An improved FENE-P k–ε $\overline{vv}$-f turbulence model for polymer-induced drag reduction

M. Masoudian[1], F.T. Pinho[1], K. Kim[2], R. Sureshkumar[3]

[1] Transport Phenomena Research Center (CEFT), University of Porto, Portugal
[2] Department of Mechanical Engineering, Hanbat National University, South Korea
[3] Department of Biomedical and Chemical Engineering, Department of Physics, Syracuse University, USA

Motivation and objective

The addition of small amounts of long chain polymer molecules to Newtonian solvents leads to dramatic drag reduction in wall-bounded turbulent flows. This phenomenon has been investigated experimentally and by Direct Numerical Simulation (DNS), but the availability of robust turbulence closures is still fairly limited. In the scope of Reynolds-averaged Navier-Stokes (RANS) models there are some recent k-epsilon based closures for viscoelastic fluids described by the FENE-P rheological constitutive equation, but either they are limited to low and intermediate drag reductions [1,2], or they do not allow the computation of the full polymer stress tensor, and viscoelastic stress work [3]. Clearly, all models require further improvements.

In this work a new turbulence model is developed and to deal with the turbulence anisotropy near-wall effects we adopt the k-ε $\overline{vv}$-f framework introduced by Durbin [4]. for Newtonian fluids and chosen also by Iaccarino et al. [3] for viscoelastic fluids. A new closure is presented for the cross correlation between the conformation and rate of strain fluctuating tensors across the whole range of drag reductions to allow computations of all components of the polymer stress tensor required if the closures are to be used in the future to model flows in geometries other than the fully-developed channel flows. The model has been validated against data from DNS simulations pertaining to low, intermediate and high drag reductions.

Governing equations

Time-averaged Momentum equation:

$$\rho\frac{\partial U_i}{\partial t}+\rho U_k\frac{\partial U_i}{\partial x_k}=-\frac{\partial \bar{p}}{\partial x_i}+\eta_s\frac{\partial^2 U_i}{\partial x_k\partial x_k}-\frac{\partial}{\partial x_k}(\rho\overline{u_i u_k})+\frac{\partial\overline{\tau_{ik,p}}}{\partial x_k}$$

$$\overline{\tau_{ij,p}}=\frac{\eta_p}{\lambda}\left[f(C_{kk})C_{ij}-f(L)\delta_{ij}\right]+\frac{\eta_p}{\lambda}\overline{f(C_{kk}+c_{kk})c_{ij}}$$

Time-averaged Constitutive equation:

$$\lambda\left[\overline{u_k\frac{\partial c_{ij}}{\partial x_k}}-\left(C_{kj}\frac{\partial U_i}{\partial x_k}+C_{ik}\frac{\partial U_j}{\partial x_k}\right)-\underbrace{\left(\overline{c_{kj}\frac{\partial u_i}{\partial x_k}}+\overline{c_{ik}\frac{\partial u_j}{\partial x_k}}\right)}_{\text{NLT}}\right]=-\left[f(C_{kk})C_{ij}-f(L)\delta_{ij}\right]$$

Turbulent kinetic energy and its dissipation:

$$U_j\frac{\partial k}{\partial x_j}=P_{kk}-\epsilon+\frac{\partial}{\partial x_j}\left(\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right)-\underbrace{\left(\tau^p_{ik}\frac{\partial u_j}{\partial x_k}+\tau^p_{jk}\frac{\partial u_i}{\partial x_k}\right)}_{\varepsilon_p}+\frac{\partial}{\partial x_k}\overline{\left(\tau^p_{ik}u_j+\tau^p_{jk}u_i\right)}$$

$$U_j\frac{\partial \epsilon}{\partial x_j}=\frac{C_{\epsilon1}P_{kk}-C_{\epsilon2}\epsilon}{T_t}-\epsilon+\frac{\partial}{\partial x_j}\left(\left(\nu+\frac{\nu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_j}\right)$$

