A k-ω LOW REYNOLDS NUMBER TURBULENCE MODEL FOR TURBULENT CHANNEL FLOW OF FENE-P FLUIDS

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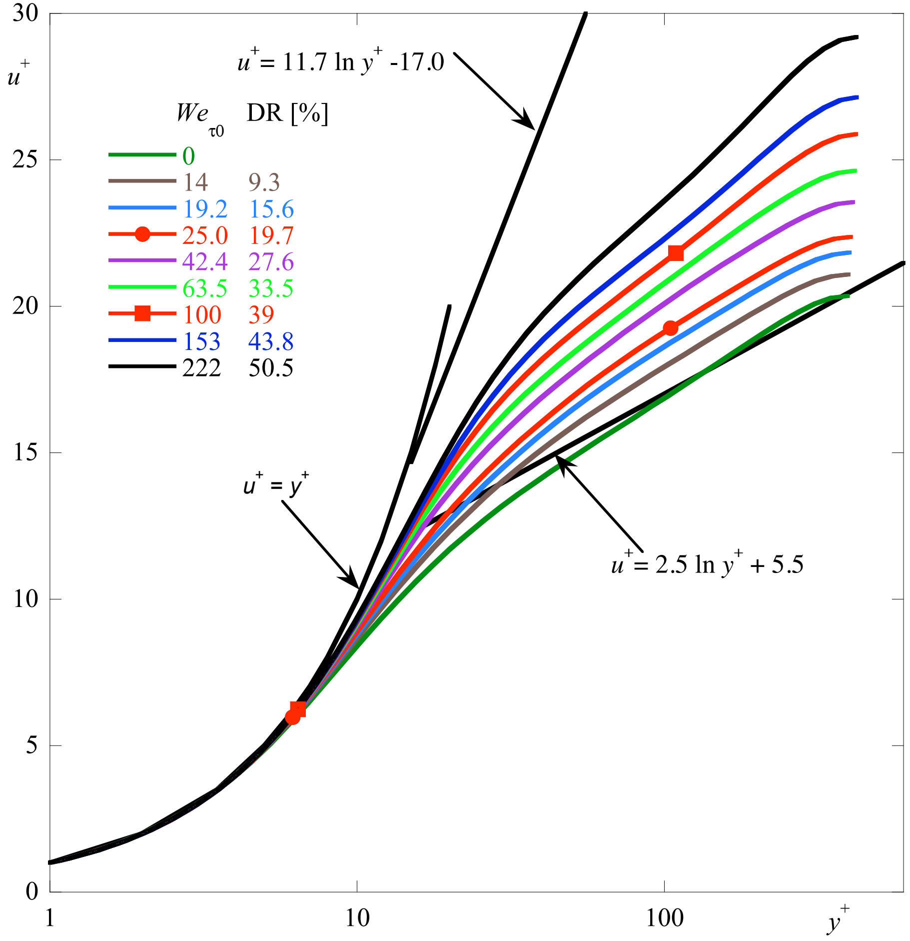
Drag reduction: motivation

Drag reduction in fully-developed channel flow



Existing models (1st order)

