On Extensibility Effects in the Cross-slot Flow Bifurcation

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Abstract. Viscoelastic flow in a cross-slot geometry are known to give rise to purely-elastic flow instabilities even under inertia-less flow conditions. Here, the flow of the finite extensibility FENE-CR model in a two-dimensional planar cross-slot configuration is studied numerically, using a finite-volume method, with a view to quantifying the influences of the level of extensibility (L^2), concentration parameter (β) and sharpness of corners (R), on the occurrence of a bifurcated flow pattern. The results show the phenomena to be largely controlled by the elongational properties of the constitutive model, with the critical Deborah number for bifurcation tending to be reduced as extensibility increases, and the sharpness or otherwise of the corners to have only a marginal influence on the triggering mechanism leading to the pitchfork bifurcation, which seems essentially to be restricted to the central, stagnation-point region.

Keywords: Flow instabilities, Bifurcation phenomenon, FENE-CR model, Finite-volume method, 2D cross-slot.
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INTRODUCTION

While Newtonian flows usually become unstable only at "high" Reynolds numbers (*Re*), flows of viscoelastic fluids give rise to interesting instabilities even under conditions of negligible inertia. These instabilities are usually associated with tension along curved streamlines and appear in rheometry of complex fluids as well as many other applications [1-2]. Two recent studies of viscoelastic flow instabilities through cross-slot geometries by Arratia *et al.* [3] and Poole *et al.* [4] have boosted a renewal of interest in this flow configuration. First, Arratia *et al.* [3] have found in the laboratory with a microfluidic apparatus that, even in a symmetric geometry under perfectly symmetric flow conditions, a polyacrylamide (PAA) polymer solution tended to evolve to a non-symmetric pattern while the corresponding Newtonian flow remained symmetric. As the incoming flow rate in the cross-slot was varied at low Reynolds number ($Re < 10^{-2}$), two new flow instabilities arose in the planar extensional flow of the dilute flexible PAA polymer solution: in the first instability the velocity field becomes strongly asymmetric, but remains steady; the second instability occurs at higher strain rates and leads to a velocity field fluctuating non-periodically in time. Later, Poole *et al.* [4] simulated numerically the complete cross-slot geometry using arguably the simplest differential viscoelastic model, the upper-convected Maxwell (UCM) model, at zero Reynolds number and were able to reproduce the main features of the experiments. At low Deborah numbers (*De*) the flow remained steady and symmetric, but above a critical Deborah number ($De_{cr} \approx 0.31$) the flow remained steady but became asymmetric. Poole *et al.* [4] have thus demonstrated that a flow asymmetry due solely to elasticity can be numerically predicted in a perfectly symmetric geometry. They argued that the asymmetry is a consequence of the compressive nature of the flow upstream of the stagnation point (at the central position of the cross-slot channel) rather than the large elongational stresses that arise in the flow downstream of the stagnation point.

In the present study we extend the work of Poole *et al.* [4] by employing a more realistic constitutive model, the finitely-extensible FENE-CR model developed by Chilcott and Rallison [5], and investigate in detail the effect of extensibility, concentration parameter and sharpness of corners on the bifurcation phenomenon.

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GOVERNING EQUATIONS AND NUMERICAL METHOD

We consider the full planar cross-slot geometry, as shown in Fig. 1(a), where a description of relevant variables is also provided.

