A FIRST-ORDER TURBULENCE MODEL FOR VISCOELASTIC FENE-P FLUIDS

F. T. Pinho

Centro de Estudos de Fenómenos de Transporte, FEUP & Universidade do Minho, Portugal

C. F. Li

Dep. Energy, Environmental and Chemical Engineering, Washington University of St. Louis, St Louis, MO, USA

Currently at School of Energy and Power Engineering, Zhenjiang, Jiangsu, P. R. China

B. A. Younis

Dep. Civil and Environmental Engineering, University of California, Davis, USA

R. Sureshkumar

Dep. Energy, Environmental and Chemical Engineering, Washington University of St. Louis, St Louis, MO, USA

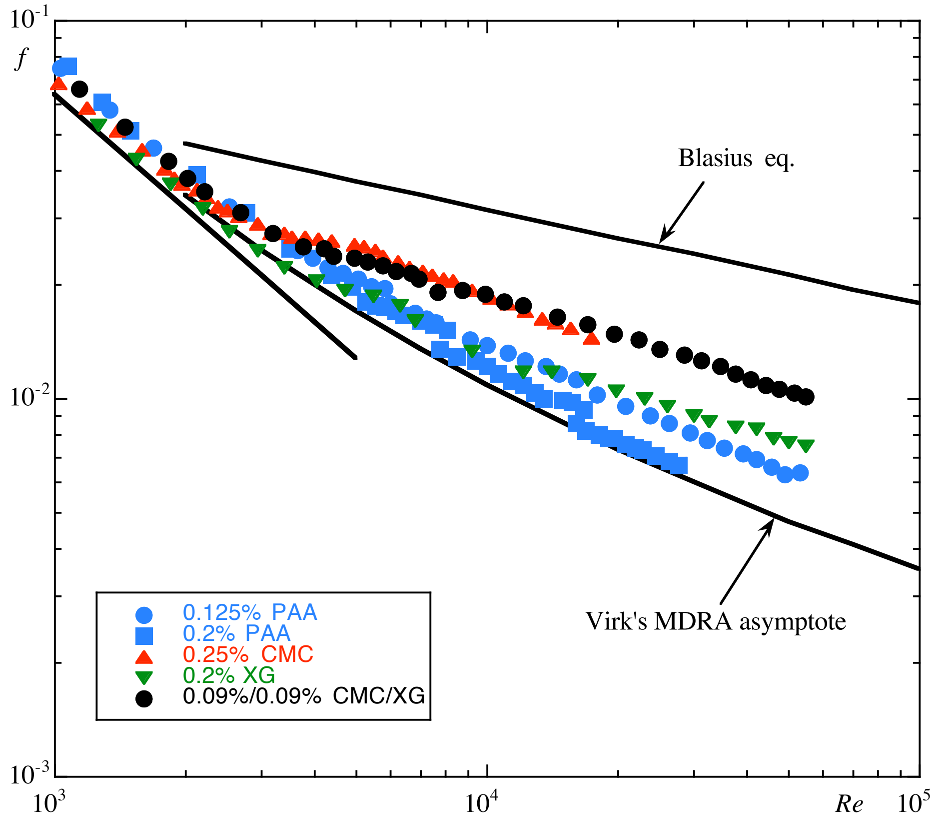
Acknowledgments:FCT (POCI/EQU/56342/2004);Gulbenkian Foundation