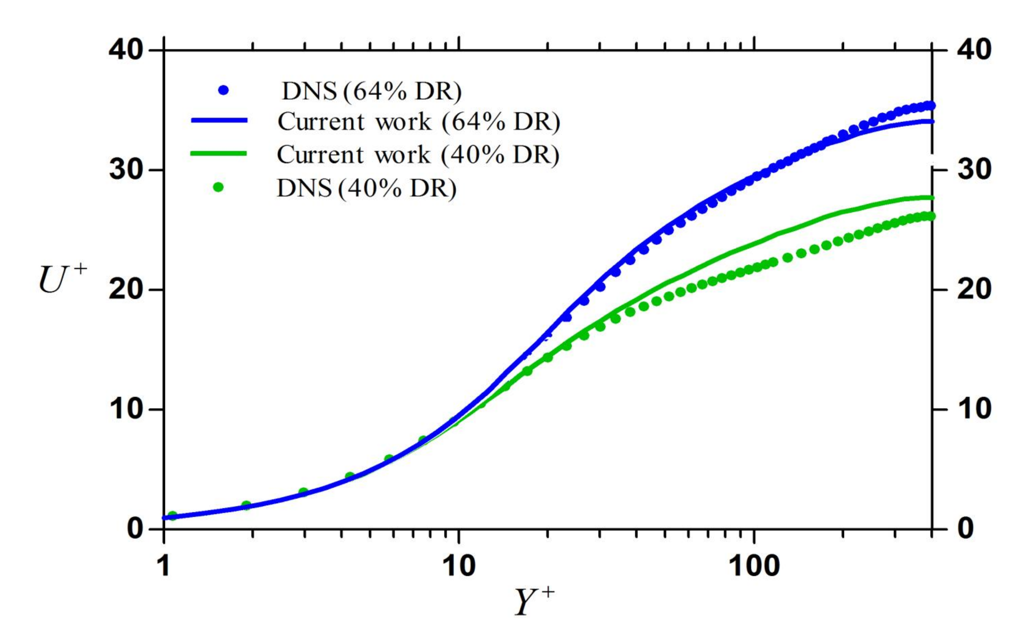
***$\overline{vv}$ and f* equations:**

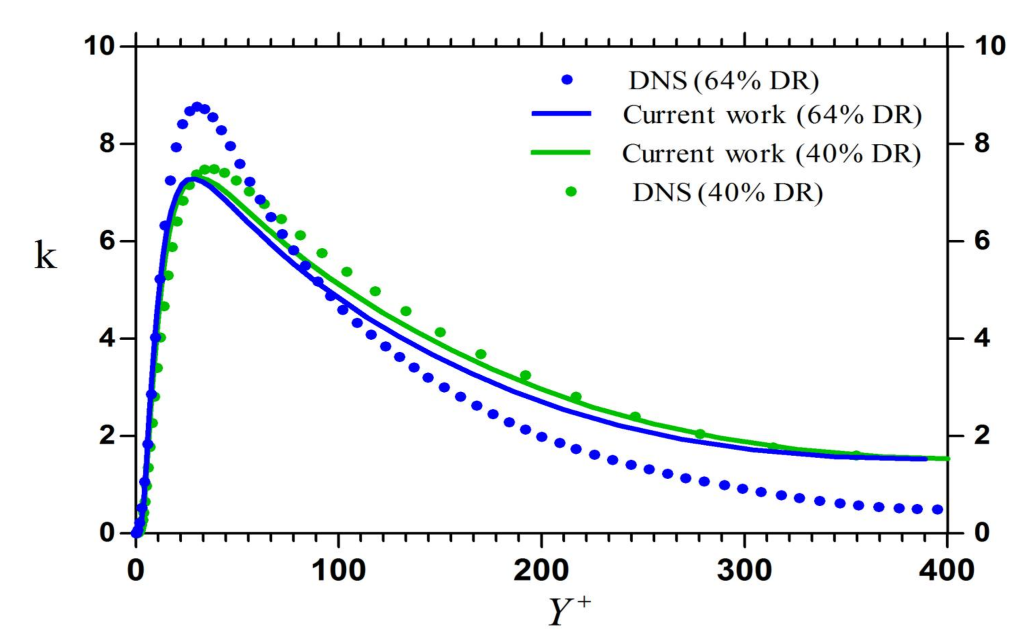
$$U_j\frac{\partial\overline{vv}}{\partial x_j}=kf+\frac{\partial}{\partial x_j}\left(\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial\overline{vv}}{\partial x_j}\right)-6\frac{\epsilon}{k}\overline{vv}-\epsilon_{yy,v}+D_{yy,v}$$