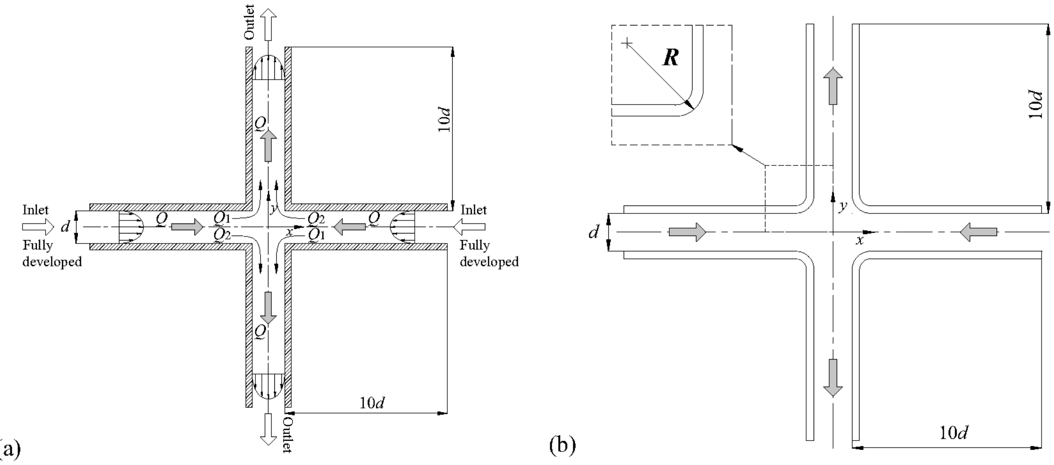

FIGURE 1. (a) Schematic of the cross-slot geometry. Minimum cell spacing $\Delta x_{\min} \approx \Delta y_{\min} \approx 0.02d$, total number of cells = 12801. (b) Solution domain for rounded-corners geometry.

We assume the flow is two-dimensional, incompressible and isothermal, with negligible inertial effects (i.e. $Re = 0$). The equations governing this laminar flow of a polymer solute in a Newtonian solvent are:

$$\nabla \cdot \boldsymbol{u} = 0 \,,\, -\nabla p + \eta_s \nabla^2 \boldsymbol{u} + \nabla \cdot \boldsymbol{\tau} = \boldsymbol{0} \tag{1}$$

expressing conservation of mass and momentum, and a rheological constitutive equation (FENE-CR model [5]) for the elastic contribution to the stress tensor $\boldsymbol{\tau}$:

$$\boldsymbol{\tau} + \lambda \overset{\nabla}{\left(\boldsymbol{\tau}/f\right)} = 2\eta_p \boldsymbol{D} \,, \text{ where } f = \frac{L^2 + \lambda/\eta_p \, tr(\boldsymbol{\tau})}{L^2 - 3} \tag{2}$$

The symbol ∇ denotes the upper-convected derivative and $\boldsymbol{D}$ is the strain rate tensor. The relevant dimensionless parameters are: L^2, the extensibility parameter of the FENE-CR model; $\beta = \eta_s/\eta_0$, the solvent viscosity ratio (where $\eta_0 = \eta_s + \eta_p$); $Re = \rho U d/\eta_0$ (taken as 0 in this study), the Reynolds number; and $De = \lambda U/d$, the Deborah number. A fully implicit finite-volume method is used to solve Eqs. (1) – (2) which has been explained in previous works [6]. We impose fully-developed velocity (average value U) and stress profiles at inlet, Neumann boundary conditions at the outlets and no-slip conditions at the walls.

RESULTS AND DISCUSSION

A non-dimensional flow-rate imbalance parameter, $DQ = (Q_2 - Q_1)/Q$, was defined to quantify the degree of asymmetry of the flow (see Poole *et al.* [4]). For a symmetric flow $Q_1 = Q_2$ and $DQ = 0$, while for an asymmetric flow $Q_1 \neq Q_2$ and $DQ \neq 0$ (completely asymmetric flow $DQ = \pm 1$). Fig. 2(a)-(b) show the predictions of the degree of asymmetry for the sharp-corner case at three values of L^2 (= 50, 100 and 200) and constant β = 0.1, and three values of β (= 0.05, 0.10 and 0.20) and constant L^2 = 100. Elasticity was seen to directly drive the instability and for De values lower than a critical Deborah number the flow remains symmetric, while for values above De_{cr} the flow becomes asymmetric but remains steady. Hence De_{cr} defines the transition point from a symmetric to an asymmetric state and is seen from Fig. 2(a)-(b) to depend on both β and L^2. In this geometry the asymmetry of the flow is triggered by elasticity, and since Re in the present simulations is exactly zero it is again confirmed that the bifurcation is a purely-elastic phenomenon. The results show that an increase of L^2 tends to accentuate the bifurcation phenomenon upon the base flow and to shift the occurrence of the transition point to asymmetry to lower *De*. From Fig. 2(b) we can note that increasing β leads to a delay of the critical bifurcation point. Polymer concentration ($c \propto \beta^{-1}$) was seen to have a strong effect and for $\beta = 0.20$ the flow was symmetric up to $De_{cr} \approx 0.68$, but if β was decreased to 0.10 (a 2.2 fold increase of polymer concentration) the asymmetry phenomenon appears at much lower $De \approx 0.46$.

