Dynamics of entry flows at high Weissenberg numbers

a numerical study

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Outline

1. Introduction
   - Benchmark flow description
2. Governing Equations
3. Numerical Method
4. Results
   - Low Deborah number flows
   - Non-linear dynamics at high Deborah number flows
   - Inertial effects: constant Elasticity
5. Conclusions

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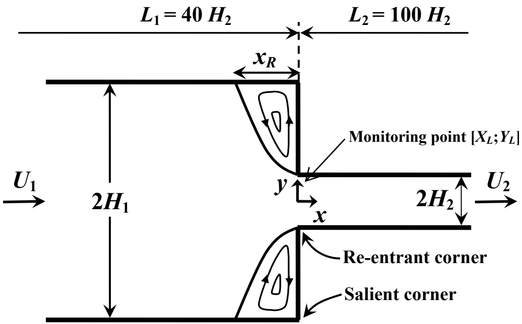
Benchmark flow description

Creeping flow in a 4:1 planar contraction

The study of this Benchmark problem started more than 25 years ago[1]! Is it still relevant?



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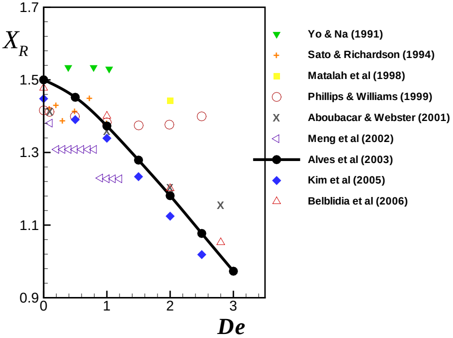

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Governing Equations

Mass & Momentum Conservation

Mass Conservation:

$$\nabla \cdot \mathbf{u} = 0$$

Momentum Conservation:

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \beta \eta_o \nabla^2 \mathbf{u} + \frac{\eta_o}{\lambda}(1-\beta)\nabla \cdot \mathbf{A}$$

Viscosity ratio

$$\beta \equiv \frac{\eta_s}{\eta_o} = \frac{\eta_s}{\eta_s+\eta_p} = \frac{1}{9}$$

Oldroyd-B (StrT)

$$\lambda \overset{\nabla}{\mathbf{A}} = (\mathbf{I} - \mathbf{A})$$

$$\overset{\nabla}{\mathbf{A}} = \frac{D\mathbf{A}}{Dt} - (\mathbf{RA} - \mathbf{AR}) - 2\mathbf{EA}$$

$$\boldsymbol{\tau} = \frac{\eta_p}{\lambda}(\mathbf{A} - \mathbf{I})$$

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$$\lambda \overset{\nabla}{\boldsymbol{\Theta}} = (e^{-\boldsymbol{\Theta}} - \mathbf{I})$$

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$$\boldsymbol{\Theta} = \log \mathbf{A}$$

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Governing Equations

Constitutive Equation: Standard stress/conformation tensor formulation (*StrT*)

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Finite Volume Method[1]

- Structured, collocated and non-orthogonal meshes.
- Discretization (formally 2nd order)
  - Diffusive terms: central differences (CDS)
  - Advective terms, high resolution scheme: CUBISTA[2]
- Dependent variables evaluated at cell centers;
- Special formulations for cell-face velocities and stresses;
- Log-conformation[3] for the extra-stress tensor[4];
- 1^{st} order implicit Euler scheme for time integration.

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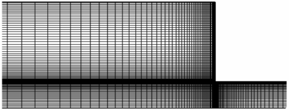
Computational meshes

	nº cells	DoF	$\triangle x_{min}/H_2$ & $\triangle y_{min}/H_2$
$M1$	5282	31692	0.02
$M2$	10587	63522	0.014
$M3$	42348	254088	0.0071
$M3C$	84696	508176	0.0071



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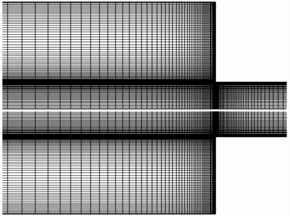
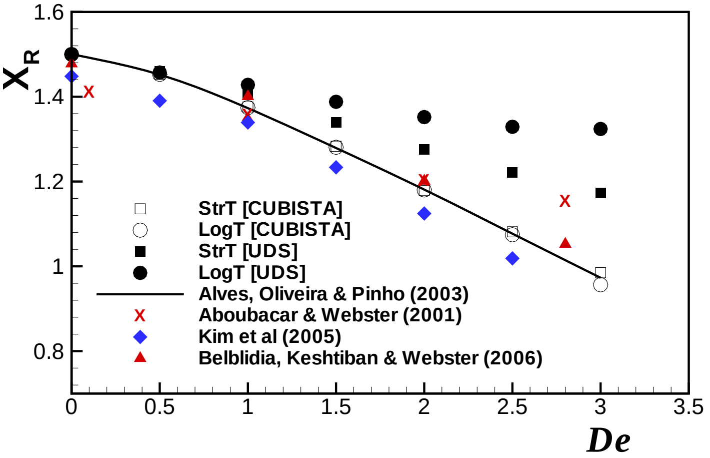