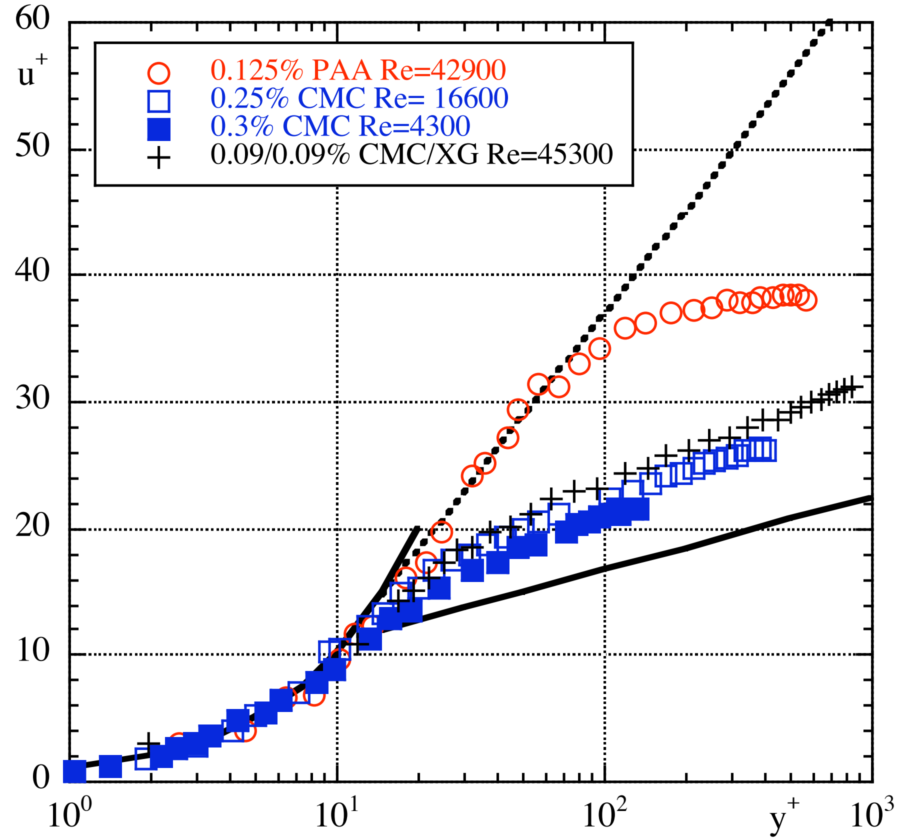
4th Annual European Rheology Conference 2007

12th-14th April 2007

Napoli, Italy

Relevance: drag reduction in turbulent pipe flow





Escudier et al JNNFM (1999)

- Reduction of shear Reynolds stress (DR)
- Increase of normal streamwise Reynolds stress
- Dampening of normal radial and tangential Reynolds stress

Deficit of Reynolds stress

Time-average governing equations: turbulent flow & FENE-P

^- instantaneous
Overbar or capital letter- time-average
` or small letter- fluctuations

Continuity: $$\frac{\partial U_i}{\partial x_i}=0$$

Momentum balance:

$$\rho\frac{\partial U_i}{\partial t}+\rho U_k\frac{\partial U_i}{\partial x_k}=-\frac{\partial \overline{p}}{\partial x_i}+\eta_s\frac{\partial^2 U_i}{\partial x_k \partial x_k}-\frac{\partial}{\partial x_k}\left(\rho\overline{u_i u_k}\right)+\frac{\partial \overline{\tau}_{ik,p}}{\partial x_k}$$

$$\overline{\tau}_{ij}=2\eta_s S_{ij}+\overline{\tau}_{ij,p}$$

Rheological constitutive equation: **FENE-P**

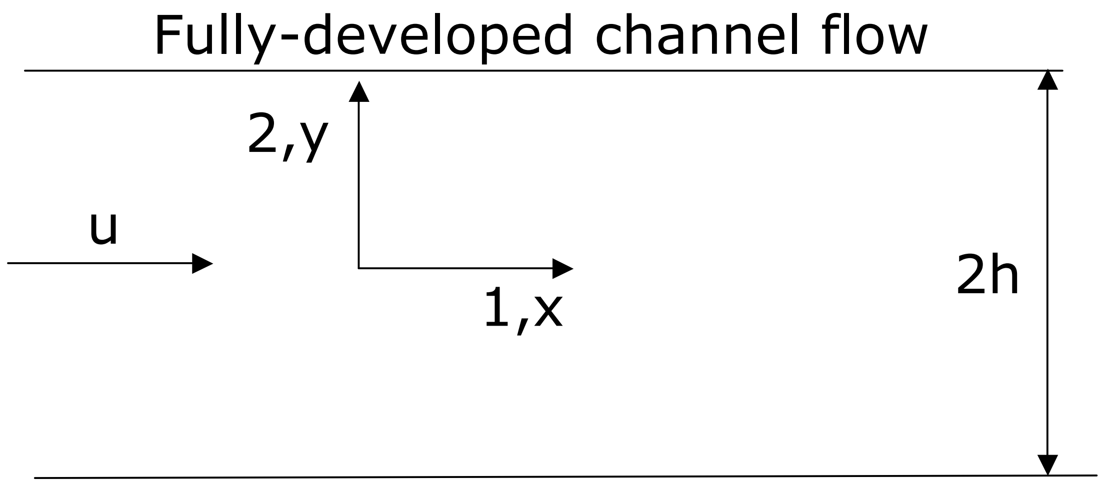
$$\hat{\tau}_{ij,p}=\frac{\eta_p}{\lambda}\left[f\left(\hat{C}_{kk}\right)\hat{C}_{ij}-f\left(L\right)\delta_{ij}\right]$$

