Purely elastic instabilities in a cross-slot flow

R. J. Poole
Dept. Engineering, Mechanical Engineering, University of Liverpool
Liverpool L69 3GH, UK, robpoole@liv.ac.uk

M. A. Alves
Departamento de Engenharia Química, CEFT, Faculdade de Engenharia da Universidade do Porto, Portugal, mmalves@fe.up.pt

A. Afonso
Departamento de Engenharia Química, CEFT, Faculdade de Engenharia da Universidade do Porto, Portugal, aafonso@fe.up.pt

F. T. Pinho
Escola de Engenharia, Universidade do Minho, Portugal, fpinho@dem.uminho.pt
CEFT, Faculdade de Engenharia Universidade do Porto, Portugal, fpinho@fe.up.pt

P. J. Oliveira
Departamento de Eng. Electromecânica, Universidade da Beira Interior,
Covilhã, Portugal, pjpo@ubi.pt

The Society of Rheology 79th annual meeting, 7th to 11th October 2007
Salt Lake City, USA

Outline

- Motivation and Previous work (UCM)
- Governing equations/numerical method
- Effect of solvent viscosity (Oldroyd-B) and inertia
- Effect of extensional viscosity (PTT)
- Conclusions
**Motivation**


Microfluidic flow in a “cross channel” geometry

---

Newtonian:  
Re < $10^{-2}$

PAA Boger fluid:  
Re < $10^{-2}$ (De=4.5)

---

**Motivation and Previous work**


Successfully used a numerical technique with a *simple* viscoelastic constitutive equation (**UCM**) to model this *steady asymmetry* under *creeping flow* conditions.

---

Newtonian  
De=0.4
**Motivation and Previous work**

Poole, Alves and Oliveira, accepted in *Physical Review Letters* (2007)

**Purely-elastic**: inertia decreases the degree of asymmetry and stabilizes the flow

\[
DQ = \frac{q_2 - q_1}{q_2 + q_1} = \frac{q_z - q_i}{Q}
\]

- \(DQ = 0\) → symmetric
- \(DQ = \pm 1\) → completely asymmetric

Re-De Map →
Governing equations

- Incompressible Viscoelastic fluid

\[ \nabla \cdot \mathbf{u} = 0 \quad (\text{Mass conservation}) \]

\[ \rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \beta \eta_o \nabla \cdot (\nabla \mathbf{u} + \nabla \mathbf{u}^T) + \frac{\eta_o}{\lambda} (1 - \beta) \nabla \cdot \mathbf{A} \quad (\text{Momentum conservation}) \]

\[ \lambda \left( \frac{DA}{Dt} - (\nabla \mathbf{u}) \mathbf{A} - \mathbf{A} (\nabla \mathbf{u})^T \right) = f(A) \quad (\text{Constitutive equation, based on the conformation tensor, A}) \]

- Examples:

\[ f(A) = \begin{cases} 
(A - I), & \text{Upper Convected Maxwell - UCM } (b=0) \\
(A - I), & \text{Oldroyd-B } (0 < b < 1) \\
-Y(\text{trA})(A - I), & \text{Phan-Thien and Tanner } (0 < b < 1), \text{ with} \\
\end{cases} \]

\[ Y(\text{trA}) = \begin{cases} 
1 + \epsilon (\text{trA} - 3) & \text{(linear)} \\
\exp[\epsilon (\text{trA} - 3)] & \text{(exponential)} \\
\end{cases} \]

Numerical method - brief description

- Finite Volume Method

- Structured, collocated and non-orthogonal meshes.
- Discretization (formally 2nd order)
  - Diffusive terms: central differences (CDS)
  - Advective terms, high resolution scheme: CUBISTA

- Dependent variables evaluated at cell centers;
- Special formulations for cell-face velocities and stresses;
Computational domain, boundary conditions

\[ Q = UD \]
\[ De = \lambda U/D \]

**Inlet** Boundary Conditions:
Fully-developed \( u(y) \) and \( t(y) \)

**Outlet** Boundary Conditions:
\[ \frac{\partial \phi}{\partial y} = 0 \]

---

Effect of mesh refinement

<table>
<thead>
<tr>
<th>NC</th>
<th>DOF</th>
<th>( \frac{(\Xi x_{MIN})}{D} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>12 801</td>
<td>76806</td>
</tr>
<tr>
<td>M2</td>
<td>50 601</td>
<td>303606</td>
</tr>
</tbody>
</table>

---

R. J. Poole, M.A. Alves, A. Afonso, F.T. Pinho and P.J. Oliveira, 79th SoR Meeting, 2007
Effect of mesh refinement (Oldroyd-B, \( b=1/9 \), \( De=0.35 \) and \( Re=0 \))