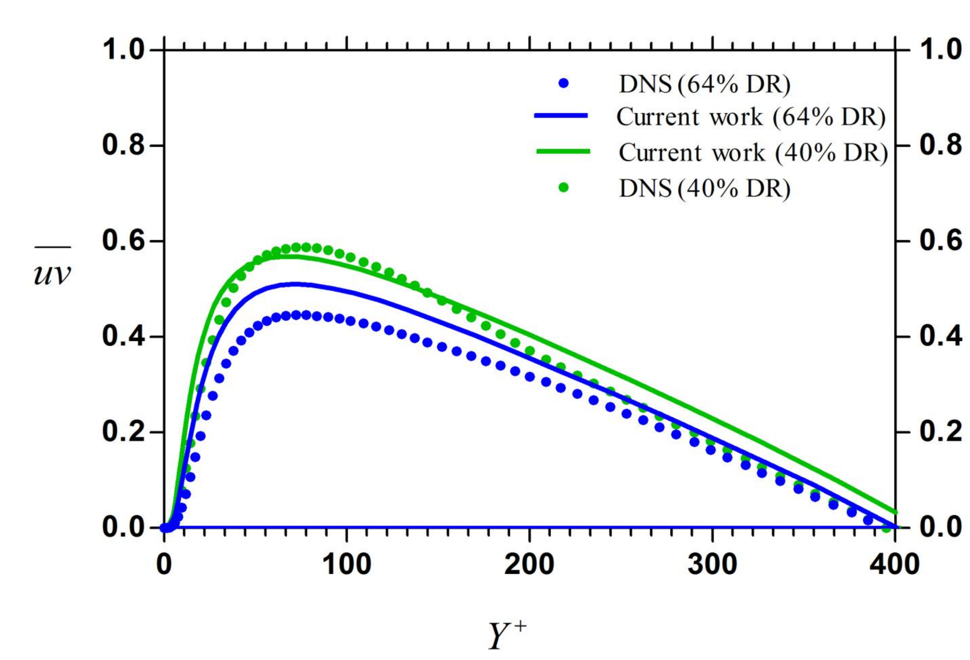
$$f-L_t^2\frac{\partial^2 f}{\partial x_j\partial x_j}=C_1\frac{\left(\frac{2}{3}-\frac{\overline{vv}}{k}\right)}{T_t}+C_2\frac{P_k}{k}-5\epsilon\frac{\overline{vv}}{k}$$

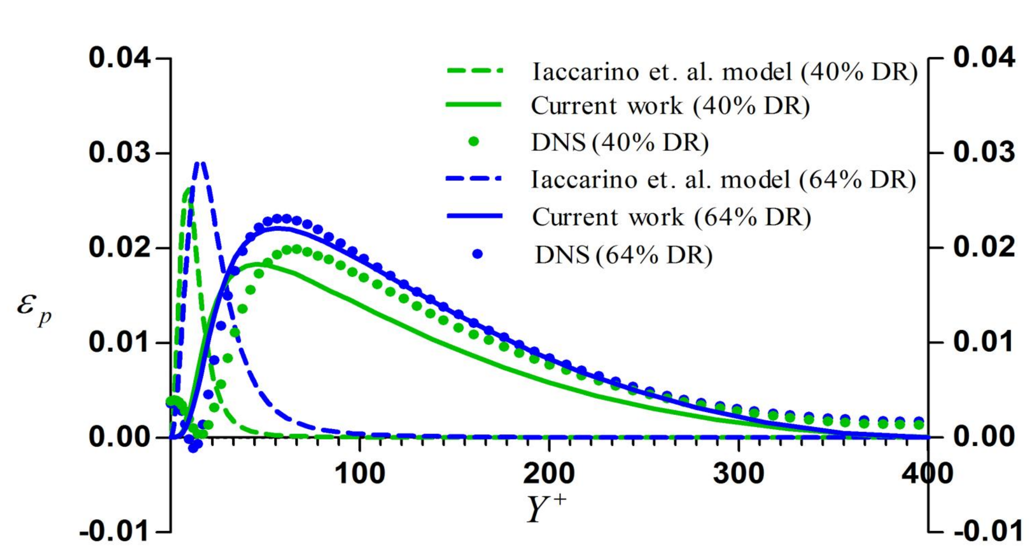
The problem is characterized by the Reynolds number (based on the friction velocity) Re=395, the Weissenberg number (ratio of polymer relaxation time and viscous time scale) $We_\tau=\lambda u_\tau^2/\nu$ =100, and polymer maximum length for 64% DR case is 14400 and for 40% DR case is 3600.