$$f\left(\hat{C}_{kk}\right)\hat{C}_{ij}+\lambda\left(\frac{\partial \hat{C}_{ij}}{\partial t}+\hat{U}_k\frac{\partial \hat{C}_{ij}}{\partial x_k}-\hat{C}_{jk}\frac{\partial \hat{U}_i}{\partial x_k}-\hat{C}_{ik}\frac{\partial \hat{U}_j}{\partial x_k}\right)=f(L)\delta_{ij}$$

$$\left(\frac{\partial \hat{C}_{ij}}{\partial t}+\hat{U}_k\frac{\partial \hat{C}_{ij}}{\partial x_k}-\hat{C}_{jk}\frac{\partial \hat{U}_i}{\partial x_k}-\hat{C}_{ik}\frac{\partial \hat{U}_j}{\partial x_k}\right)=\overset{\nabla}{\hat{C}}_{ij}=-\frac{\hat{\tau}_{ij,p}}{\eta_p}$$

DNS case: LDR

DNS, DR=18% (LDR)



$$We_\tau = 25, Re_\tau = 395$$
$$\beta = 0.9, L^2 = 900$$

$$We_\tau = \frac{\lambda u_\tau^2}{\nu_0} \qquad Re_\tau = \frac{h u_\tau}{\nu_0}$$

Reynolds decomposition of conformation tensor

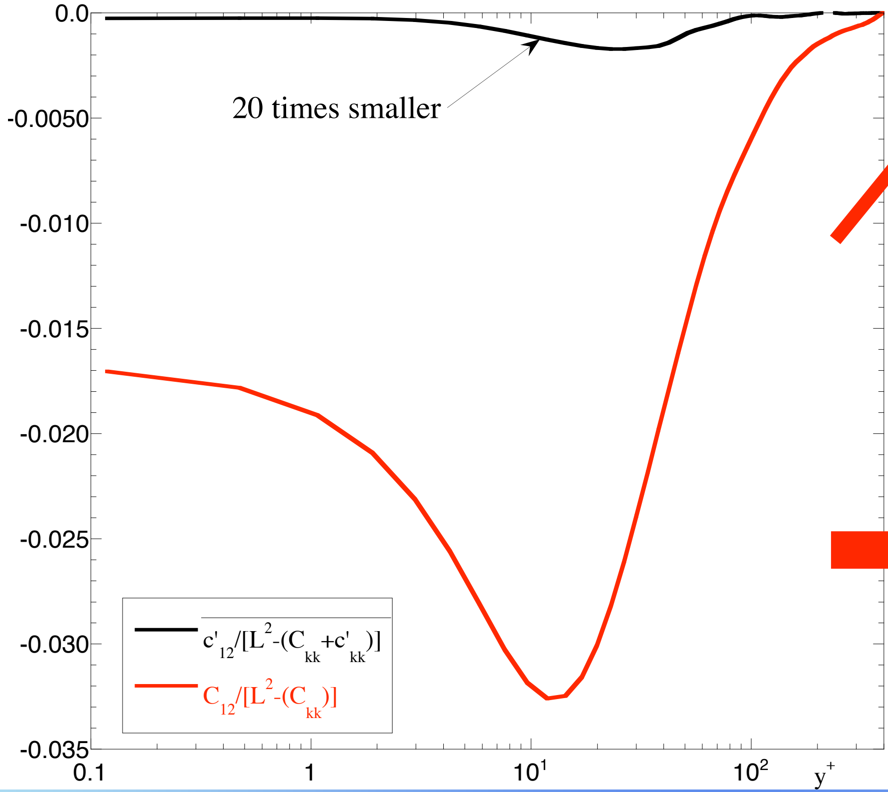
$$\hat{B} = B + b' \quad \text{where} \quad \overline{b'} = 0$$