k-ε: Pinho et al, JNNFM 154 (2008) 89

k-ε improved: Pinho et al (2010) in prep

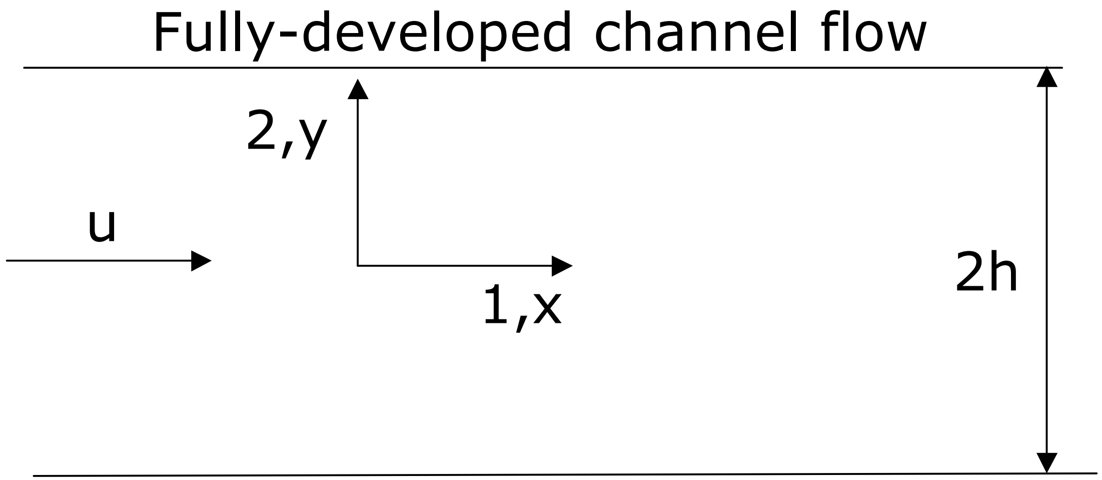
k-ε-v^2-f: Iaccarino et al,165(2010)376

Can a k-ω model improve on k-ε?

Advantages:
Valid across all BL (no damping)
Better in BL with adverse pres. grad.

Disadvantages:
Too sensitive to ω in free stream

DNS cases: channel flow



$$We_\tau = \frac{\lambda u_\tau^2}{\nu_0}$$

$$Re_\tau = \frac{h u_\tau}{\nu_0}$$

DNS test/calibration cases

$$Re_\tau = 395, \beta = 0.9, L^2 = 900$$

Low Drag Reduction

$$We_\tau = 25, DR = 18\%$$

High Drag Reduction

$$We_\tau = 100, DR = 37\%$$

New model: Governing Equations

Reynolds decomposition: $\hat{B} = B + b'$

Overbar & upper-case: time-averaged quantities
Lower-case: fluctuating quantities

Continuity: $\frac{\partial U_i}{\partial x_i} = 0$

Momentum balance:

$$\rho \frac{\partial U_i}{\partial t} + \rho U_k \frac{\partial U_i}{\partial x_k} = -\frac{\partial \bar{p}}{\partial x_i} + \eta_s \frac{\partial^2 U_i}{\partial x_k \partial x_k} - \frac{\partial}{\partial x_k}\left(\rho \overline{u_i u_k}\right) + \frac{\partial \bar{\tau}_{ik,p}}{\partial x_k}$$

$$\bar{\tau}_{ij} = 2\eta_s S_{ij} + \bar{\tau}_{ij,p}$$

Rheological constitutive equation: **FENE-P**

$$\bar{\tau}_{ij,p} = \frac{\eta_p}{\lambda}\left[f(C_{kk})C_{ij} - f(L)\delta_{ij}\right] + \frac{\eta_p}{\lambda}\overline{f(C_{kk} + c_{kk})c_{ij}}$$

RACE →

$$\overset{\nabla}{C}_{ij} + \overline{u_k \frac{\partial c_{ij}}{\partial x_k}} - \left(\overline{c_{kj}\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik}\frac{\partial u_j}{\partial x_k}}\right) = -\frac{\bar{\tau}_{ij,p}}{\eta_p}$$

M_{ij} CT_{ij} NLT_{ij}

Closures required

Independent of turbulence model

Conformation (RACE) equation

$$\lambda \overset{\nabla}{C}_{ij} + \lambda\left[u_k \frac{\partial c_{ij}}{\partial x_k} - \left(c_{kj}\frac{\partial u_i}{\partial x_k} + c_{ik}\frac{\partial u_j}{\partial x_k} \right) \right] = -\left[f(C_{kk})C_{ij} - f(L)\delta_{ij} \right] - f(C_{kk} + c_{kk})c_{ij}$$

M_{ij} $\quad CT_{ij}$ $\quad NLT_{ij}$

Model for NLT_{ij} essentially identical to that for k-ε, except in some coefficients/ functions

$$f(C_{mm})\frac{NLT_{ij}}{\lambda} = \frac{f(C_{mm})}{\lambda}\left\{ f_{N_1} C_{ij}\frac{f(C_{mm})}{\lambda} - f_{N_2}\left[C_{kj}\frac{\partial U_i}{\partial x_k} + C_{ik}\frac{\partial U_j}{\partial x_k} \right] \right\}$$

$$+ f_{N_3}\left[\frac{C_{kn}}{\nu_0\sqrt{2S_{pq}S_{pq}}}\left[\overline{u_i u_m}\frac{\partial U_j}{\partial x_k}\frac{\partial U_m}{\partial x_n} + \overline{u_j u_m}\frac{\partial U_i}{\partial x_k}\frac{\partial U_m}{\partial x_n} \right] + \frac{1}{\nu_0\sqrt{2S_{pq}S_{pq}}}\left[\frac{\partial U_k}{\partial x_n}\frac{\partial U_m}{\partial x_k}\left(C_{jn}\overline{u_i u_m} + C_{in}\overline{u_j u_m} \right) \right] \right]$$