### Oldroyd-B Results – \( b \) effect

- Effect of increasing the solvent viscosity (creeping flow)

Flow asymmetry:

\[
DQ = \frac{q_2 - q_1}{q_2 + q_1}
\]

- \( DQ=0 \) → symmetric
- \( DQ=\pm 1 \) → completely asymmetric

Increases the critical \( De_{CR} \)

For \( b>3/9 \) flow became asymmetric unsteady;

<table>
<thead>
<tr>
<th>NC</th>
<th>DOF</th>
<th>( (x/x_{MIN})/D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>12 801</td>
<td>76806</td>
</tr>
<tr>
<td>M2</td>
<td>50 601</td>
<td>303606</td>
</tr>
</tbody>
</table>

R. J. Poole, M.A. Alves, A. Afonso, F.T. Pinho and P.J. Oliveira, 79th SoR Meeting, 2007
Oldroyd-B Results – $b$ effect

- Streamlines (creeping flow and $b=1/9$)

Oldroyd-B Results – Inertia effect

- Effect of increasing the Reynolds number ($b=1/9$)

Increases the critical $De_{CR}$

Decreases the degree of asymmetry;

For $Re>2$, asymmetric unsteady flow.

R. J. Poole, M.A. Alves, A. Afonso, F.T. Pinho and P.J. Oliveira, 79th SoR Meeting, 2007
Oldroyd-B Results – Inertia effect

• $Re \text{ vs. } De$ map ($b=1/9$)

![Diagram showing $Re$ vs. $De$ map with critical points for symmetric, steady asymmetric, and unsteady asymmetric flows.]

PTT Results

• Effect of varying $\varepsilon$ parameter in PTT model (creeping flow and $b=1/9$)

![Diagram showing the effect of varying $\varepsilon$ parameter on the PTT model with critical points for creeping flows.]

- Increases the critical $De_{cr}$
- Decreases the degree of asymmetry ($\varepsilon<0.04$);
- Increases the degree of asymmetry and in $De$ extension ($\varepsilon>0.04$);
- For $\varepsilon>0.08$, asymmetric stable flow disappears.
PTT Results

• $e$ vs. $De$ stability map (Creeping flow and $b=1/9$)

R. J. Poole, M.A. Alves, A. Afonso, F.T. Pinho and P.J. Oliveira, 79th SoR Meeting, 2007

PTT Results

• Streamlines (creeping flow, $e=0.02$ and $b=1/9$)

R. J. Poole, M.A. Alves, A. Afonso, F.T. Pinho and P.J. Oliveira, 79th SoR Meeting, 2007
Conclusions

- Increasing the solvent viscosity (increasing $b$) increases the critical De. For $b > 2/9$ the first steady instability disappears and the flow becomes unsteady (but still asymmetric);

- Increasing the level of inertia (increasing Re) shifts the onset of the instability to higher De and decreases the degree of asymmetry. Essentially inertia appears to stabilize the flow.

- For the PTT model, decreasing the extensional viscosity (increasing $e$) increases the critical De.

- We propose that this flow would make a good “benchmark” case for purely-elastic instabilities.

Acknowledgments

- Fundação para a Ciência e Tecnologia:
  - PROJECT: BD/28288/2007
  - PROJECT POCI/EME/59338

- Fundação Luso-Americana:
  - PROJECT: 544/2007

- The Society of Rheology:
  - Student Travel Grant
THE SOCIETY OF RHEOLOGY

79TH ANNUAL MEETING
PROGRAM AND ABSTRACTS

Hilton Salt Lake City Center
Salt Lake City, Utah
October 7 - 11, 2007

Program Committee:

Mataz Alcoutlabi
University of Utah
Patrick Anderson
Eindhoven University of Technology
Ralph Colby
Pennsylvania State University
John Dorgan
Colorado School of Mines
Pat Doyle
Massachusetts Institute of Technology
Eric Furst
University of Delaware
Michael Graham (Co-Chair)
University of Wisconsin
Erik Hobbie
National Institute of Standards & Technology
Ravi P. Jagadeeshan
Monash University
Rajesh Khare
Texas Tech University
Daniel Klingenberg (Co-Chair)
University of Wisconsin

Matt Liberatore
Colorado School of Mines
James Oberhauser
University of Virginia
Jai Pathak
US Naval Research Lab
Bob Prud’homme
Princeton University
Jonathan Rothstein
University of Massachusetts
Jay Schieber
Illinois Institute of Technology
Nina Shapley
Columbia University
Phil Sullivan
Schlumberger Technology Corp
Sachin Velankar
University of Pittsburgh
Henning Winter
University of Massachusetts