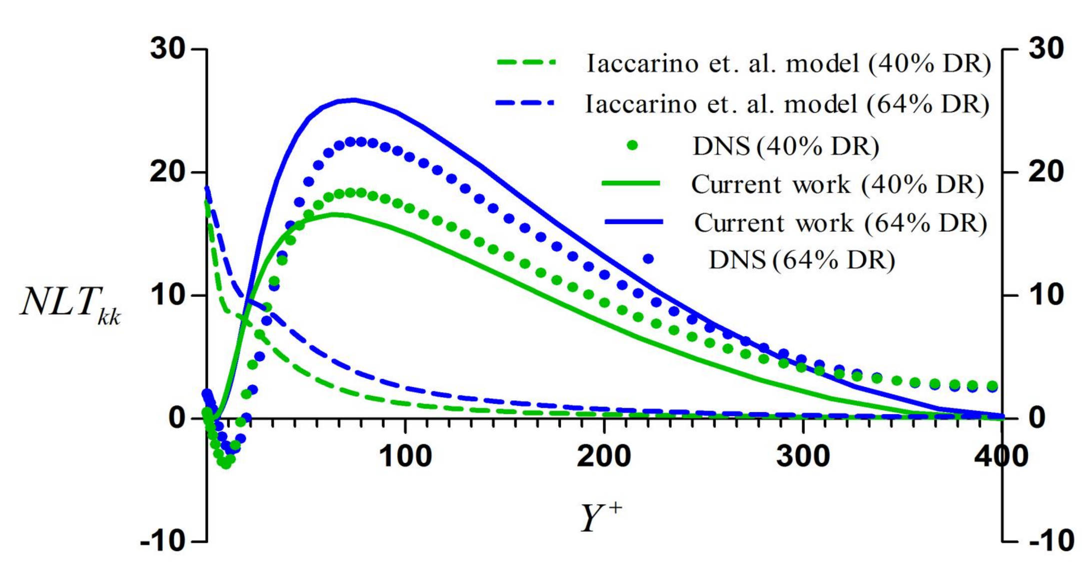
Results

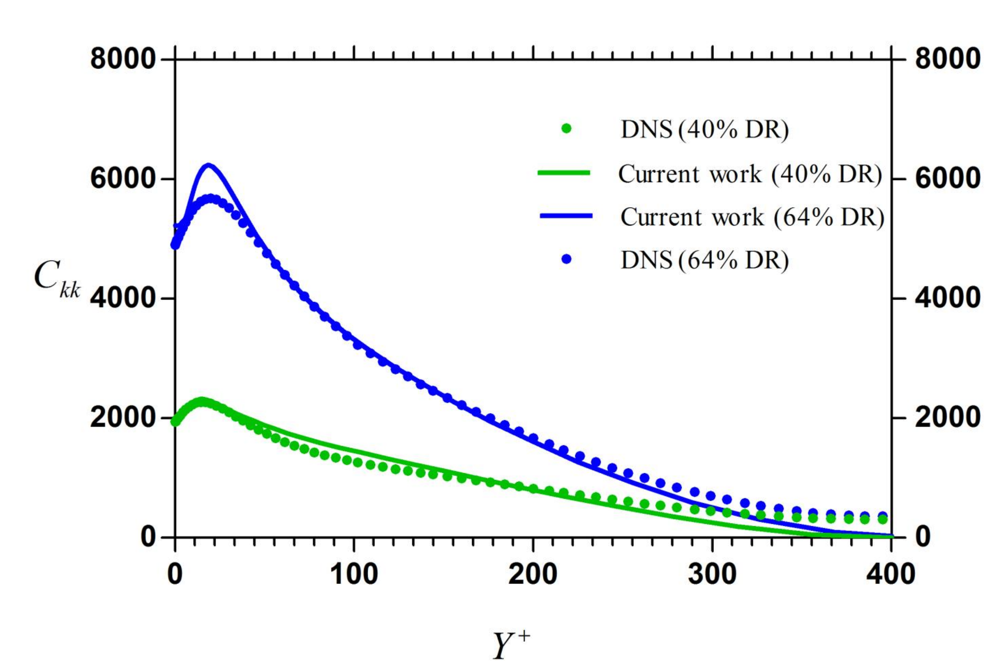












Conclusion

A model for predicting turbulent flows of homogeneous polymer solutions described by FENE-P has been developed. The equilibrium of production and dissipation of turbulence, typical of wall-bounded flows, is altered by the presence of the polymer chains, and this leads to suppression of wall-normal fluctuations, the thickening of the viscous sublayer and, eventually, to considerable drag reduction. Channel flow predictions that have been carried out for a combination of rheological parameters and the results have very good agreement with DNS data in terms of drag reduction percentage, the mean velocity, turbulent kinetic energy and polymer elongation profiles in the channels for low, intermediate and high drag reductions.

References

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