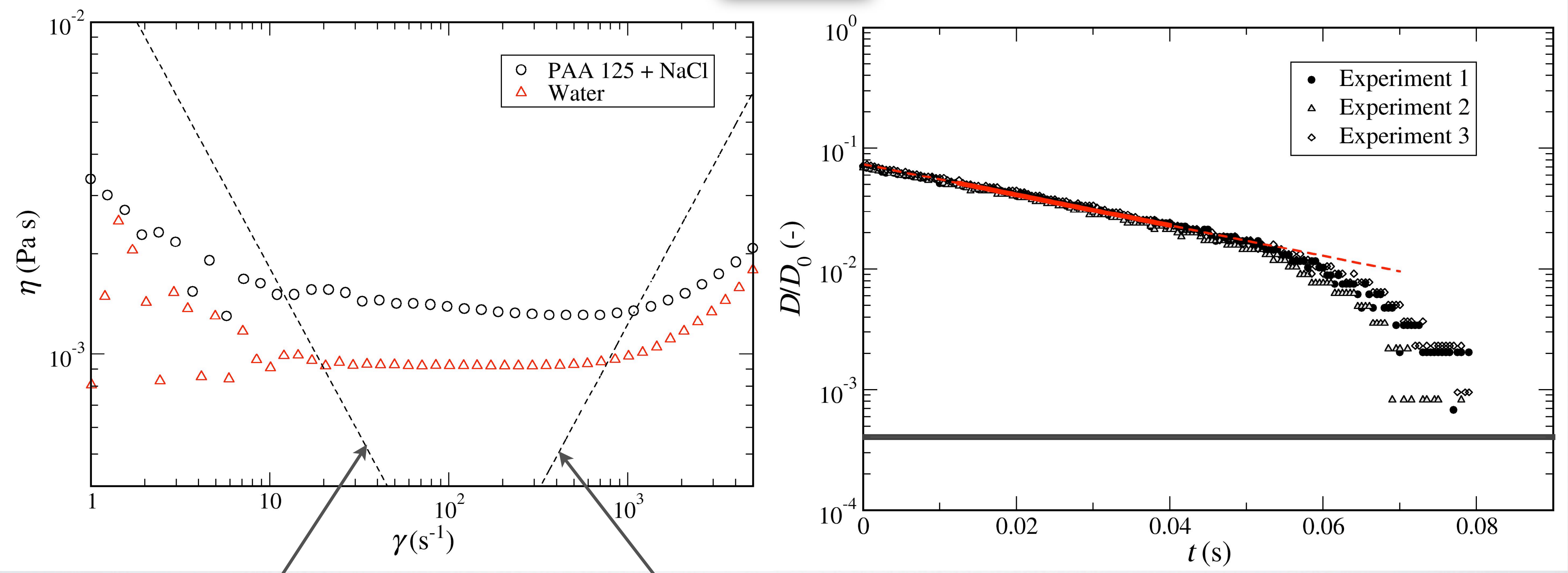
Oliveira et al. JNNFM 160 (2009) 31-39

Extensional rheology, Haake CaBER 1

D_p= 6 mm; AR=0.33 & 1.56; ε=1.53

$$\eta_0 = 0.00131 \text{ Pa.s} \qquad \rho = 1005 \text{ kg/m}^3$$

$$\lambda_{CaBER} = 12.4 \pm 0.2 \text{ s}$$



20X the minimum
measurable torque (1x 10^{-7} Nm)

Onset of inertial
instabilities

GOVERNING EQUATIONS (I)

- Continuity: $\frac{\partial u_i}{\partial x_i} = 0$
- Momentum: $\rho \frac{\partial u_i}{\partial t} + \rho u_k \frac{\partial u_i}{\partial x_k} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ik}}{\partial x_k}$
- Constitutive equation: $\tau_{ij} = \underbrace{2\eta_s D_{ij}}_{\text{Newtonian solvent}} + \tau_{ij,p}$

$$D_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Polymer

$$f\left(\tau_{kk,p}\right)\tau_{ij,p} + \lambda\left[\frac{\partial \tau_{ij,p}}{\partial t} + \frac{\partial\left(u_k \tau_{ij,p}\right)}{\partial x_k} - \tau_{jk,p}\frac{\partial u_i}{\partial x_k} - \tau_{ik,p}\frac{\partial u_j}{\partial x_k} + \xi\left(\tau_{jk,p} D_{ik} + \tau_{ik,p} D_{jk}\right)\right] + \frac{\alpha\lambda}{\eta_p}\tau_{ik,p}\tau_{kj,p} = \eta_p\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$f\left(\tau_{kk,p}\right) = \exp\left(\frac{\varepsilon\lambda}{\eta_p}\right)$$

$$f\left(\tau_{kk,p}\right) = 1 + \frac{\varepsilon\lambda}{\eta_p}\tau_{kk,p}$$

$\alpha=0$; $\xi=0$, $\varepsilon=0$: UCM or Oldroyd-B

$\alpha=0$: PTT model

$\alpha=0$; $\xi=0$: Simplified PTT model

$\xi=0$, $\varepsilon=0$: Giesekus model

GOVERNING EQUATIONS (2)

- Scalar (energy, species): $$\frac{\partial(\rho\phi)}{\partial t}+\frac{\partial(\rho u_i\phi)}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\Gamma\frac{\partial\phi}{\partial x_i}\right)+S$$

Modifications for standard conformation and log-conformation

$$\rho\frac{\partial u_i}{\partial t}+\rho u_k\frac{\partial u_i}{\partial x_k}=-\frac{\partial p}{\partial x_i}+\eta_s\frac{\partial^2 u_i}{\partial x_k\partial x_k}+\frac{\eta_p}{\lambda}\frac{\partial A_{ik}}{\partial x_k}$$

$$\boxed{\tau_{ij,p}=\frac{\eta_p}{\lambda}\left(A_{ij}-\delta_{ij}\right)}$$

↓

$$\lambda\overset{\nabla}{A}_{ij}=-Y\left(A_{kk}\right)\left(A_{ij}-\delta_{ij}\right)$$

$$Y\left(A_{kk}\right)=1+\varepsilon\left(A_{kk}-3\right)$$

$$\boxed{\frac{\partial\Theta_{ij}}{\partial t}+u_k\frac{\partial\Theta_{ij}}{\partial x_k}-\left(R_{ik}\Theta_{kj}-\Theta_{ik}R_{kj}\right)-2E_{ij}=-\frac{Y\left(e^{\Theta_{kk}}\right)}{\lambda}\left(e^{-\Theta_{ij}}-\delta_{ij}\right)}$$

Fattal & Kupferman JNNFM, 123 (2004) 281-285.

$$\Theta_{ij}=\log A_{ij}$$

More details for FVM:
Afonso et al. JNNFM 157 (2009) 55-65

NUMERICAL METHODS: SOLUTION OF THE GOVERNING EQUATIONS

- Finite-volume method (in-house code)
- Collocated block-structured mesh
- Non-orthogonal coordinates (Cartesian velocity and stress tensor)
- Diffusion: central differences (2nd order in uniform mesh)
- SIMPLEC algorithm
- Rhie-and-Chow to couple velocity and pressure
- Special scheme to couple velocity and extra stress

Oliveira et al. JNNFM, 79 (1998) 1-43.

- Advection: CUBISTA high-resolution scheme (based on QUICK, 3rd order)

Alves et al. IJNMF, 41 (2003) 47-75.

- Standard formulation and log-conformation formulation (allows higher De)

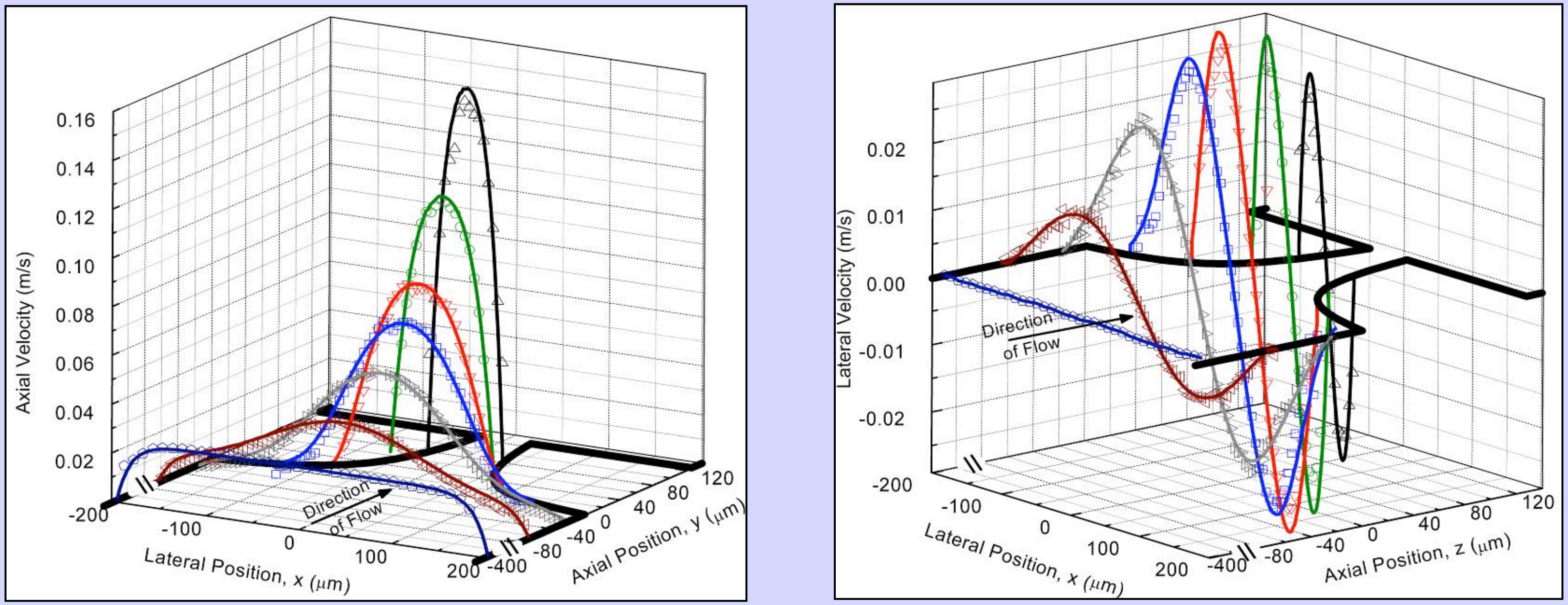
Fattal & Kupferman JNNFM, 123 (2004) 281-285.

More details for FVM: Afonso et al. JNNFM 157 (2009) 55-65

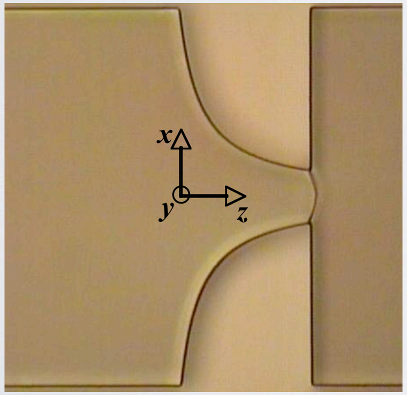
HYPERBOLIC SINGLE CHANNEL FLOW
Newtonian & Viscoelastic

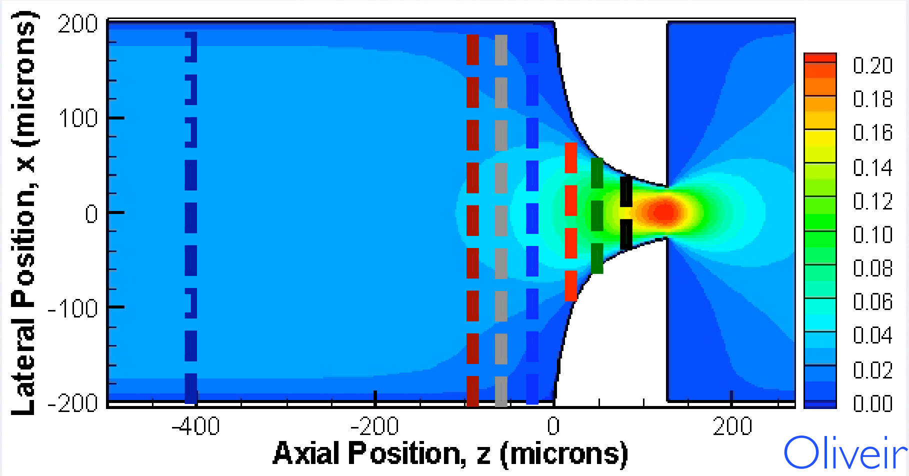
HYPERBOLIC CONTRACTION: NEWTONIAN FLUIDS (I)

Centre plane (y=0): experimental versus numerical



Velocity Profiles along the Lateral Direction





Water

Q **= 1 ml/h**

Re **= 3.21**

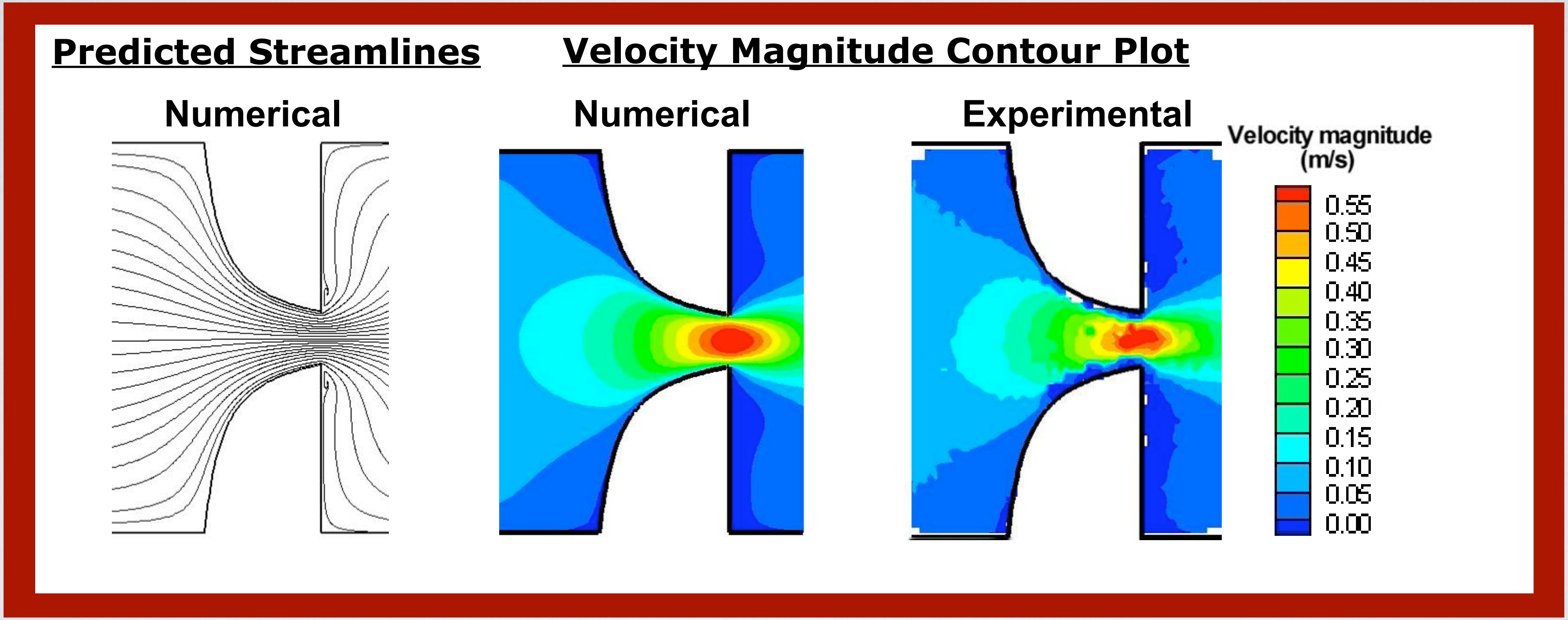
Oliveira et al. EiF, 43 (2007) 437-451.

HYPERBOLIC CONTRACTION: NEWTONIAN FLUIDS (2)

Centre plane (y=0): experimental versus numerical

Q = 3ml/h

Re = 9.62



Predicted Streamlines

Numerical

Velocity Magnitude Contour Plot

Numerical

Experimental

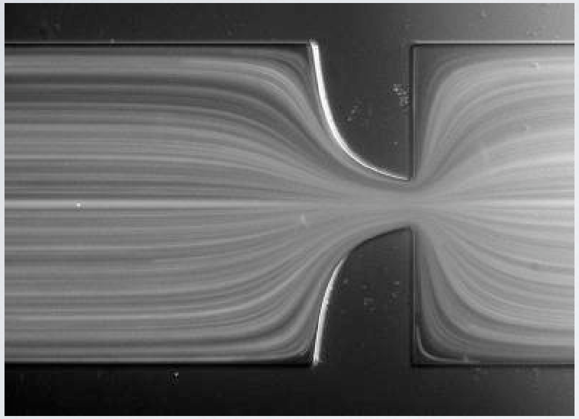
Nearly constant acceleration on centreline → **Purely extensional flow**

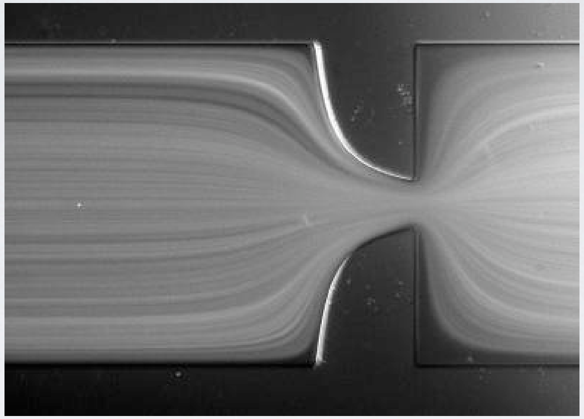
Oliveira et al. EiF, 43 (2007) 437-451.

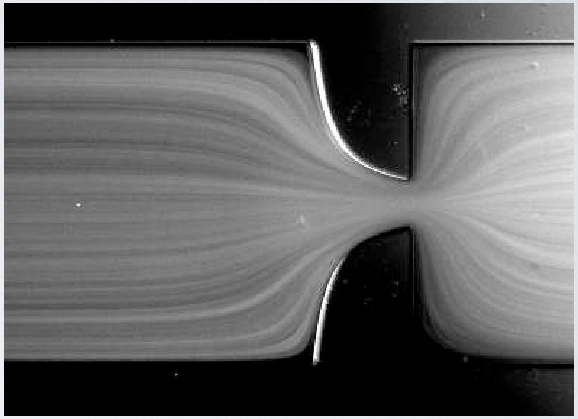
HYPERBOLIC CONTRACTION: VISCOELASTIC FLUIDS (I)

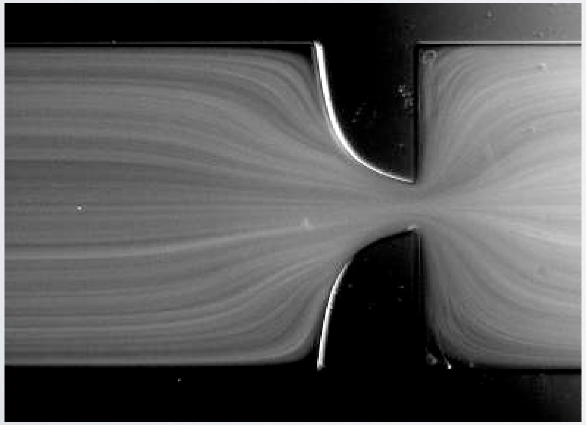
0.3% PEO

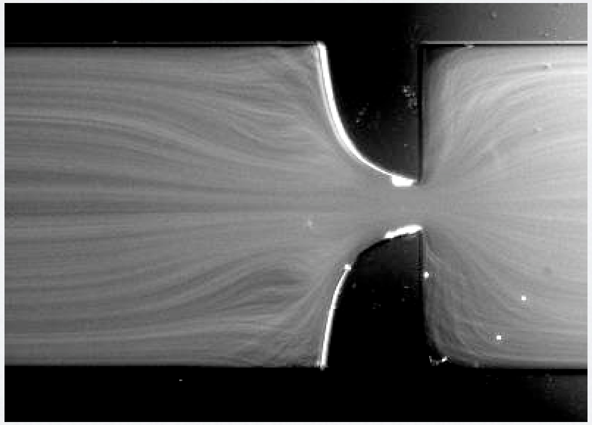
Hencky Strain $e_H = 2$

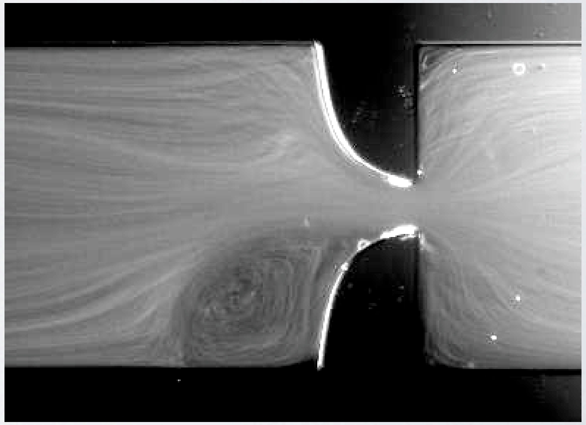

Q = 1 ml/h, Re = 13.2
De = 1.13


Q = 3 ml/h, Re = 39.6
De = 3.40


Q = 5 ml/h, Re = 66.0
De = 5.66


Q = 7 ml/h, Re = 92.3
De = 7.93


Q = 9 ml/h, Re = 119
De = 10.2


Q = 11 ml/h, Re = 145
De = 12.5

HYPERBOLIC CONTRACTION: VISCOELASTIC FLUIDS (2)

0.3% PEO: Q=0-0.2 ml/h

0.3% PEO: Q=7 ml/h

HYPERBOLIC FLUID RECTIFIER

FLUIDIC DIODE: HYPERBOLIC CONTRACTION

Sousa et al. JNNFM 165 (2010) 652-671

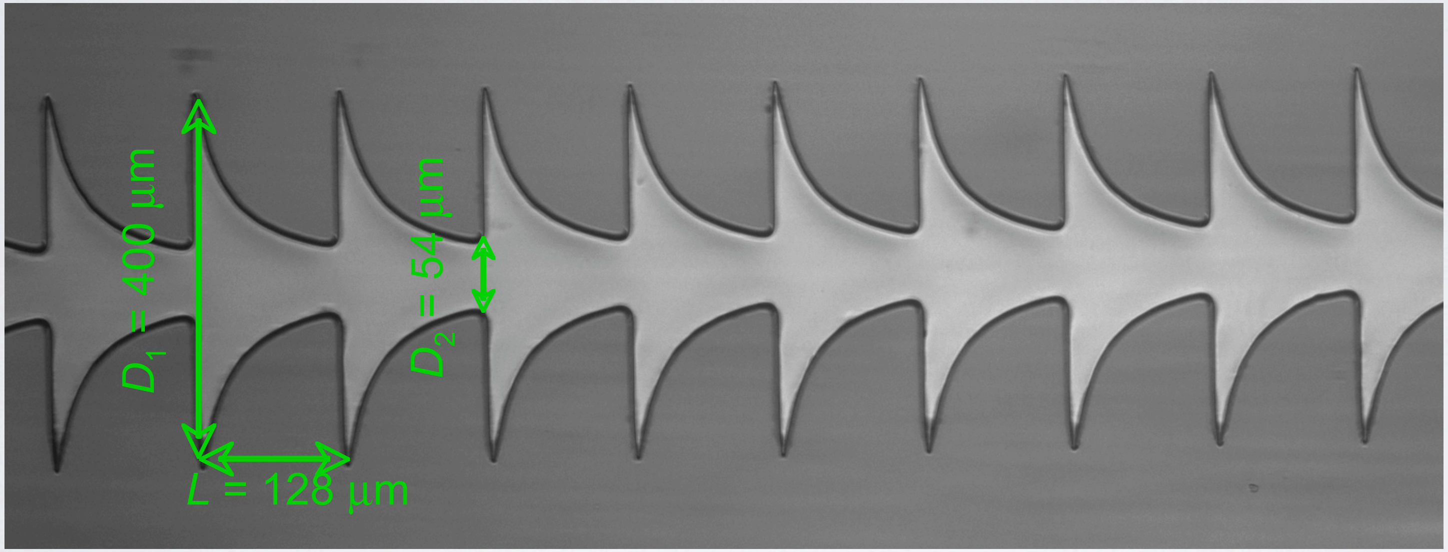
Planar geometry with hyperbolic shape

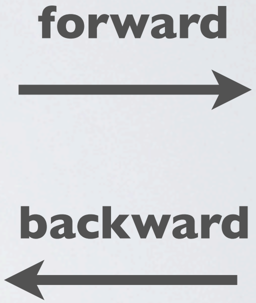
↓

Nearly constant acceleration on centreline
Oliveira et al., Exp Fluids 43 (2007) 437-451