$$- f_{N_4}\left[C_{jn}\frac{\partial U_k}{\partial x_n}\frac{\partial U_i}{\partial x_k} + C_{in}\frac{\partial U_k}{\partial x_n}\frac{\partial U_j}{\partial x_k} + C_{kn}\left(\frac{\partial U_j}{\partial x_n}\frac{\partial U_i}{\partial x_k} + \frac{\partial U_i}{\partial x_n}\frac{\partial U_j}{\partial x_k} \right) \right] + f_{N_5}\frac{4}{15}\frac{\varepsilon^N}{\beta \nu_s}C_{mm}\delta_{ij}$$

$$f_{N_i} = f(We_{\tau_0}, y^+)$$

The specific dissipation rate: ω

$$-\rho\overline{u_i u_j} = 2\mu_T S_{ij} - \frac{2}{3}\rho k \delta_{ij}$$

Prandtl- Kolmogorov closure for Reynolds Stress

$$\mu_T = \rho\sqrt{k}\,l$$

$k \longrightarrow$ Transport equation

How to determine *l* ? Generally difficult ! Various alternatives

1) Estimate of dissipation (large scale)

$$\varepsilon \sim \frac{k^{3/2}}{l} \qquad [\varepsilon] = \frac{length^2}{time^3}$$

Chou (1945) $\longrightarrow$ $\mu_T \sim \rho\frac{k^2}{\varepsilon}$

2) Specific dissipation rate: $\omega \sim \frac{\varepsilon}{k}$ Kolmogorov (1942) $\longrightarrow$ $\boxed{\mu_T \sim \rho\frac{k}{\omega}}$

$$[\omega] = \frac{1}{time}$$

ω is better behaved near walls,

but more sensitive far from walls $\qquad \omega \to \frac{2\nu}{C_k \cdot y^2}$

Reynolds stress closure: eddy viscosity model (k-ε & k-ω)

Prandtl-Kolmogorov model

$$-\overline{u_i u_j} = 2\nu_T S_{ij} - \frac{2}{3} k \delta_{ij}$$

Pinho et al (2010): k-ε

$$\nu_T = \nu_T^N - \nu_T^P$$

$$\nu_T^N = C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}^N}$$

$$\nu_T^P = C_\mu f_\mu C_\mu^P f_\mu^P C_{kk} \frac{k^2}{\tilde{\varepsilon}^N}$$

New model with $\omega^N = \dfrac{\varepsilon^N}{C_k k}$

Note: $C_k = C_\mu$

$$\nu_T^N = f_\mu \frac{k}{\omega^N}$$

$$\nu_T^P = f_\mu C_\mu^P f_\mu^P C_{kk} \frac{k}{\omega^N}$$

Transport equation for *k*

$$\rho\frac{Dk}{Dt} = -\rho\overline{u_i u_k}\frac{\partial U_i}{\partial x_k} - \rho\overline{u_i\frac{\partial k'}{dx_i}} - \frac{\partial\overline{p'u_i}}{\partial x_i} + \eta_s\frac{\partial^2 k}{\partial x_i\partial x_i} - \eta_s\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_k}} + \frac{\partial\overline{\tau'_{ik,p}u_i}}{\partial x_k} - \overline{\tau'_{ik,p}\frac{\partial u_i}{\partial x_k}}$$

0	P_k	D^T	D^N	$-\varepsilon^N$	D^V	$-\varepsilon^V$
	exact	Unchanged (Newtonian)	exact		Previous Model	Previous Model

$$-\varepsilon^N = -C_k k\omega^N$$

$$D^V = \frac{\eta_p}{\lambda}\frac{\partial}{\partial x_k}\left[C_{ik}\overline{f(C_{mm}+c_{mm})u_i} + \overline{c_{ik}f(C_{mm}+c_{mm})u_i}\right]$$

$$\approx \frac{\eta_p}{\lambda}\frac{\partial}{\partial x_k}\left[f(C_{mm})\frac{C_{ik}(FU)_i + (CU)_{ijk}}{2}\right]$$