Function: $$f\left(C_{kk}\right) = \frac{L^2 - 3}{L^2 - C_{kk}}$$

Time average polymeric stress

$$\overline{\tau}_{ij,p} = \frac{\eta_p}{\lambda}\left[f\left(C_{kk}\right) C_{ij} - f\left(L\right)\delta_{ij}\right] + \frac{\eta_p}{\lambda} \overline{f\left(C_{kk} + c_{kk}\right) c_{ij}} \rightarrow \overline{f' c_{ij}}$$

Negligible



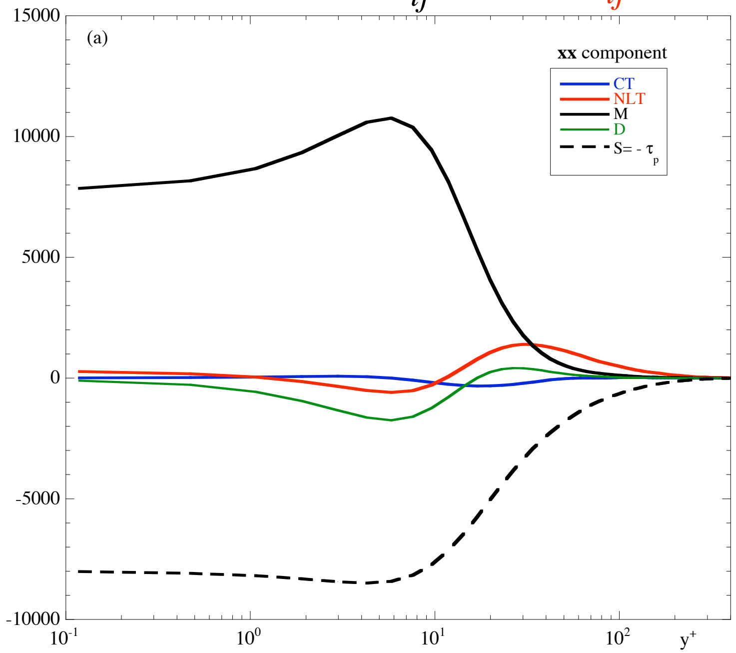
$$f\left(C_{kk}\right) C_{12} >> \overline{f' c_{12}}$$

Time-average conformation tensor equation

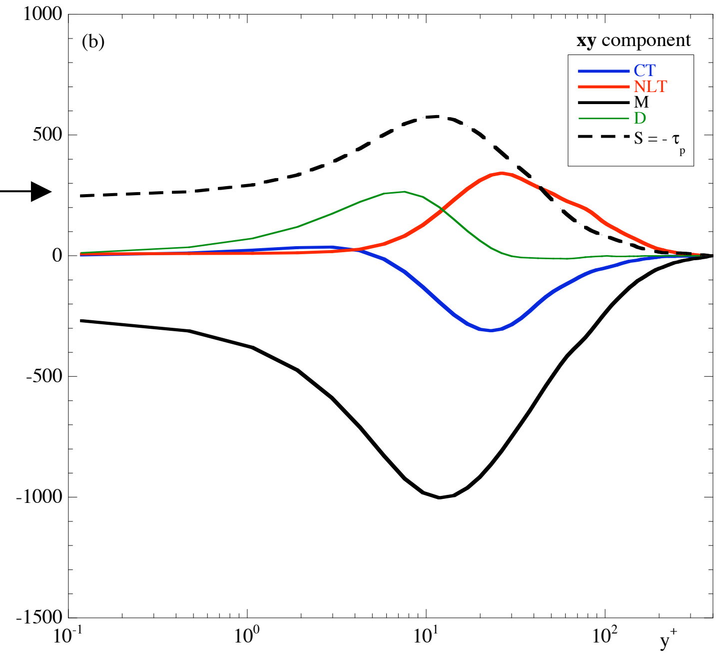
$$\lambda \overset{\nabla}{C}_{ij} + \lambda\left[\overline{u_k \frac{\partial c_{ij}}{\partial x_k}} - \left(\overline{c_{kj}\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik}\frac{\partial u_j}{\partial x_k}}\right)\right] = -\left[f(C_{kk})C_{ij} - f(L)\delta_{ij}\right]$$