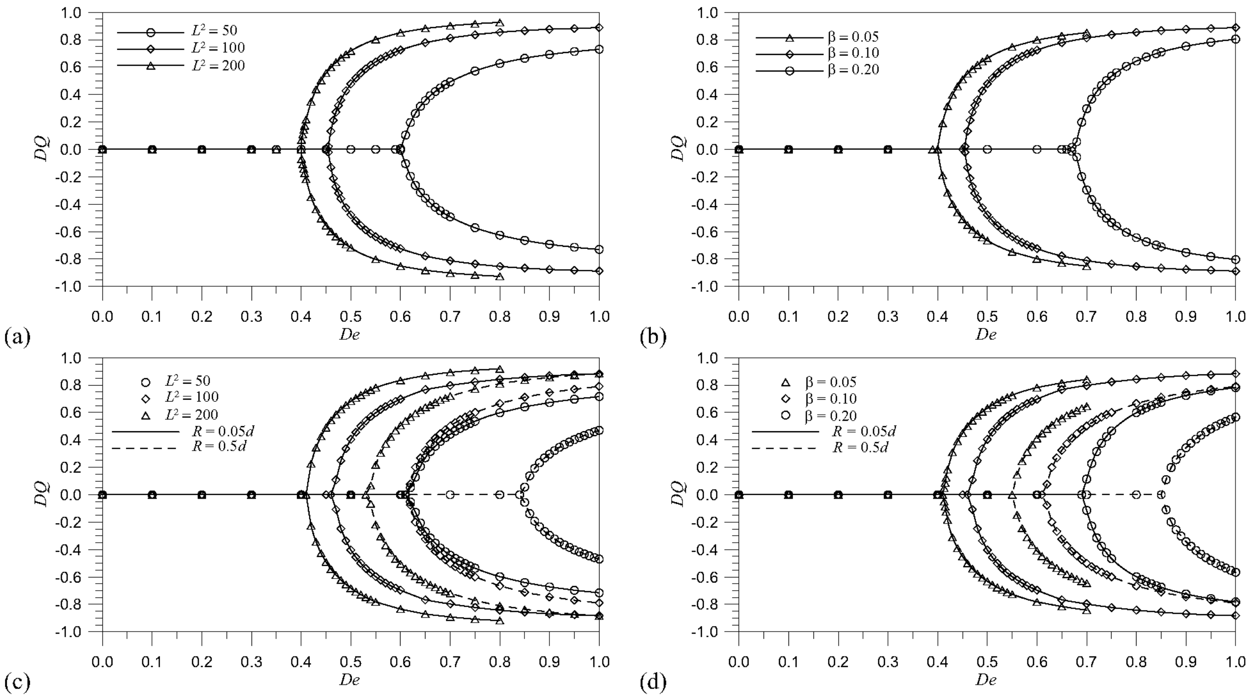

FIGURE 2. Variation of asymmetry parameter *DQ vs De* for sharp-corner case: (a) Influence of L^2 ($\beta = 0.1$); (b) Influence of β ($L^2 = 100$). Effect of rounding corners $R = 0.05d$ and $0.5d$: (c) Influence of L^2 ($\beta = 0.1$); (d) Influence of β ($L^2 = 100$).

In order to test if rounding the corners would significantly affect the bifurcation phenomenon, we used two geometries with rounded corners $R = 0.05d$ and $0.5d$ (see Fig. 1(b)) and generated meshes having essentially the same characteristics as the sharp-corner ones. The bifurcation plots are shown in Fig. 2(c)-(d) for varying L^2 (at fixed $\beta = 0.1$) and β (at fixed $L^2 = 100$), respectively. It is clear from the results with $R = 0.05d$ (slightly rounded case) that both the De_{cr} and the variation of *DQ* remain unchanged from the sharp-corner case (contrast solid-line curves of Fig. 2(c) with Fig. 2(a) above it; the same for Fig. 2(d) and 2(b)). However, for $R = 0.5d$ (large curvature case) a delay of the instability is predicted, certainly as a consequence of the velocity field being significantly altered, with an anticipated reduction of strain rate along the channel's centerline on approaching and leaving the internal stagnation point at the cross-slot centre.

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