Essentially **unchanged**
Coefficients & functions

$$C_{ik}(FU)_i \simeq f_{FU}C_{kn}\frac{\overline{u_i u_i}}{\partial x_n}$$

$$f_{FU} = f_{FU}(We)$$

$$f_{\beta_1}, f_{\beta_7} = f_\beta(We)$$

$$\frac{f(C_{mm})CU_{ijk}}{\lambda} = -f_{\beta_1}\left(\overline{u_i u_m}\frac{\partial C_{kj}}{\partial x_m} + \overline{u_j u_m}\frac{\partial C_{ik}}{\partial x_m}\right) - \frac{f_{\beta_7}f(C_{mm})}{\lambda}\left[\pm\sqrt{\overline{u_j^2}}C_{ik} \pm \sqrt{\overline{u_i^2}}C_{jk}\right]$$

Viscoelastic stress work: ε^V

$$\varepsilon^V \equiv \frac{1}{\rho}\overline{\tau'_{ik,p}\frac{\partial u_i}{\partial x_k}} \approx \frac{\eta_p}{\rho\lambda}\left[\overline{c_{ik} f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}}\right]$$

$$\overline{f' c'_{ik}\frac{\partial u_i}{\partial x_k}} \approx f_{\varepsilon^V} \times f\left(C_{mm}\right)\overline{c_{ik}\frac{\partial u_i}{\partial x_k}}$$

$\overline{c_{ik}\frac{\partial u_i}{\partial x_k}} \rightarrow$ ***NLT_{ii}***

$$f_{\varepsilon^V} = f_{\varepsilon^V}\left(We\right)$$

Same model as in k-ε

$$\varepsilon^V = f_{\varepsilon^V}\frac{\eta_p}{\rho\lambda} f\left(C_{mm}\right)\frac{NLT_{ii}}{2}$$

Unchanged

Transport equation of *k*: final modeled form

Based on Newtonian model of Nagano & Hishida (1984)

$$0=\frac{d}{dy}\left[\left(\eta_p+\eta_s+\frac{\rho f_T \nu_T}{\sigma_k}\right)\frac{dk}{dy}\right]+P_k-\rho C_k \omega^N k+\frac{\eta_p}{\lambda}\frac{d}{dy}\left[f\left(C_{mm}\right)\frac{C_{nk}\left(FU\right)_n+CU_{nny}}{2}\right]-\eta_p\frac{f\left(C_{mm}\right)}{\lambda}\frac{NLT_{nn}}{2}$$

New form

$$\sigma_k=1.1$$

$$f_T=1+3.5\exp\left[-\left(R_T/150\right)^2\right]$$

Variable Prandtl numbers: Nagano & Shimada (1993), Park and Sung (1995)

Specific rate of deformation: transport equation

Destruction — Turbulent diffusion — Viscoelastic interaction

$$\frac{D\varepsilon^N}{Dt} = P_{\varepsilon^N} - \Phi_{\varepsilon^N} + \Pi_{\varepsilon^N} + D^T_{\varepsilon^N} + D^N_{\varepsilon^N} + E^V_{\varepsilon^N}$$

Production — Redistribution — Molecular diffusion

$$\frac{Dk}{Dt} = P_k - \varepsilon^N + \Pi_k + D^T_k + D^N_k + D^V_k - \varepsilon^V$$

$$\omega^N = \frac{\varepsilon^N}{C_\mu k} \longrightarrow \frac{D\omega^N}{Dt} = \frac{1}{C_\mu k}\frac{D\varepsilon^N}{Dt} - \frac{\omega^N}{k}\frac{Dk}{Dt}$$

$$\frac{D\omega^N}{Dt} = P_{\omega^N} - \Phi_{\omega^N} + \Pi_{\omega^N} + D^T_{\omega^N} + D^N_{\omega^N} + E^V_{\omega^N}$$

$$\rho\frac{D\omega^N}{Dt} = C_{\omega_1}\frac{\omega}{k}P_k + \frac{\partial}{\partial x_i}\left[\left(\eta_s + \eta_p + \rho\frac{\nu_T}{\sigma_\varepsilon}\right)\frac{\partial\omega^N}{\partial x_i}\right] - C_{\omega_2}\rho\omega^2 + \rho\frac{C_\omega}{k}\left(\frac{\eta_s}{\rho} + \nu_T\right)\frac{\partial k}{\partial x_i}\frac{\partial\omega}{\partial x_i} + E^V_{\omega^N}$$

Viscous cross-diffusion (Bredberg et al. 2002)

Viscoelastic contribution to ω: model

Definition and model

$$E^V_{\omega^N} = \frac{1}{C_\mu k} E^V_{\varepsilon^N} - \frac{\omega}{k} D^V_k + \frac{\omega}{k} \varepsilon^V$$