→ **Purely extensional flow**

Hencky strain, $\varepsilon_H = \ln(D_1/D_2) = 2.18$

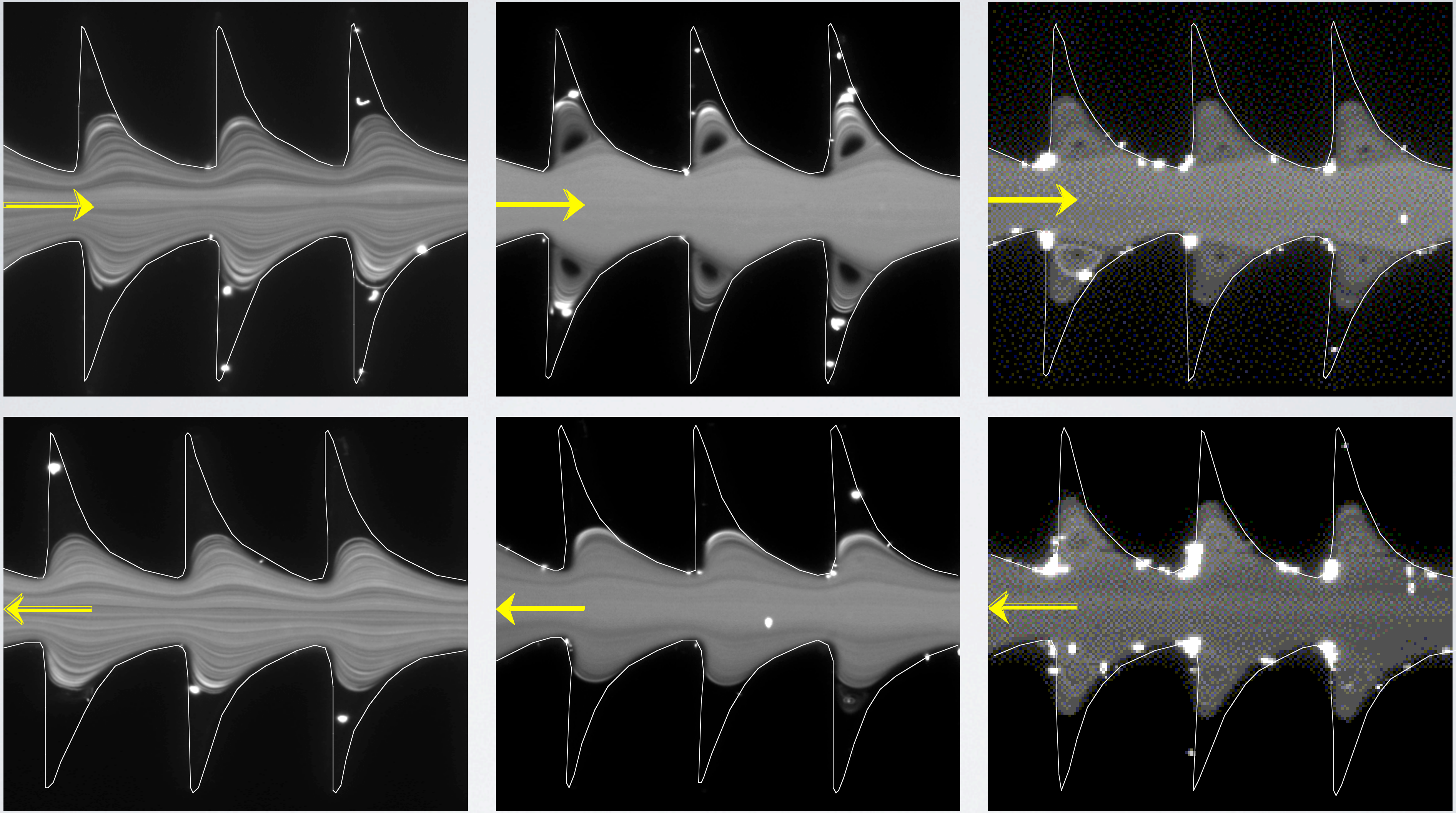




42 identical elements, uniform depth= 50 µm

HYPERBOLIC FLUIDIC DIODE: NEWTONIAN FLUID (I)

Sousa et al. JNNFM 165 (2010) 652-671



$Q = 0.1 \text{ ml h}^{-1}$ $Re = 0.594$

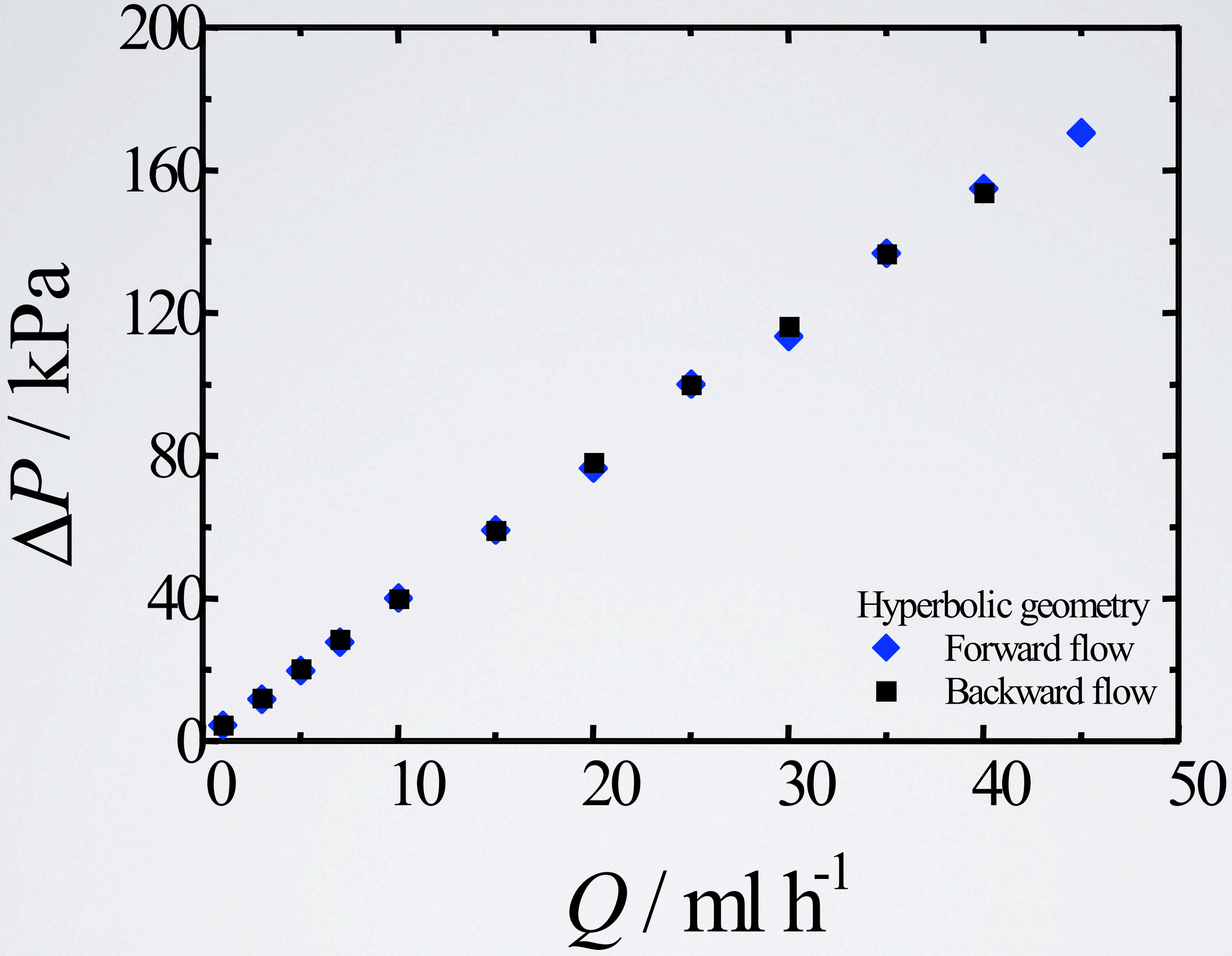
$Q = 5 \text{ ml h}^{-1}$ $Re = 29.7$

$Q = 20 \text{ ml h}^{-1}$ $Re = 119$

HYPERBOLIC FLUIDIC DIODE: NEWTONIAN FLUID (2)

Sousa et al. JNNFM 165 (2010) 652-671

Pressure drop

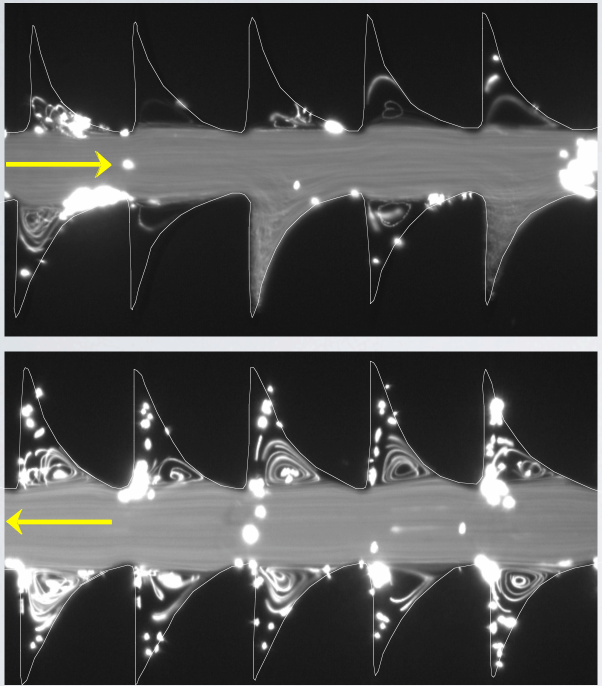


No fluidic rectification effect

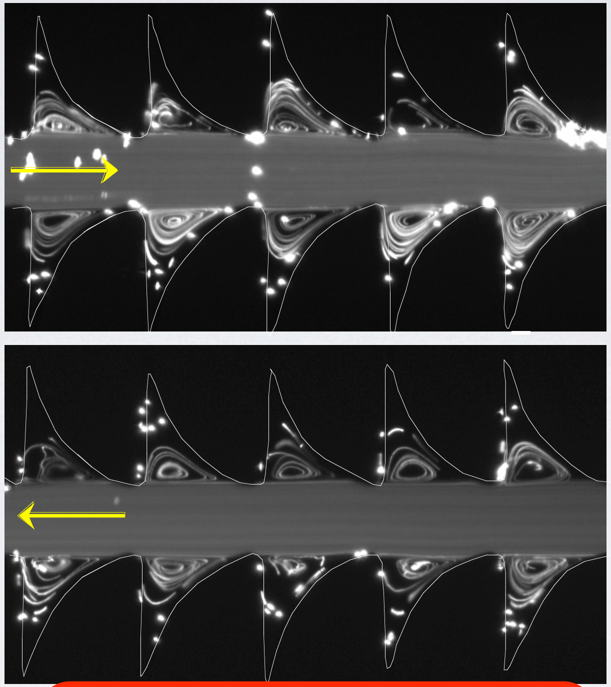
HYPERBOLIC FLUIDIC DIODE: VISCOELASTIC FLUID (I)

Sousa et al. JNNFM 165 (2010) 652-671

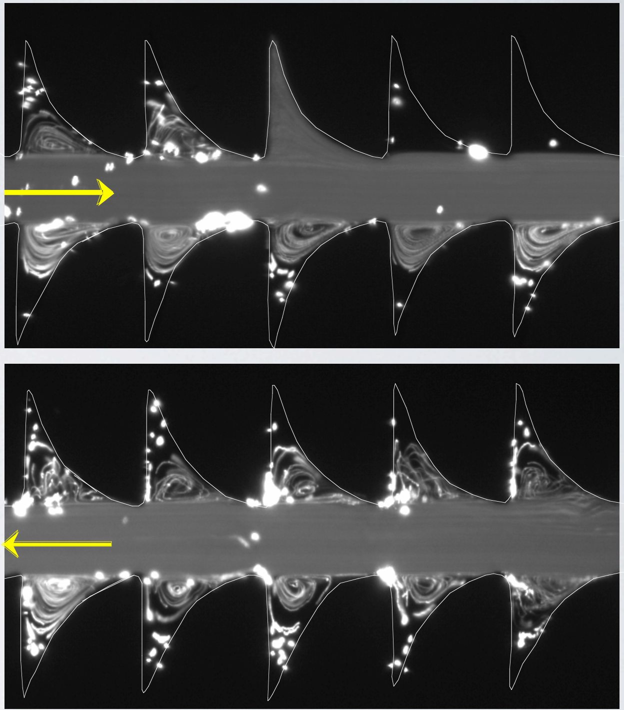
0.1% aqueous solution of PEO (M_w=8x10^6 g mol^{-1})



$Q = 0.2$ ml h^{-1} $Re = 0.149$
$De = 55.7$ $Wi = 10.2$



$Q = 0.8$ ml h^{-1} $Re = 0.598$
$De = 223$ $Wi = 40.7$



$Q = 2$ ml h^{-1} $Re = 1.49$
$De = 557$ $Wi = 102$

Flow rate
8x's higher
Similar Re

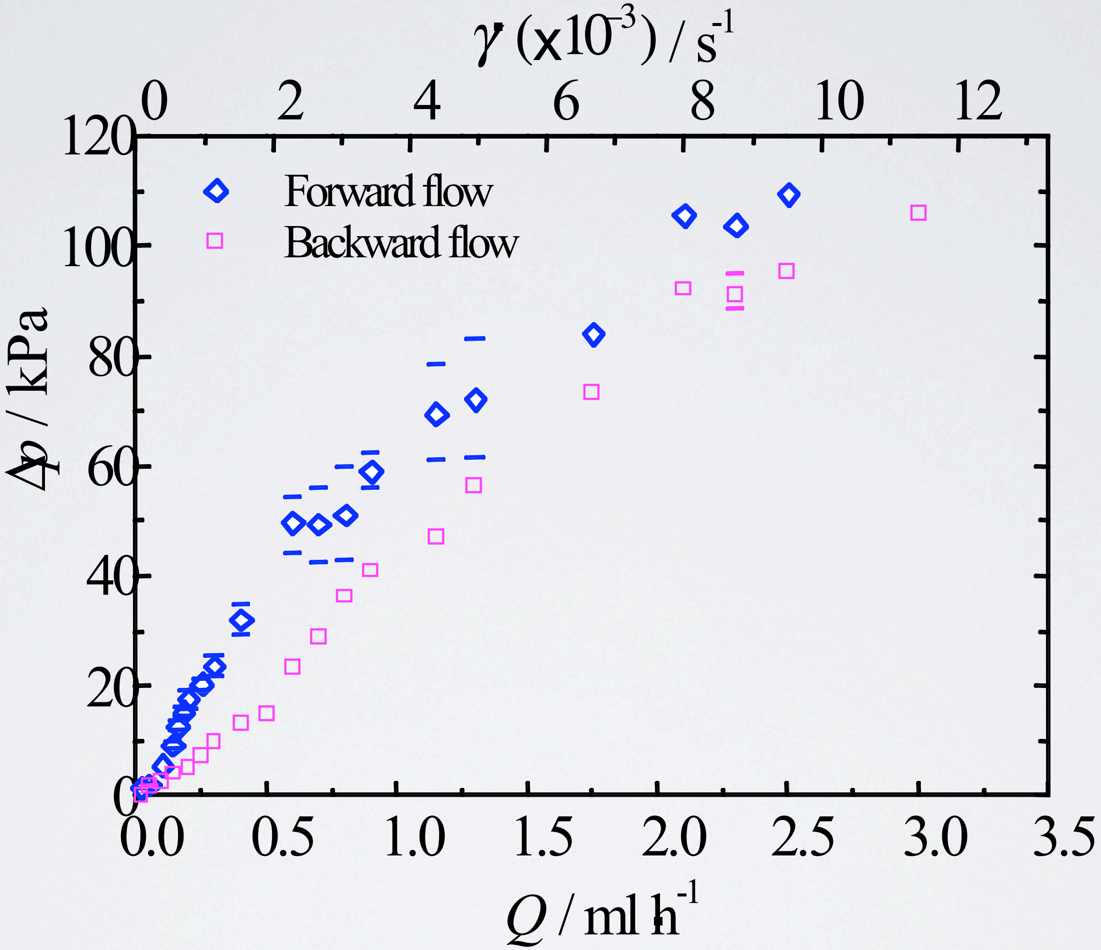
$$De = \lambda\dot{\gamma} = \frac{\lambda U_2}{D_2/2}$$

$$Wi = \lambda\dot{\varepsilon} = \frac{\lambda\left(U_2 - U_1\right)}{L}$$

HYPERBOLIC FLUIDIC DIODE: VISCOELASTIC FLUID (2)

Sousa et al. JNNFM 165 (2010) 652-671

0.1% aqueous solution of PEO (M_w=8x10^6 g mol^{-1})

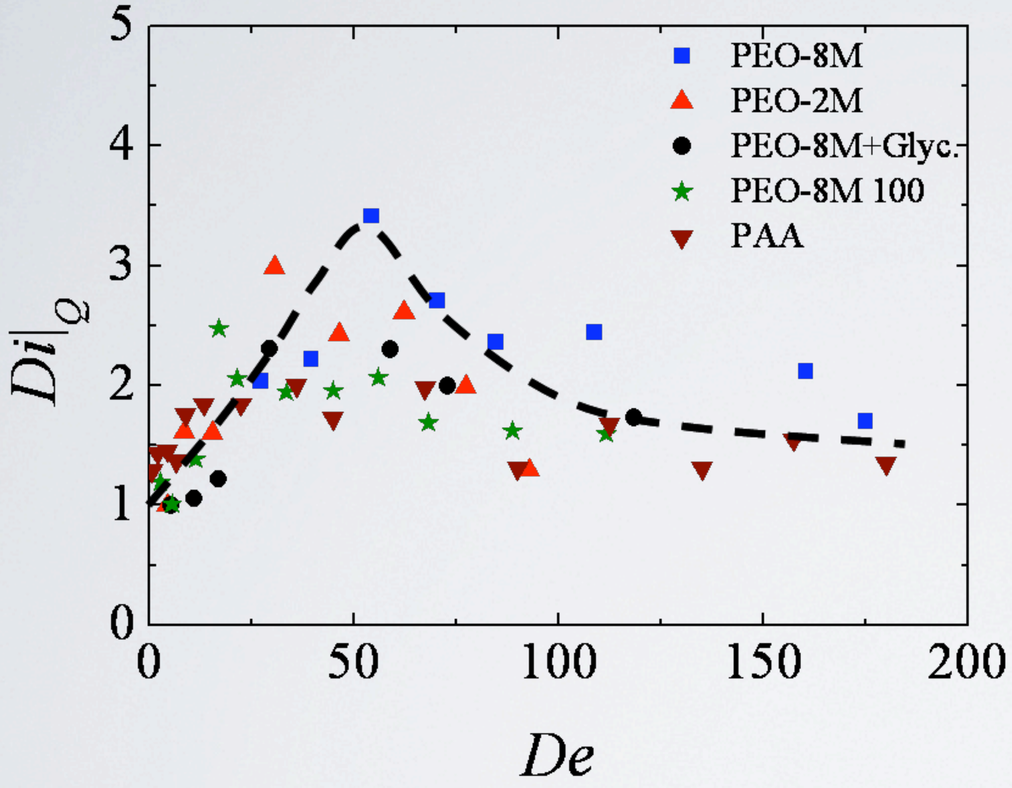


Rectifier effect: more resistance in forward

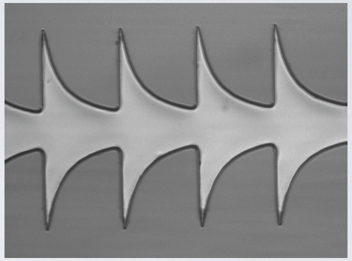
HYPERBOLIC FLUIDIC DIODE: VARIOUS FLUIDS & TRIANGULAR SHAPE

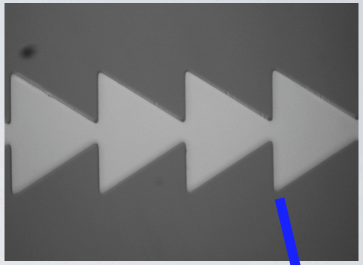
Diodicity

$$Di\big|_Q = \frac{\Delta p_{forward}}{\Delta p_{backward}}$$

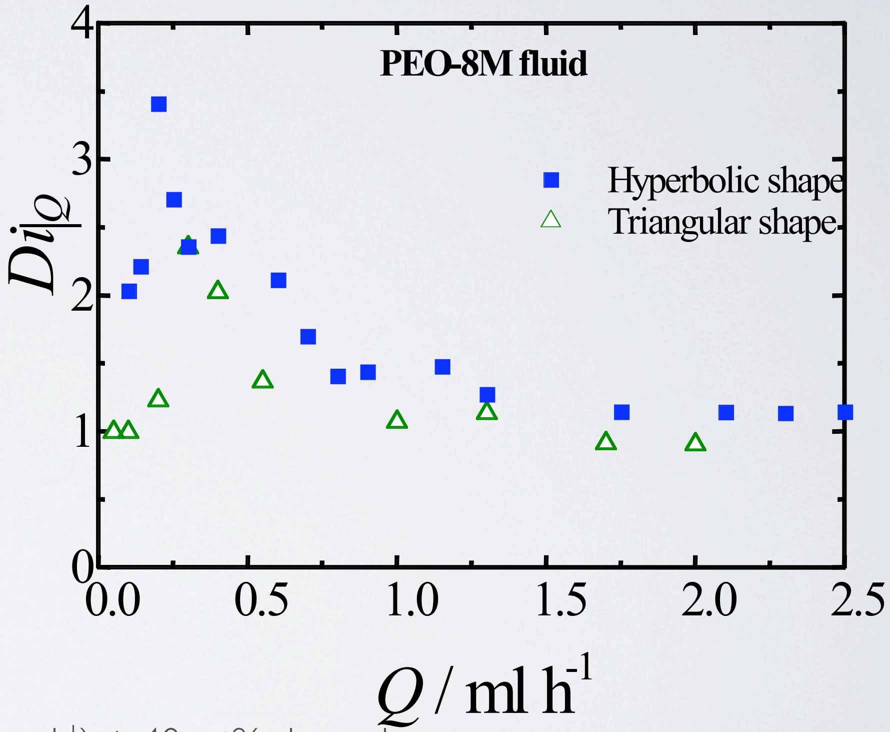


Sousa et al. JNNFM 165 (2010) 652-671





Groissman & Quake, PRL 92 (2004) 92401

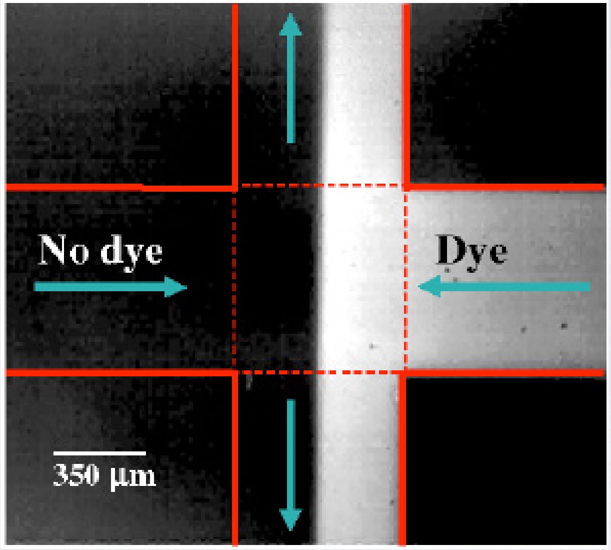


PEO-8M: 0.1% wt aqueous solution PEO (M_w=8 × 10^6 g mol^{-1})
PEO-2M: 0.1% wt aqueous solution PEO (M_w=2 × 10^6 g mol^{-1})
PEO-8M+Glyc: 0.1% wt aqueous solution PEO (M_w=8 × 10^6 g mol^{-1}) + 40 wt% glycerol
PEO-8M 100: 100 ppm aqueous solution PEO (M_w=8 × 10^6 g mol^{-1})
PAA: 100 ppm aqueous solution of PAA (M_w=8 × 10^6 g mol^{-1}) + 13 wt.% sucrose + 1 wt.% NaCl

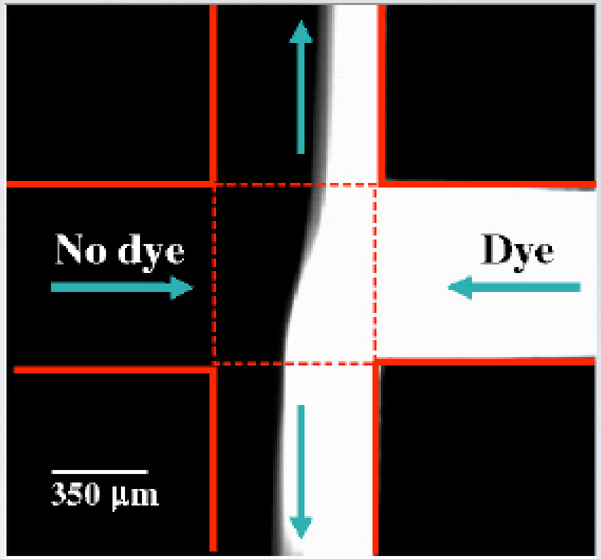
CROSS SLOT

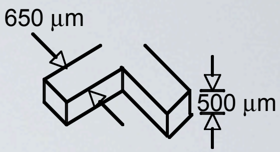
2D CROSS SLOT: MOTIVATION

Arratia et al., PRL 96 (2006) 144502



Newtonian (Re < 10^{-2})

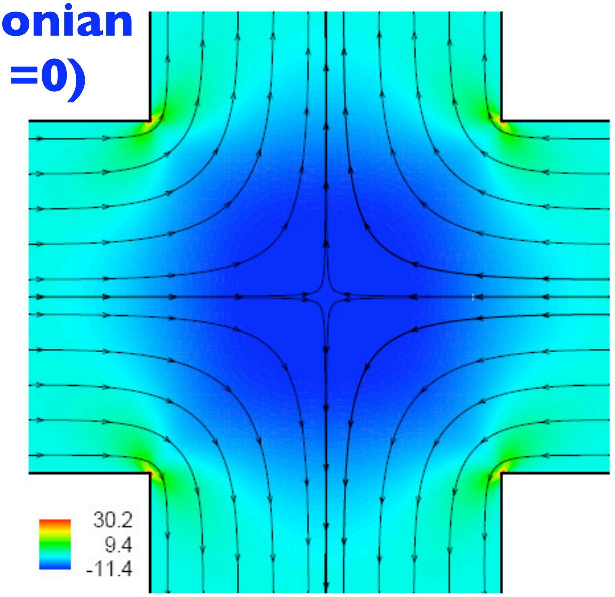




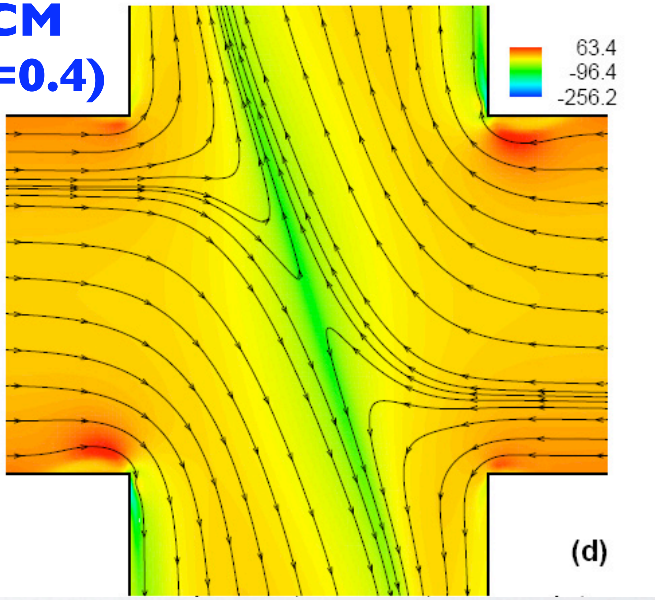
PAA Boger fluid (Re < 10^{-2}, De=4.5)

Poole et al., PRL 99 (2007) 164503

Newtonian
(Re =0)