$$\overset{\nabla}{C}_{ij} + \overline{u_k \frac{\partial c_{ij}}{\partial x_k}} - \left(\overline{c_{kj}\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik}\frac{\partial u_j}{\partial x_k}}\right) = -\frac{\bar{\tau}_{ij,p}}{\eta_p}$$

M_{ij} CT_{ij} NLT_{ij}



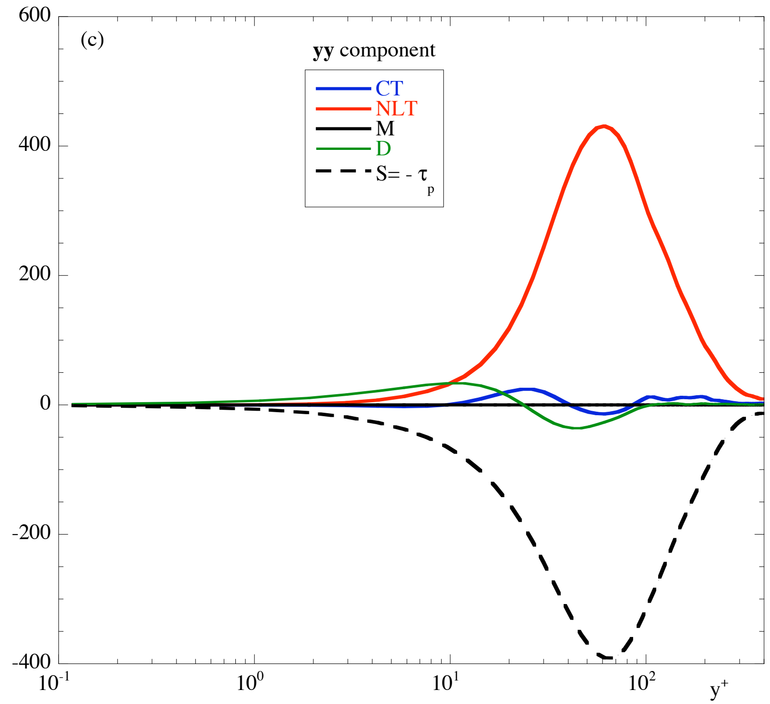
xx ← → **xy**



Polymer stress: DNS

$$\frac{\overline{\tau}_{ij,p}}{\eta_p} = -\overset{\nabla}{C}_{ij} - \overline{u_k \frac{\partial c_{ij}}{\partial x_k}} + \left(\overline{c_{kj} \frac{\partial u_i}{\partial x_k}} + \overline{c_{ik} \frac{\partial u_j}{\partial x_k}} \right)$$

yy →



Exact M_{ij}

NLT_{ij}: originates in distortion of Oldroyd derivative- not negligible
Must be modeled

CT_{ij} : originates in advective term, negligible
no need for modeling

DNS: Housiadas et al (2005), Li et al (2006) JNNFM

Modeling the Reynolds stress

1) Reynolds stresses: Prandtl-Kolmogorov model

$$-\overline{u_i u_j} = 2\nu_T S_{ij} - \frac{2}{3} k \delta_{ij} \quad \text{with} \quad \nu_T = C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}^N}$$