$E^V_{\varepsilon^N}$ → Slide 9 (D^V_k); Slide 10 (ε^V)

$$E^V_{\varepsilon^N} \equiv 2\eta_s \frac{\eta_p}{\lambda\left(L^2-3\right)} \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_k} \left\{ \frac{\partial}{\partial x_m} \left[f\left(C_{nn}\right) f\left(\hat{C}_{pp}\right) c'_{qq} C_{ik} \right] \right\}}$$

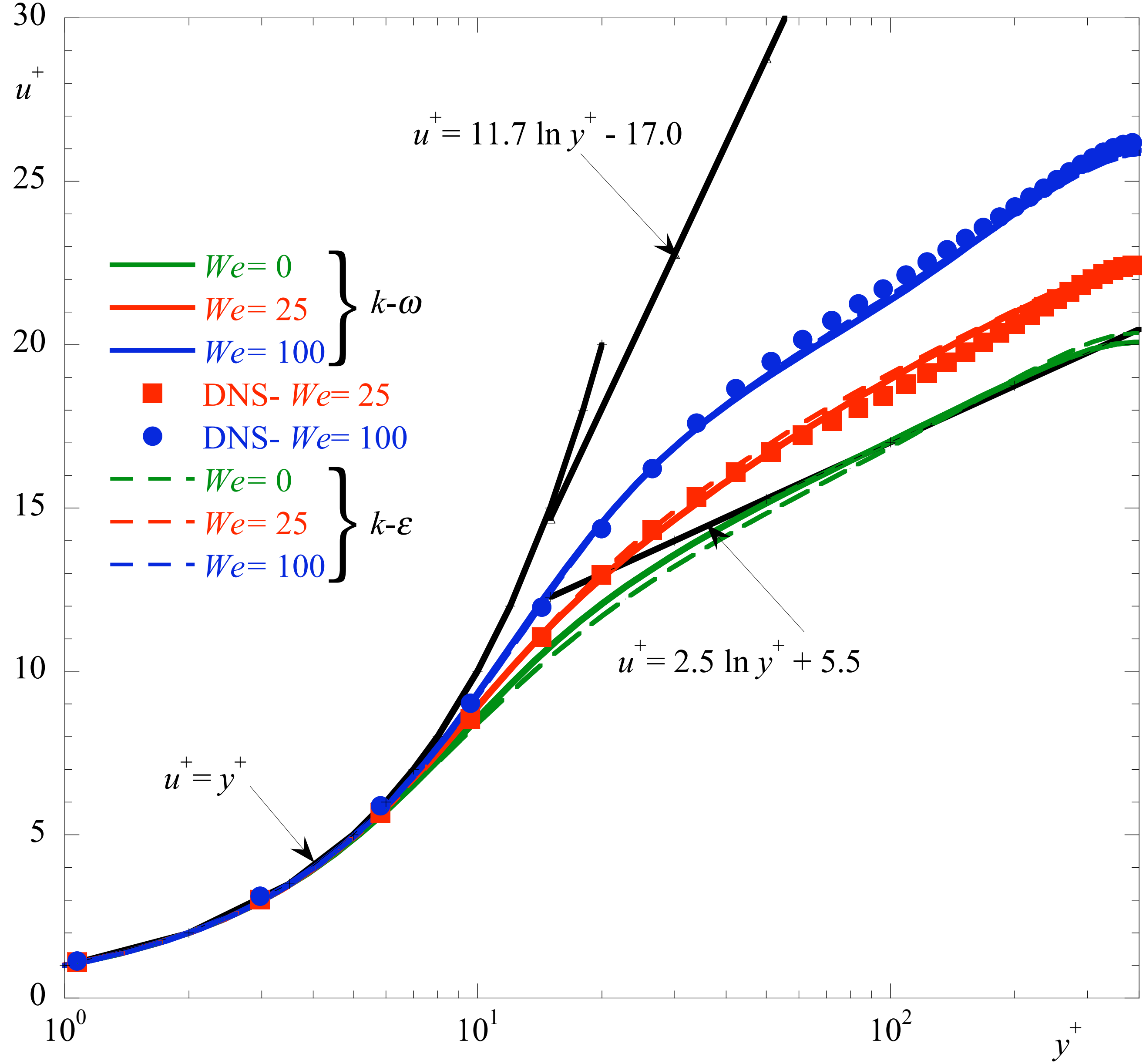
Model of $E^V_{\varepsilon^N}$

$$E^V_{\varepsilon^N} \equiv -f^{\varepsilon}_{DR} \frac{\varepsilon^{N^2}}{k} \left[C_{\varepsilon F1} \frac{\varepsilon^V}{\varepsilon^N} \left(L^2-3\right)^2 + C_{\varepsilon F2} \left[C_{ii} f\left(C_{kk}\right) \right]^2 \right]$$