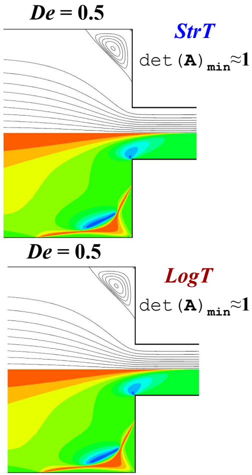
Low Deborah number flows: Results

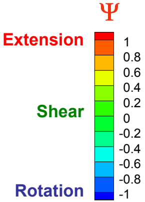
Length of primary vortex, X_R, on mesh M3



Low Deborah number flows: Results

Flow patterns on mesh M3





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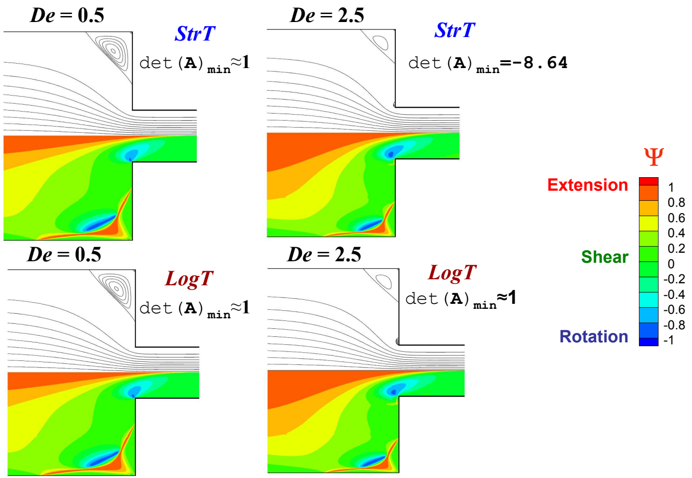

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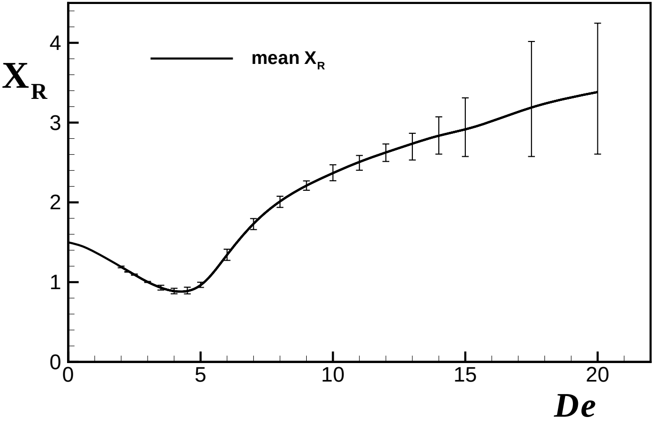
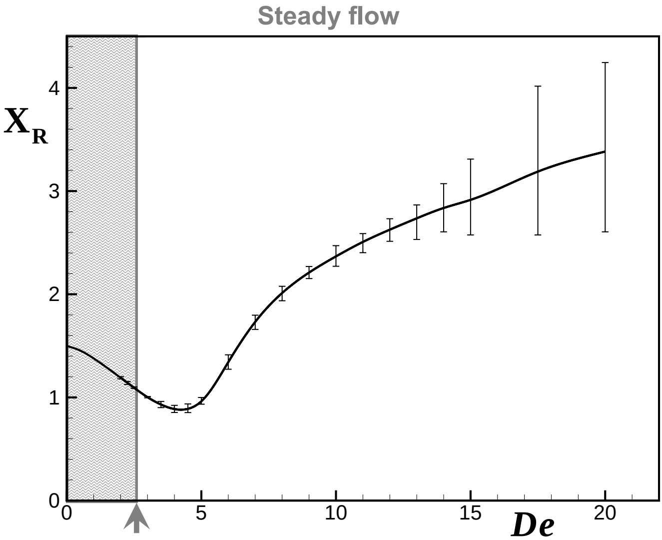