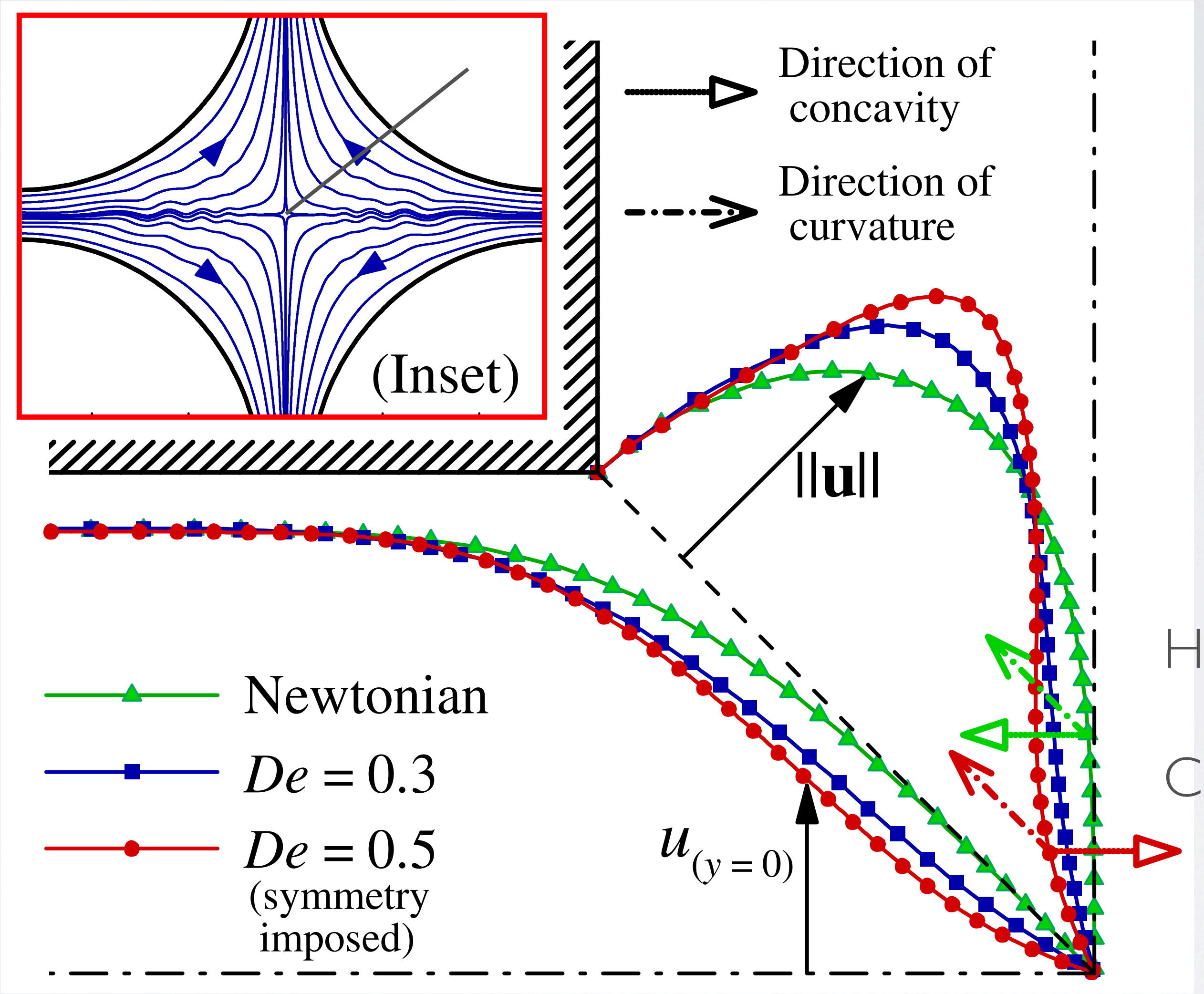


UCM
(De=0.4)



2D CROSS SLOT WITH UCM: CAUSES OF INSTABILITY

Poole et al., PRL 99 (2007) 164503

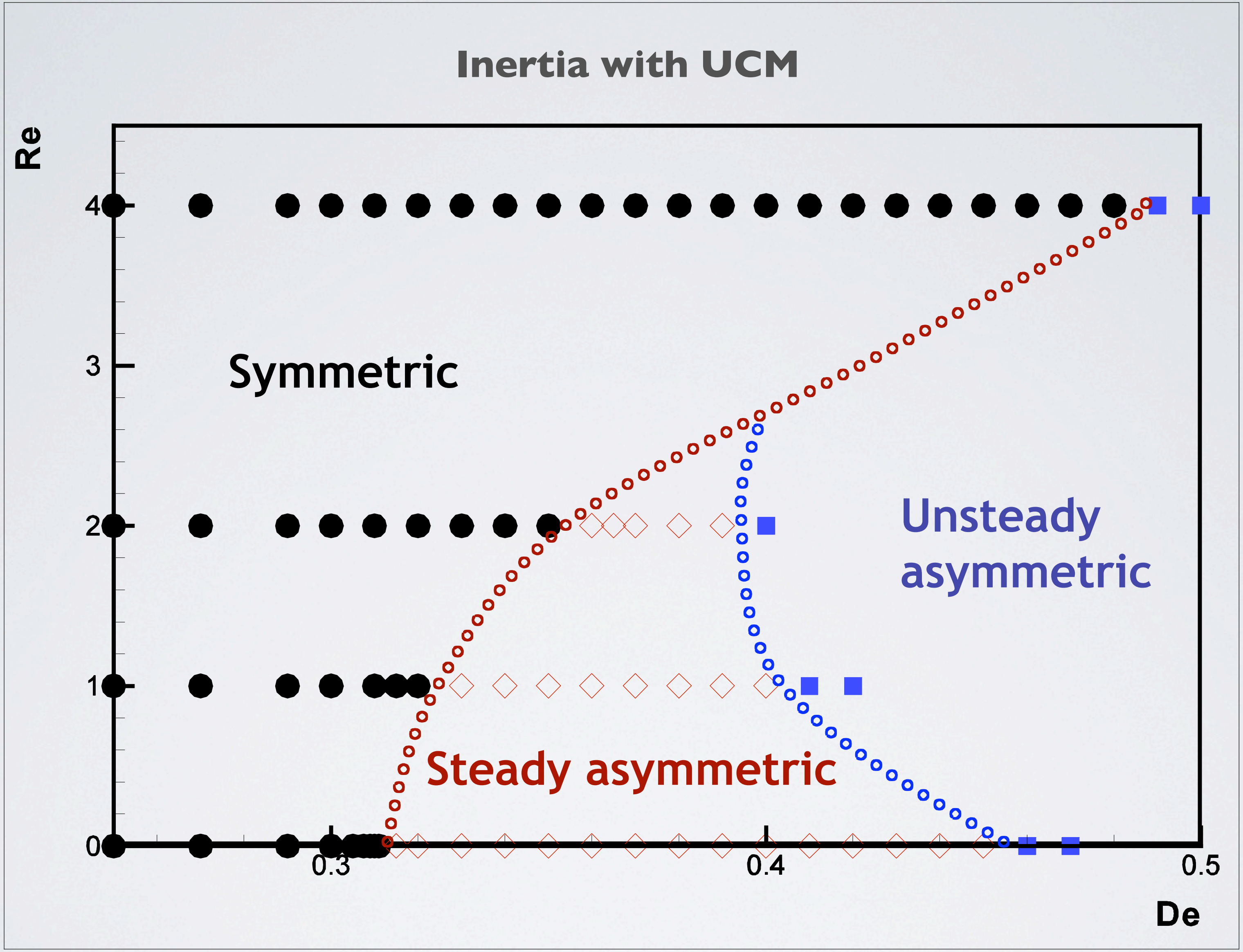


High compressive stresses

Concave profile: fluid pushed towards center (smaller R)

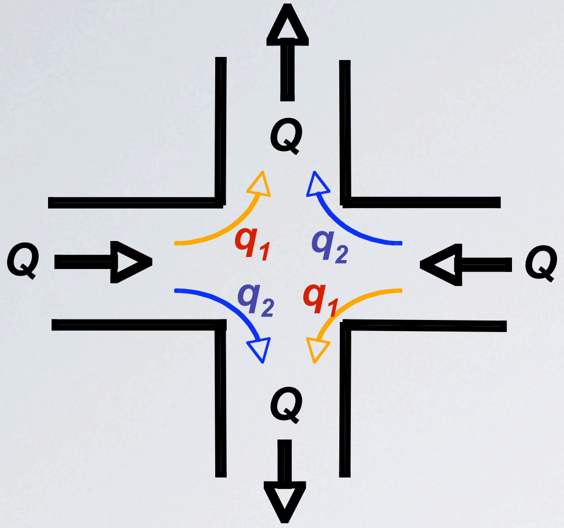
2D CROSS SLOT WITH UCM: EFFECT OF INERTIA

Poole et al., PRL 99 (2007) 164503



Inertia decreases degree of asymmetry and stabilizes the flow

2D CROSS SLOT: OLDROYD-B — EFFECT OF SOLVENT — CREEPING FLOW

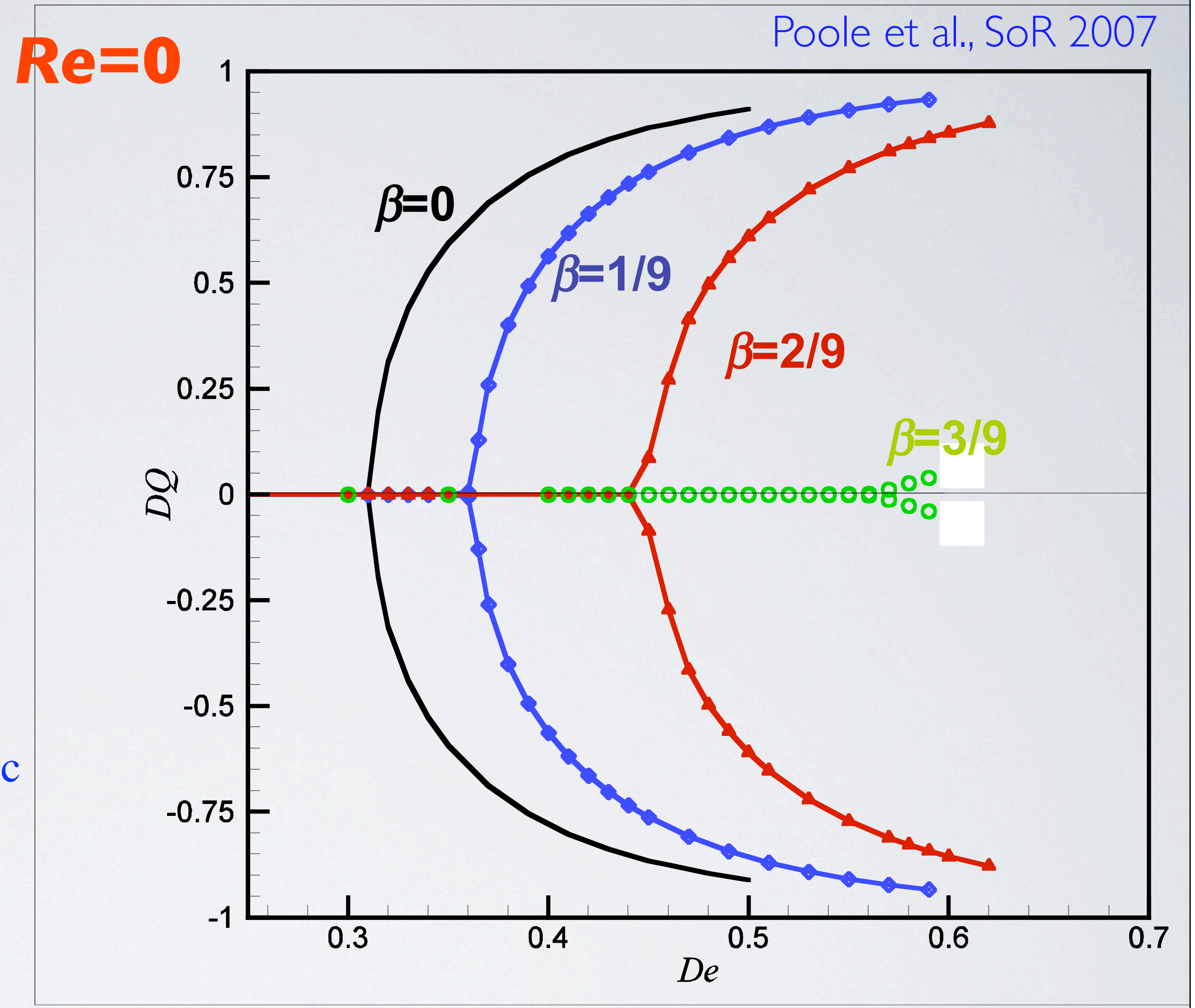


$$DQ = \frac{q_2 - q_1}{q_2 + q_1} = \frac{q_2 - q_1}{Q}$$

$DQ = 0 \rightarrow$ symmetric

$DQ = \pm 1 \rightarrow$ completely asymmetric

$$\beta = \frac{\eta_s}{\eta_s + \eta_p}$$



Poole et al., SoR 2007

Increasing the solvent viscosity

Increases De_{CR}

For β>3/9 flow becomes asymmetric unsteady (as in flow focusing)

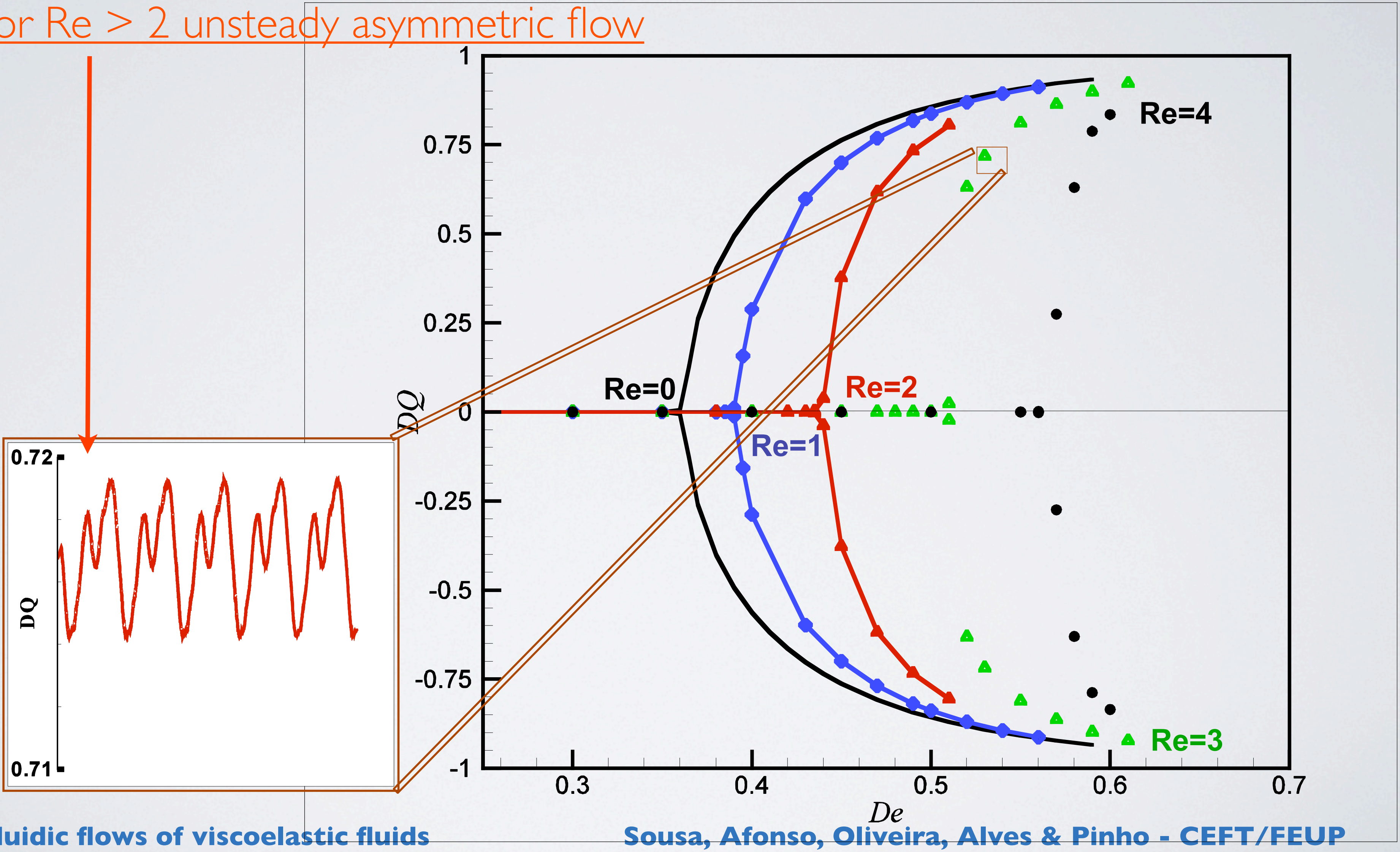
2D CROSS SLOT: OLDROYD-B — SOLVENT AND INERTIA

Poole et al., SoR 2007

$\beta = 1/9$

Increasing Re

- Increases De_{CR}
- Decreases degree of asymmetry
- For Re > 2 unsteady asymmetric flow



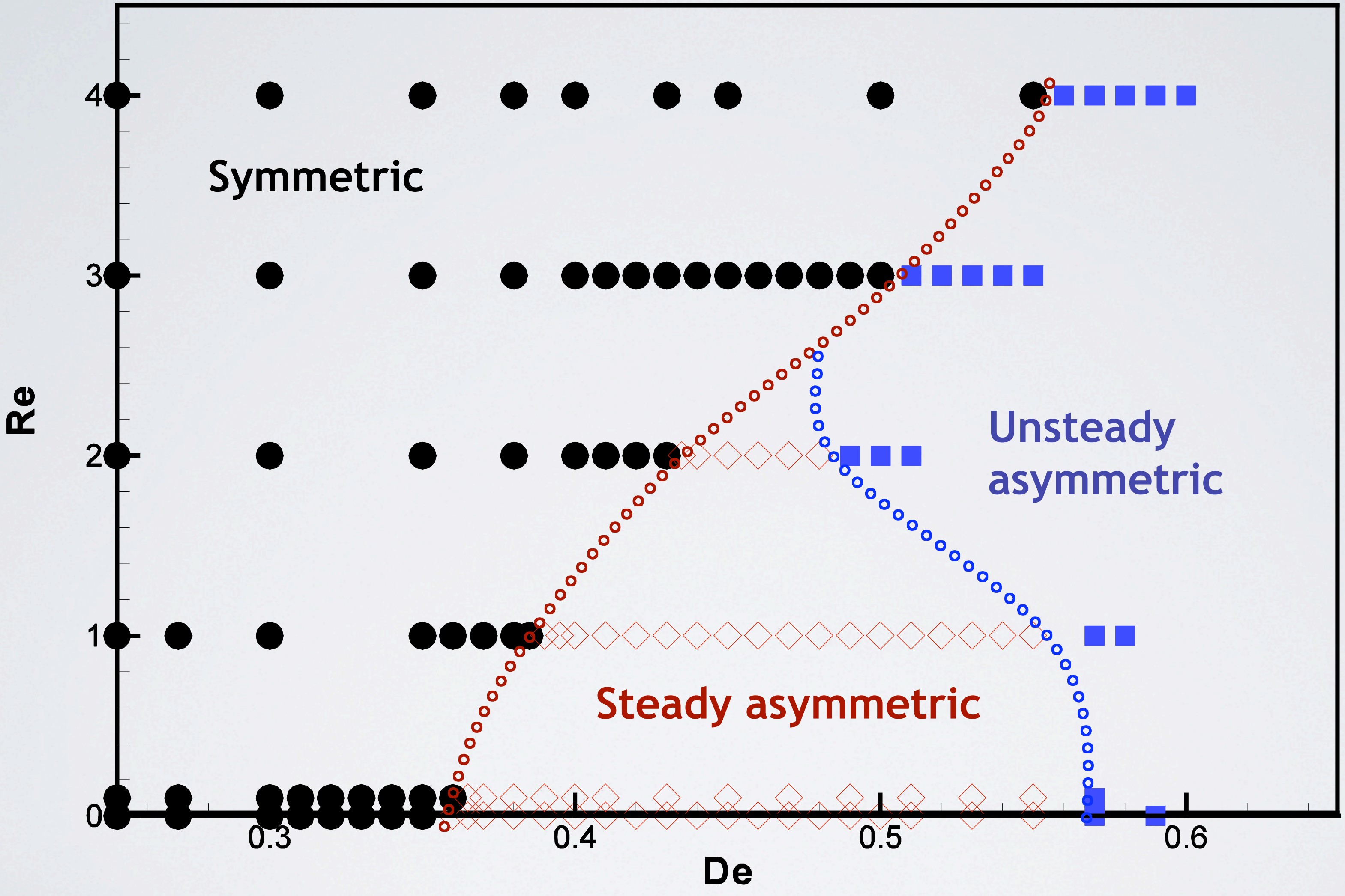
Microfluidic flows of viscoelastic fluids
V BCR 2010

Sousa, Afonso, Oliveira, Alves & Pinho - CEFT/FEUP
Rio de Janeiro, Brazil, 14-16th July 2010

2D CROSS SLOT: OLDROYD-B — STABILITY MAP

Poole et al., SoR 2007

$\beta=1/9$



2D CROSS SLOT: SPTT — EFFECT OF EPSILON

Poole et al., SoR 2007

$\beta=1/9$

Increasing ε

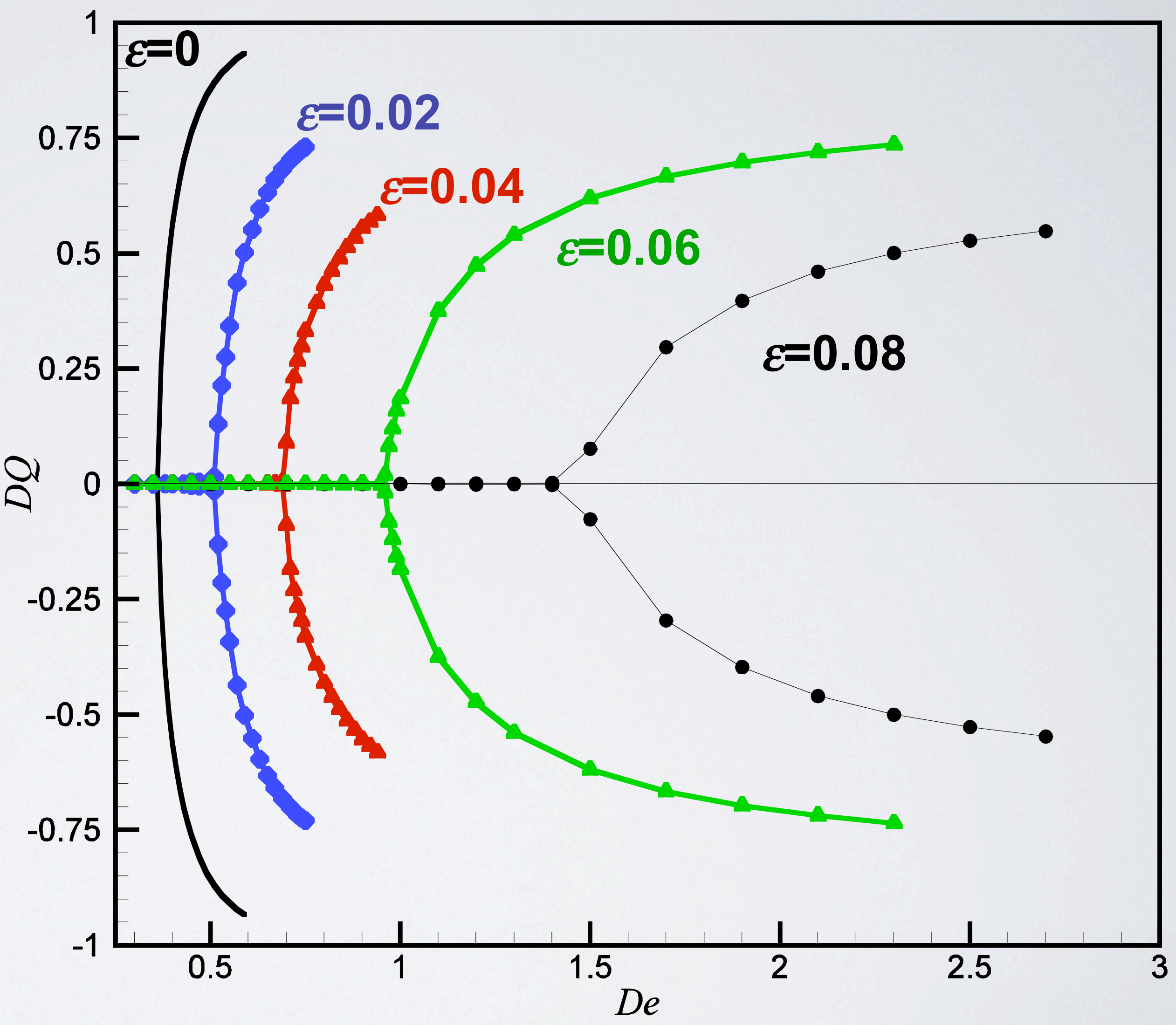
Increases De_{CR}

Decreases degree of asymmetry ($\varepsilon<0.04$)