Local Arrangements:

Jules Magda
University of Utah

Andy Kraynik
Sandia National Laboratories

Abstract Book Editor and Webmaster:

Albert Co, University of Maine
### Meeting Schedule

<table>
<thead>
<tr>
<th>Time</th>
<th>Monday, October 8, 2007</th>
<th>Tuesday, October 9, 2007</th>
<th>Wednesday, October 10, 2007</th>
<th>Thursday, October 11, 2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:30</td>
<td>F. Waleffe (PL1)</td>
<td>J. F. Brady (PL2)</td>
<td>J. A. Lewis (PL3)</td>
<td>8:05</td>
</tr>
<tr>
<td>9:20</td>
<td>Coffee</td>
<td>Coffee</td>
<td>Coffee</td>
<td>8:30</td>
</tr>
<tr>
<td>9:45</td>
<td>SC1 FM1 MR1 PS1</td>
<td>FM14 MR14 PS14</td>
<td>SC27 BE4 SM3 BS5</td>
<td>SC41 BE17 SM17 SG1</td>
</tr>
<tr>
<td>10:10</td>
<td>SC2 FM2 MR2 PS2</td>
<td>SC15 FM15 MR15 PS15</td>
<td>SC28 BE5 SM4 BS6</td>
<td>8:30</td>
</tr>
<tr>
<td>10:35</td>
<td>SC3 FM3 MR3 PS3</td>
<td>SC16 FM16 MR16 PS16</td>
<td>SC29 BE6 SM5 BS7</td>
<td>SC42 BE18 SM18 SG2</td>
</tr>
<tr>
<td>11:00</td>
<td>SC4 FM4 MR4 PS4</td>
<td>SC17 FM17 MR17 PS17</td>
<td>SC30 BE7 SM6 BS8</td>
<td>8:55</td>
</tr>
<tr>
<td>11:25</td>
<td>SC5 FM5 MR5 PS5</td>
<td>SC18 FM18 MR18 PS18</td>
<td>SC31 BE8 SM7 BS9</td>
<td>EP1 BE19 SM19 SG3</td>
</tr>
<tr>
<td>11:50</td>
<td>Lunch</td>
<td>Lunch</td>
<td>Lunch</td>
<td>9:20</td>
</tr>
<tr>
<td>1:30</td>
<td>SC6 FM6 MR6 PS6</td>
<td>SC19 FM19 MR19 PS19</td>
<td>SC32 BE9 SM8 BS10</td>
<td>SC43 BE21 SM21 SG5</td>
</tr>
<tr>
<td>1:55</td>
<td>SC7 FM7 MR7 PS7</td>
<td>SC20 FM20 MR20 PS20</td>
<td>SC33 BE10 SM9 BS11</td>
<td>10:10</td>
</tr>
<tr>
<td>2:45</td>
<td>SC9 FM9 MR9 PS9</td>
<td>SC22 FM22 MR22</td>
<td>SC35 BE12 SM11 BS13</td>
<td>11:35</td>
</tr>
<tr>
<td>3:10</td>
<td>Coffee</td>
<td>Coffee</td>
<td>Coffee</td>
<td>EP4 BE22 SM22 SG6</td>
</tr>
<tr>
<td>3:35</td>
<td>SC10 FM10 MR10 PS10</td>
<td>SC23 FM23 MR23 BS1</td>
<td>SC36 BE13 SM12 BS14</td>
<td>11:00</td>
</tr>
<tr>
<td>4:00</td>
<td>SC11 FM11 MR11 PS11</td>
<td>SC24 BE1 MR24 BS2</td>
<td>SC37 BE14 SM13 BS15</td>
<td>SC44 BE23 SM23 SG7</td>
</tr>
<tr>
<td>4:25</td>
<td>SC12 FM12 MR12 PS12</td>
<td>SC25 BE2 SM1 BS3</td>
<td>SC38 BE15 SM14 BS16</td>
<td>11:25</td>
</tr>
<tr>
<td>5:15</td>
<td>End</td>
<td>5:15 End</td>
<td>5:15 End</td>
<td>12:15</td>
</tr>
<tr>
<td></td>
<td>5:30 Business Meeting</td>
<td>5:30 Business Meeting</td>
<td>5:30 Business Meeting</td>
<td>End</td>
</tr>
<tr>
<td></td>
<td>7:00 Awards Reception</td>
<td>7:00 Awards Reception</td>
<td>7:00 Awards Reception</td>
<td>6:00 Poster Session &amp; Reception</td>
</tr>
<tr>
<td></td>
<td>8:00 Awards Banquet</td>
<td>8:00 Awards Banquet</td>
<td>8:00 Awards Banquet</td>
<td></td>
</tr>
</tbody>
</table>