Next slide

2) Dissipation of turbulent kinetic energy: ε^N

$$2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\rho \frac{Du_i}{Dt}\right)} + 2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\rho u_k \frac{\partial U_i}{\partial x_k}\right)} + 2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\rho \frac{\partial u_i u_k}{\partial x_k}\right)}$$

$$+2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\frac{\partial p'}{\partial x_i}\right)} - 2\rho\nu_s^2 \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\frac{\partial^2 u_i}{\partial x_k^2}\right)} - 2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\frac{\partial \tau'_{ik,p}}{\partial x_k}\right)} = 0$$

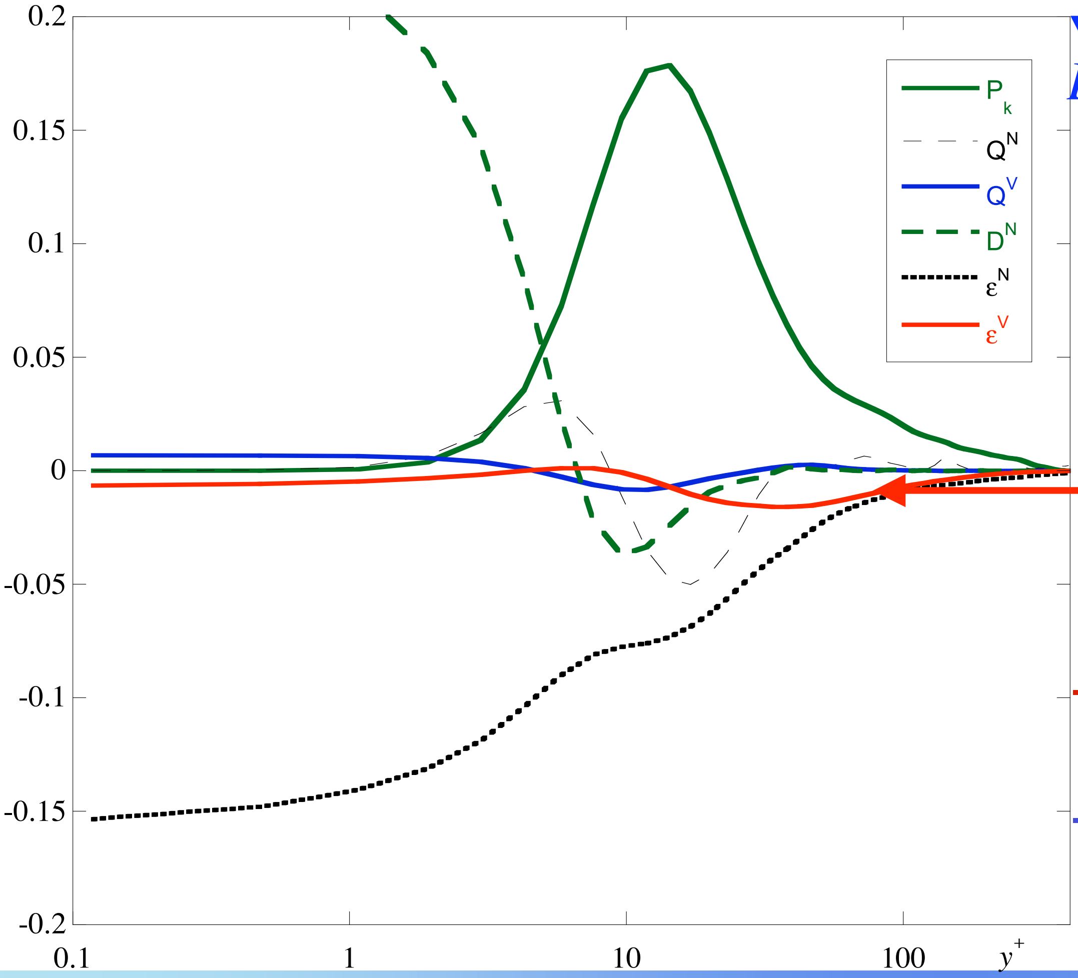
New term

As for Newtonian fluids, the whole equation will be approximated

3) Turbulent kinetic energy: its transport equation

$$\rho \frac{Dk}{Dt} + \rho \overline{u_i u_k} \frac{\partial U_i}{\partial x_k} = -\rho \overline{u_i \frac{\partial k'}{dx_i}} - \frac{\partial \overline{p' u_i}}{\partial x_i} + \eta_s \frac{\partial^2 k}{\partial x_i \partial x_i} - \eta_s \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_k}} + \frac{\partial \overline{\tau'_{ik,p} u_i}}{\partial x_k} - \overline{\tau'_{ik,p} \frac{\partial u_i}{\partial x_k}}$$