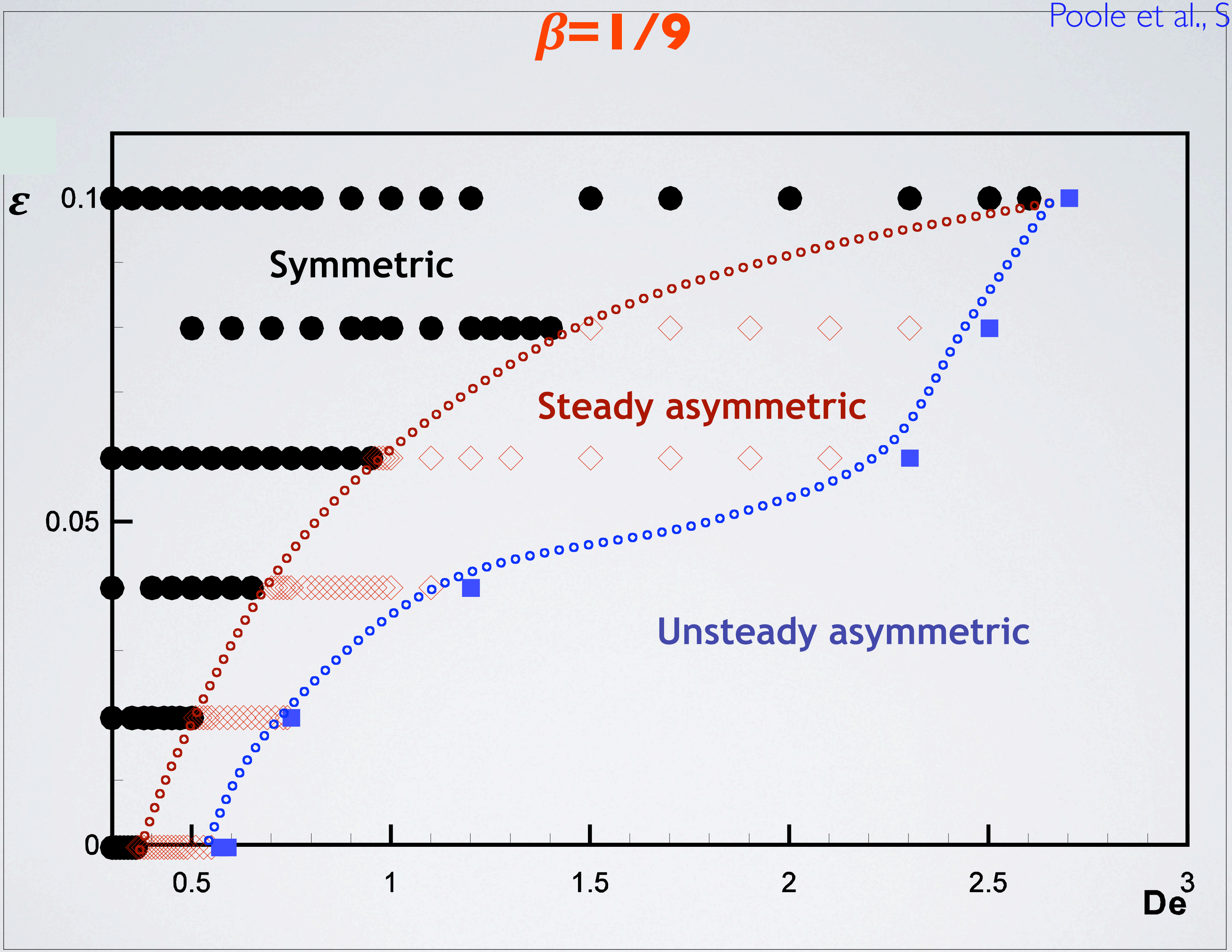
Increases degree of asymmetry and extension in De ($\varepsilon>0.04$)

Asymmetric stable flow disappears for $\varepsilon>0.08$

Qualitatively as in flow focusing



2D CROSS SLOT: SPTT — STABILITY MAP

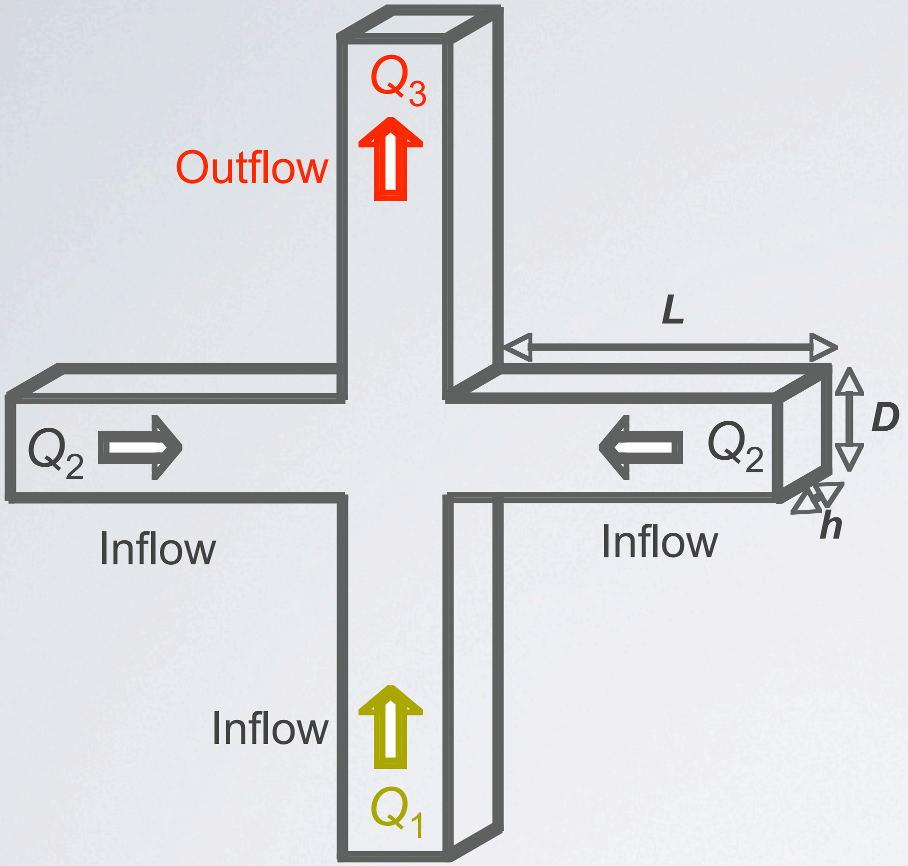


FLOW FOCUSING

(Alternative extensional flow)

FLOW FOCUSING

Oliveira et al. JNNFM 160 (2009) 31-39



Cross-slot with
3 inlets and 1 outlet

Operational Variables

$$Q_1, Q_2$$

$$Q_3 = 2 \times Q_2 + Q_1$$

Dimensionless Variables

$$FR = \frac{Q_2}{Q_1}$$

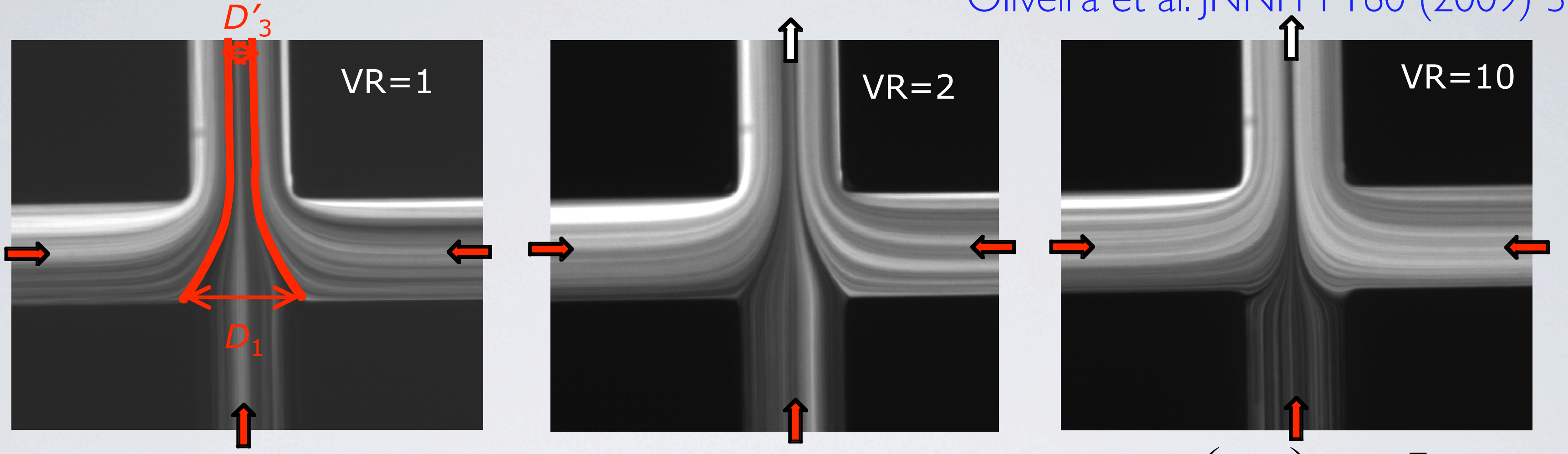
$$VR = \frac{U_2}{U_1} \quad (= FR)$$

$$\left.\begin{array}{l} \mathrm{Re} = \dfrac{\rho U_2 D}{\eta_0} \\ \\ De = \dfrac{\lambda U_2}{D} \end{array}\right\} El = \frac{De}{\mathrm{Re}}$$

All dimensions kept constant
in experiments and calculations

FLOW FOCUSING: NEWTONIAN

Oliveira et al. JNNFM 160 (2009) 31-39

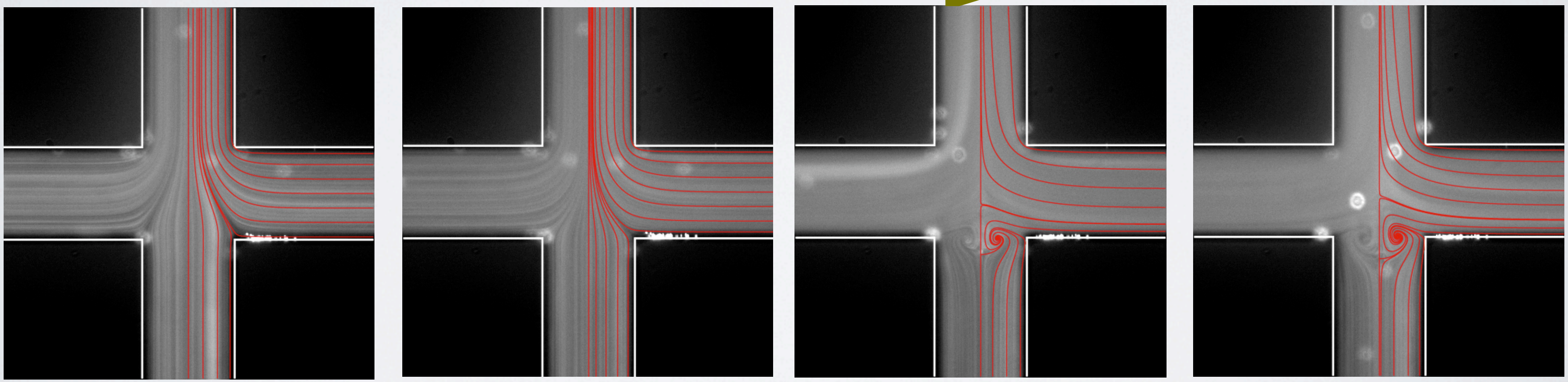


Separation streamlines: nearly hyperbolic shape

$$\varepsilon_H = \ln\left(\frac{D_1}{D_3}\right) = \ln\left[\frac{3}{2}\left(1+2VR\right)\right]$$

increasing Q_2

$Q_1 = 0.01$ ml/h



Q_2 = 0.3 ml/h
VR = 1, Re_3 = 2.8

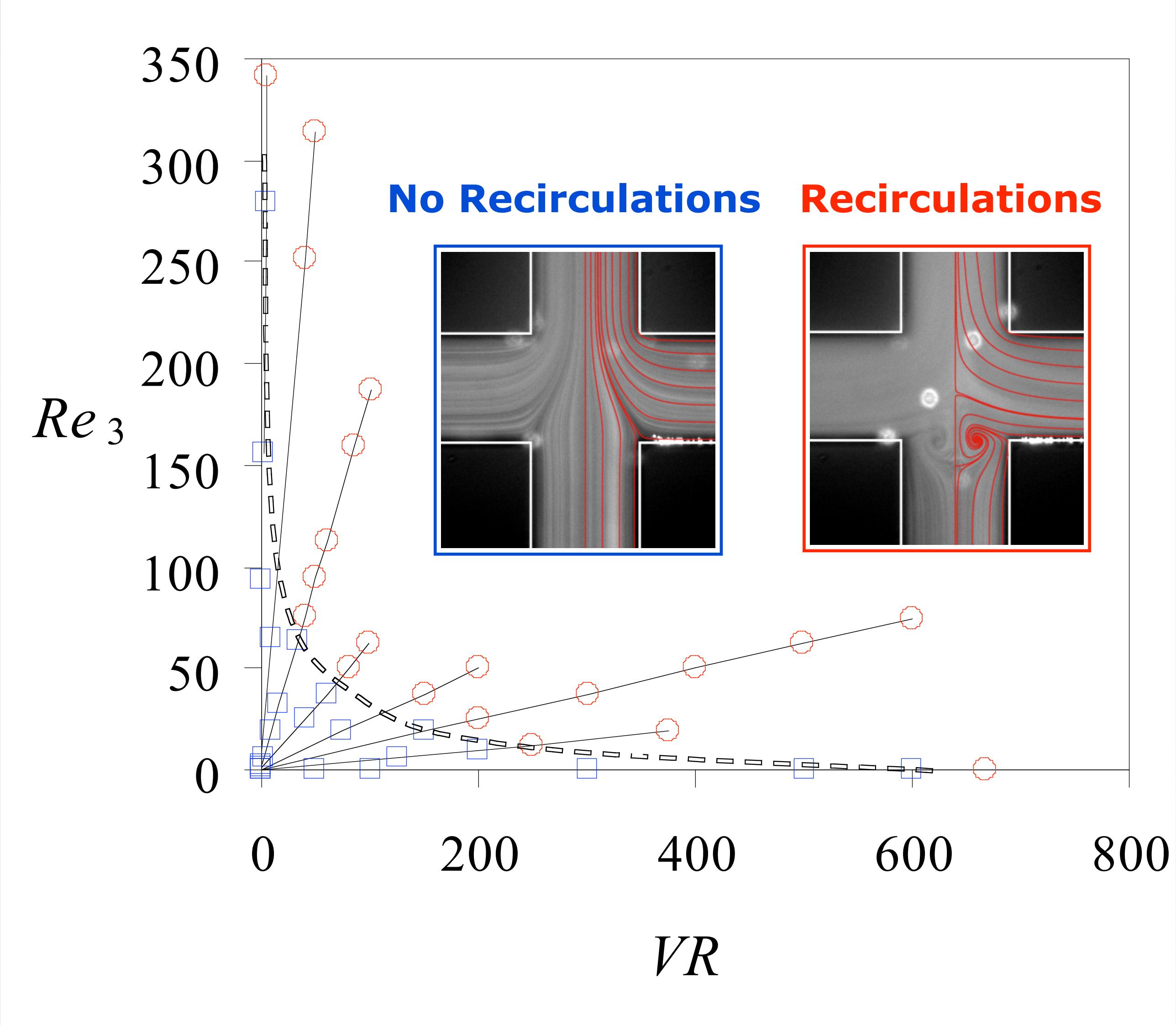
Q_2 = 0.9 ml/h
VR = 3, Re_3 = 6.5

Q_2 = 15 ml/h
VR = 50, Re_3 = 94.2

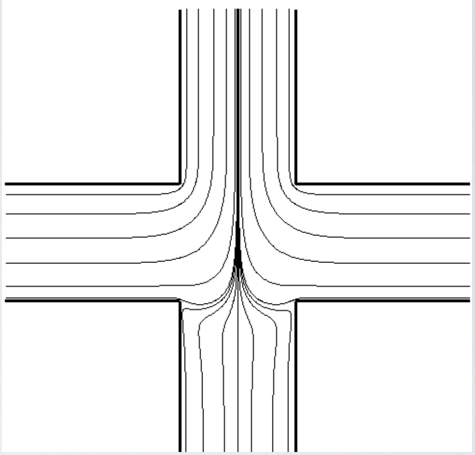
Q_2 = 18 ml/h
VR = 60, Re_3 = 112.8

FLOW FOCUSING: 3D EFFECTS & NEWTONIAN (2)

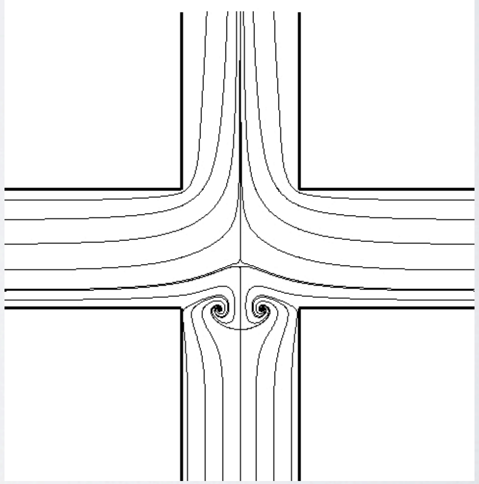
Oliveira et al. JNNFM 160 (2009) 31-39



VR= 50
Re= 0



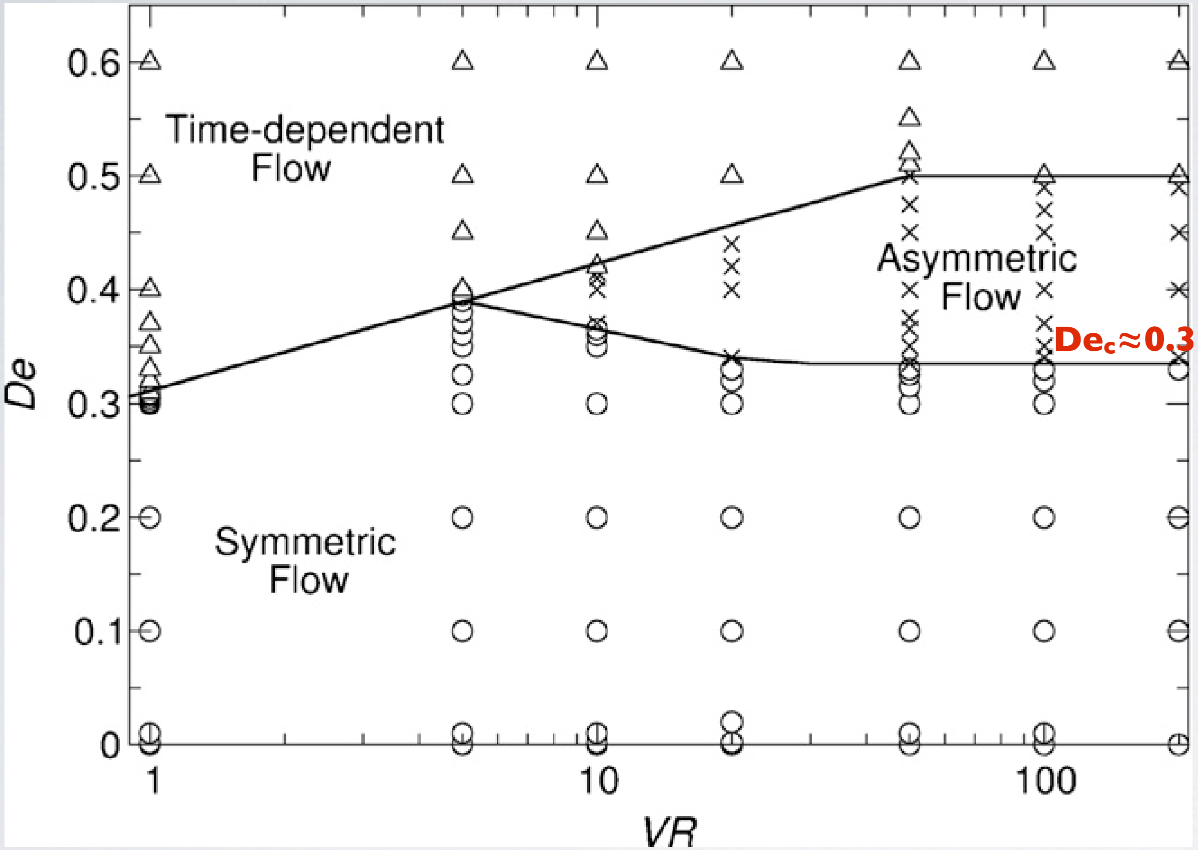
VR= 50
Re= 47

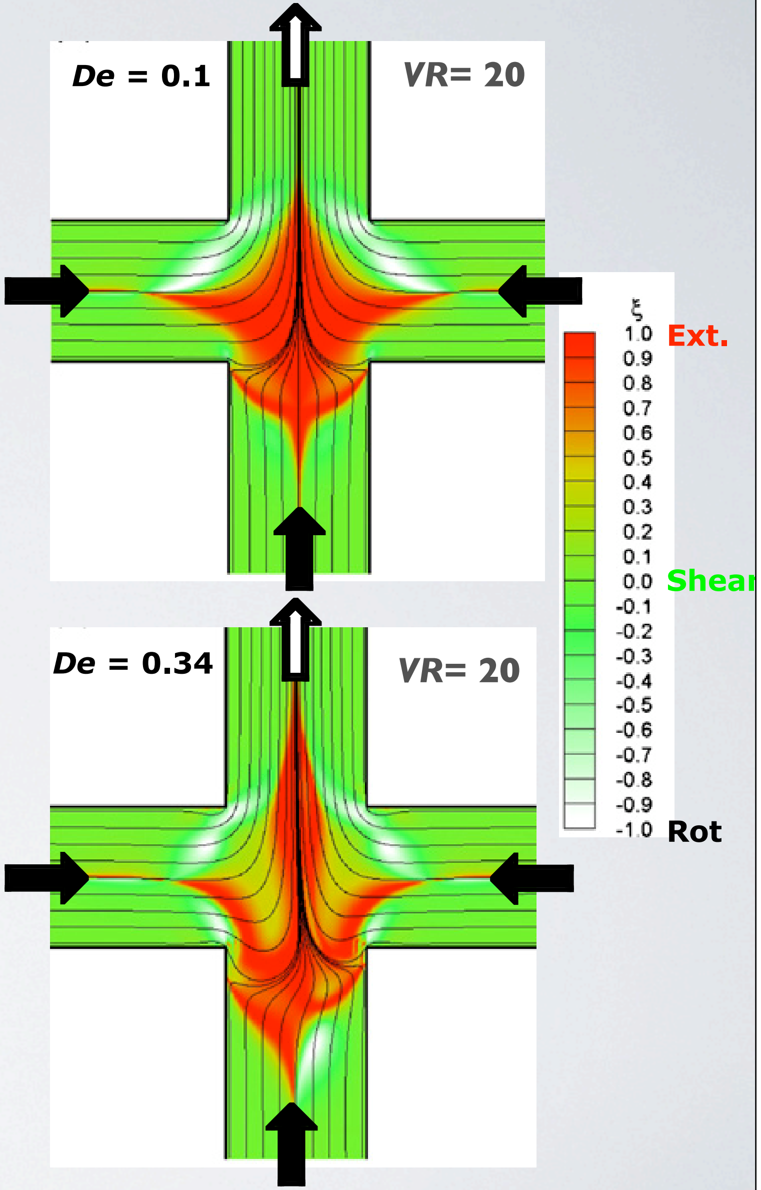


FLOW FOCUSING: VISCOELASTIC INSTABILITIES

Oliveira et al. JNNFM 160 (2009) 31-39

UCM, 2D, Re=0





$$\xi = \frac{1-R}{1+R}$$

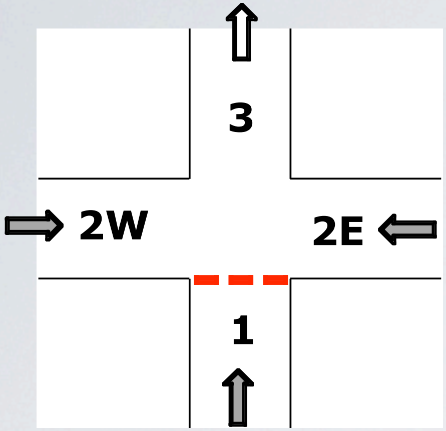
$$R = \frac{tr\tilde{\mathbf{W}}^2}{tr\mathbf{D}^2}$$

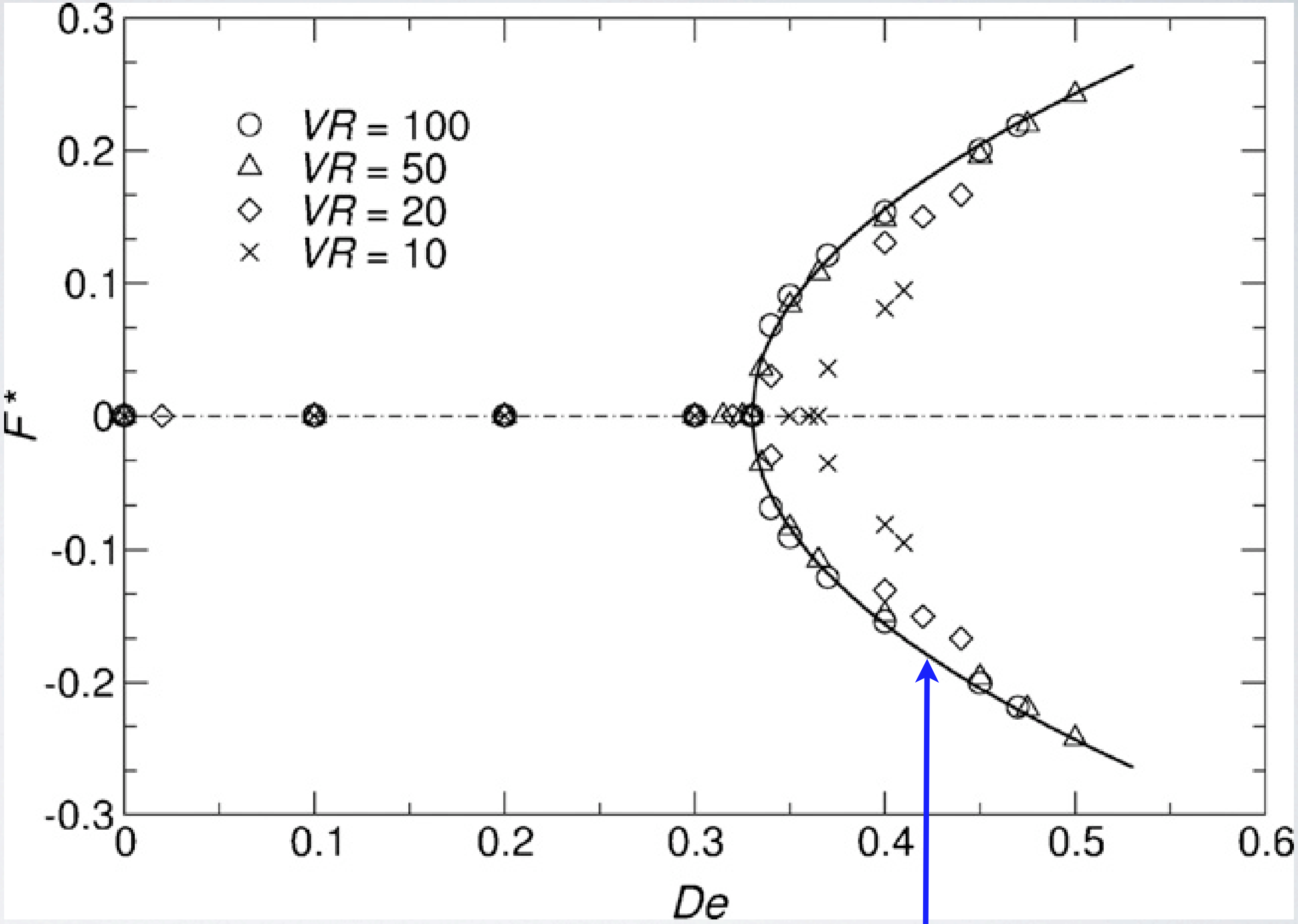
Astarita, JNNFM 6 (1979) 69
Thompson et al., JNNFM 86 (1999) 375
Mompean et al., JNNFM 111 (2003) 151

