Lorentz center

Flow Instabilities & Turbulence in Viscoelastic Fluids

Workshop: July 19 – 23 2010, Leiden, The Netherlands

Inertial instabilities in Newtonian cross-slot flow - A comparison against the viscoelastic bifurcation

Gerardo N. Rocha

Departamento de Engª Electromecânica, Universidade da Beira Interior, Portugal

Paulo J. Oliveira

Departamento de Engª Electromecânica, Universidade da Beira Interior, Portugal

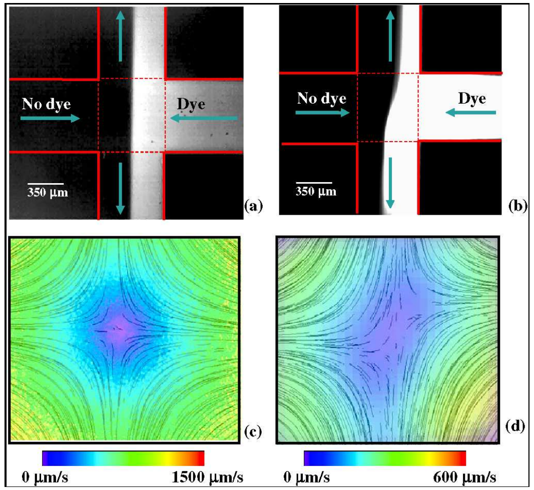
G.N. Rocha & P.J. Oliveira

Viscoelastic Flow

(inertia is neglected, $Re \approx 0$)

Arratia et al., *Phys. Rev. Lett.* **96**, 144502 (2006)

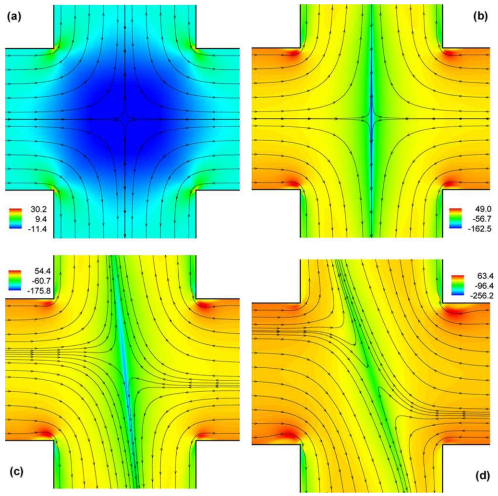
Results: Experimental



Newtonian $Re < 10^{-2}$ PAA solution $De = 4.5$; $Re < 10^{-2}$

Poole et al., *Phys. Rev. Lett.* **99**, 164503 (2007)

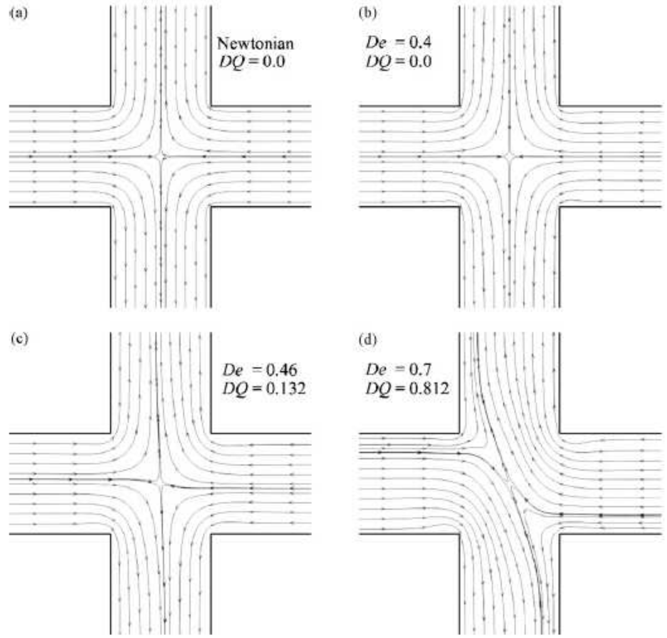
Numerical: UCM model



Contours of N_1 (τ_{xx}-τ_{yy}) with streamlines superimposed for: (a) Newtonian fluid, (b) $De = 0.3$, (c) $De = 0.32$ e (d) $De = 0.4$

Rocha et al., *J. Non-Newt. Fluid Mech.* **156**, 58-69 (2009)

Numerical: FENE-CR & FENE-P



Streamlines for: (a) newtonian fluid, (b) $De = 0.40$, (c) $De = 0.46$ e (d) $De = 0.70$ (FENE-CR, $L^2 = 100$ and $\beta = 0.1$)

DEPARTAMENTO DE ELECTROMECÂNICA
G.N. Rocha & P.J. Oliveira

Objective

- Investigate inertial effects in 2D Newtonian flow through a cross-slot

- Will a supercritical pitchfork bifurcation occur?

- Contrast the resulting instability with that for viscoelastic flow at Re=0

DEPARTAMENTO DE ELECTROMECÂNICA
G.N. Rocha & P.J. Oliveira

Equations

Assume 2D laminar, steady and incompressible flow

(1) Mass conservation:

$$\frac{\partial u_i}{\partial x_i} = 0$$

(2) Momentum conservation:

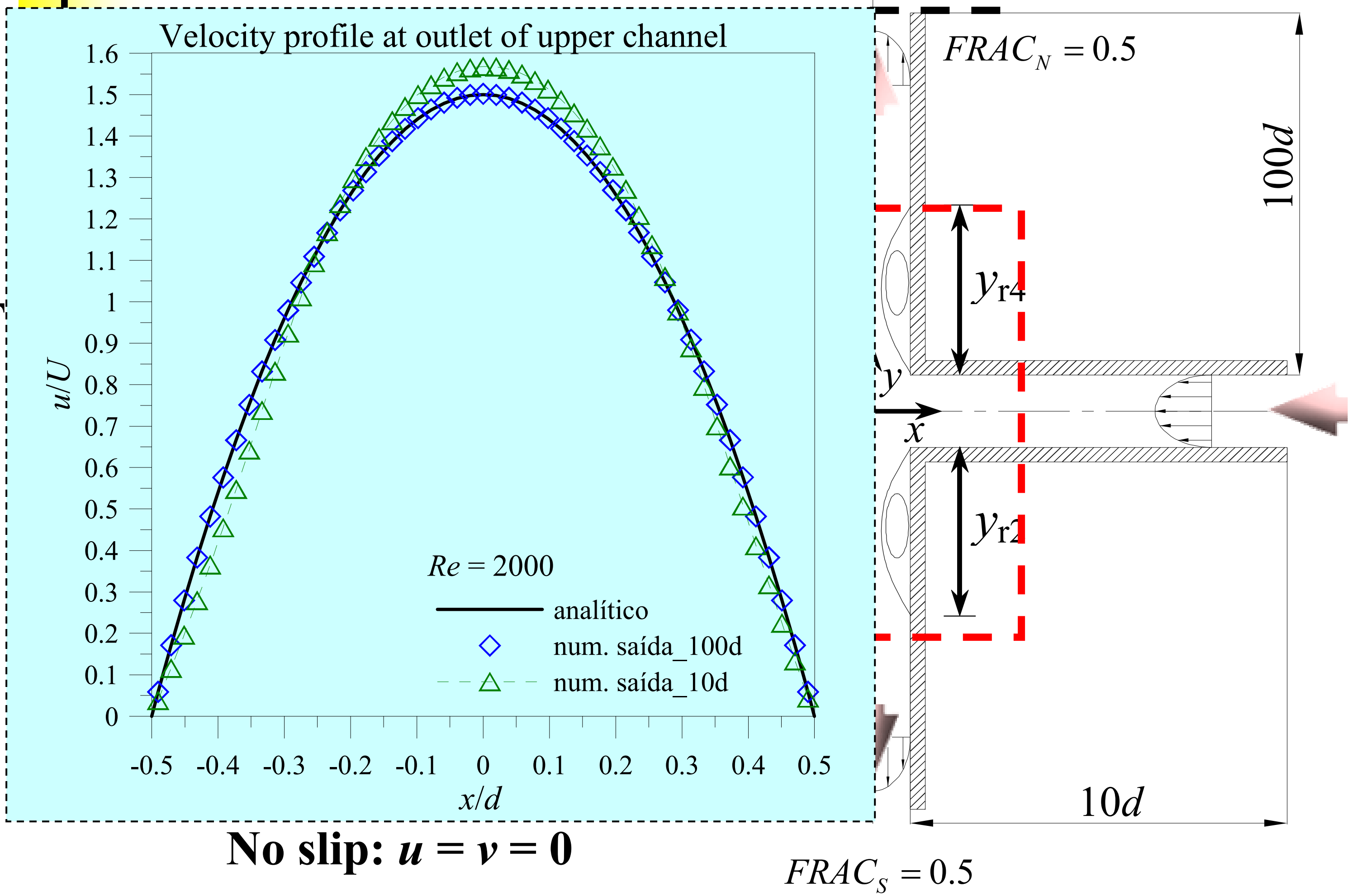
$$\rho\left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$$

(3) Constitutive Eq. (Newton law of viscosity):