### Session Codes

- **BE** = Blends, Emulsions and Multiphase Fluids
- **BS** = Biological and Self-assembled Systems
- **EP** = Rheology in Energy Production
- **FM** = Non-Newtonian Fluid Mechanics
- **MR** = Micro rheology, Microfluidics and Confined Systems
- **PL** = Plenary Lectures
- **PS** = Polymer Solutions
- **SC** = Suspensions, Colloids and Granular Media
- **SG** = Solids and Glasses
- **SM** = Entangled Solutions and Melts
## Contents

### Monday Morning

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plenary Lectures</td>
<td>1</td>
</tr>
<tr>
<td>Suspensions, Colloids and Granular Media</td>
<td>1</td>
</tr>
<tr>
<td>Non-Newtonian Fluid Mechanics</td>
<td>3</td>
</tr>
<tr>
<td>Microrheology, Microfluidics and Confined Systems</td>
<td>4</td>
</tr>
<tr>
<td>Polymer Solutions</td>
<td>6</td>
</tr>
</tbody>
</table>

### Monday Afternoon

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspensions, Colloids and Granular Media</td>
<td>9</td>
</tr>
<tr>
<td>Non-Newtonian Fluid Mechanics</td>
<td>11</td>
</tr>
<tr>
<td>Microrheology, Microfluidics and Confined Systems</td>
<td>13</td>
</tr>
<tr>
<td>Polymer Solutions</td>
<td>15</td>
</tr>
</tbody>
</table>

### Tuesday Morning

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plenary Lectures</td>
<td>19</td>
</tr>
<tr>
<td>Suspensions, Colloids and Granular Media</td>
<td>19</td>
</tr>
<tr>
<td>Non-Newtonian Fluid Mechanics</td>
<td>20</td>
</tr>
<tr>
<td>Microrheology, Microfluidics and Confined Systems</td>
<td>22</td>
</tr>
<tr>
<td>Polymer Solutions</td>
<td>23</td>
</tr>
</tbody>
</table>

### Tuesday Afternoon

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspensions, Colloids and Granular Media</td>
<td>27</td>
</tr>
<tr>
<td>Non-Newtonian Fluid Mechanics</td>
<td>29</td>
</tr>
<tr>
<td>Blends, Emulsions and Multiphase Fluids</td>
<td>31</td>
</tr>
<tr>
<td>Microrheology, Microfluidics and Confined Systems</td>
<td>32</td>
</tr>
<tr>
<td>Entangled Solutions and Melts</td>
<td>34</td>
</tr>
<tr>
<td>Polymer Solutions</td>
<td>34</td>
</tr>
<tr>
<td>Biological and Self-assembled Systems</td>
<td>35</td>
</tr>
</tbody>
</table>

### Wednesday Morning

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plenary Lectures</td>
<td>37</td>
</tr>
<tr>
<td>Suspensions, Colloids and Granular Media</td>
<td>37</td>
</tr>
<tr>
<td>Blends, Emulsions and Multiphase Fluids</td>
<td>39</td>
</tr>
<tr>
<td>Entangled Solutions and Melts</td>
<td>40</td>
</tr>
<tr>
<td>Biological and Self-assembled Systems</td>
<td>42</td>
</tr>
</tbody>
</table>

### Wednesday Afternoon

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspensions, Colloids and Granular Media</td>
<td>45</td>
</tr>
<tr>
<td>Blends, Emulsions and Multiphase Fluids</td>
<td>47</td>
</tr>
<tr>
<td>Entangled Solutions and Melts</td>
<td>50</td>
</tr>
<tr>
<td>Biological and Self-assembled Systems</td>
<td>52</td>
</tr>
</tbody>
</table>
This publication was generated with scripts developed by Albert Co. The contents of this publication were extracted from the database of The Society of Rheology abstract submission web site at http://www.rheology.org/sorabst/. Online version is available at http://www.rheology.org/sor07a/.
Purely elastic instabilities in a cross-slot flow

Robert J. Poole, Manuel A. Alves, Alexandre Afonso, Fernando T. Pinho, and Paulo J. Oliveira

Department of Engineering, University of Liverpool, Liverpool L69 3GH, United Kingdom; Departamento de Engenharia Química, CEFT, Faculdade de Engenharia da Universidade do Porto, Porto 4200-465, Portugal; Centro Estudos de Fenómenos de Transporte, Faculdade de Engenharia da Universidade do Porto, Porto 4200-465, Portugal; Electromechanical Engineering Department, University of Beira Interior, Covilhã, Castelo Branco 6201-001, Portugal.