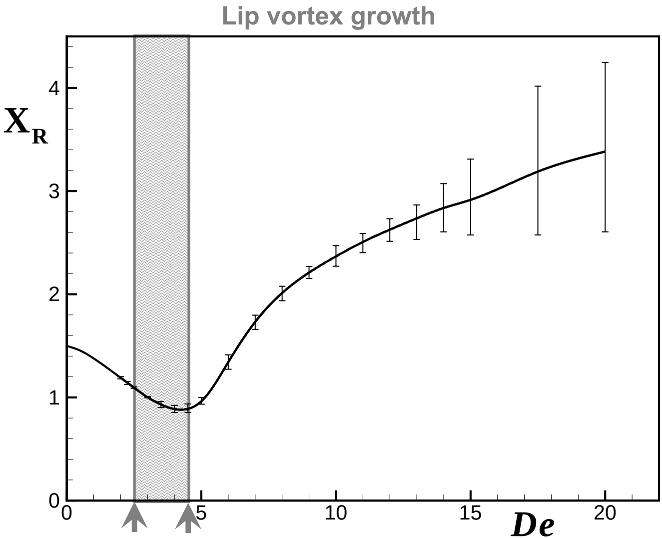
- For $De > 2.5$ the flow became unsteady;
- Simulations with *StrT* diverged at $De = 3$.



Steady flow
X_R
De
0
1
2
3
4
0
5
10
15
20

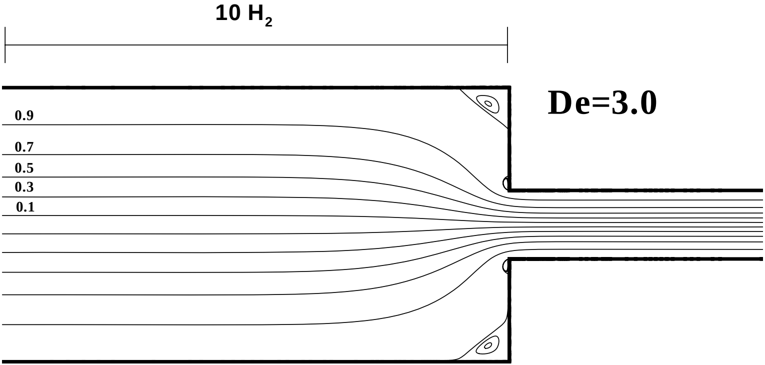
High Deborah number flows: Results

Flow dynamics: lip vortex growth regime [$2.5 < De < 4.5$]

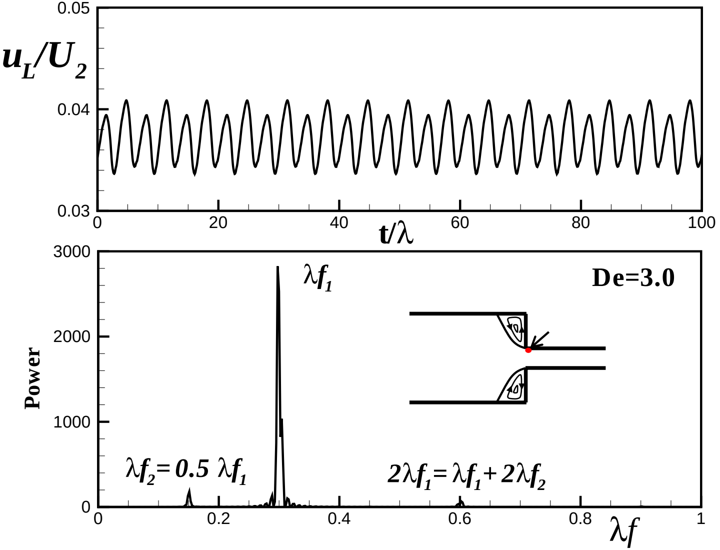


High Deborah number flows: Results

Flow dynamics: lip vortex growth regime [$De = 3$]



Flow dynamics: lip vortex growth regime [$De = 3$]



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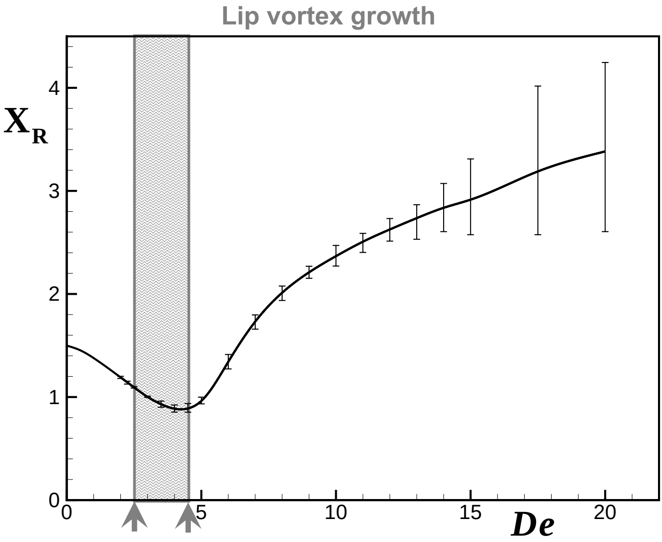
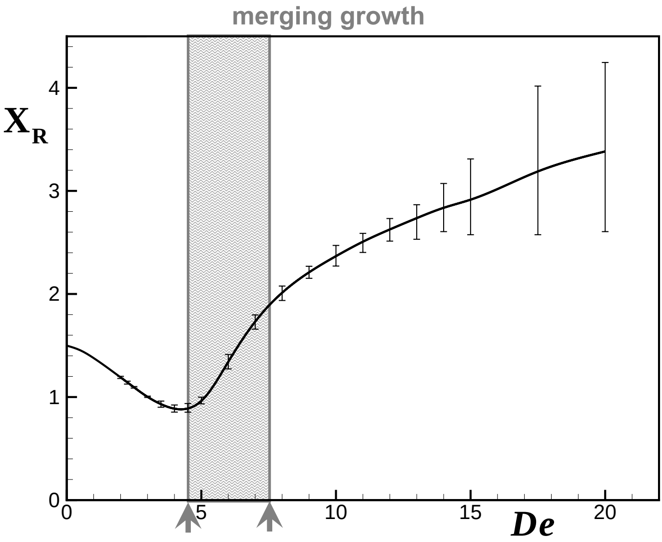
High Deborah number flows: Results