FLOW FOCUSING: EFFECT OF VR

Oliveira et al. JNNFM 160 (2009) 31-39

$$F^* = \frac{F_W - F_E}{F_3}$$





$$F^* = 0.59\sqrt{De - 0.33}$$

Bistable flow

High VR:

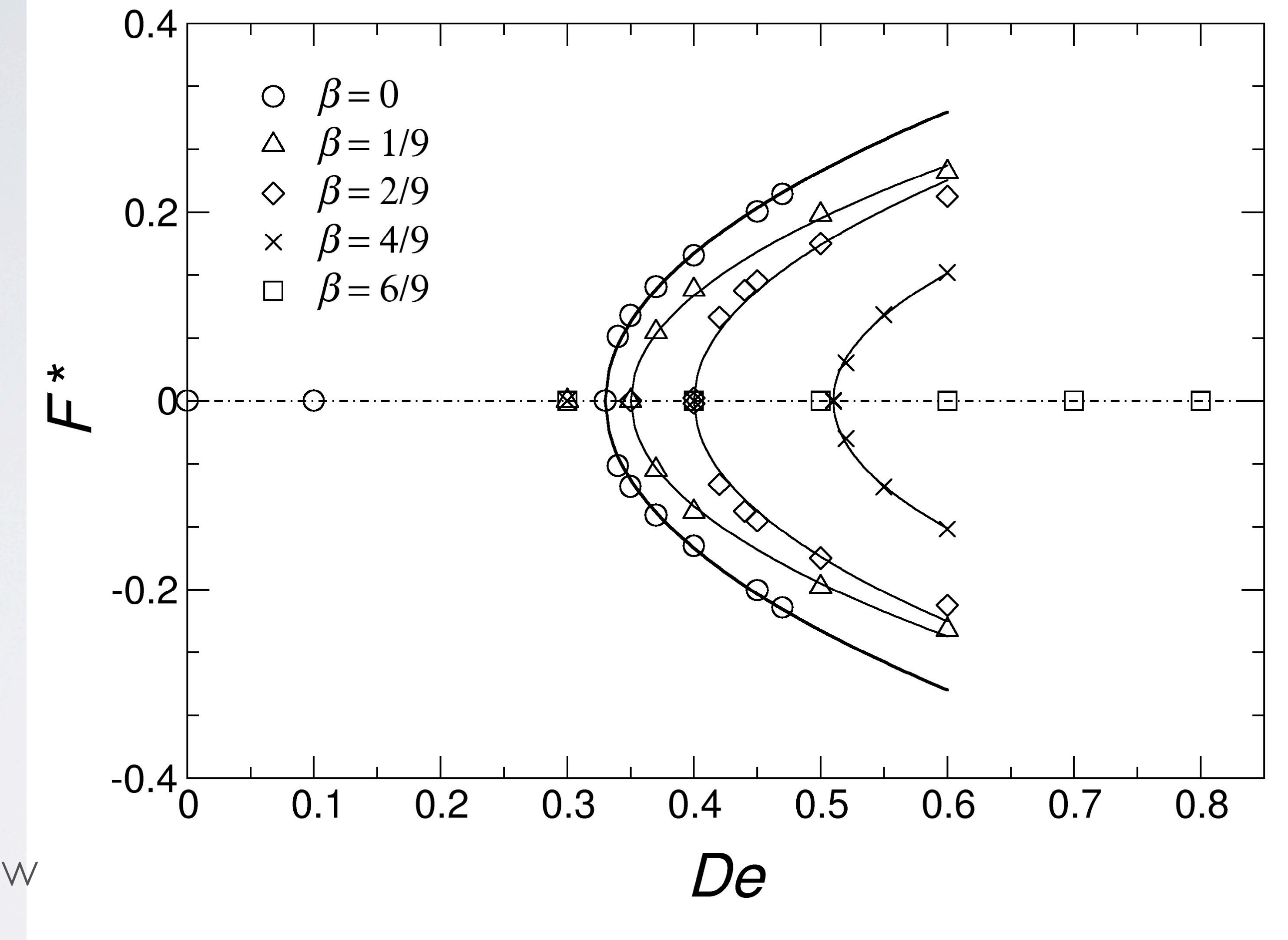
- constant De_c
- evolution independent of VR
- supercritical pitchfork bifurcation

FLOW FOCUSING: EFFECT OF β

Oliveira et al. JNNFM 160 (2009) 31-39

$$\beta = \frac{\eta_s}{\eta_s + \eta_p}$$

Oldroyd-B



β stabilizes the flow

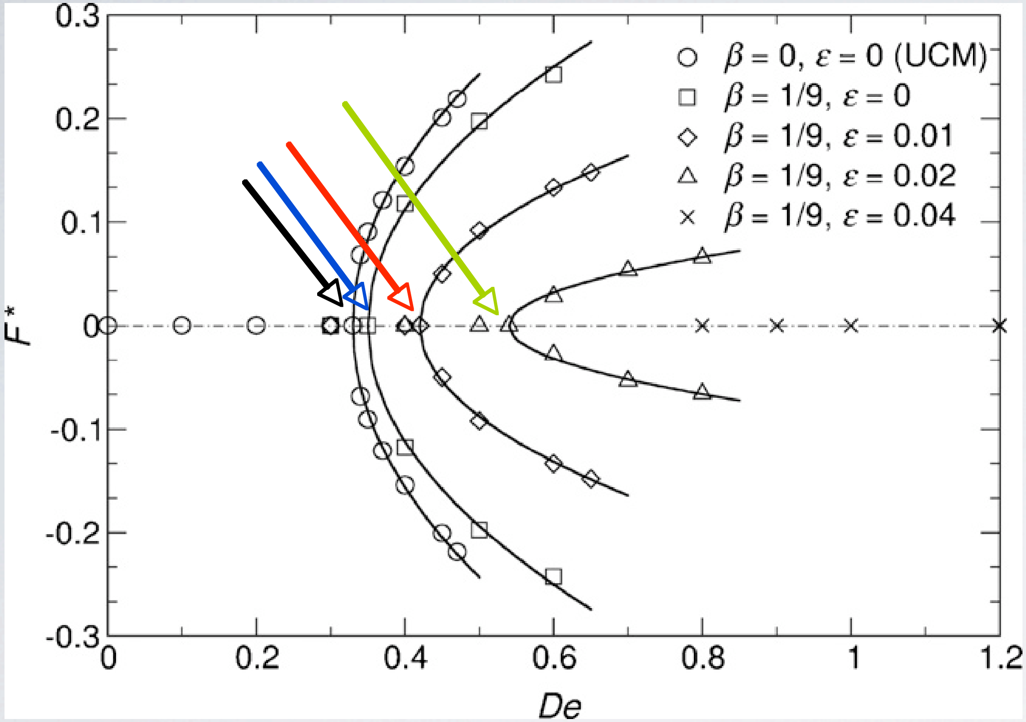
increases De_c

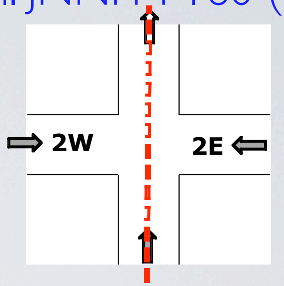
$\beta \geq 6/9$, <u>no steady</u> asymmetry

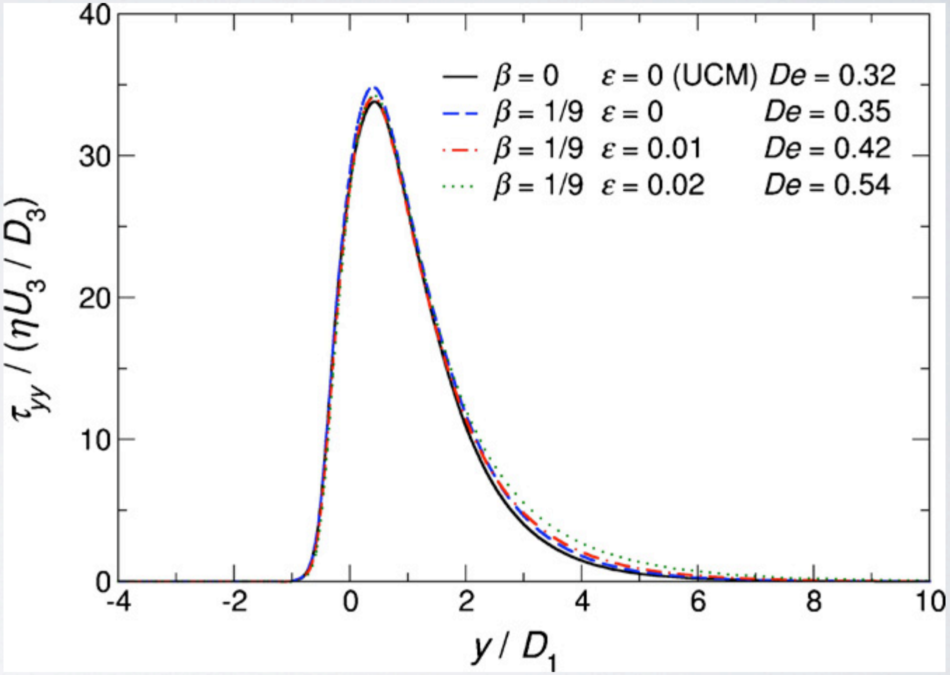
FLOW FOCUSING: EFFECT OF ε

Oliveira et al. JNNFM 160 (2009) 31-39

sPTT







- ε stabilizes the flow
  - increases De_c
  - decreases degree of asymmetry
  - $\varepsilon \geq 0.04$ steady asymmetry disappears
  - **(Transition directly to unsteady flow)**

- Similar levels of normal stresses achieved near critical conditions
- Extensional properties decisive for onset of flow asymmetry

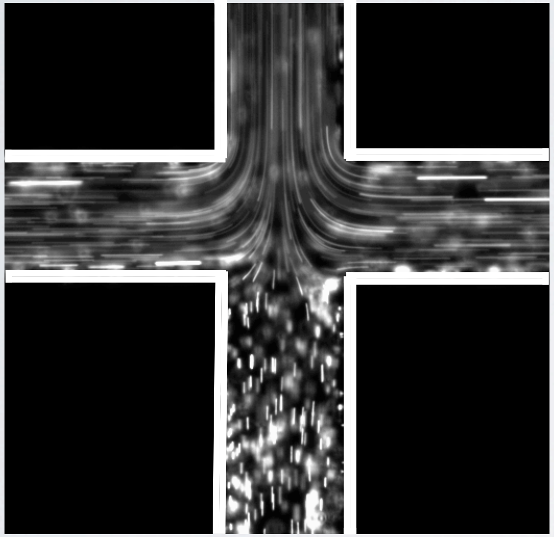
FLOW FOCUSING: EXPERIMENTS FOR PAA125 (I)

Oliveira et al. JNNFM 160 (2009) 31-39

$$Q_1 = 0.01 \text{ ml/h}$$

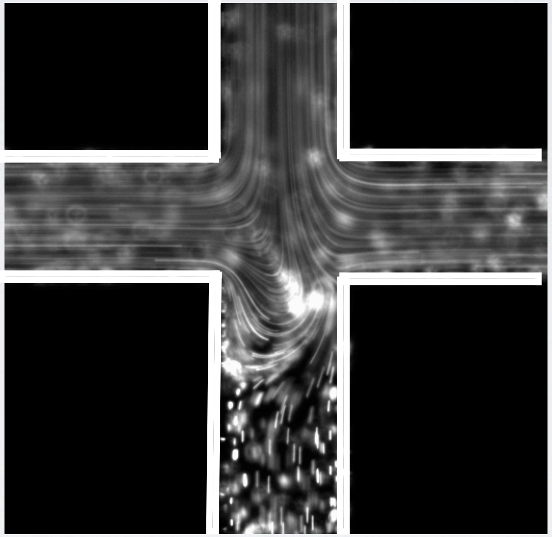
increasing Q_2 →

Viscoelastic



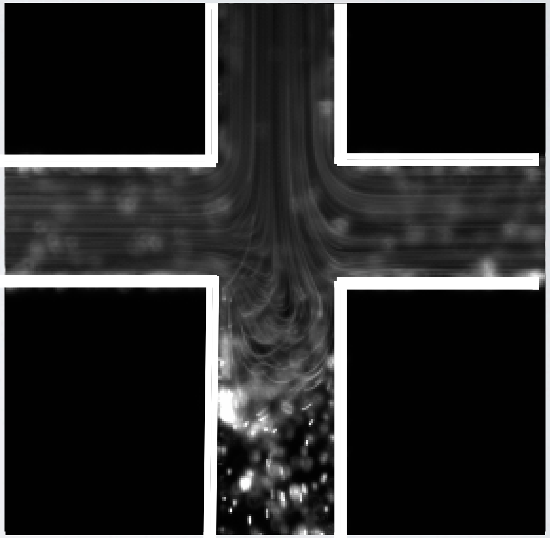
$Q2$ = 0.05 ml/h, *VR* = 5
Re = 0.23, *De* = 0.38

Symmetric



$Q2$ = 0.2 ml/h, VR = 20
Re = 0.87, *De* = 1.41

Steady Asymmetric



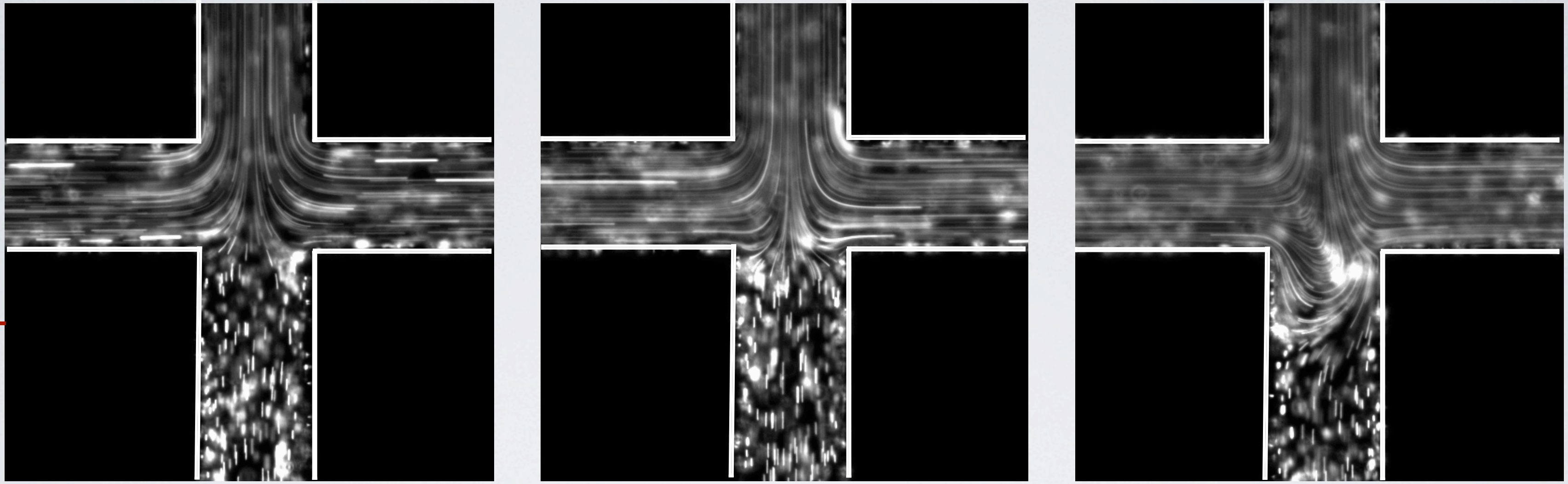
$Q2$ = 0.5 ml/h, VR = 50
Re = 2.15, *De* = 3.479

Unsteady 3D

FLOW FOCUSING: NUMERICAL VERSUS EXPERIMENTS (PAA 125)

$Q_1 = 0.01$ ml/h

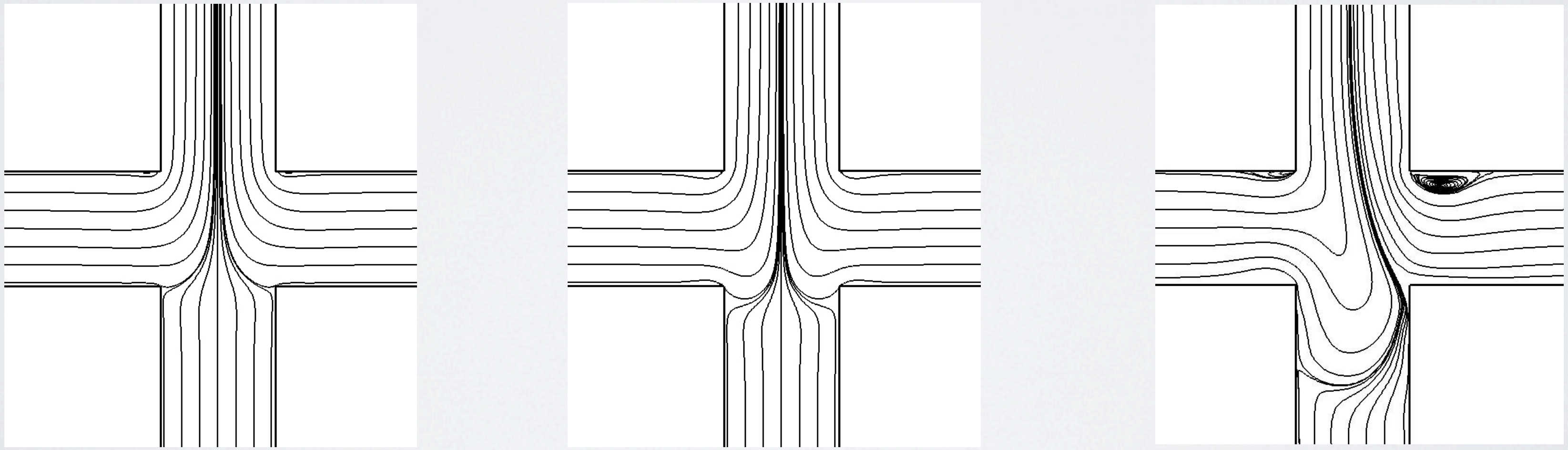
Oliveira et al. JNNFM 160 (2009) 31-39



$Q2$ = 0.05 ml/h, $VR = 5$
$Re = 0.23$, $De = 0.38$

$Q2$ = 0.1 ml/h, VR = 10
$Re = 0.45$, $De = 0.723$

$Q2$ = 0.2 ml/h, VR = 20
$Re = 0.87$, $De = 1.41$

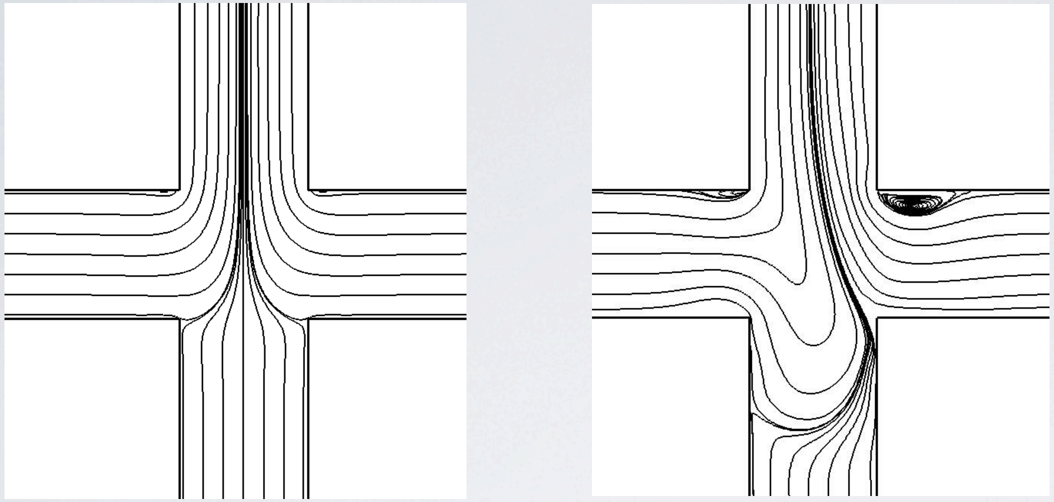


FLOW FOCUSING: UCM VERSUS OLDROYD-B

Oliveira et al. JNNFM 160 (2009) 31-39

$Q_1 = 0.01\ \text{ml/h}$

UCM
2D Calculations



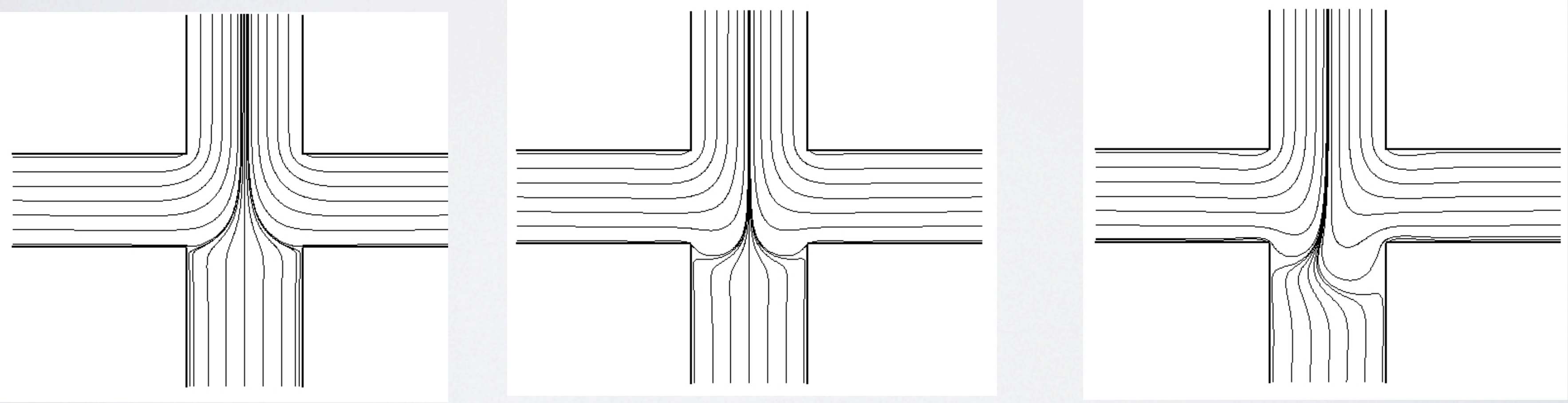
Unsteady 3D

Q_2 = 0.05 ml/h, *VR* = 5
Re = 0.23, *De* = 0.38

Q_2 = 0.2 ml/h, VR = 20
Re = 0.87, *De* = 1.41

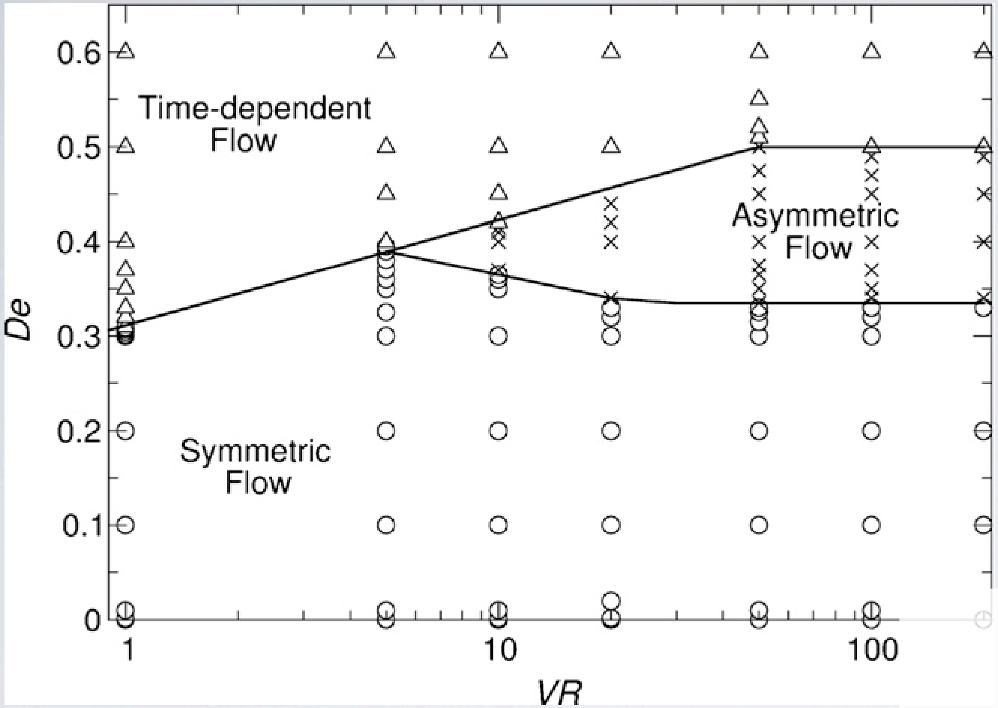
Q_2 = 0.35 ml/h, VR = 35
Re = 0.87, *De* = 1.41

Oldroyd-B
2D Calculations



FLOW FOCUSING: NUMERICAL VERSUS EXPERIMENTS (PAA 125)

Oliveira et al. JNNFM 160 (2009) 31-39

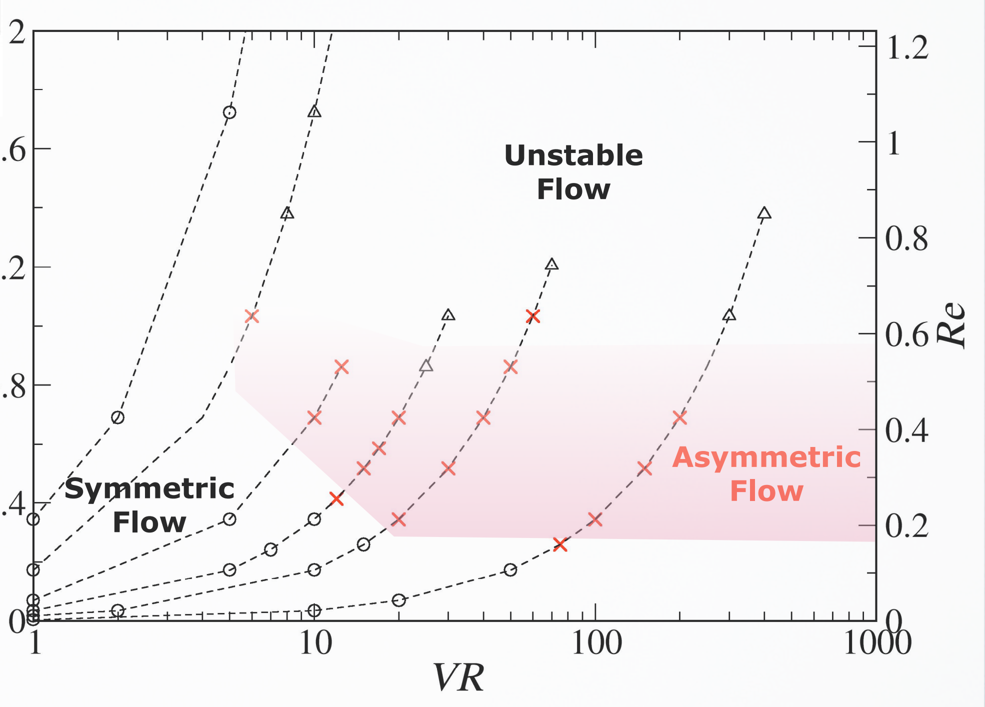


Numerical

UCM, 2D, Re=0

Experimental

PAA 125 + NaCl



Microfluidic flows of viscoelastic fluids
V BCR 2010

Sousa, Afonso, Oliveira, Alves & Pinho - CEFT/FEUP
Rio de Janeiro, Brazil, 14-16th July 2010