In a recent paper Arratia et al. [Phys. Rev. Lett. Vol. 96(14) (2006)] demonstrated experimentally that the low Reynolds number flow of a viscoelastic polymer solution in a microfluidic cross-slot geometry can produce two types of instabilities. In the first instability above a critical Deborah number the flow becomes asymmetric, but remains steady. Upon increasing the Deborah number still further a second instability sets in and the flow becomes strongly time dependent.

In the current paper we demonstrate numerically, using a finite-volume method, that both these instabilities can be predicted using the upper-convected Maxwell model under creeping flow conditions, in so doing demonstrating that both instabilities are purely elastic in nature. We also show the stabilizing effect of inertia in reducing the flow asymmetry, and we highlight these effects in a De-Re map, identifying the regions of steady symmetric, steady asymmetric and unsteady (asymmetric) flow regimes. The effects of rounding the corners and the use of more realistic viscoelastic constitutive equations will also be discussed.

A mechanism for oscillatory instability in viscoelastic cross-slot flow

Li Xi and Michael D. Graham

Department of Chemical and Biological Engineering, University of Wisconsin-Madison, Madison, WI 53706-1691, United States.

Interior stagnation point flows of viscoelastic liquids arise in a wide variety of applications including extensional viscometry, polymer processing and microfluidics. Experimentally, these flows have long been known to exhibit instabilities, but the mechanisms underlying them have not previously been elucidated. We computationally demonstrate the existence of a supercritical oscillatory instability of low-Reynolds number viscoelastic flow in a two-dimensional cross-slot geometry. The fluctuations are closely associated with the "birefringent strand" of highly stretched polymer chains associated with the outflow from the stagnation point at high Weissenberg number. Additionally, we describe the mechanism of instability, which arises from the coupling of flow with extensional stresses and their steep gradients in the stagnation point region.

Low inertia mixing of viscous fluids by a chemically triggered shear flow instability

Teodor I. Burghela, Kerstin Wielage-Burchard, Ian A. Frigaard, and Mark D. Martinez

Department of Mathematics, University of British Columbia, Vancouver, British Columbia V6T 1Z2, Canada; University of British Columbia, Vancouver, Canada; Department of Chemical and Biological Engineering, University of British Columbia, Vancouver, British Columbia V6T 1Z4, Canada.

We study experimentally the mixing of two viscous fluid streams by a low Reynolds number shear flow instability triggered by a fast chemical reaction. The viscous fluids are evenly injected side by side in a T-shaped straight channel. An acid-base reaction taking place at the interface between a Newtonian fluid and a Carbopol-940 solution leads to a strong viscosity stratification, which locally destabilizes the flow. As one advances down stream, complex secondary flow patterns develop resulting in efficient mixing. We also present preliminary analysis of the flow and instability, via a simplified model based on averaging through the channel height.

The effects of poly(ethylene oxide) on the stability boundaries of flow regimes in co- and counter-rotating Taylor-Couette flow

Cari S. Dutcher and Susan J. Muller

Department of Chemical Engineering, University of California, Berkeley, CA 94720, United States.

The presence of just a few parts per million of high molecular weight linear polymers is known to have a significant impact on turbulent features in flow, namely polymer-induced turbulent drag reduction. To contribute to the quantitative understanding of this dramatic phenomenon, the authors have studied the influence of dilute polyethylene oxide (PEO) solutions on isolated secondary flow features by mapping the stability boundaries of multiple flow regimes in a Taylor-Couette (TC) geometry. This work expands on previous work involving PEO solutions by mapping the boundaries for both co- and counter-rotational flows, with a focus on modification of higher order transitions (up to $Re_{inert} \sim O(10^3)$). The elasticity, defined as the ratio of the polymer relaxation time to the inertial time scale, for the PEO solutions in a viscosified aqueous solvent, ranges from $O(10^{-4})$ to $O(10^{-3})$. The polymeric solutions are characterized by a number of independent techniques, including dynamic and steady shear flows, extensional flows (CaBER), sessile drop experiments and dynamic light scattering. Changes in stability in TC experiments were found during adiabatic increases of the inner cylinder Reynolds number using spectral analysis and flow visualization in 2D planes of radial, axial, projected azimuthal and time dimensions. The resultant flow state transitions are compared to previous stability maps for Newtonian fluids obtained in our TC geometry of radius ratio 0.912 and aspect ratio 60.7. As a result, the effect of elasticity on the critical