Flow dynamics: lip vortex growth regime [$2.5 < De < 4.5$]



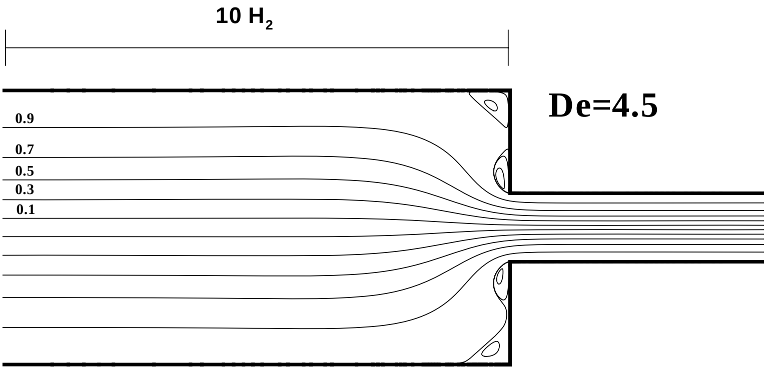
High Deborah number flows: Results

Flow dynamics: merging vortex growth regime [$4.5 \leq De < 8$]



High Deborah number flows: Results

Flow dynamics: lip vortex growth regime [$De = 4.5$]



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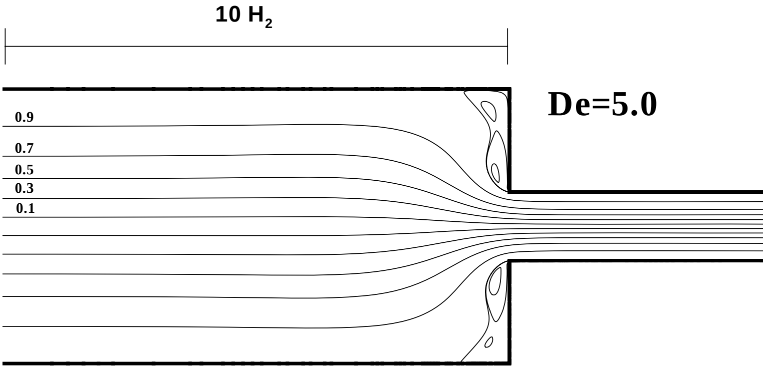
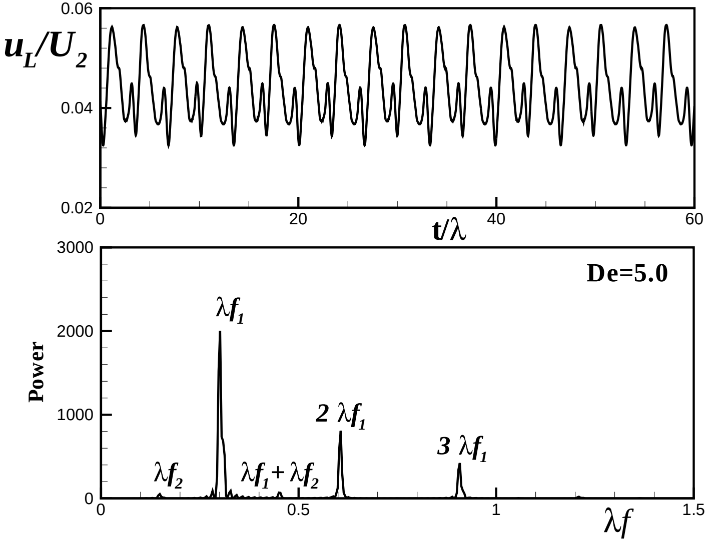
High Deborah number flows: Results