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

G.N. Rocha & P.J. Oliveira

Geometry and boundary conditions



Outlet conditions

⬇

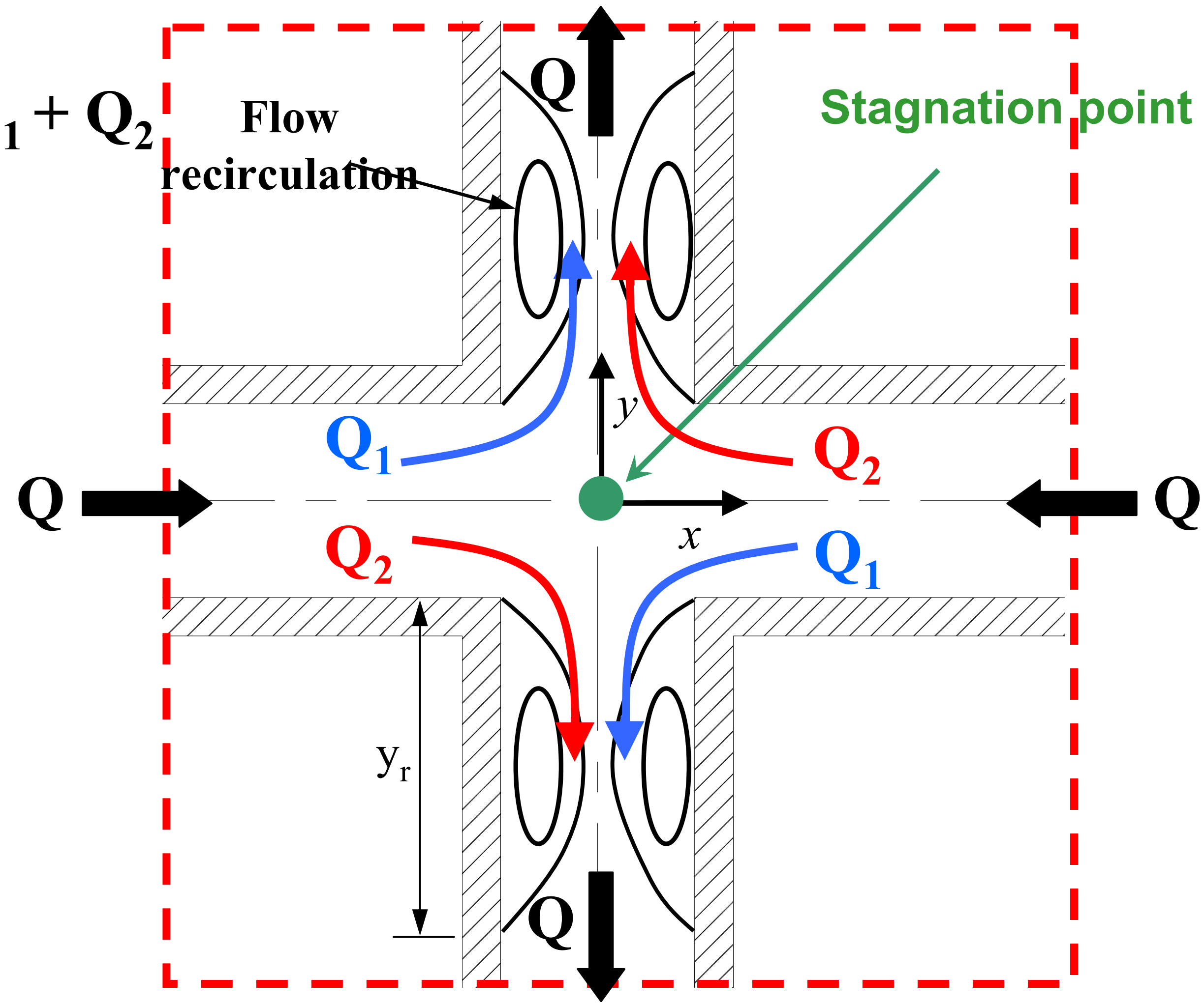
Neumann conditions

No slip: $u = v = 0$

G.N. Rocha & P.J. Oliveira

Geometry and boundary conditions

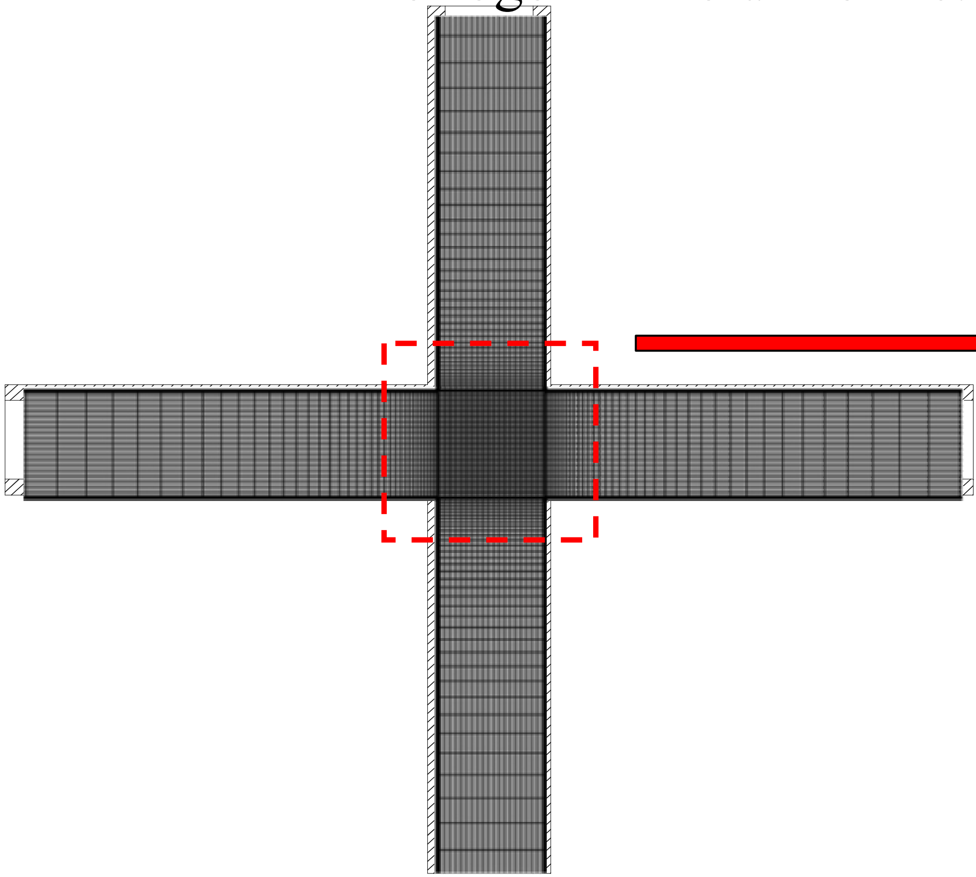
$$Q = Ud = Q_1 + Q_2$$

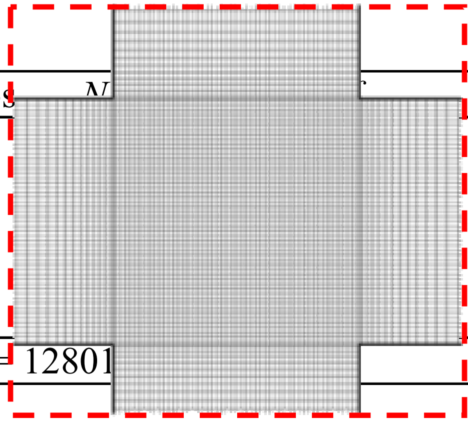


G.N. Rocha & P.J. Oliveira

Numerical method

- **Finite volume method** (Oliveira *et al.*, JNNFM (1998)):
  - Equations discretized over a mesh;
  - Malha ortogonal não uniforme: $\Delta x_{\min} = \Delta y_{\min} \approx 0.02d$





Blocos	N…	f_y
I		1.000000
II		1.000000
III		0.929296
IV		1.075369
V		1.000000
NVC = 12801		

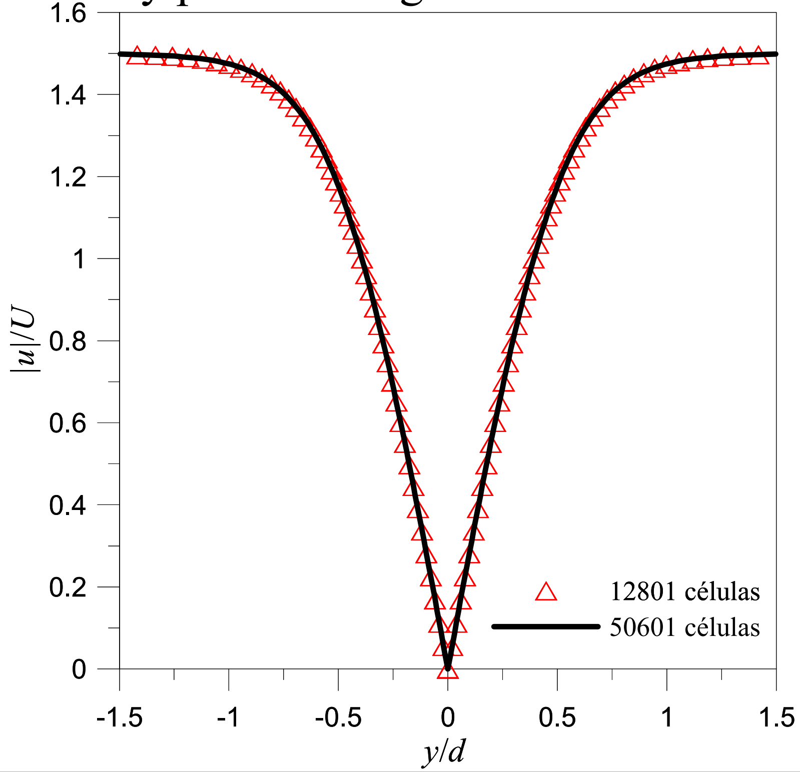
G.N. Rocha & P.J. Oliveira

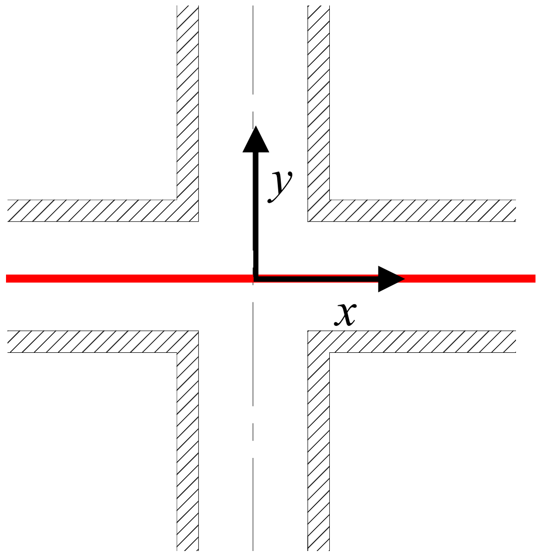
Mesh refinement

Controlling parameter:

$$Re = \frac{\rho U d}{\mu}$$

Velocity profile along central horizontal line ($y = 0$) at ***Re* = 1400**.



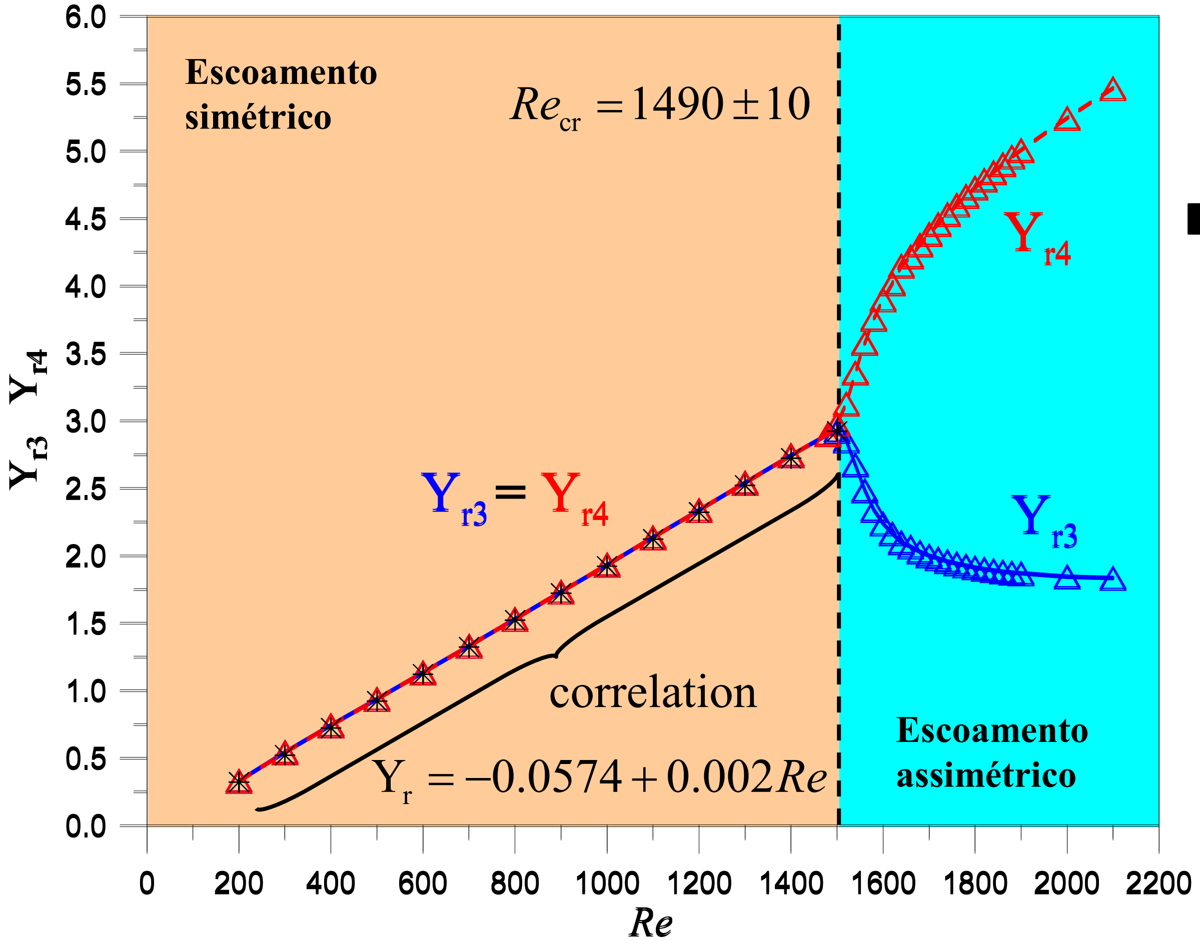


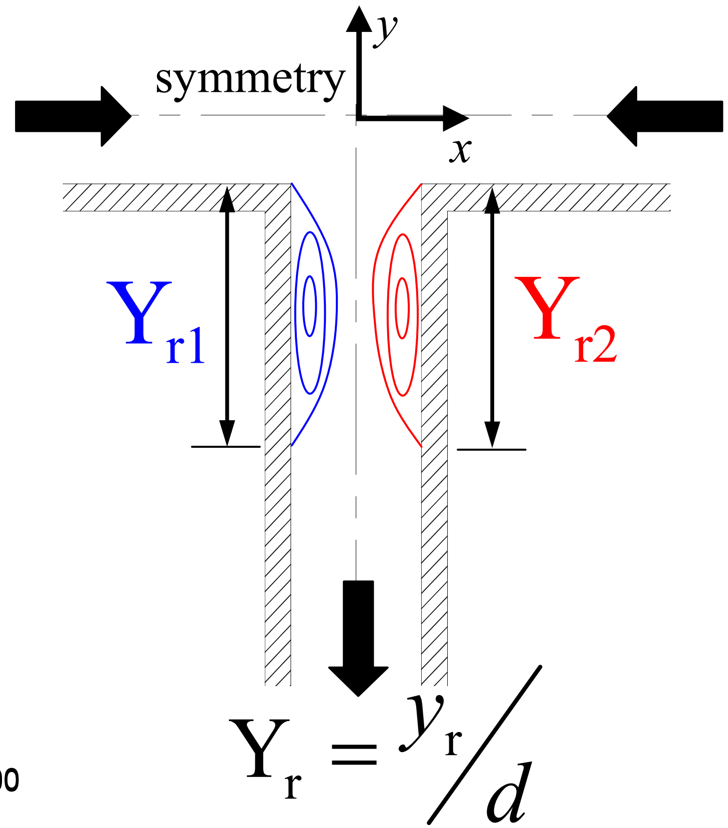
	Volumes controlo	Tamanho mínimo células
Malha 1	12801	$\Delta x_{\min} = \Delta y_{\min} \approx 0.02d$
Malha 2	50601	$\Delta x_{\min} = \Delta y_{\min} \approx 0.01d$

G.N. Rocha & P.J. Oliveira

Results

Influence of inertia (*Re*) upon the size of the two eddies on the lower outlet channel





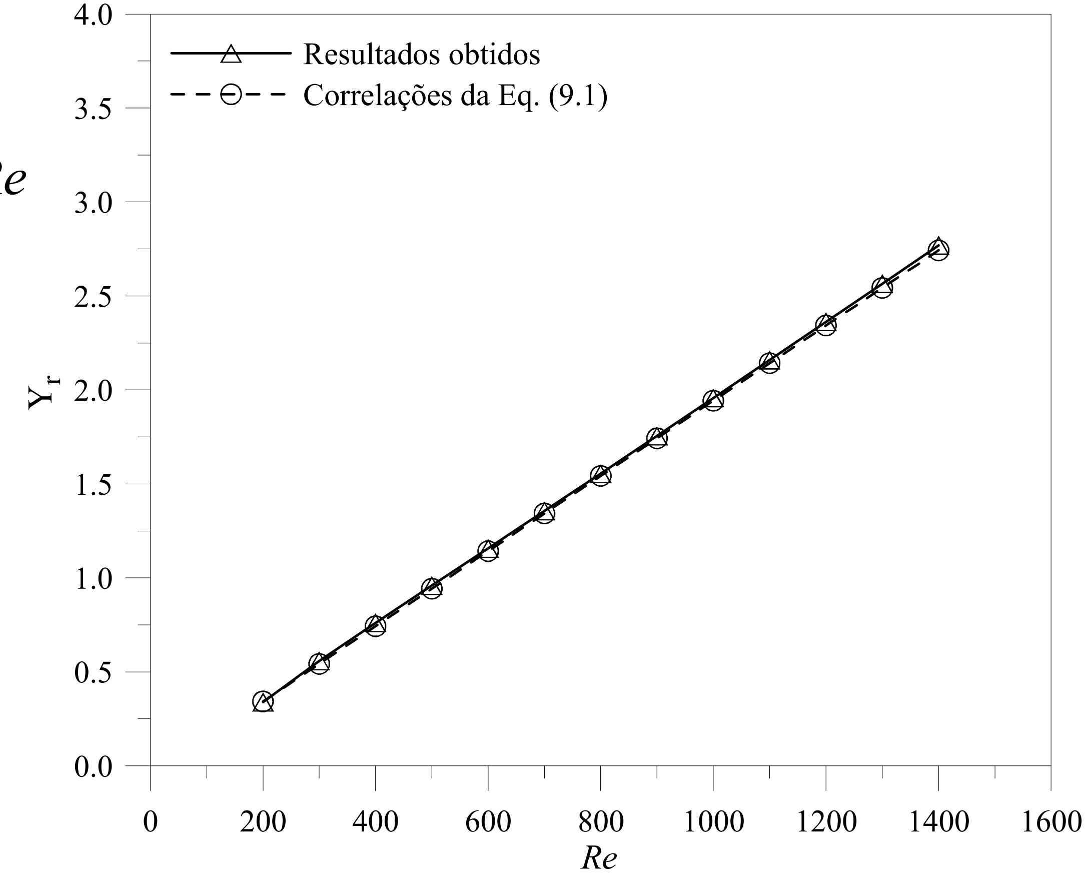
DEPARTAMENTO DE ELECTROMECÂNICA
G.N. Rocha & P.J. Oliveira