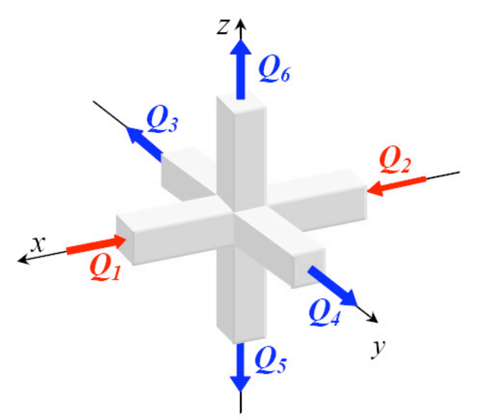
3D CROSS SLOT
Uniaxial and biaxial

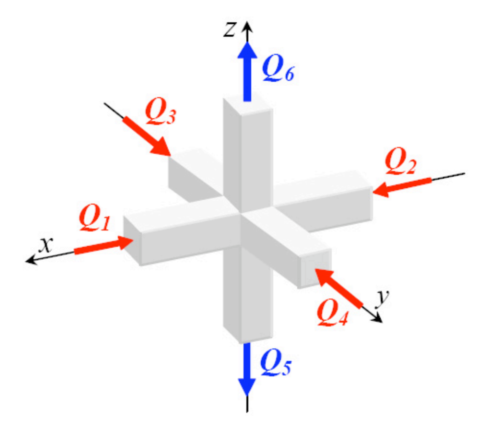
3D CROSS SLOT: FLOW CONFIGURATIONS

Afonso et al., JNNFM 165 (2010) 743-751

Planar extension

Uniaxial extension





I_o=2:4

I_o=4:2

$$m = 1$$

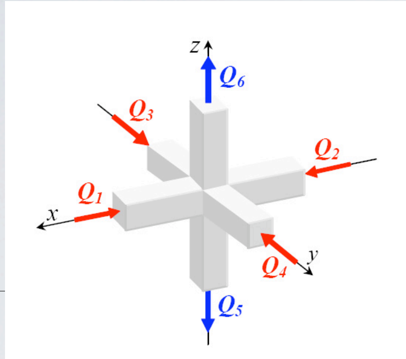
$$m = -\frac{1}{2}$$

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_0 \begin{bmatrix} -(m+1) & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

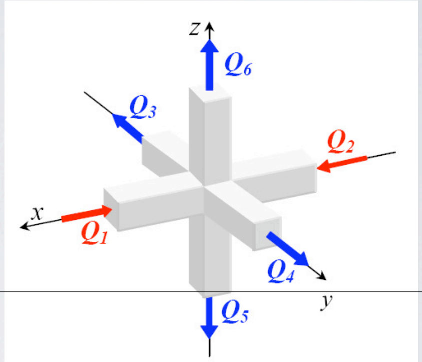
3D CROSS SLOT: UNIAXIAL VERSUS BIAXIAL EXTENSION

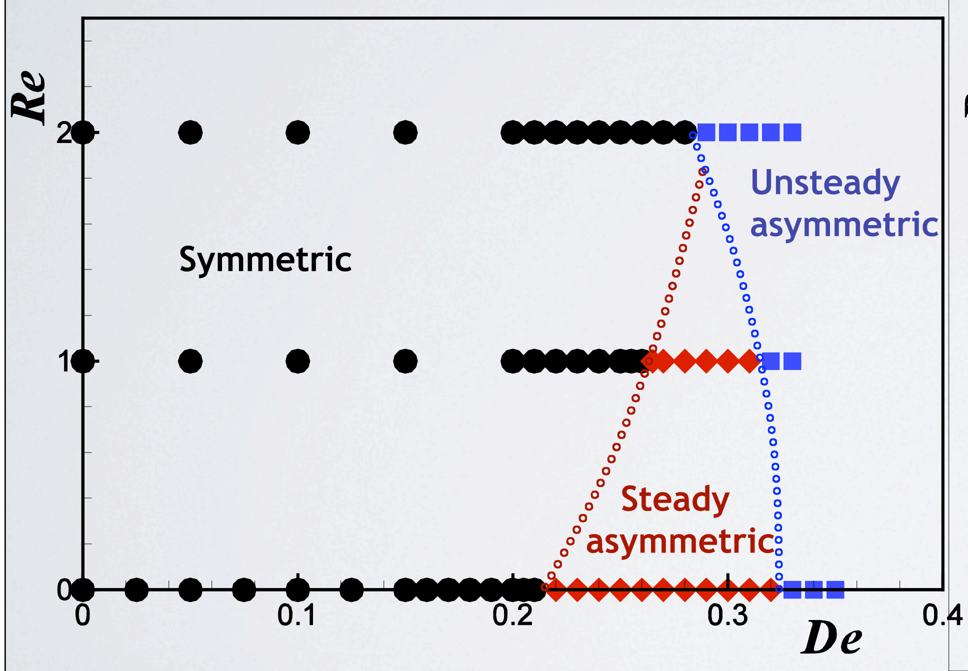
Afonso et al., JNNFM 165 (2010) 743-751

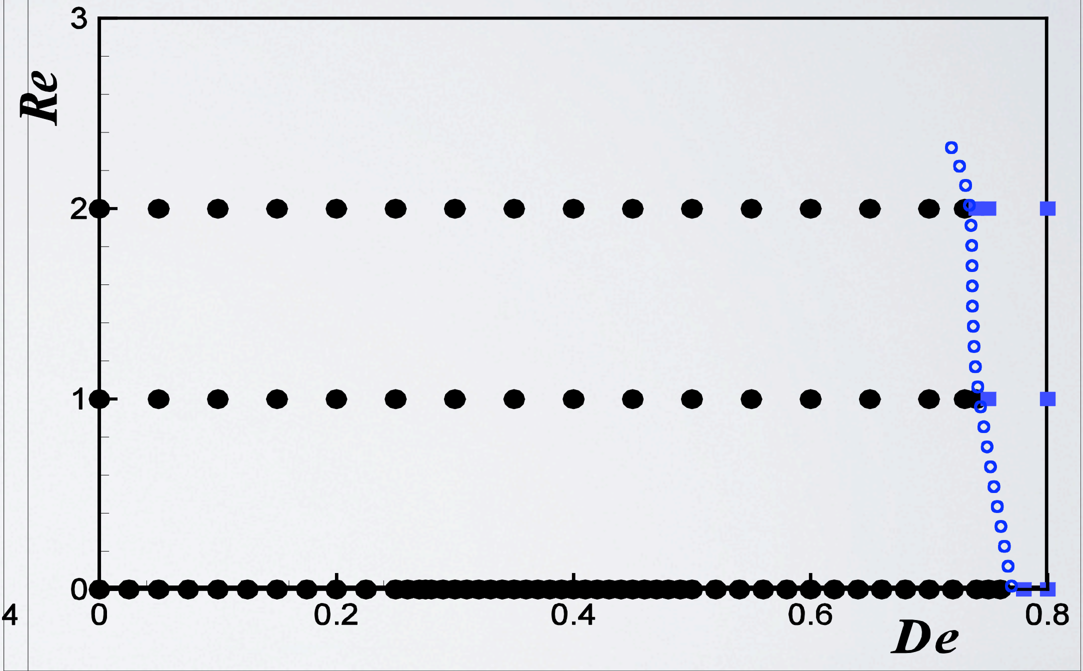
Uniaxial



Biaxial





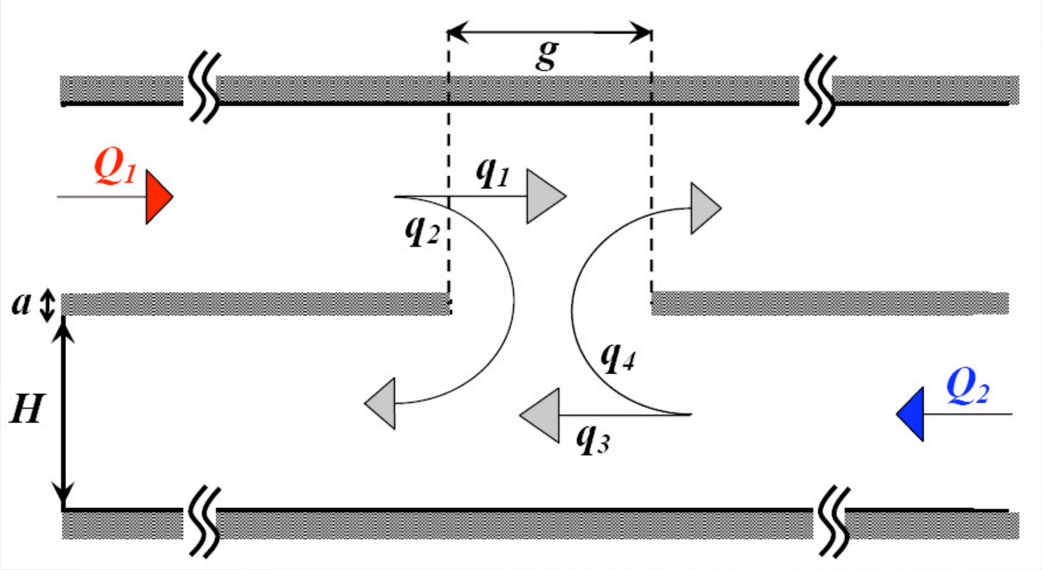


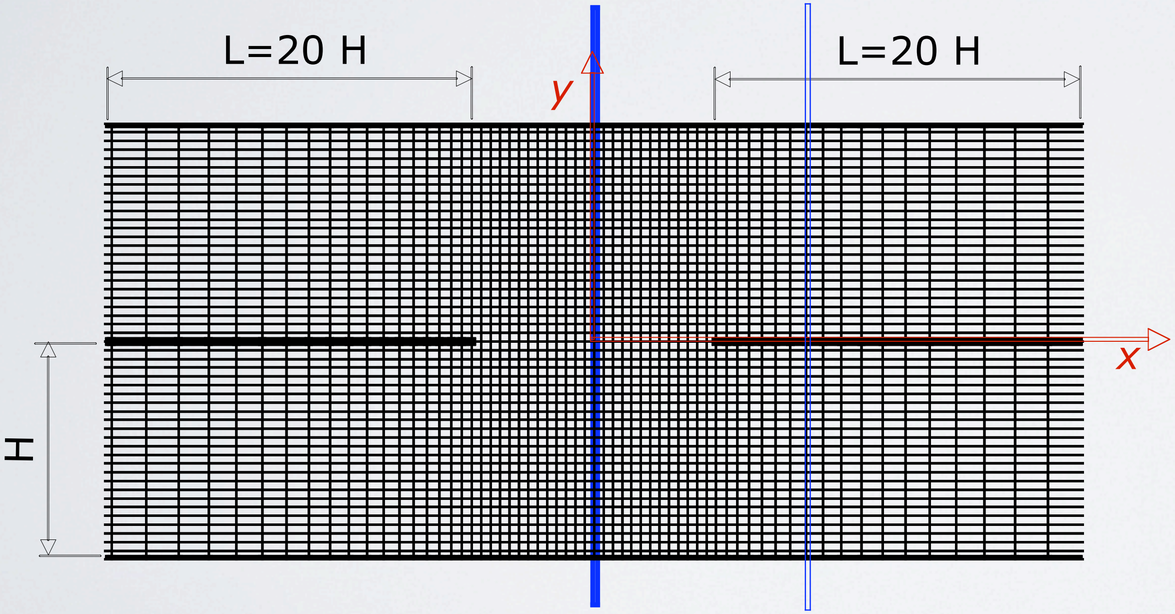
No steady asymmetric flow

MIXING SEPARATING CHANNEL

MIXING-SEPARATING FLOW: FLOW CONFIGURATION

Afonso et al., J Eng. Math (2010) in press





- Non-dimensional gap size

$$\theta = {g}/{H}$$

- Non-dimensional thickness:

$$\alpha = {a}/{H}$$

- Deborah number:

$$De = {\lambda U}/{g}$$

- Reynolds number:

$$Re = {\rho U H}/{\eta}$$

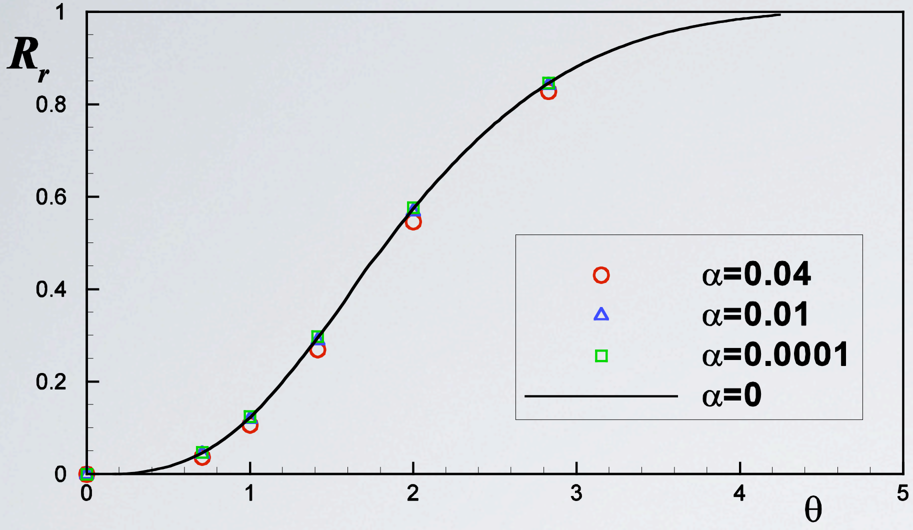
- Degree of flow reversal:

$$R_r = {q_2}/{Q_1}$$

Experimental: Walters et al. (1981, 1982) Proc. R. Soc.
Numerical: Baloch et al. JNNFM (1995); Fétier (2002) PhD UPFL ⟵ **Macroscopic flow studies**

MIXING-SEPARATING FLOW: NEWTONIAN CREEPING FLOW & RE

Afonso et al., J Eng. Math (2010) in press

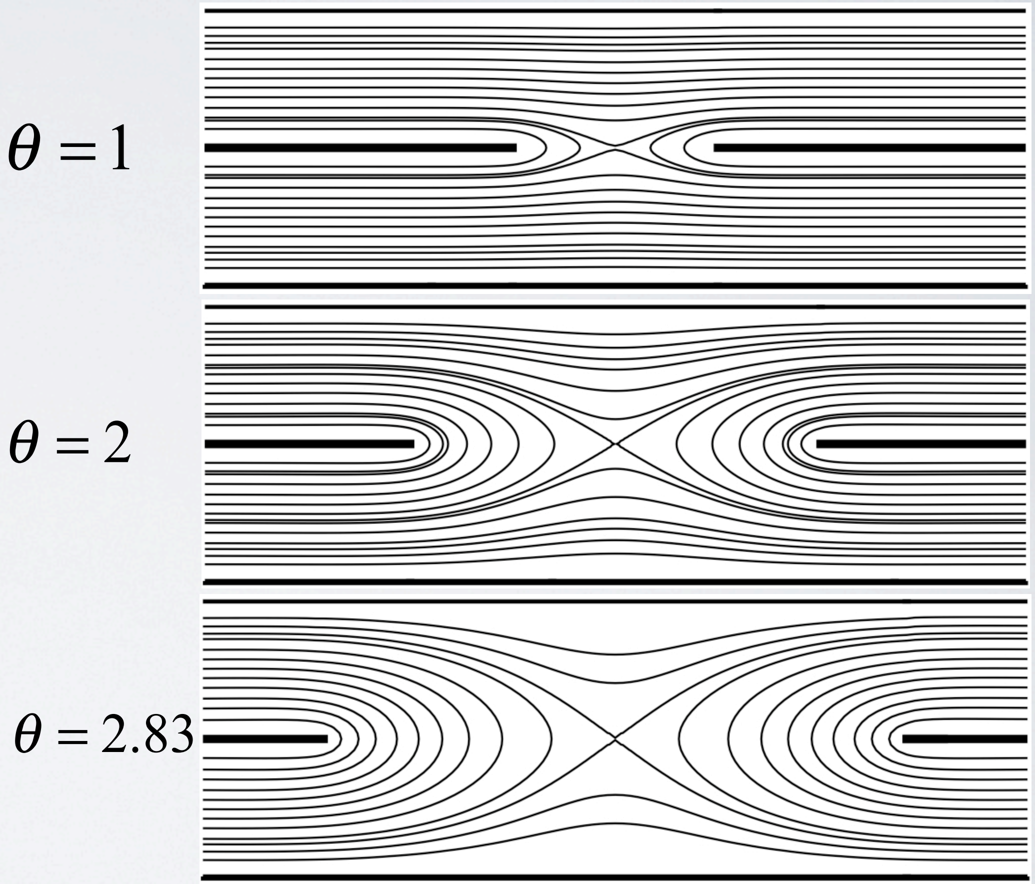


Weak effect of thickness (thin plates)

Reversed flow increases with α (gap size)

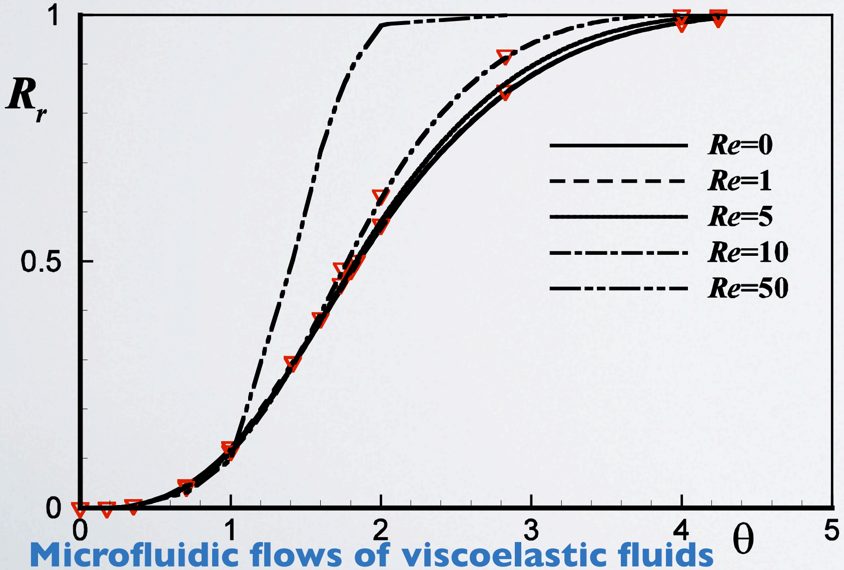
Agrees with experiments

Streamlines



Symmetry of streamlines

Inertia enhances flow reversal (Re>1)



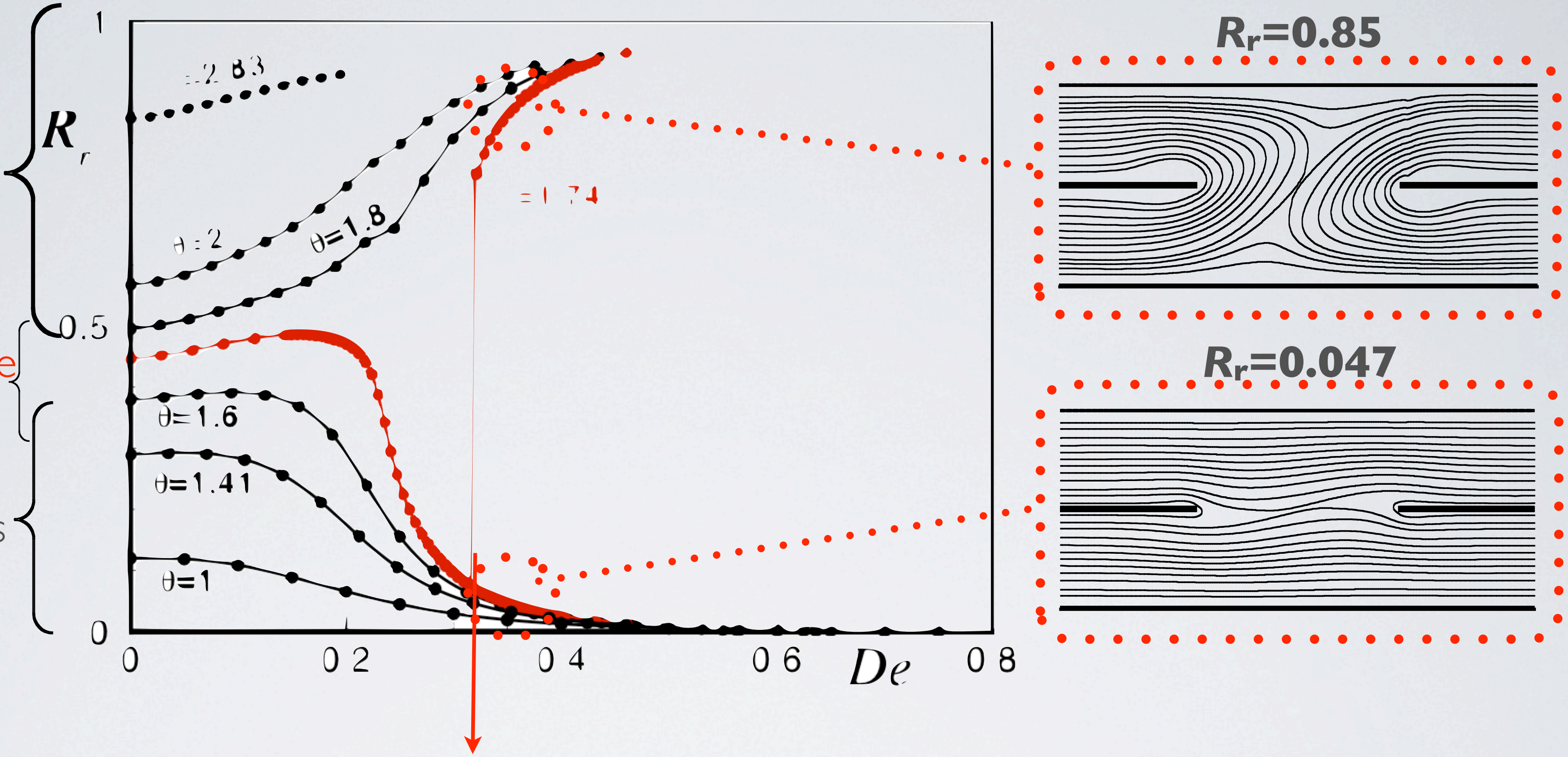
MIXING-SEPARATING FLOW: VISCOELASTIC CREEPING FLOW (I)

Afonso et al., J Eng. Math (2010) in press

θ>2
R_r increases
with ***De***

1.7<*θ*<2
Steady bistable
bifurcation

θ<1.7
R_r decreases
with ***De***



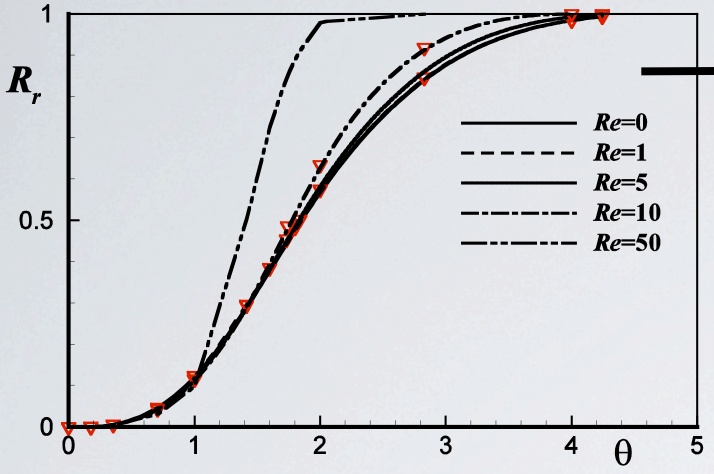
De_c varies slightly with *θ*

Purely elastic instability and supercritical behavior never seen before, probably because of inertia

MIXING-SEPARATING FLOW: VISCOELASTIC CREEPING FLOW (2)