Flow dynamics: lip vortex growth regime [$De = 5$]



High Deborah number flows: Results

Flow dynamics: lip vortex growth regime [$De = 5$]



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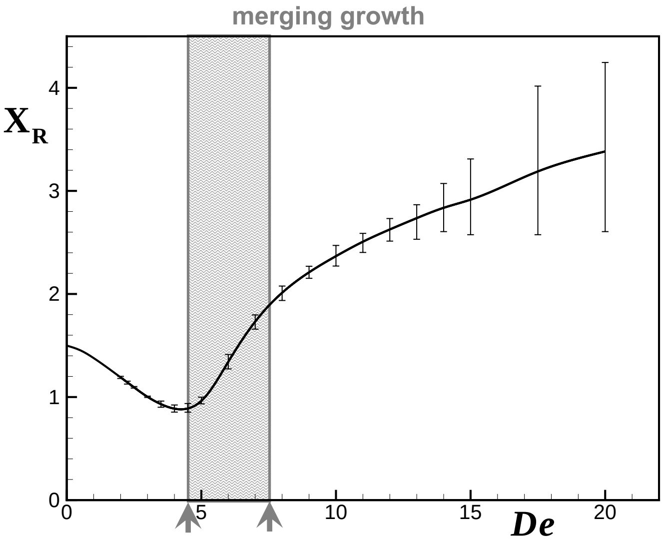
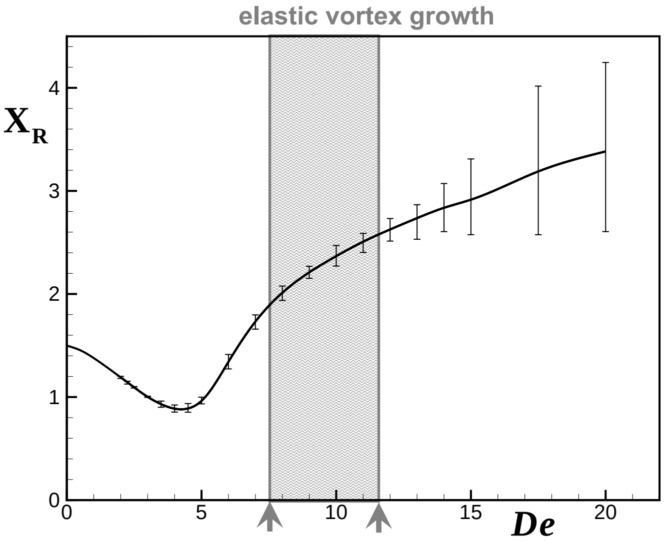


High Deborah number flows: Results

Flow dynamics: elastic vortex growth regime $[8 \leq De < 12]$



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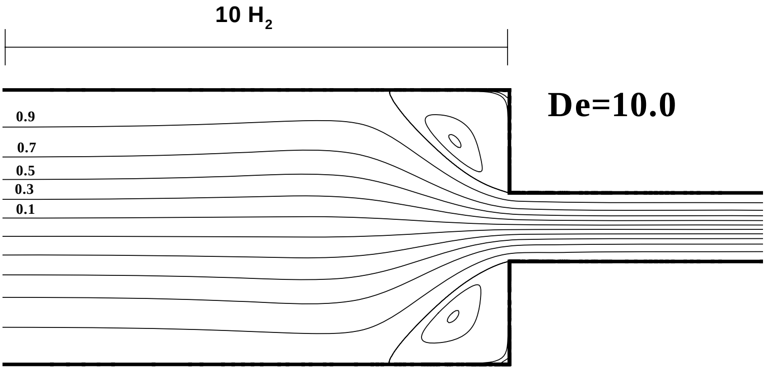

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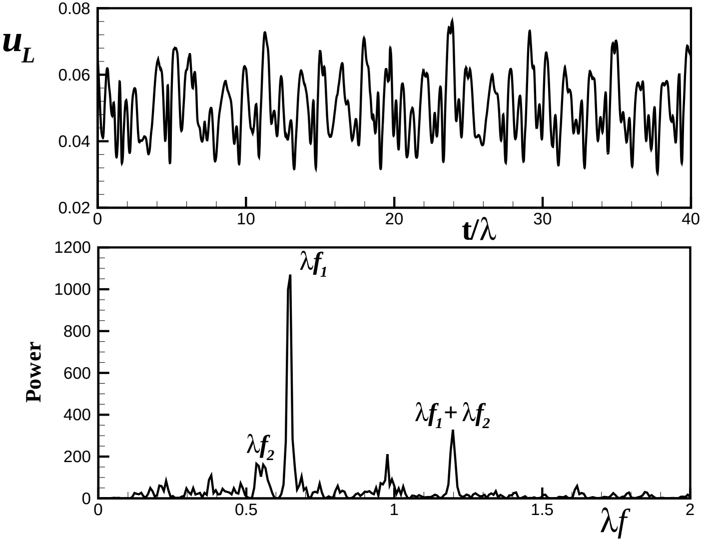
Flow dynamics: elastic vortex growth regime [$De = 10$]