Results: correlation for eddy size

Correlation:

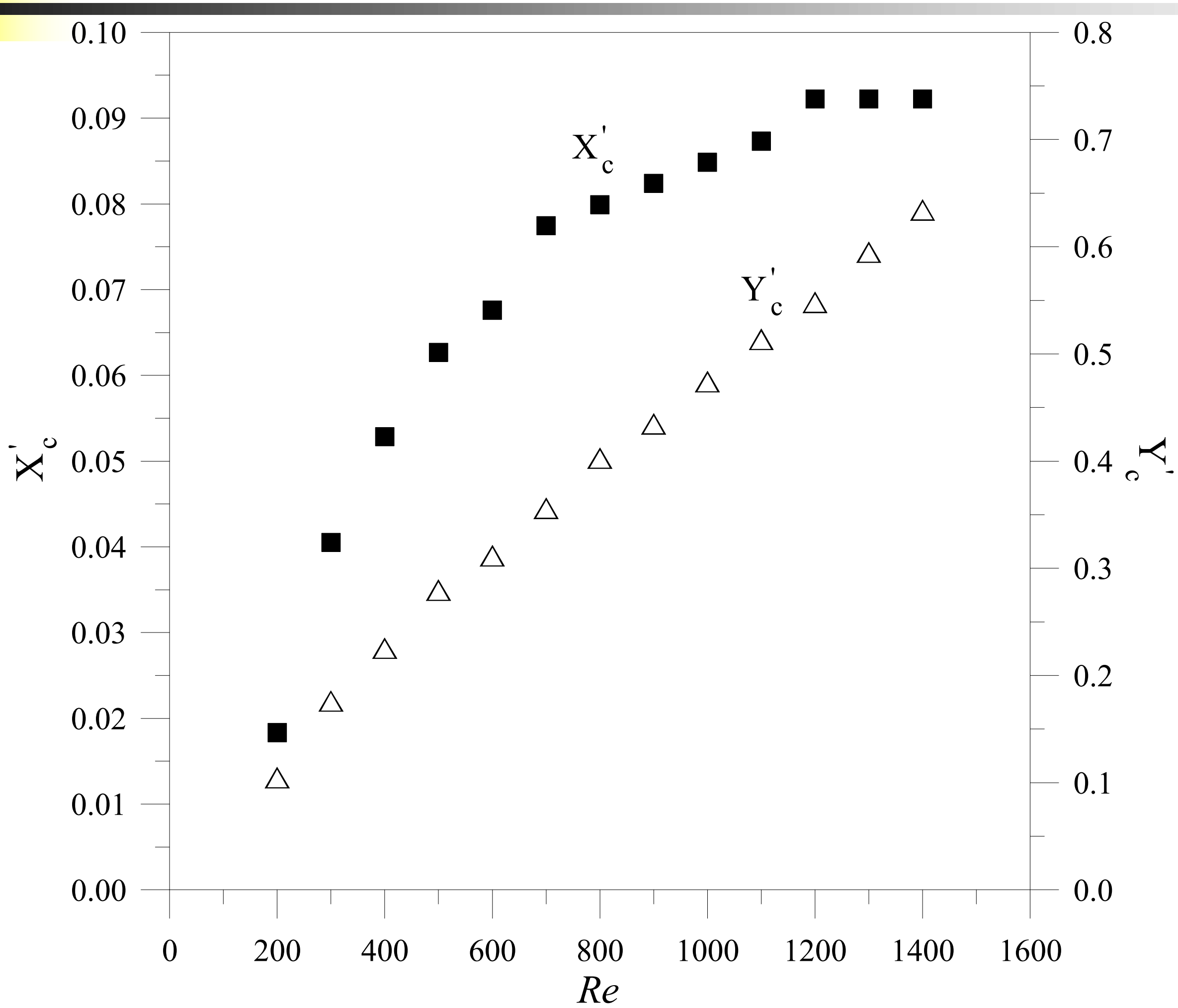
$$Y_r = -0.0574 + 0.002\,Re$$

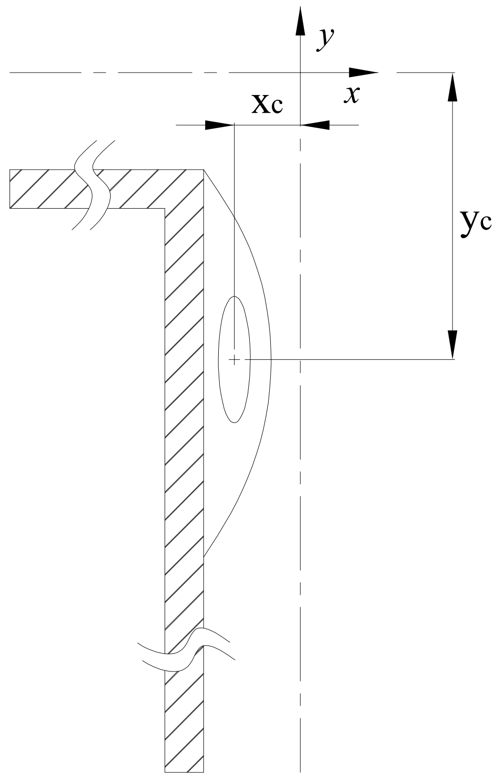
$$200 \le Re \le 1500$$



G.N. Rocha & P.J. Oliveira

Results: eddy centre position (sym. flow)





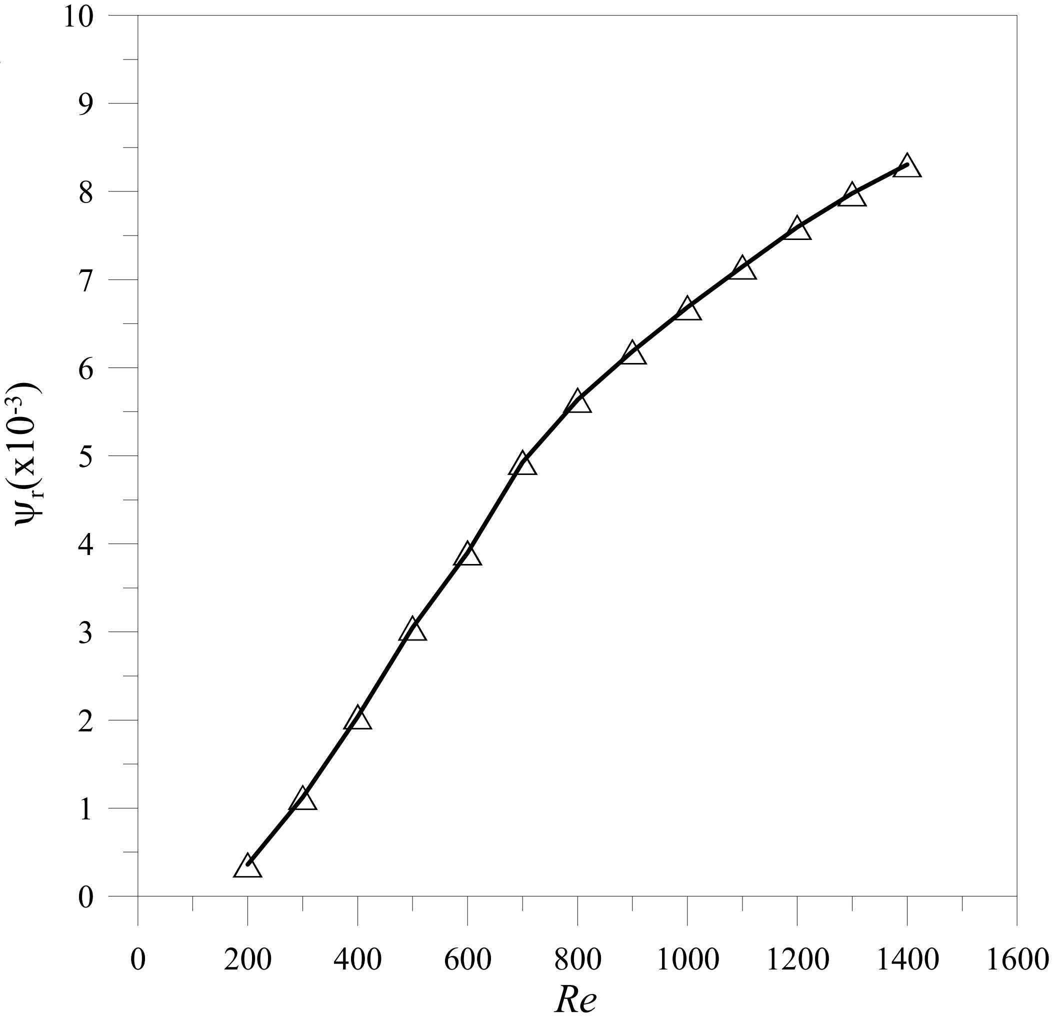
$$X'_c = X_c - (-0.5)$$

$$Y'_c = -(Y_c - (-0.5))$$

DEPARTAMENTO DE ELECTROMECÂNICA
G.N. Rocha & P.J. Oliveira

Results: eddy strength (symmetric flow)