Afonso et al., J Eng. Math (2010) in press

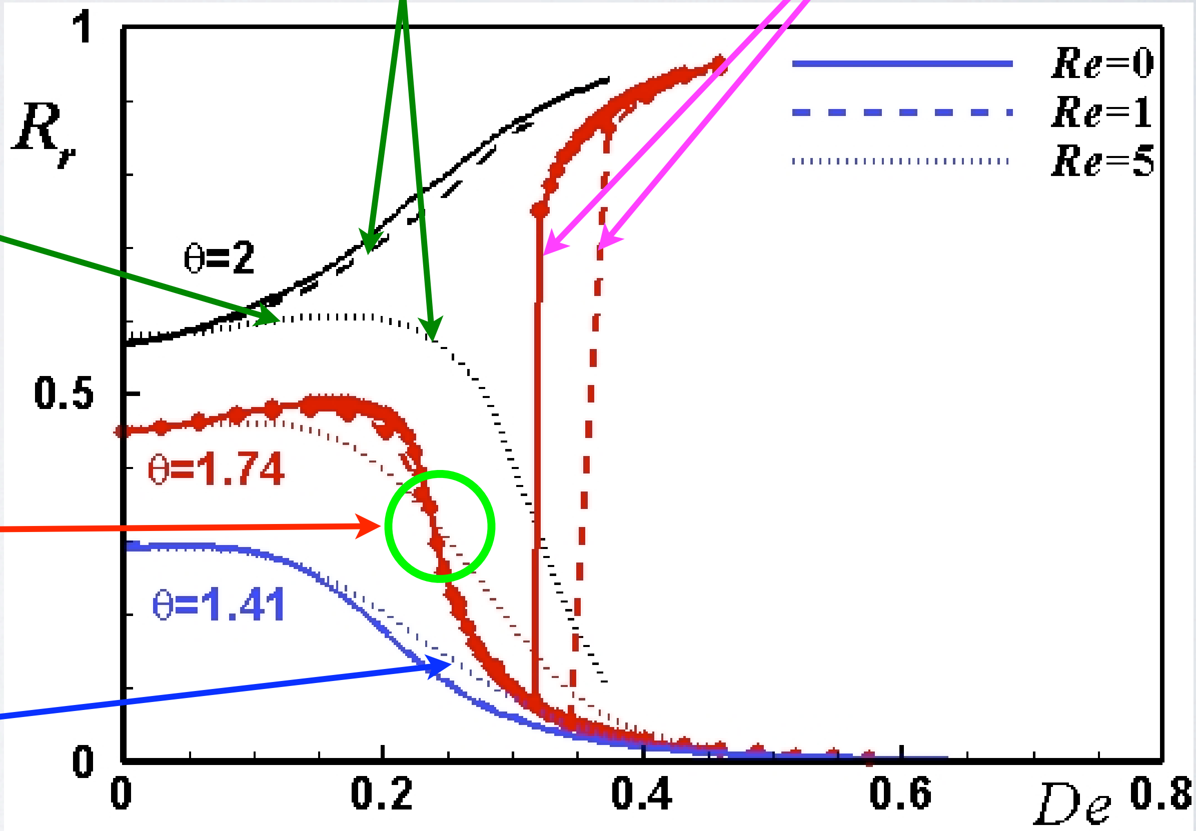
Effect of inertia



At *De*=0, inertia raised R_r

Inertia delays bifurcation to higher *De*

Dramatic effect at large θ
Trend reversed



θ>2 & large De
Inertia reduces R_r

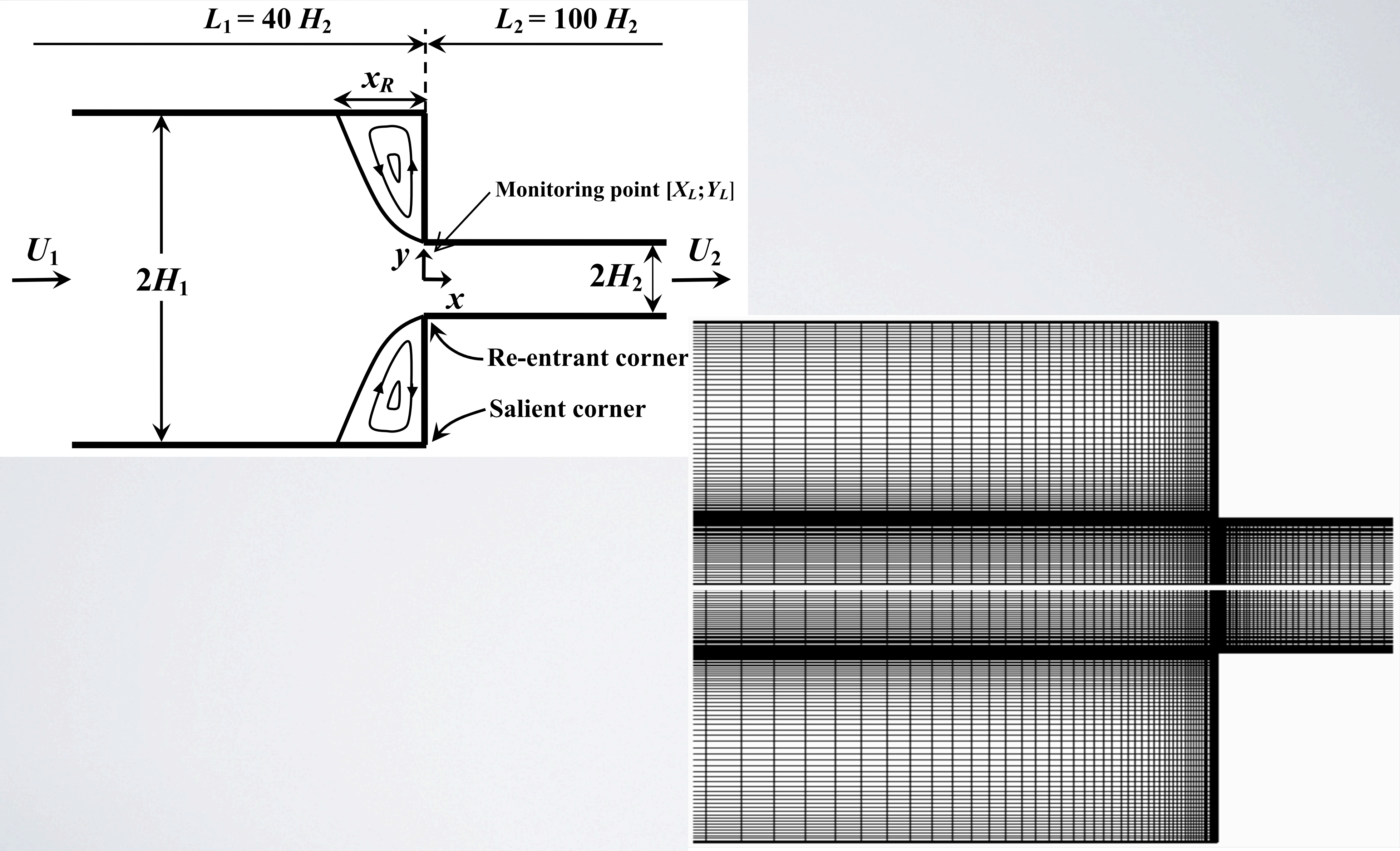
1.7<θ<2
Mixed behavior

θ<1.7
Inertia raises R_r

PLANAR SUDDEN CONTRACTION
Very high Deborah number flows

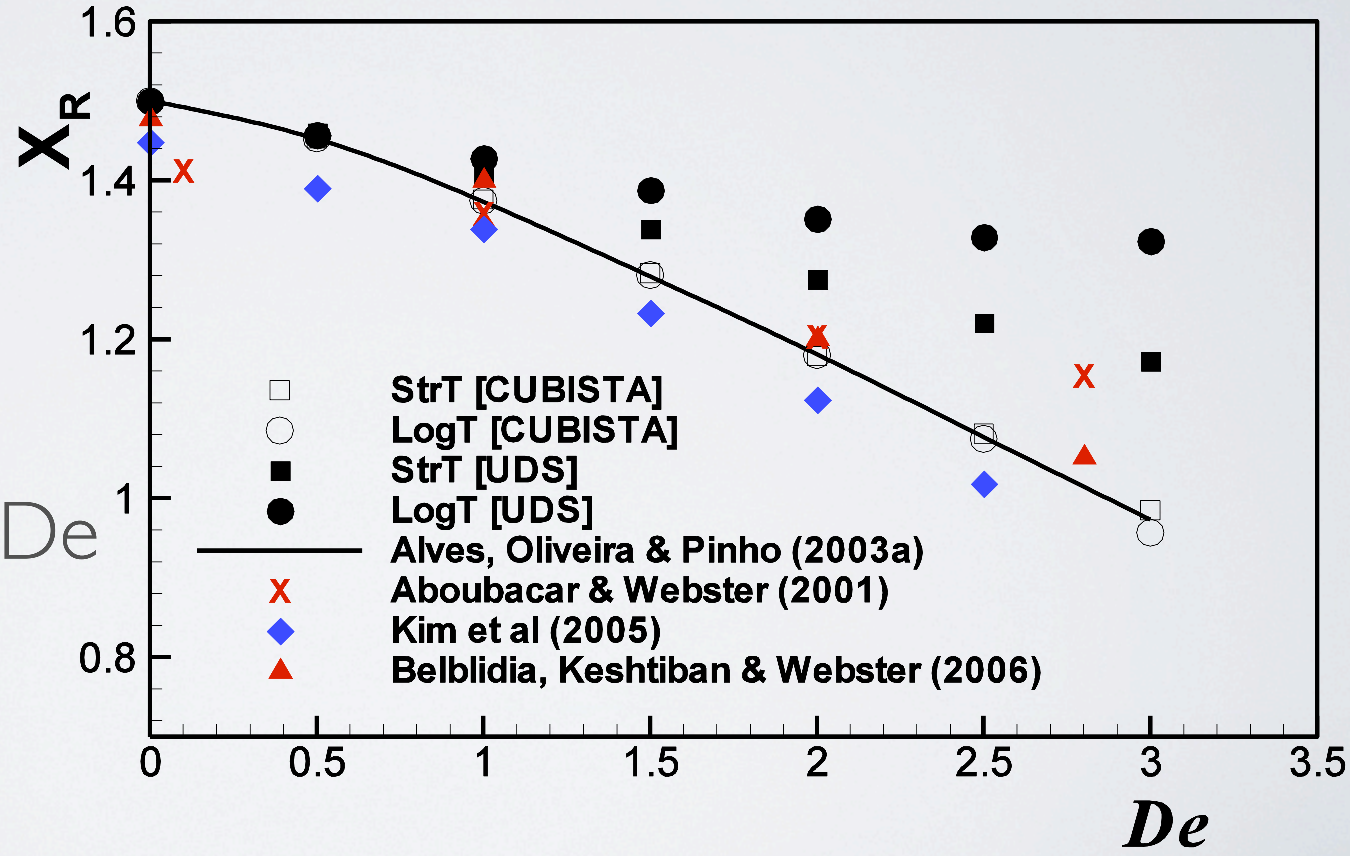
2D 4:1 SUDDEN CONTRACTION: GEOMETRY

Benchmark flow case (25 years ago)



2D 4:1 SUDDEN CONTRACTION: LOW DEBORAH NUMBER

	nº cells	DoF	$\triangle x_{min}/H_2$ & $\triangle y_{min}/H_2$
$M1$	5282	31692	0.02
$M2$	10587	63522	0.014
$M3$	42348	254088	0.0071
$M3C$	84696	508176	0.0071

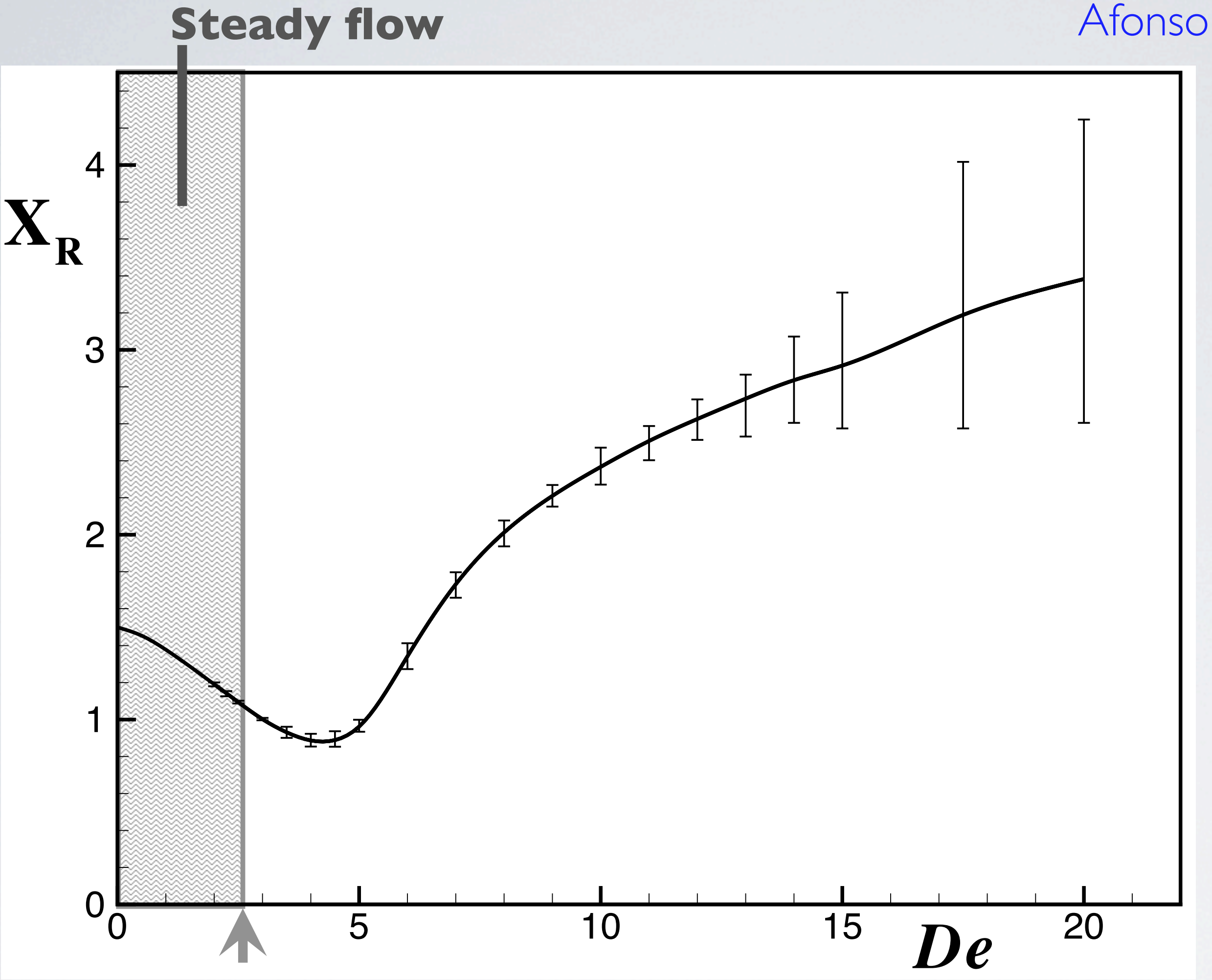


Two formulations:

- StrT: low De
- LogT: low & high De

2D 4:1 SUDDEN CONTRACTION: WHOLE RANGE

Afonso et al., J Fluid Mech (2010) submit.



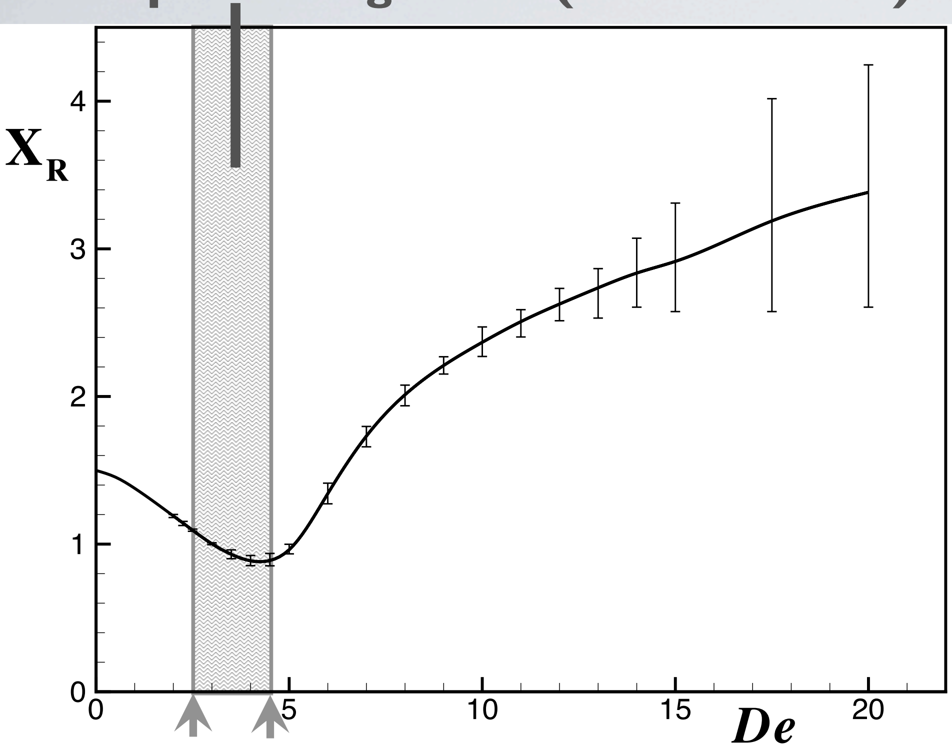
De> 2.5: unsteady flow (StrT diverges at *De*=3)

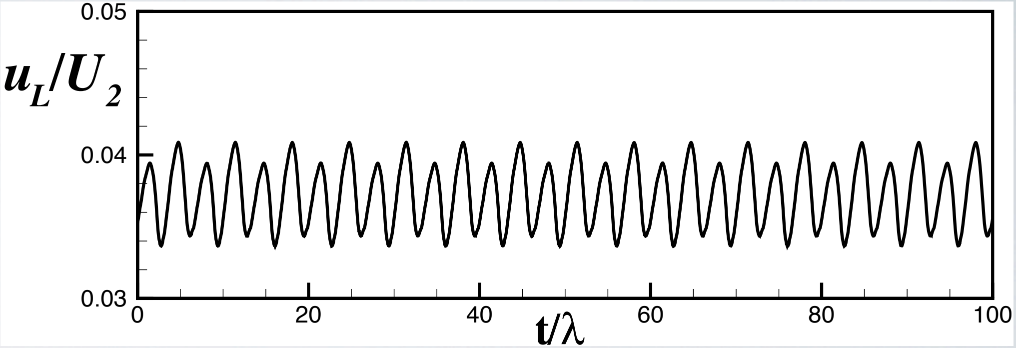
Very small Δt: $\Delta t = 10^{-5} \rightarrow \frac{n^\circ\ time\ steps}{\lambda} = [5000; 25000]$

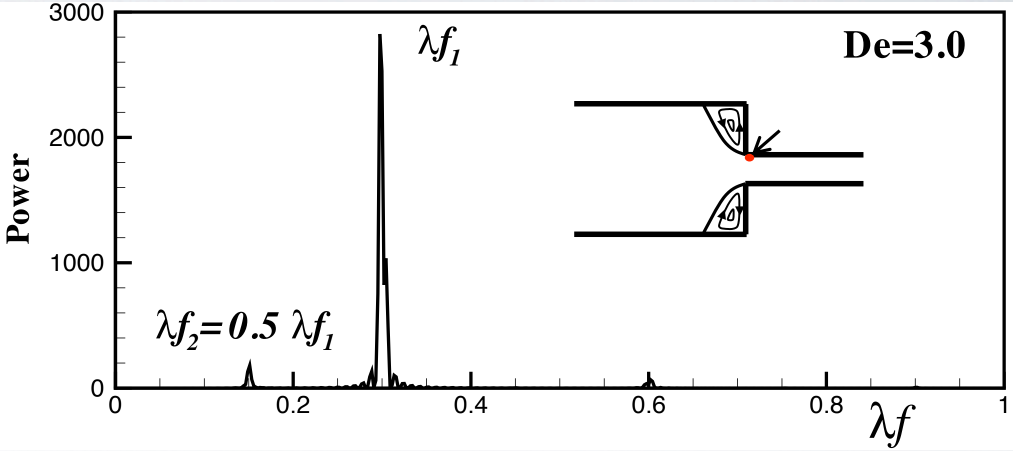
2D 4:1 SUDDEN CONTRACTION: LIP VORTEX GROWTH REGIME

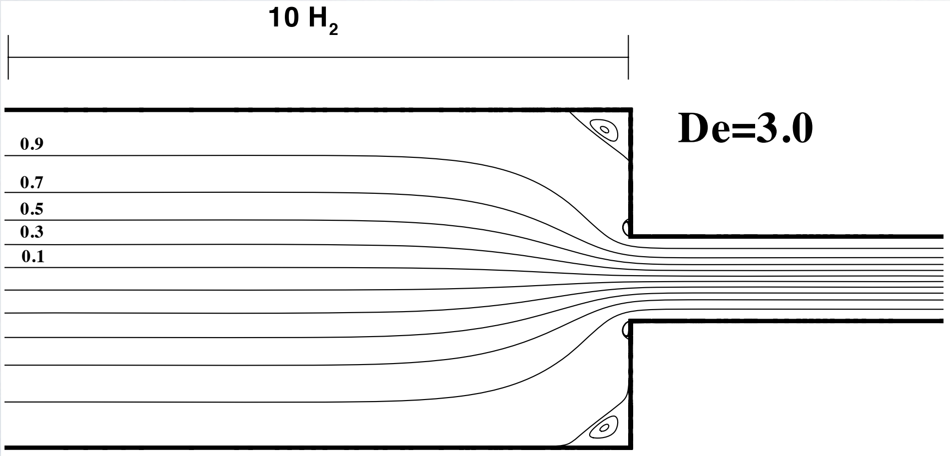
Lip vortex growth (2.5 < *De* < 4.5)

Afonso et al., J Fluid Mech (2010) submit.





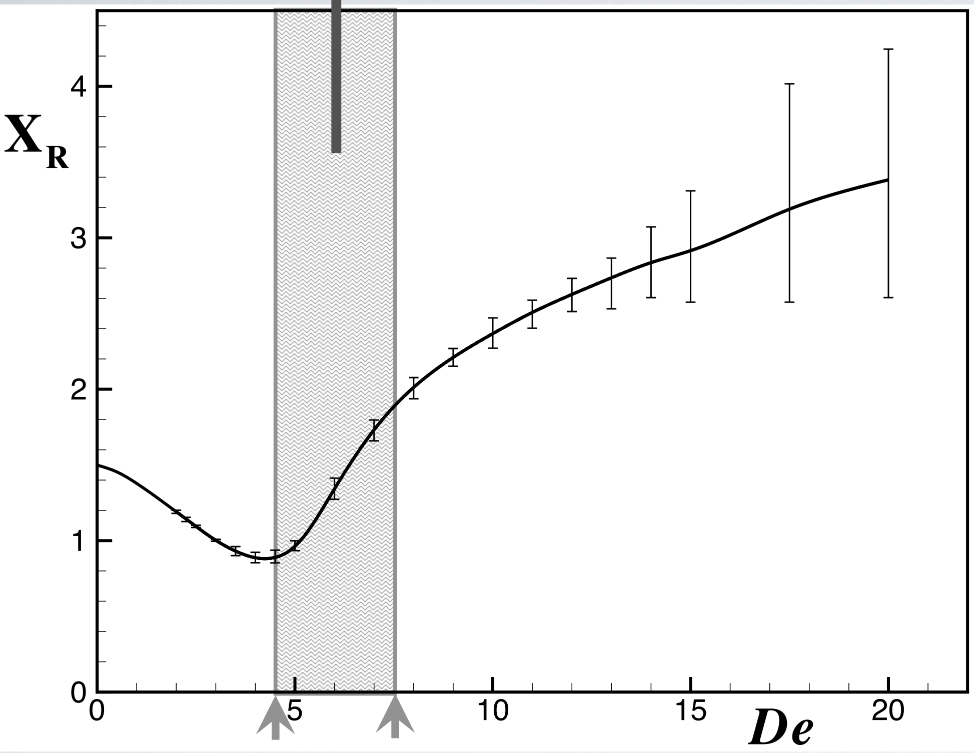


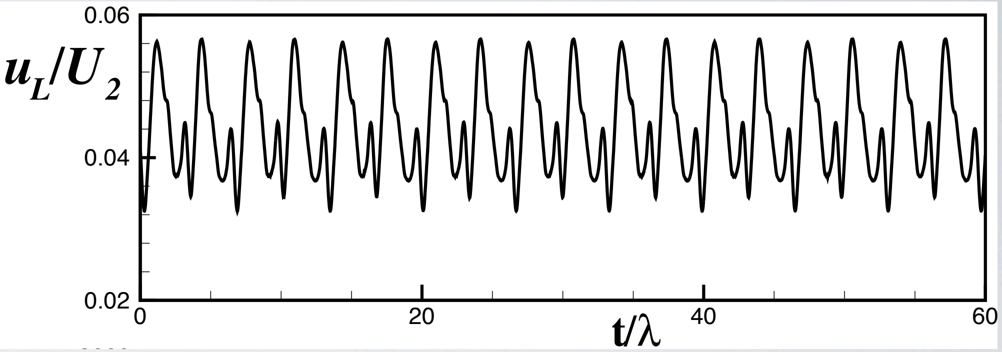


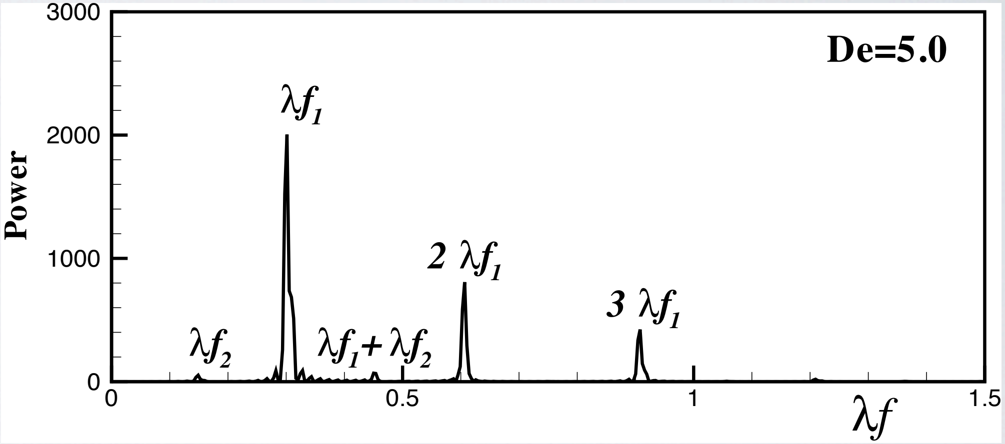
2D 4:1 SUDDEN CONTRACTION: MERGING GROWTH REGIME

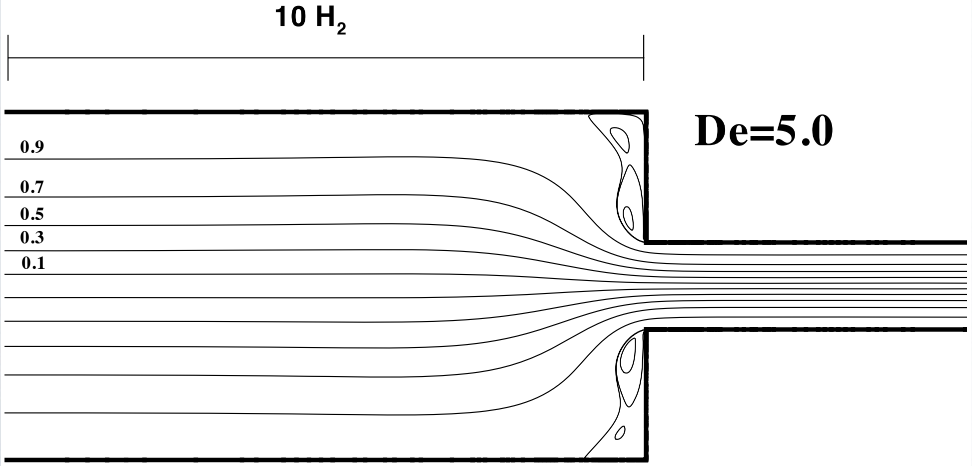
Merging growth (4.5 < *De* < 8)

Afonso et al., J Fluid Mech (2010) submit.