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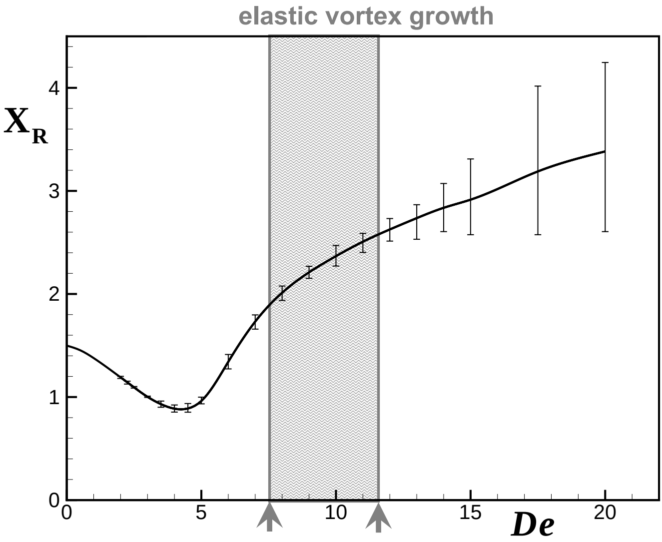
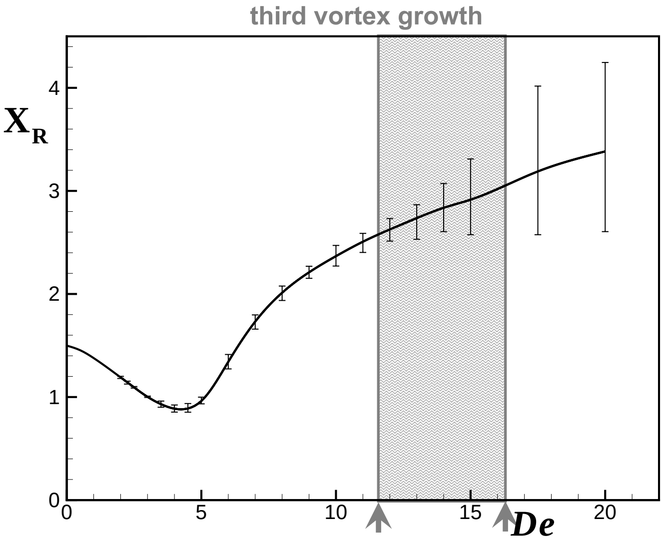
High Deborah number flows: Results

Flow dynamics: elastic vortex growth regime [$8 \leq De < 12$]



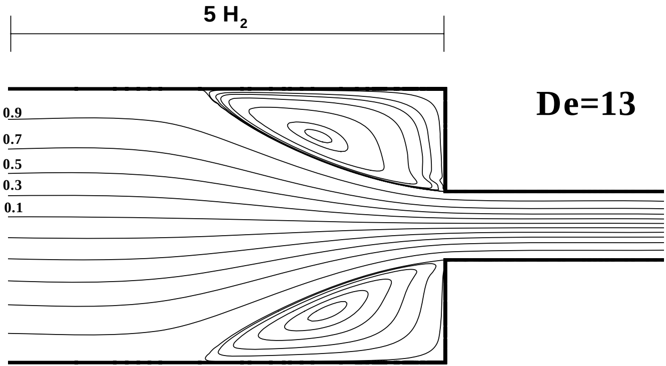
High Deborah number flows: Results

Flow dynamics: third vortex growth regime [$12 \leq De < 17.5$]



High Deborah number flows: Results

Flow dynamics: third vortex growth regime [$De = 13$]



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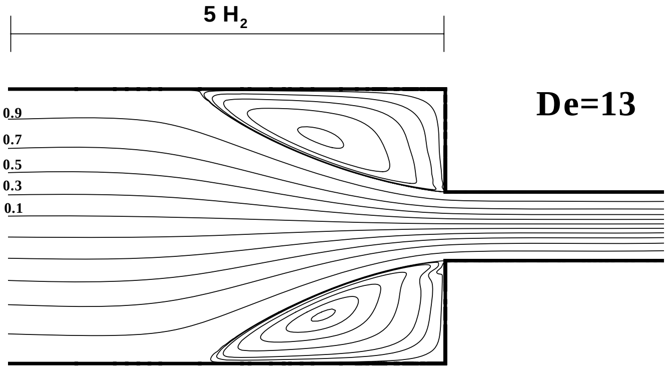