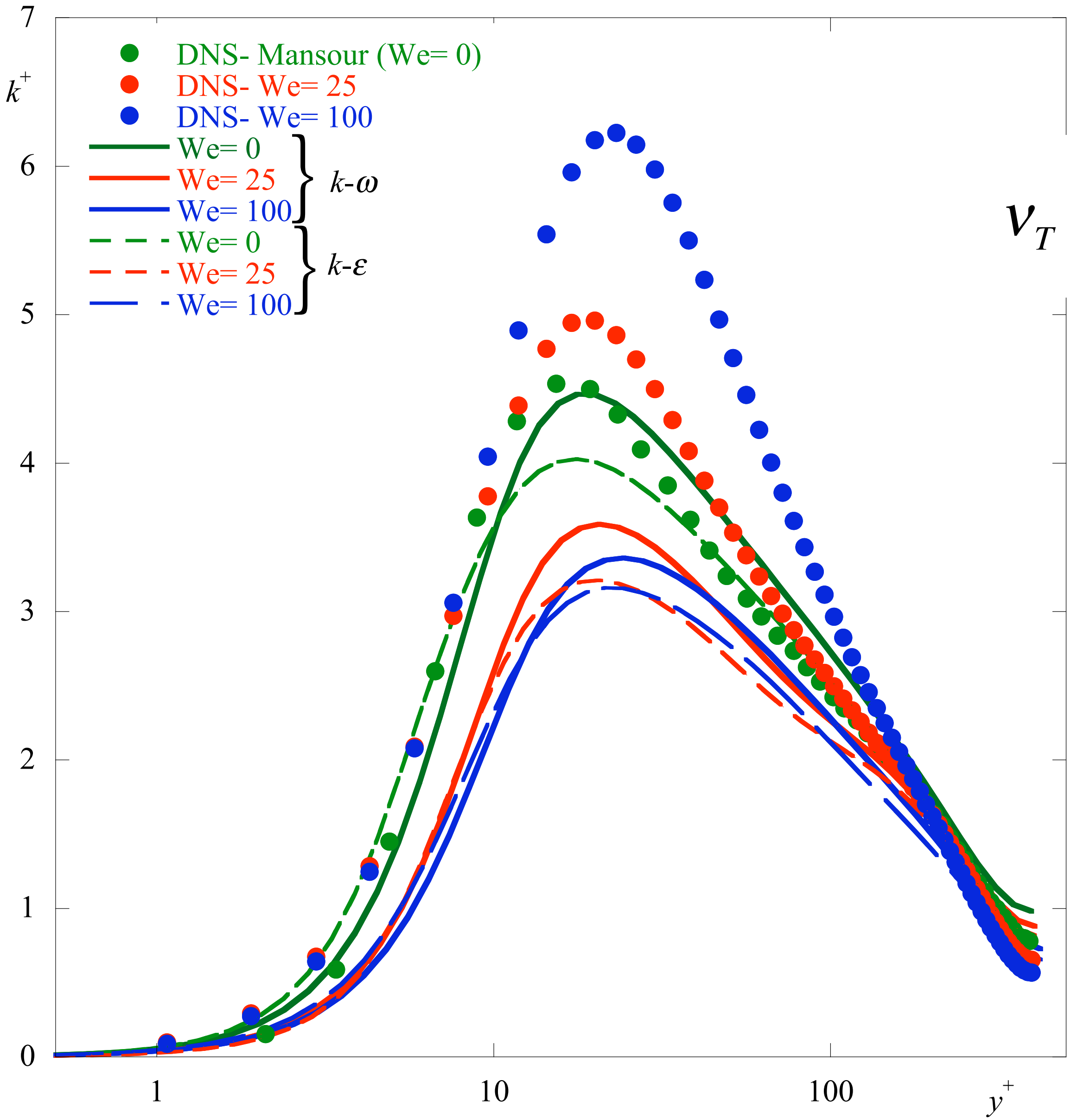
$$f^{\varepsilon}_{DR} = f^{\varepsilon}_{DR}\left(We_0, \beta, L^2\right)$$