D^N ε^N Q^V ε^V



-**Need to model viscoelastic stress work**

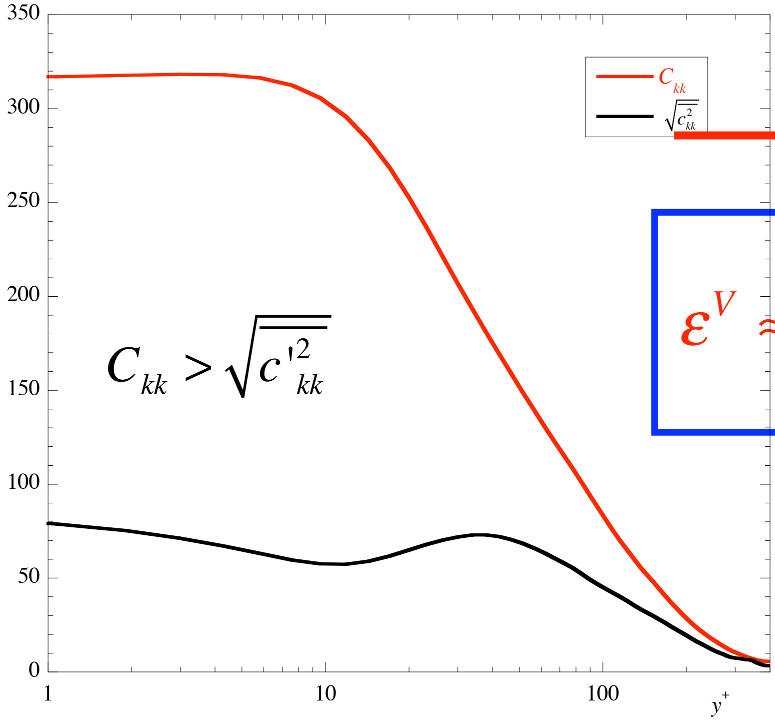
- **Viscoelastic turbulent transport is small**

Viscoelastic stress work: ε^V

$$\varepsilon^V \equiv \frac{1}{\rho}\overline{\tau'_{ik,p}\frac{\partial u_i}{\partial x_k}} = \frac{\eta_p}{\rho\lambda}\left[\overline{C_{ik} f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik} f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}}\right]$$

Assumptions & DNS:

$$\overline{C_{ik} f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}} < \overline{c_{ik} f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}}$$