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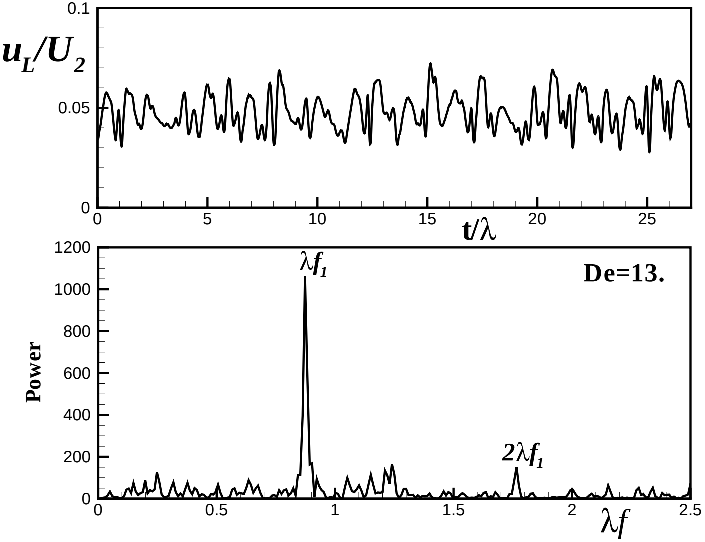

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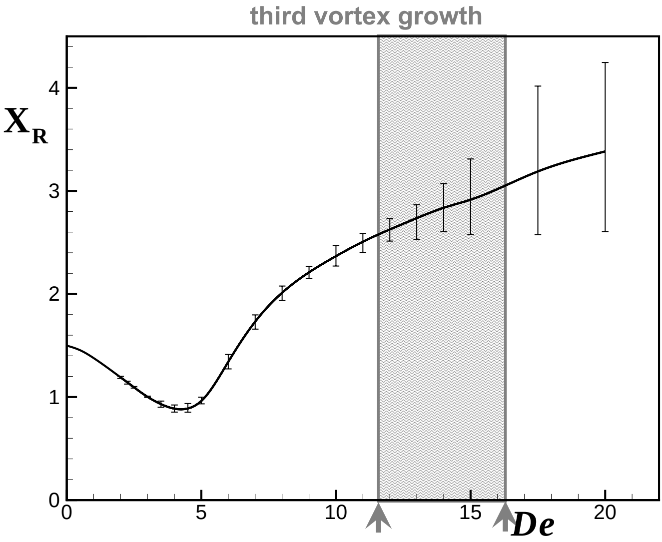
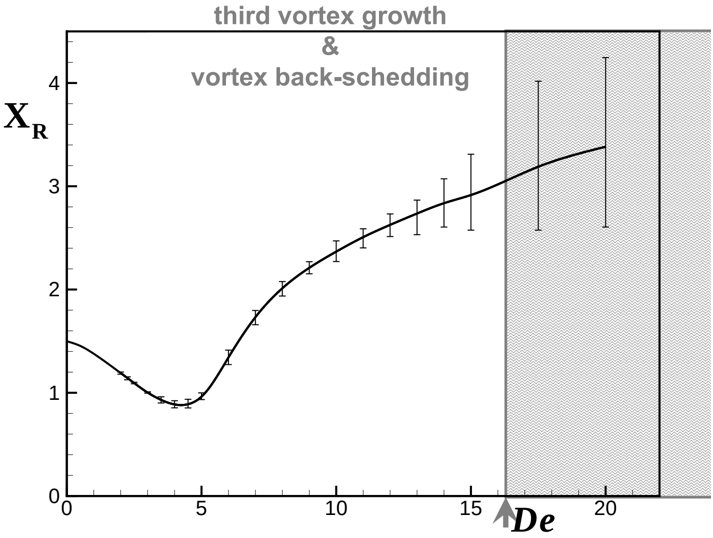


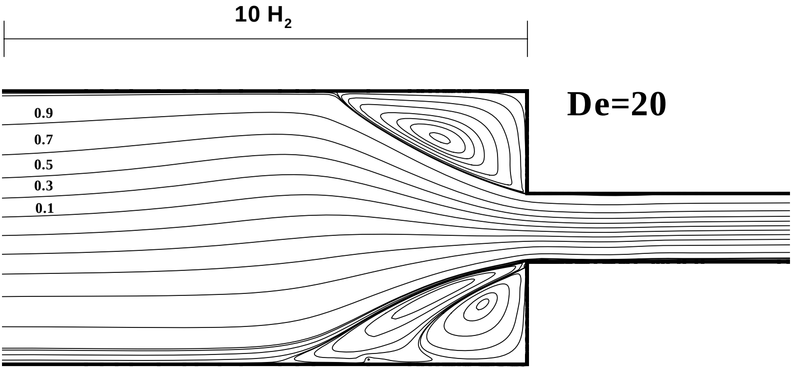
High Deborah number flows: Results

Flow dynamics: vortex back-schedding [$17.5 \leq De \leq 100$]