improved version relative to k-ε, it also incorporates effects of β & L^2

Mean velocity 1: $Re_{\tau 0}$= 395; β=0.9, L^2=900

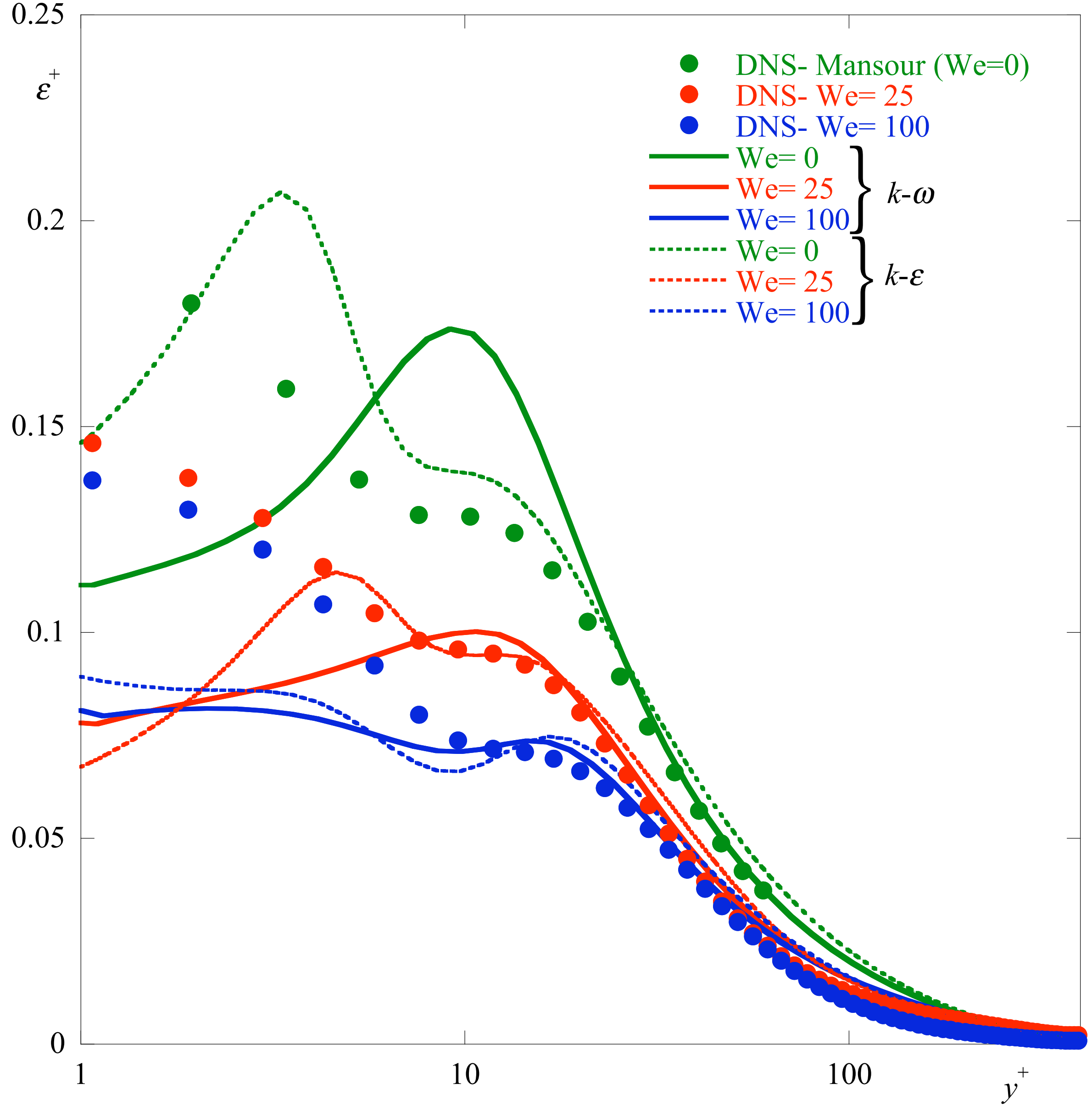


Turbulent kinetic energy: $Re_{\tau 0}$= 395; β=0.9, L^2=900

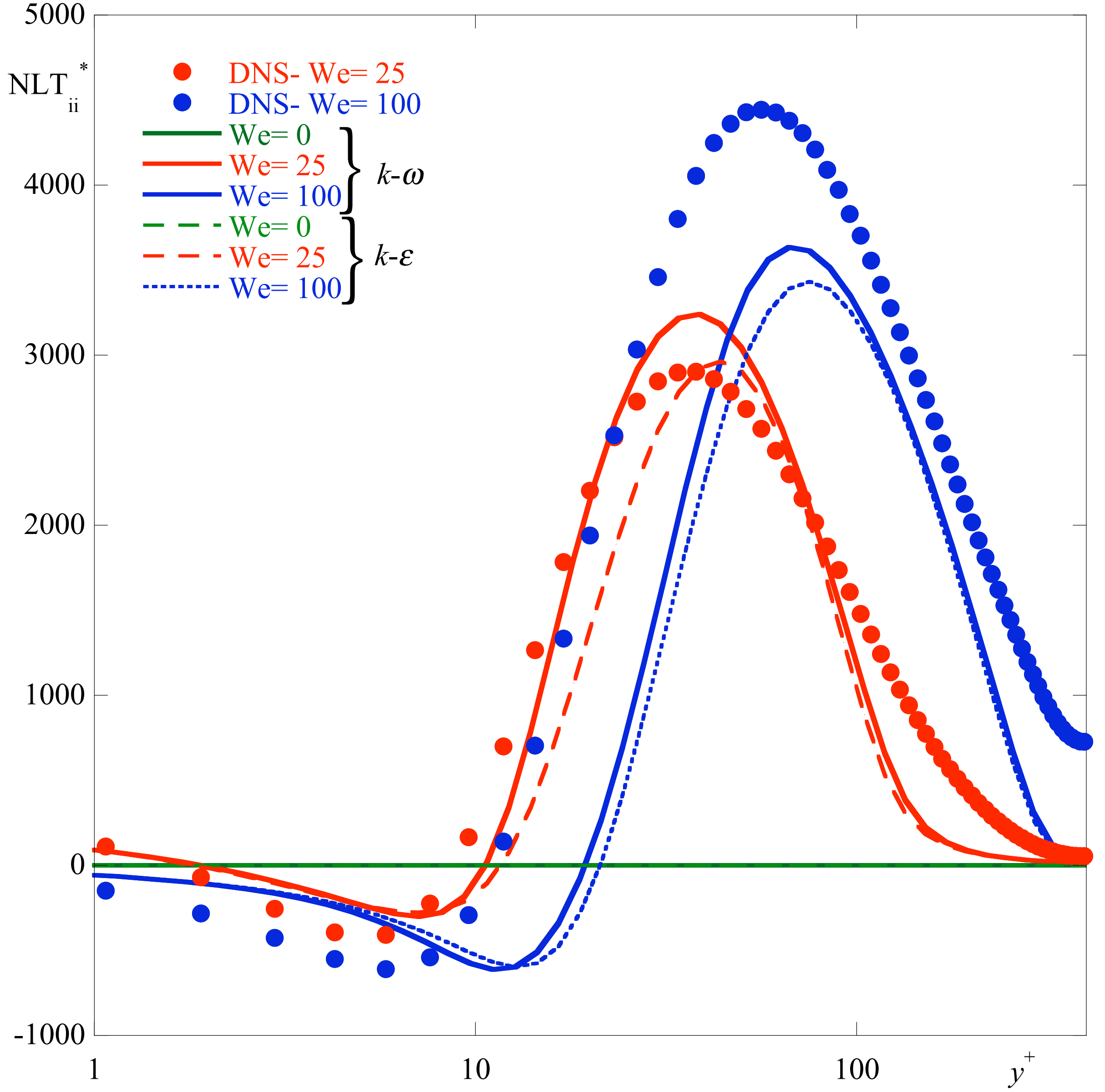


$$\nu_T = C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}^N}\left(1 - C_\mu^P f_\mu^P C_{kk}\right)$$

Dissipation of *k* by solvent: $Re_{\tau 0}$= 395; β=0.9, L^2=900



NLT_{ii}: $Re_{\tau 0}$= 395; β=0.9, L^2=900



Conclusions, Future Work and Acknowledgments

- k-ω model developed, it works well at Low DR and High DR (50%)
- Closure for elastic terms: similar to corresponding in k-ε
- Slightly better than k-ε
- More stable (easier convergence)

- Need for 2nd order Reynolds stress closures

- Need to extend models to Maximum DR, & β & L^2

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