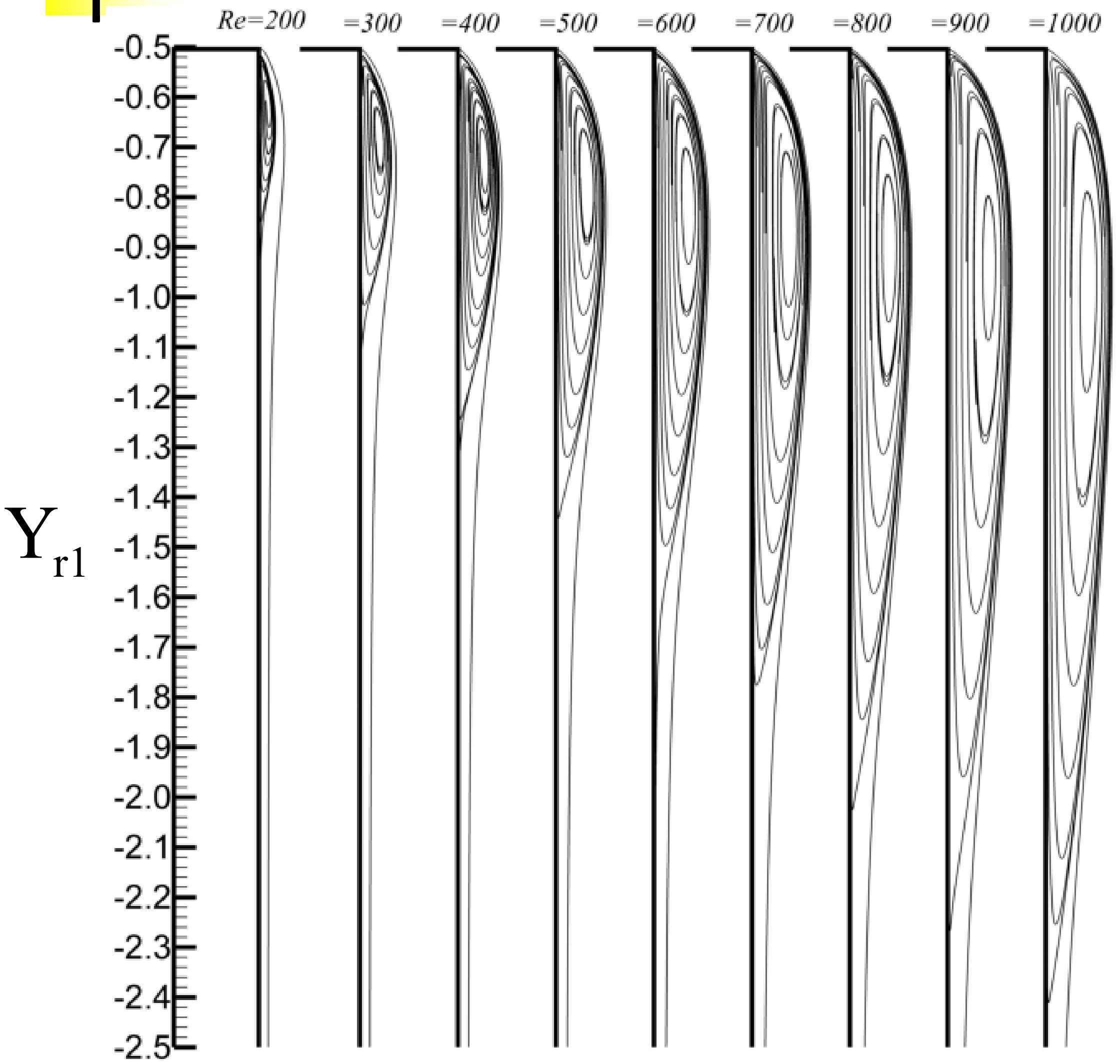
Intensity as a function of Reynolds number:



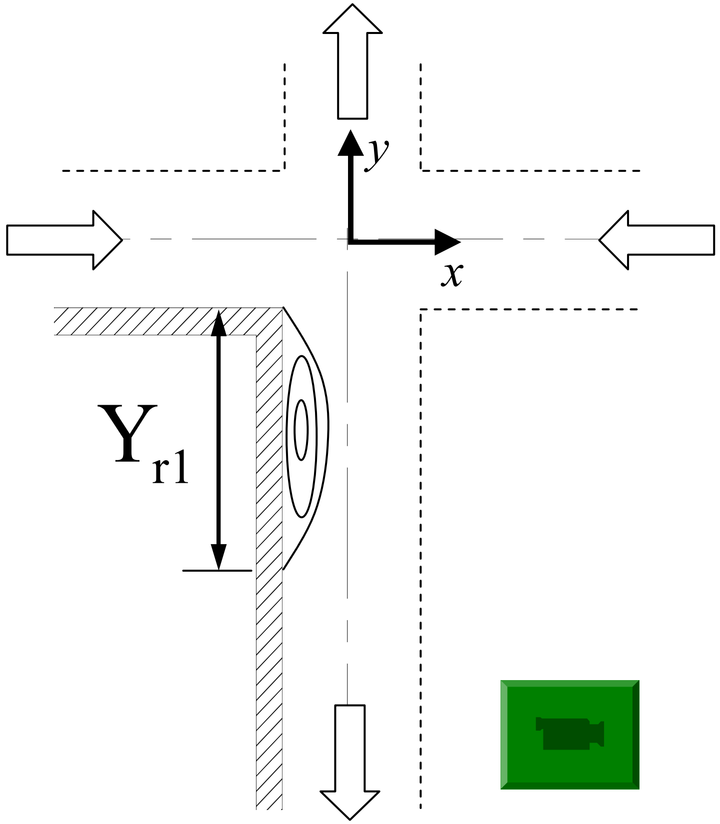
$200 \leq Re \leq 1400$

G.N. Rocha & P.J. Oliveira

Results: streamlines (symmetric flow)



Detail of the eddy attached to the left wall of the lower outlet channel for increasing values of Reynolds number



dem DEPARTAMENTO DE ELECTROMECÂNICA
G.N. Rocha & P.J. Oliveira

Results: bifurcation parameter

To quantify the degree of asymmetry:

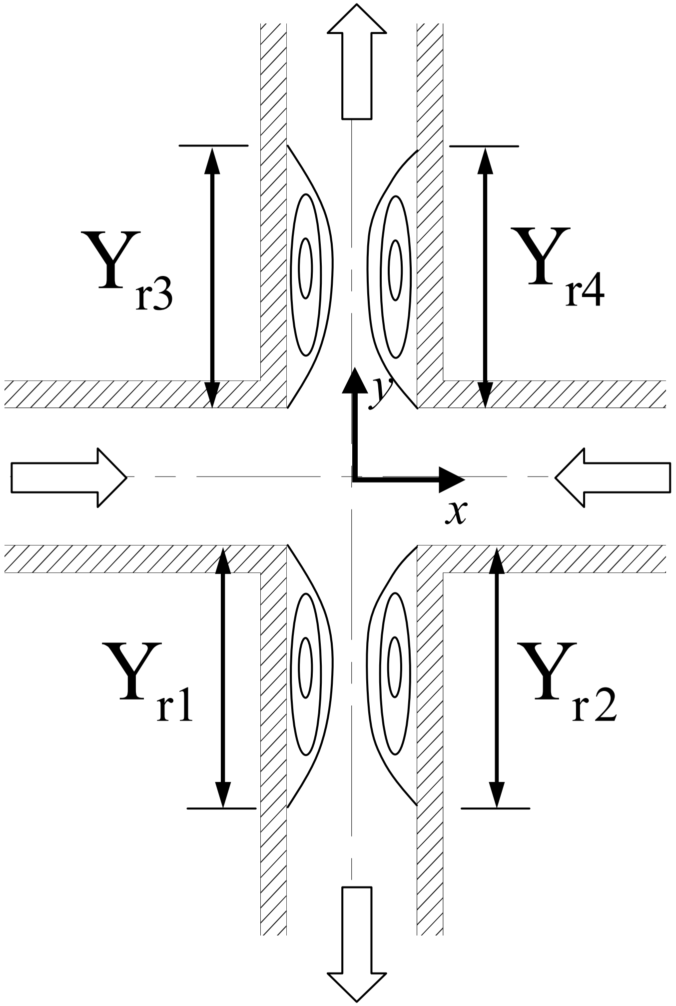
$$DY = -\frac{Y_{r2} - Y_{r1}}{\frac{1}{2}\left(Y_{r2} + Y_{r1}\right)} = \frac{Y_{r4} - Y_{r3}}{\frac{1}{2}\left(Y_{r4} + Y_{r3}\right)}$$