High Deborah number flows: Results

Flow dynamics: vortex back-schedding regime [$De = 20$]



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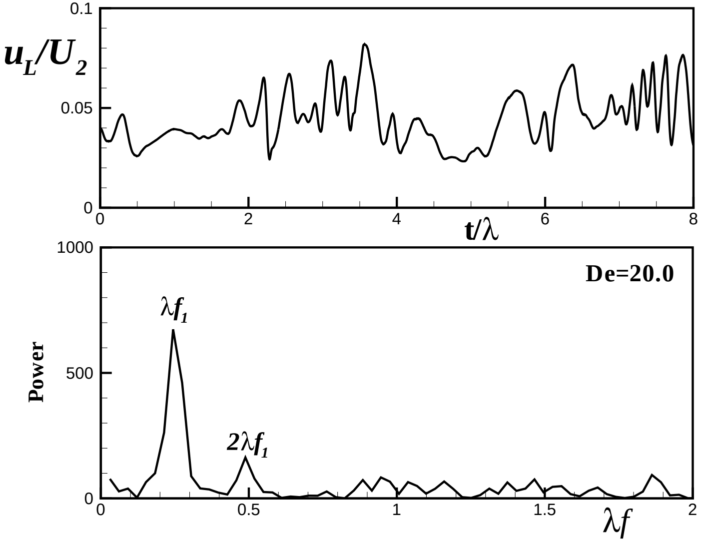
Flow dynamics: vortex back-schedding regime [$De = 20$]



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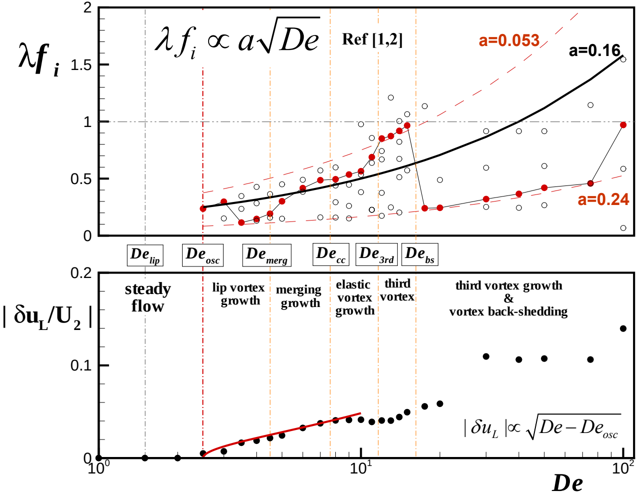
High Deborah number flows: Results

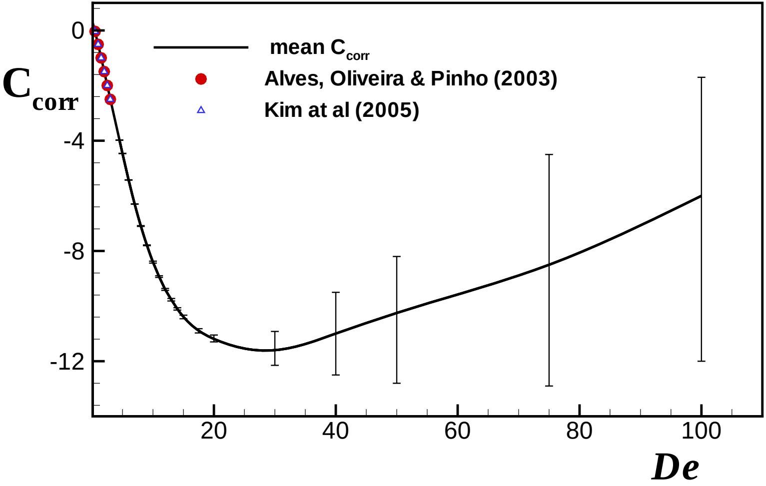
Flow dynamics: Dominant frequencies and velocity amplitude



[1] El Hadj and Tanguy, *JNNFM* (1990); [2] Fortin and Esselaoui, *IJNMF* (1987).

High Deborah number flows: Results

Flow dynamics: Couette correction



- C_{corr} increases for $De > 20$.

High Deborah number flows: Results

Inertial effects: Re.vs.De

Conclusions

Low De numbers

- Excellent agreement between the *StrT* and *LogT* formulations and other benchmark data.

High De numbers

- For $De > 2.5$ the flow became unsteady;
- Simulations with *StrT* diverged at $De = 3$;
- Vortex size, X_R, and pressure drop, C_{corr}, show the typical upturn shape seen in experimental data;
- A rich sequence of elastic transitions was mapped, such the back-shedding of vorticity from the two pulsating vortices;
- The dimensional frequency is inversely proportional to the square root of relaxation time.

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Thanks!

Questions?