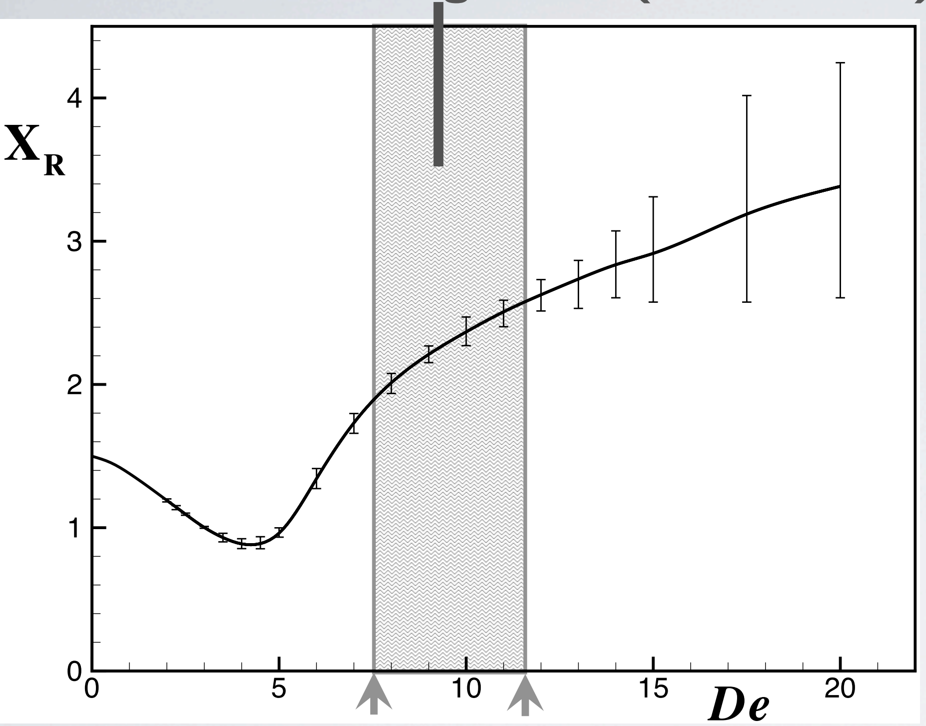
Microfluidic flows of viscoelastic fluids
V BCR 2010

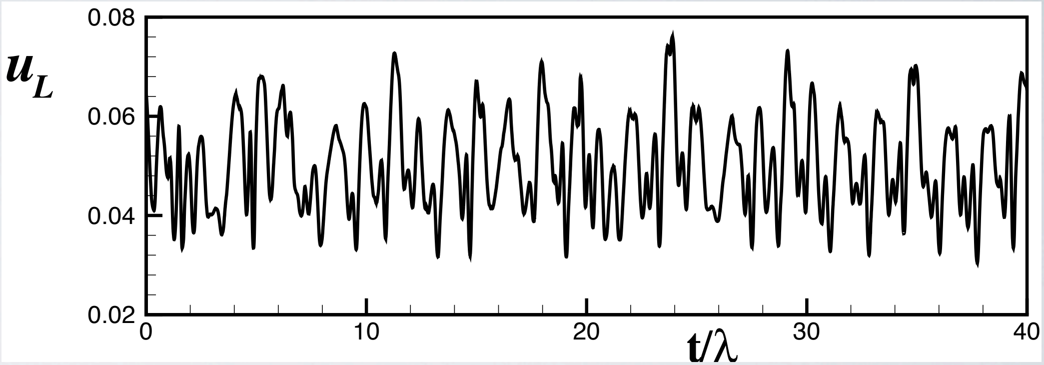
Sousa, Afonso, Oliveira, Alves & Pinho - CEFT/FEUP
Rio de Janeiro, Brazil, 14-16th July 2010

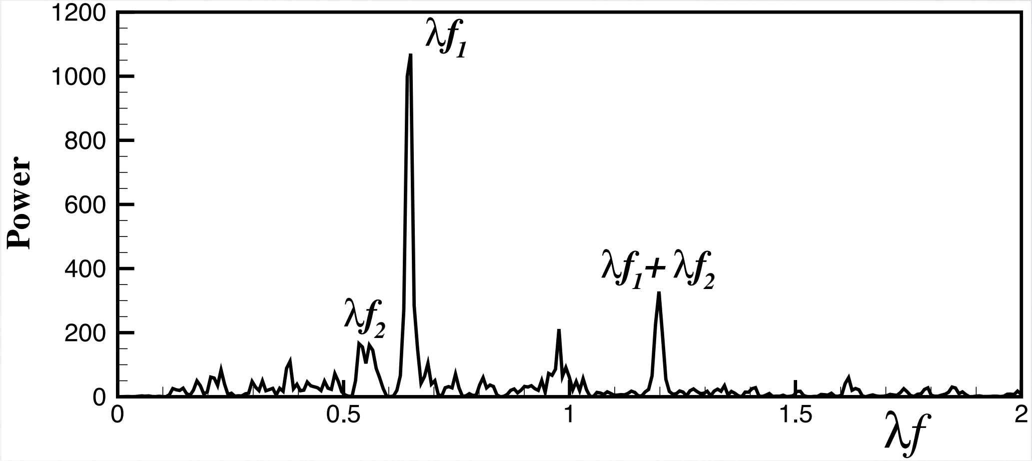
2D 4:1 SUDDEN CONTRACTION: ELASTIC VORTEX GROWTH REGIME

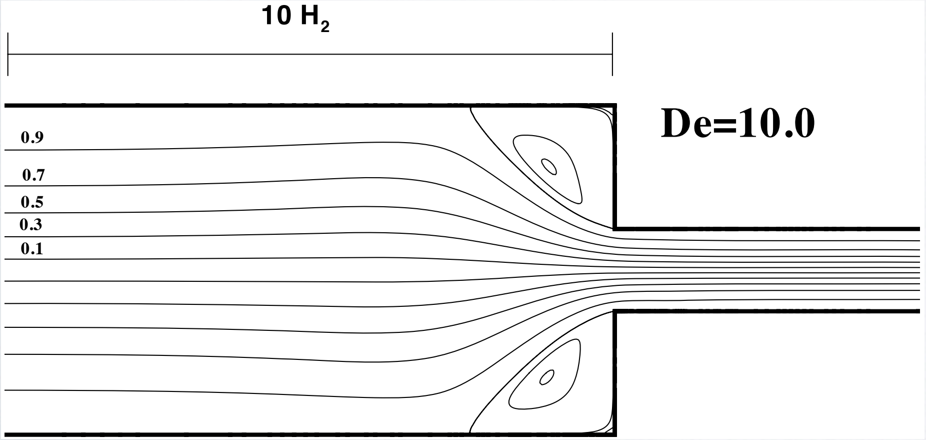
Elastic vortex growth (8 < *De* < 12)

Afonso et al., J Fluid Mech (2010) submit.





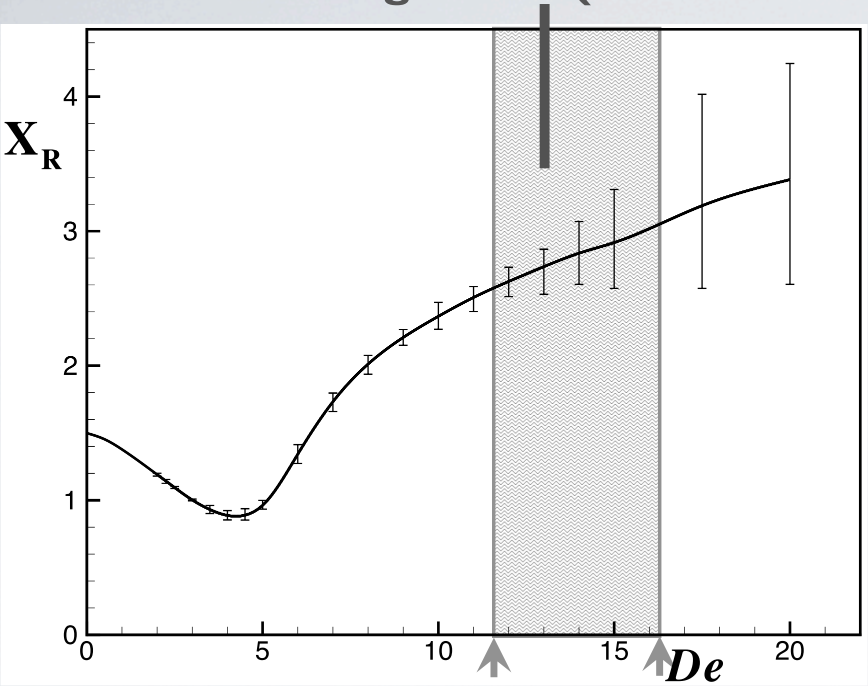


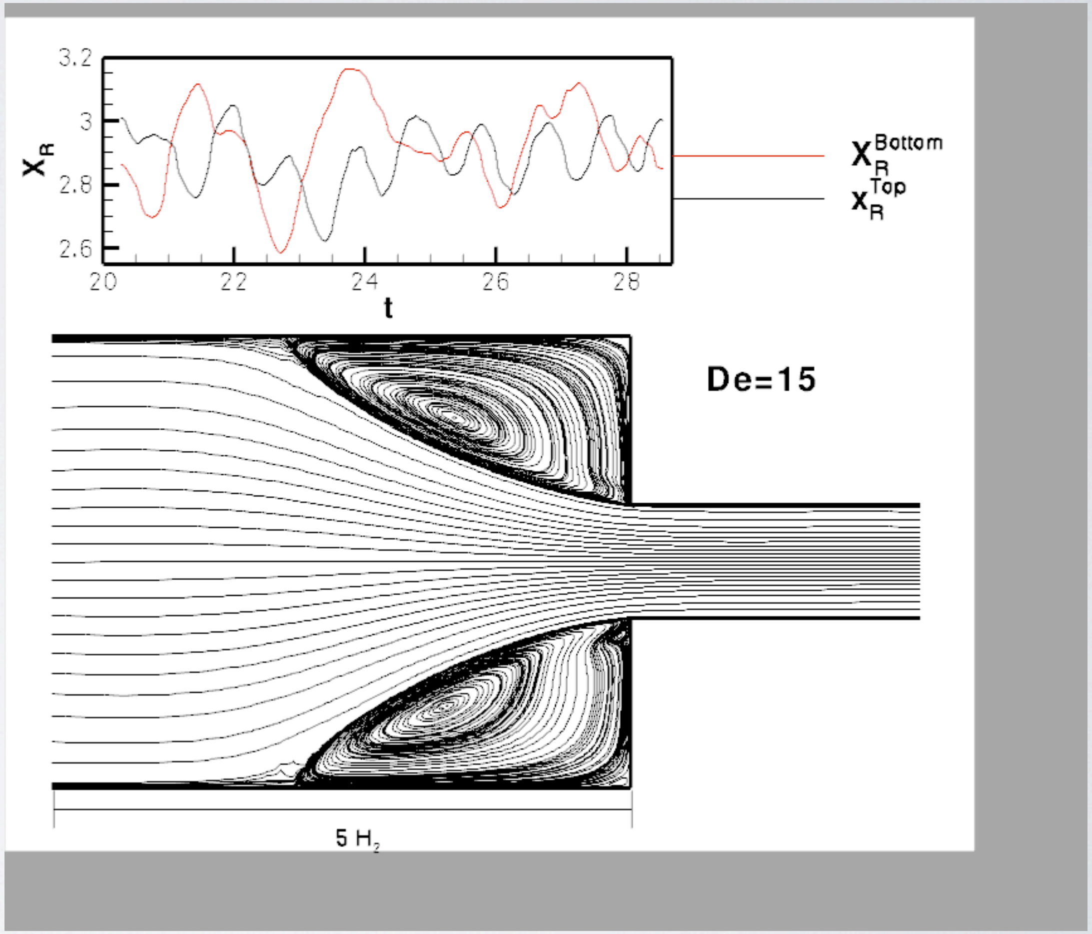


2D 4:1 SUDDEN CONTRACTION: THIRD VORTEX GROWTH REGIME

Third vortex growth (12 < *De* < 17.5)

Afonso et al., J Fluid Mech (2010) submit.

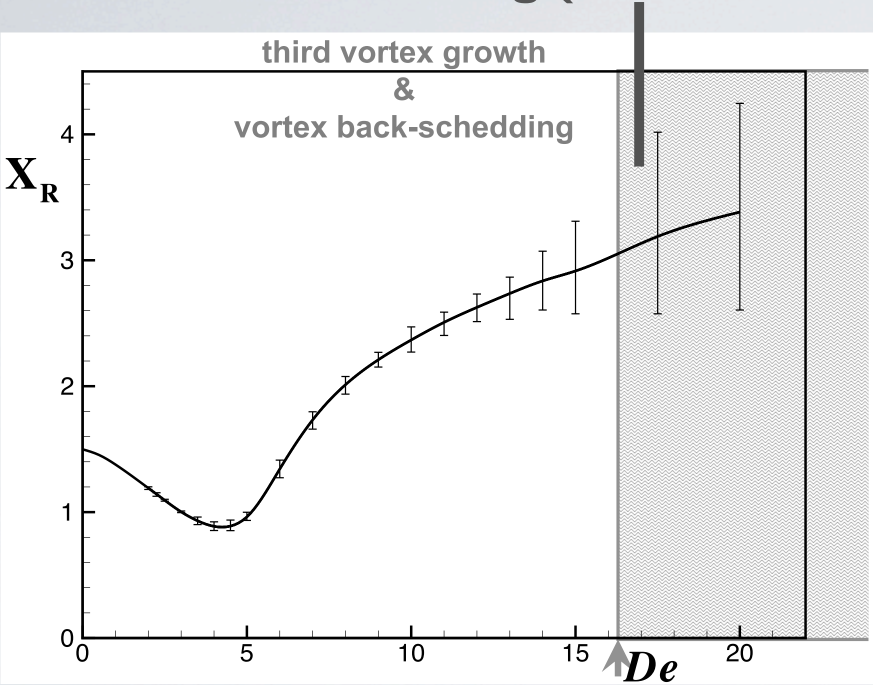


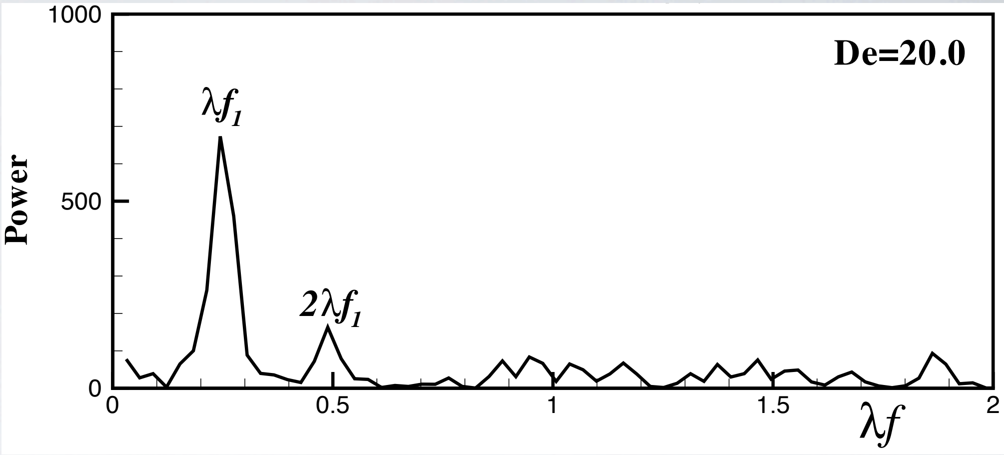


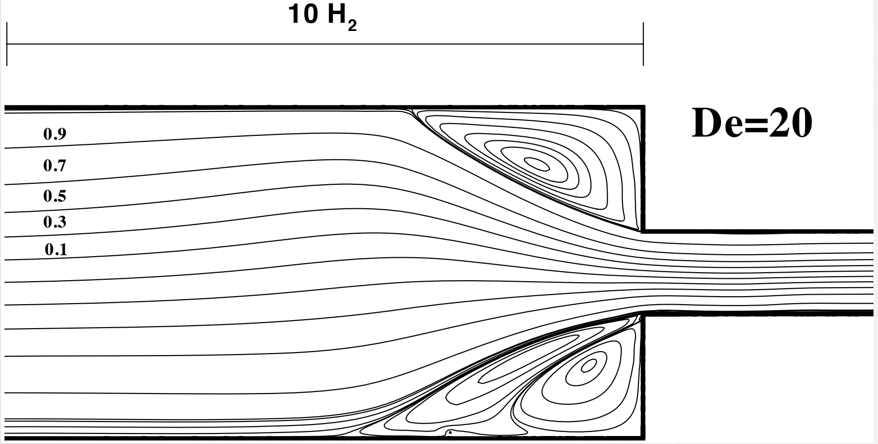
2D 4:1 SUDDEN CONTRACTION: VORTEX BACK SHEDDING

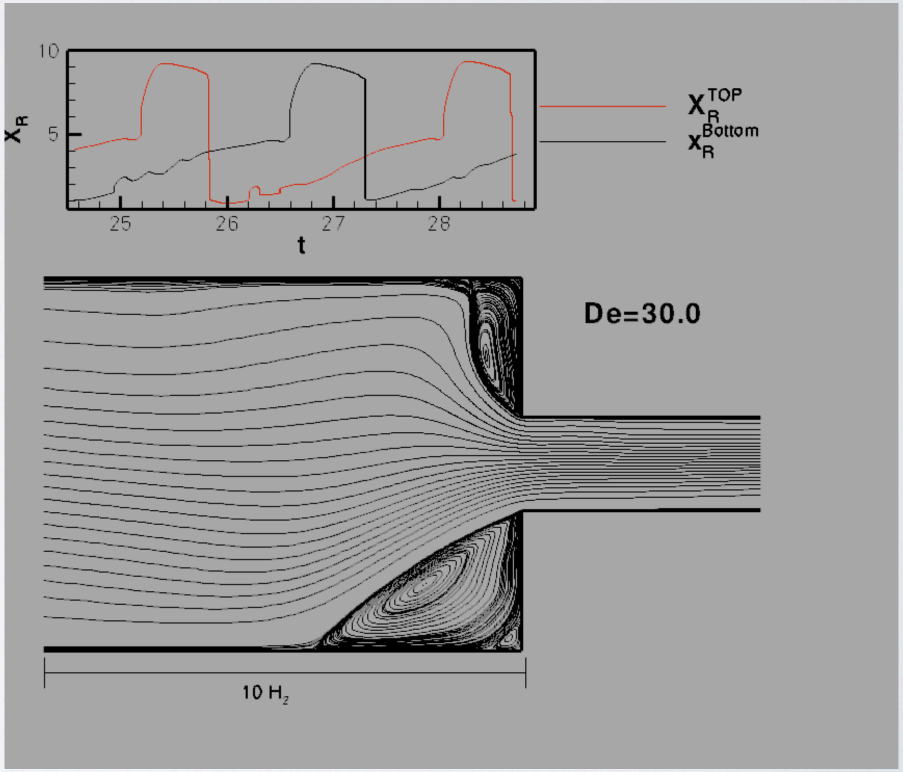
Vortex back-shedding (17.5 < *De* < 100 ?!?)

Afonso et al., J Fluid Mech (2010) submit.



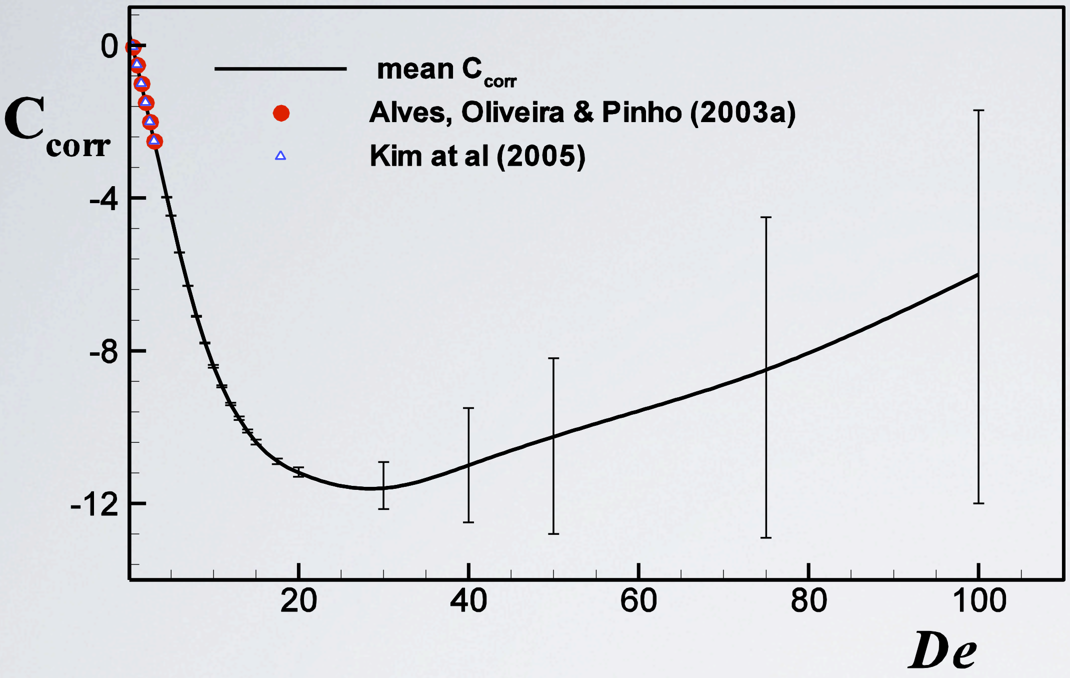






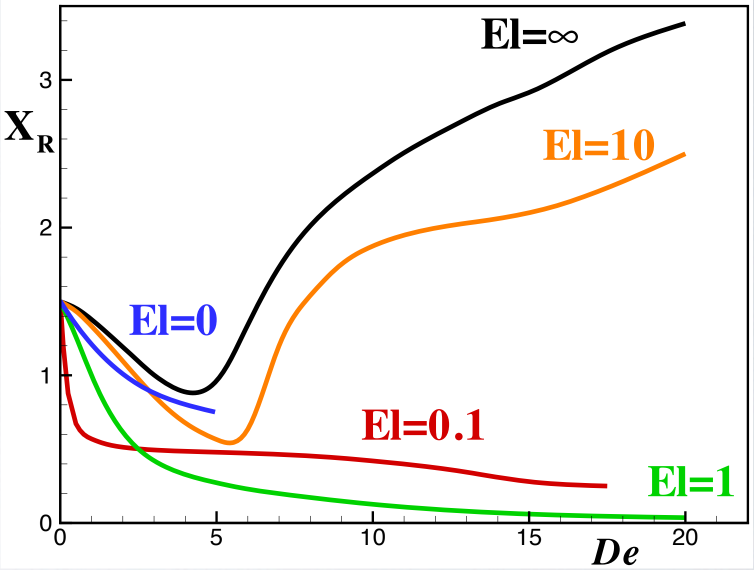
2D 4:1 SUDDEN CONTRACTION: OTHER RESULTS

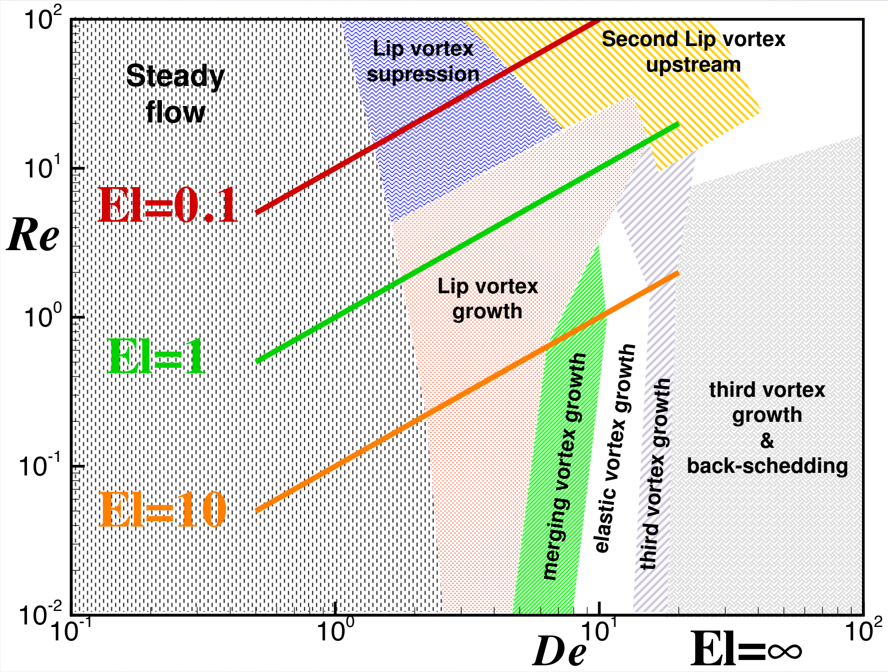
Couette correction



$$El = \frac{De}{\mathrm{Re}} = \frac{\lambda \nu}{L^2}$$

Effect of inertia





CLOSURE

- Microfluidics: low *Re* & large *De* (contrasts with macro fluid dynamics)
- Need to micro-fabrication of high quality: requires clean environments
- Elastic instabilities observed & calculated at $Re \approx 0$ ⟶ **improved mixing**
- Distinct transitions: steady symmetric to steady asymmetric; steady asymmetric to unsteady flow; steady symmetric to unsteady
- Log-conformation allows numerical calculations at very high *De*/*Wi* flows
- Rich transitions in plane sudden contraction: path to elastic turbulence?

- Other challenges: complex fluids with electrokinetic effects, surface tension gradients, surface patterning

ACKNOWLEDGEMENTS

- Centro de Estudos de Fenómenos de Transporte
- Departamento de Engenharia Química at FEUP
- Fundação para a Ciência e a Tecnologia & Feder: PTDC/EQU-FTT/71800/2006, PTDC/EQU-FTT/70727/2006, PTDC/EME-MFE/70186/2006, PPCDT & POCI/EME/59338/2004, REEQ/262/EME/2005, REEQ/928/EME/2005
- Hatsopoulos Microfluidics Laboratory at MIT: Gareth McKinley, Chris Pipe, Soulages
- Dr. Rob Poole at University of Liverpool
- Prof. Paulo Oliveira at Universidade da Beira Interior