For symmetric flow:

$$Y_{r1} = Y_{r2} = Y_{r3} = Y_{r4} \Rightarrow DY = 0$$

For asymmetric flow:

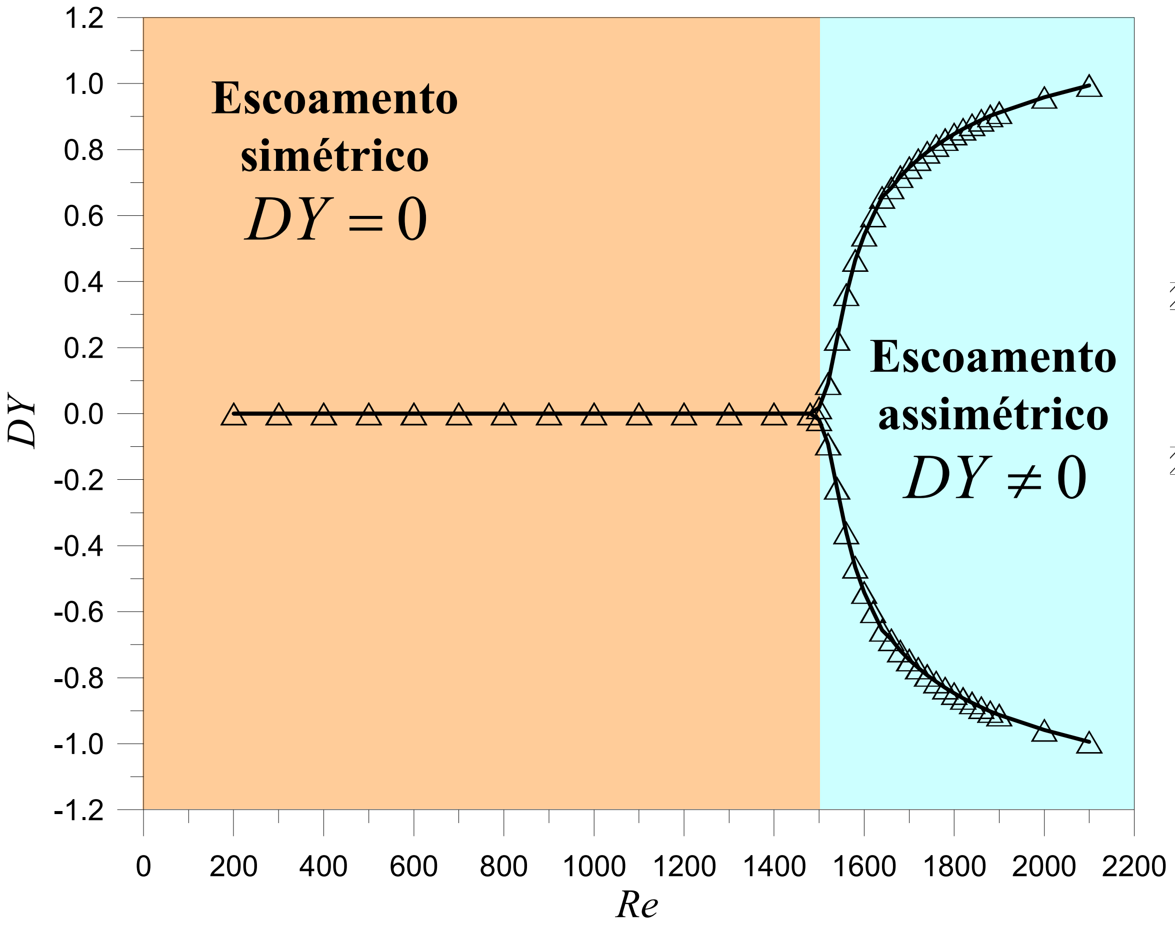
$$Y_{r1} = Y_{r3} \text{ e } Y_{r2} = Y_{r4} \Rightarrow DY \neq 0$$

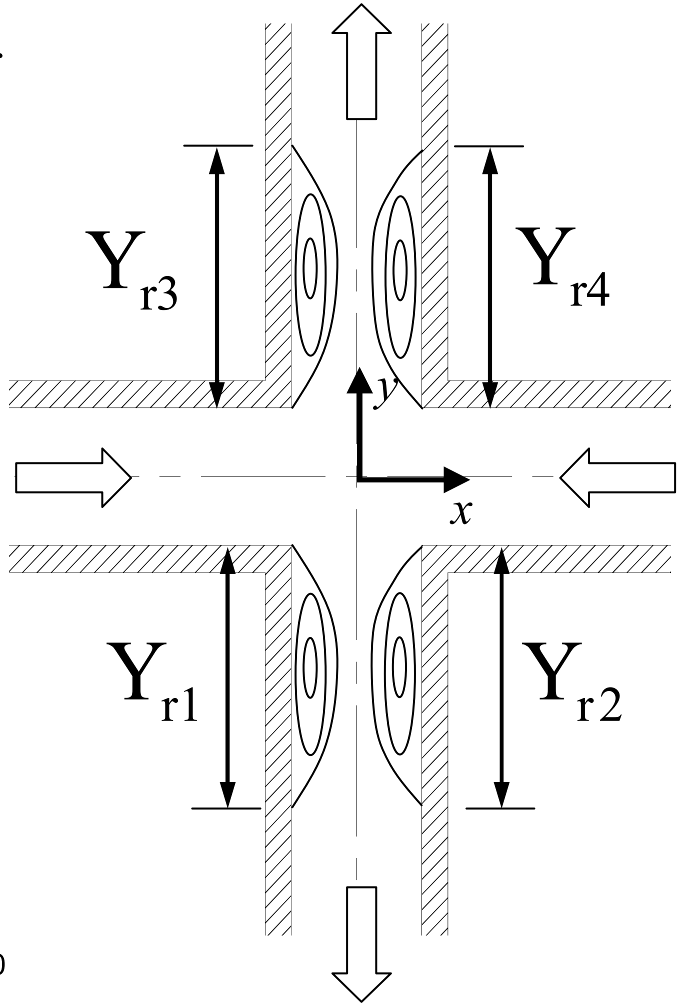


G.N. Rocha & P.J. Oliveira

Results: bifurcation diagram

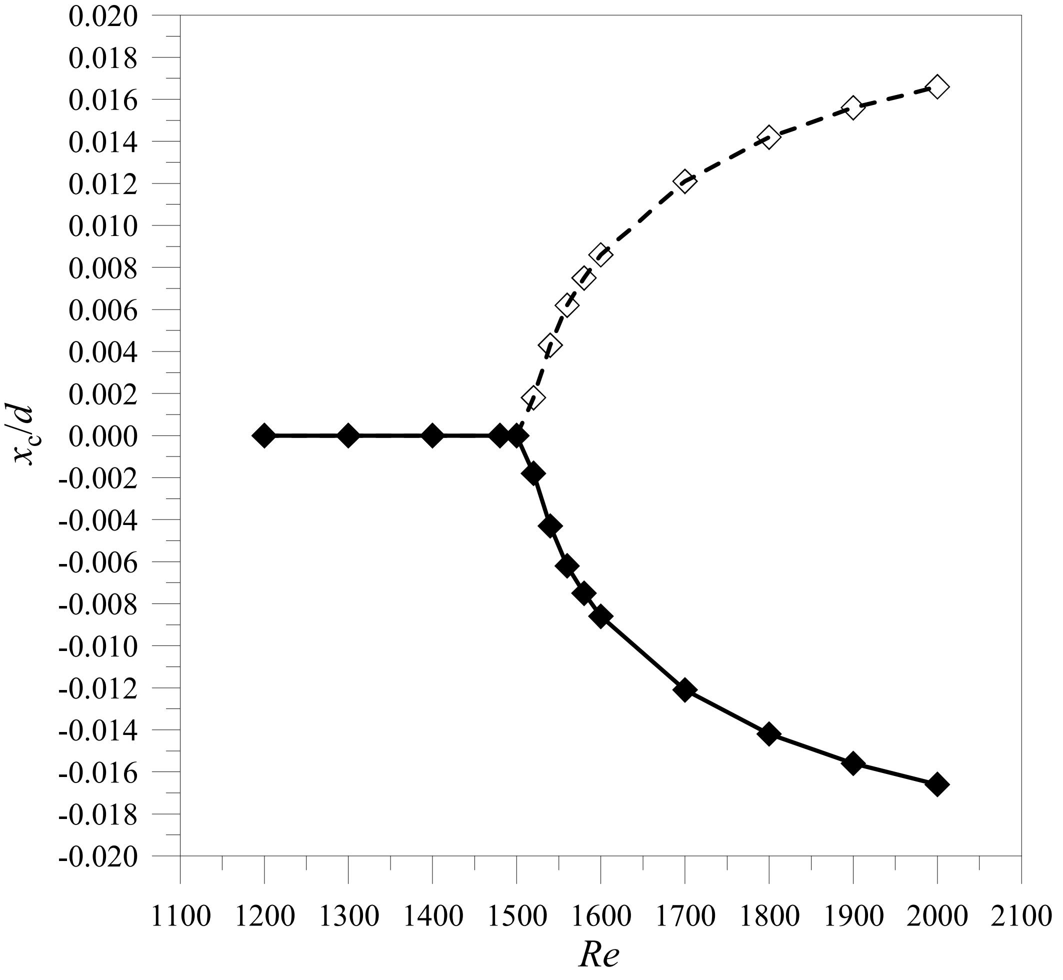
Bifurcation diagram: *DY* as a function of Reynolds number





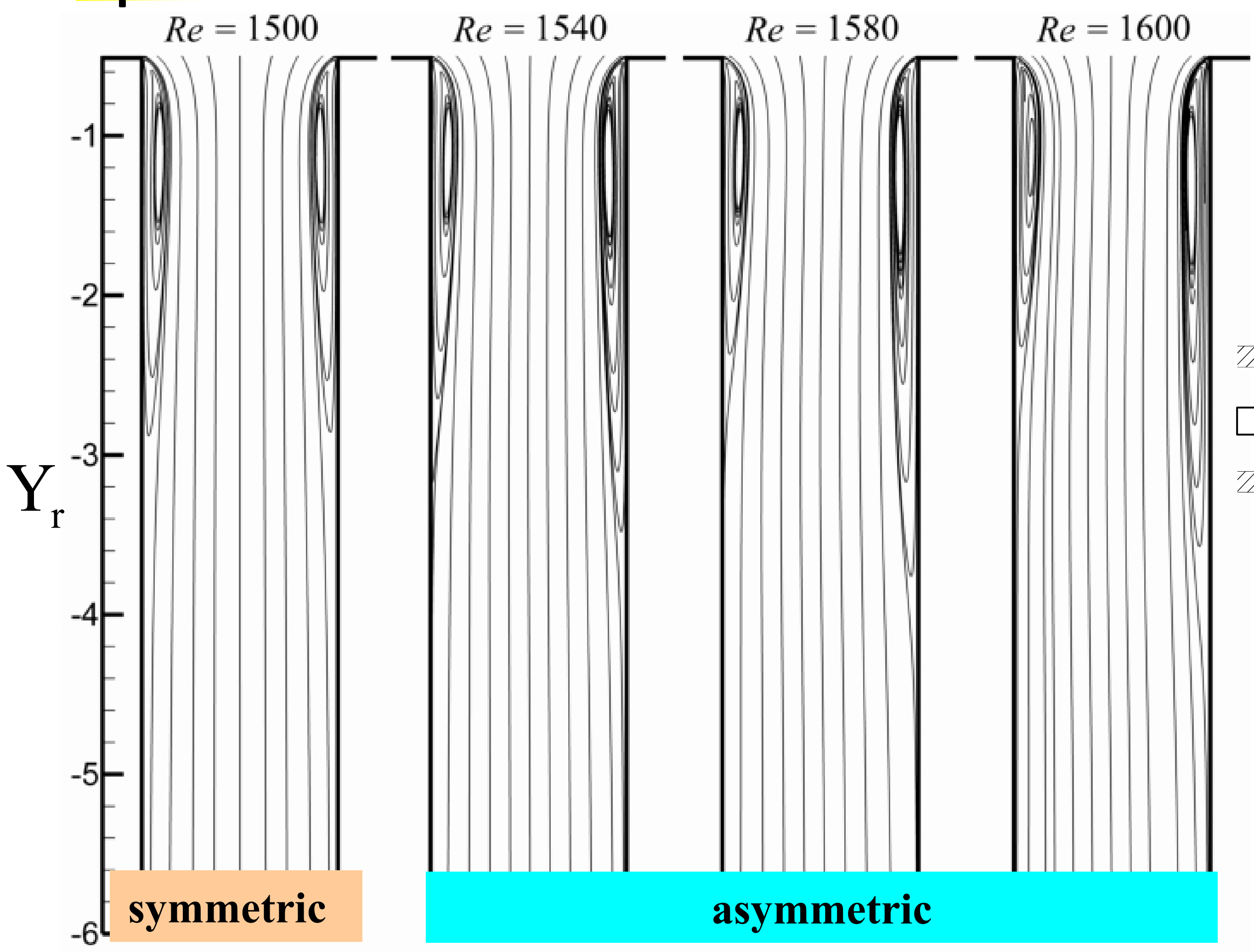
G.N. Rocha & P.J. Oliveira

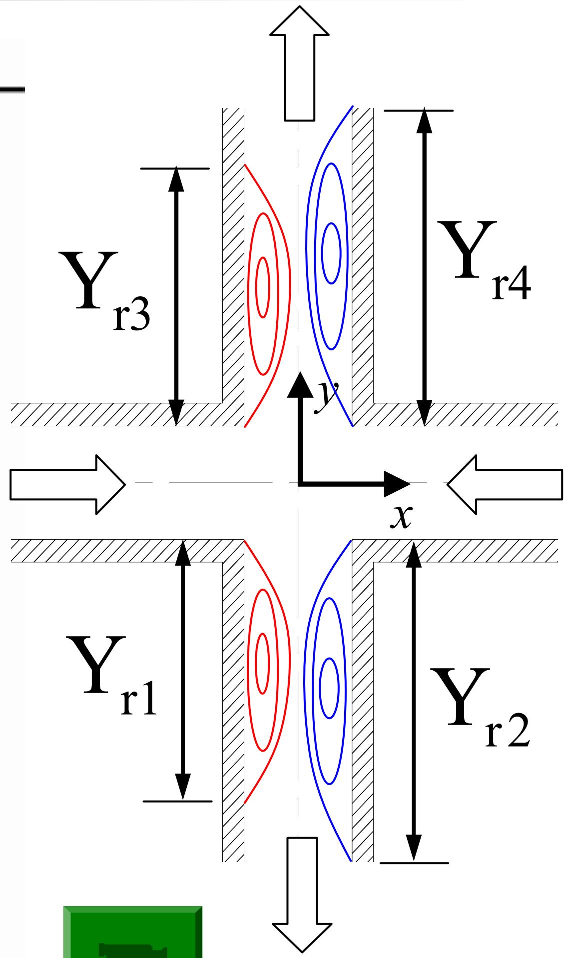
Results: stagnation point position



G.N. Rocha & P.J. Oliveira

Results: streamlines (asymmetric flow)





G.N. Rocha & P.J. Oliveira

Results: pressure variation (1)

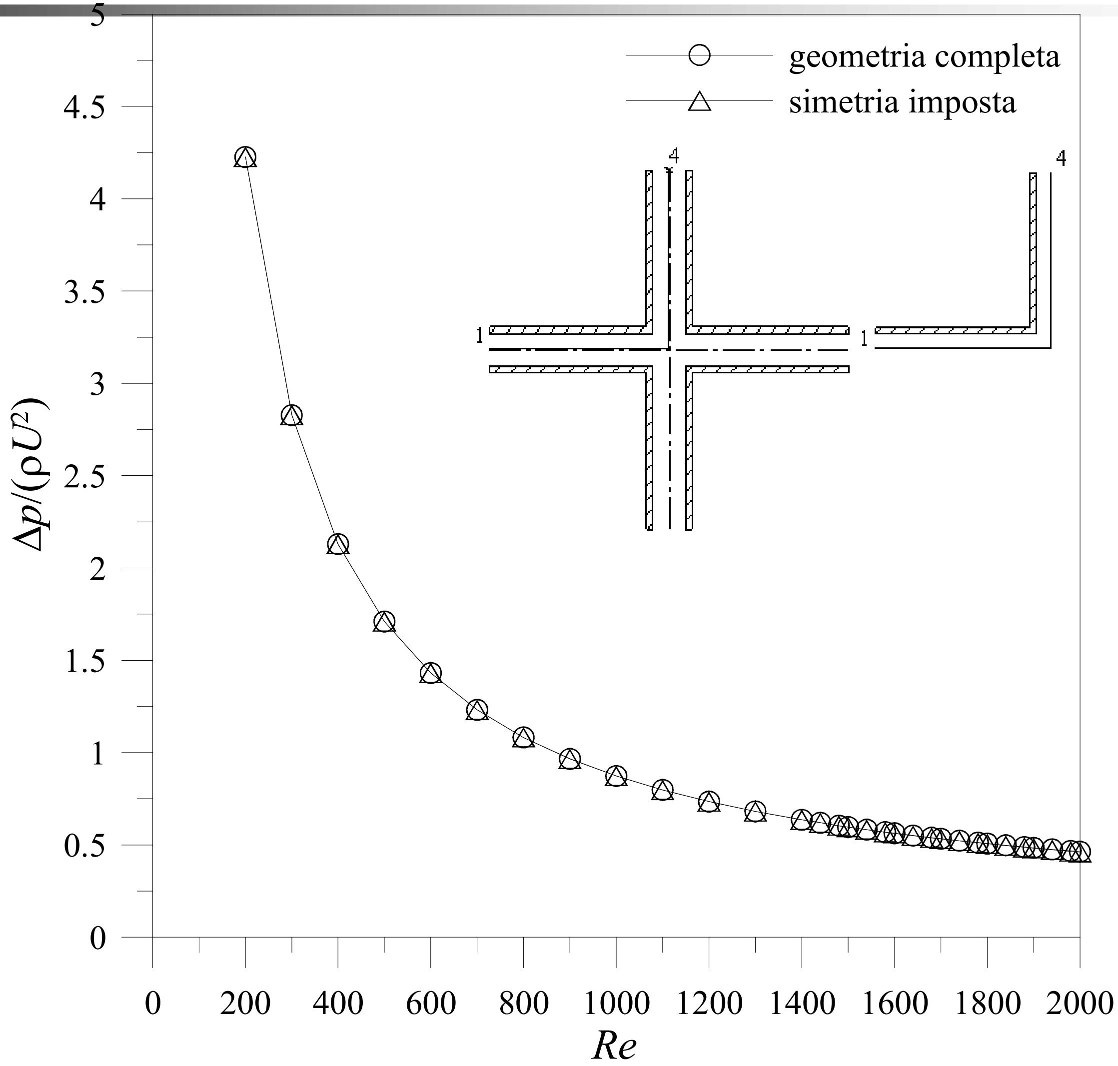
Compare full geometry against quarter of geometry (full symmetry imposed)

$$\Delta p = p_1 - p_4$$

Note: large part is

$$\frac{\Delta p_{FD} / \rho U^2}{L/d} = f$$

$$f = 12/Re$$

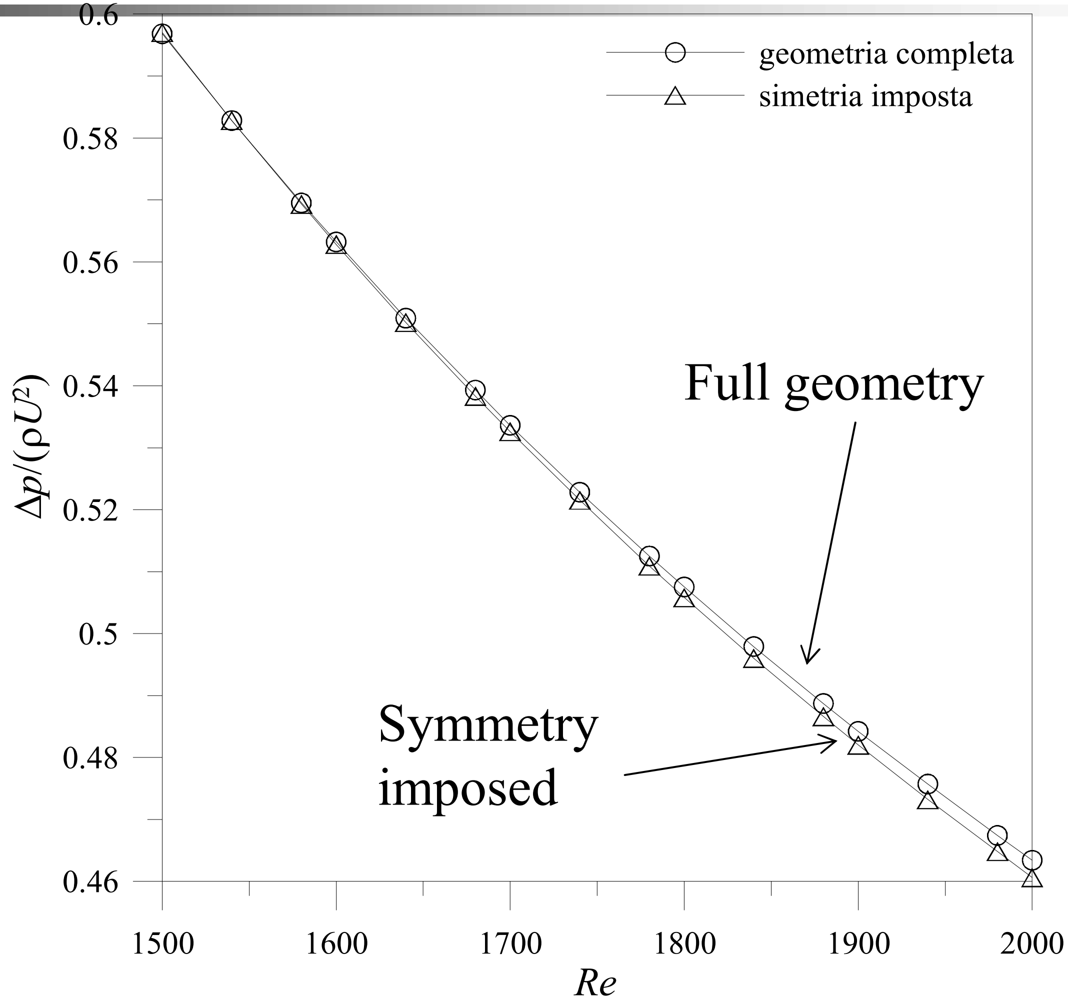


G.N. Rocha & P.J. Oliveira

Results: pressure variation (2)

Detail for
1500<*Re*<2000

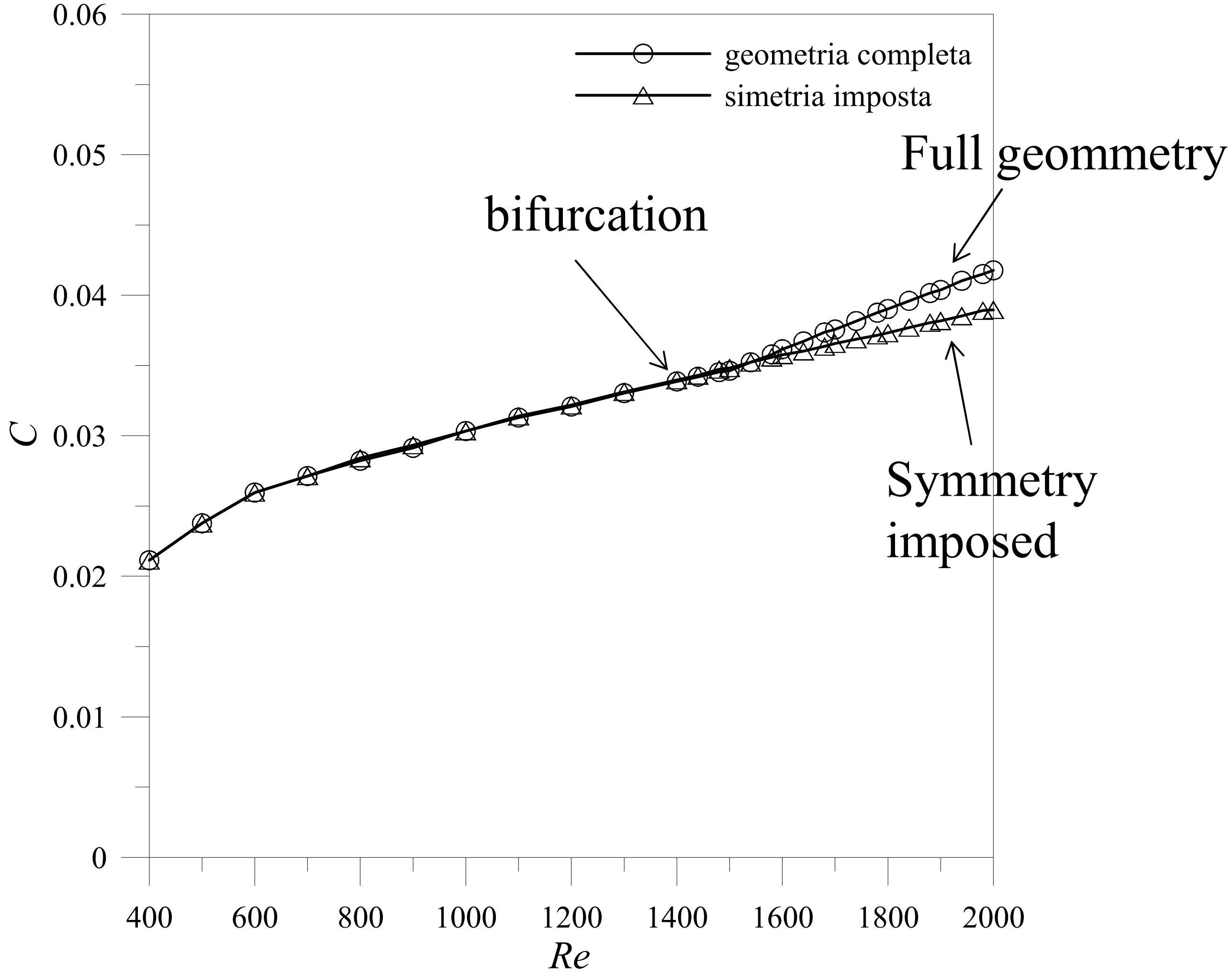
$$\Delta p_{asym} > \Delta p_{sym}$$



DEPARTAMENTO DE ELECTROMECÂNICA
G.N. Rocha & P.J. Oliveira

Results: Couette correction

$$C = \left(\Delta p - \Delta p_{FD}\right) / \rho U^2$$



DEPARTAMENTO DE ELECTROMECÂNICA
G.N. Rocha & P.J. Oliveira

Conclusions

- **Inertia** provokes the appearence (Re>190) of attached eddies, after the corners on the walls of outlet channels.

- The **size of those eddies** increases linearly with *Re,* while the flow remains symmetric (up to $Re_{cr} = 1490 \pm 10$) .

- When $\boldsymbol{Re > Re_{cr}}$ the flow becomes **asymmetric**, with larger eddies on one side of the walls, as compared to the opposite wall.

- The size of the smaller eddies tend to remain constant with *Re*, while the larger eddies keep increasing in size.

- The ***symmetry*** of the flow pattern after bifurcation is different from the viscoelastic case at low *Re*.

Acknowledgments

FCT Fundação para a Ciência e a Tecnologia

MINISTÉRIO DA CIÊNCIA, INOVAÇÃO E DO ENSINO SUPERIOR Portugal

- **Projectos:**
  - **SFRH/BD/22644/2005 (G.N. Rocha)**
  - **PTDC/EME-MFE/70186/2006**

Unidade de Investigação
Materiais Têxteis e Papeleiros
Unit of Textile and Paper Materials

Universidade da Beira Interior