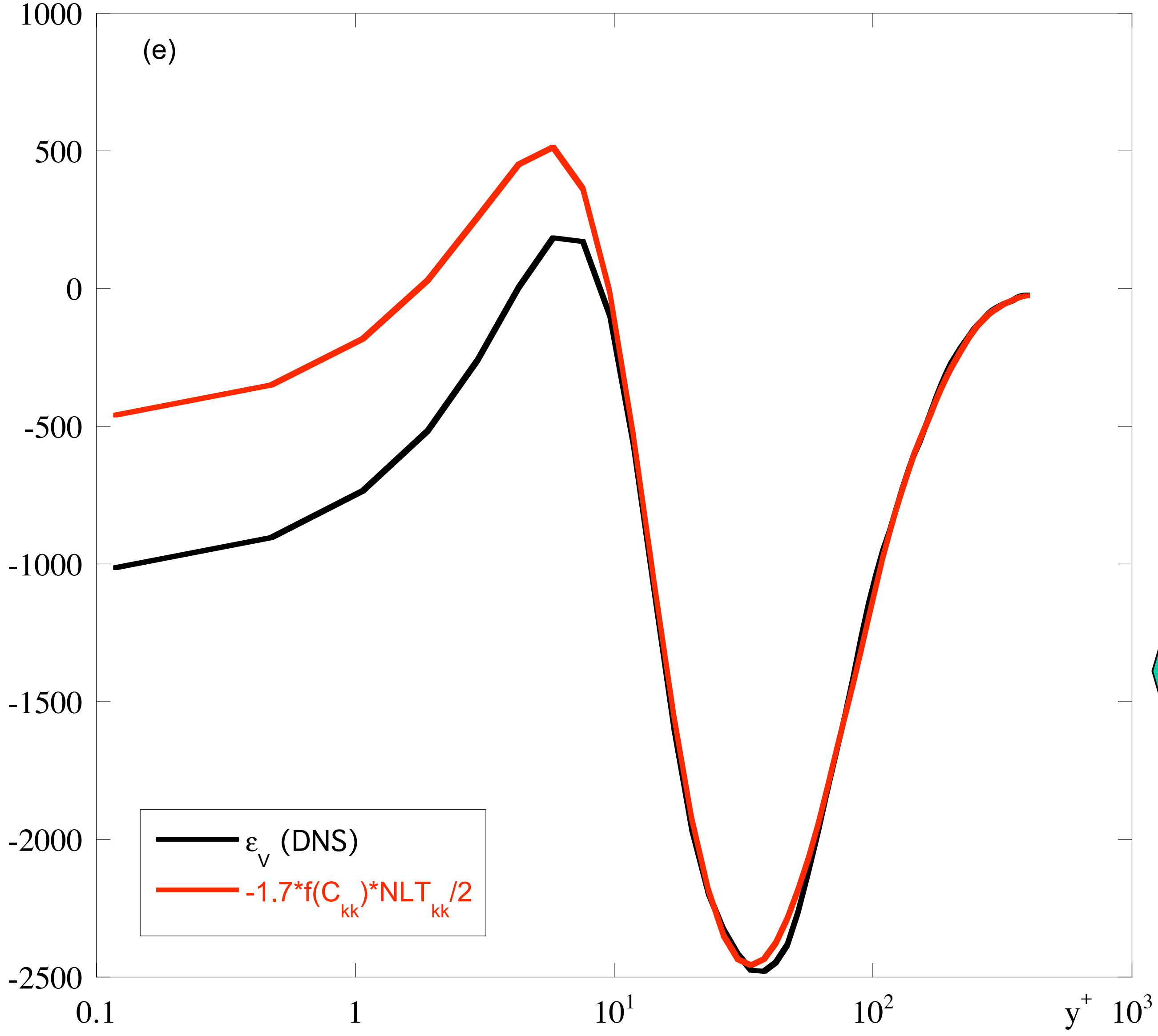
$$\varepsilon^V \approx \frac{\eta_p}{\rho\lambda} f\left(C_{mm}\right)\overline{c_{ik}\frac{\partial u_i}{\partial x_k}} = C_{\varepsilon^V}\frac{\eta_p}{\rho\lambda} f\left(C_{mm}\right)\frac{NLT_{nn}}{2}$$

Needs model

$C_{\varepsilon^V} = 1.076$

(from DNS- next slide)

Performance of the viscoelastic work model



$$\varepsilon^V > 0$$

Negative sign:
different
definitions

$$\varepsilon^{V+}\left(\mathrm{Re}_{\tau_0}\right)^2$$

versus

$$f\left(C_{kk}\right)NLT_{ij}^{*}$$

Modeling NLT_{ij}

Key ideas:

1) Write down exact equation- complex 4 lines long
2) Assumptions, physical insight, trial-and-error
3) Do *a priori* testing of each term
4) Select appropriate combination and dimensional homogeneity
5) Test in code and **modify**- under investigation

$$\overline{u_i u_m \frac{\partial c_{kj}}{\partial x_m}} + \overline{u_i u_m}\frac{\partial C_{kj}}{\partial x_m} \approx Coef \times \overline{u_i u_m}\frac{\partial C_{kj}}{\partial x_m}$$

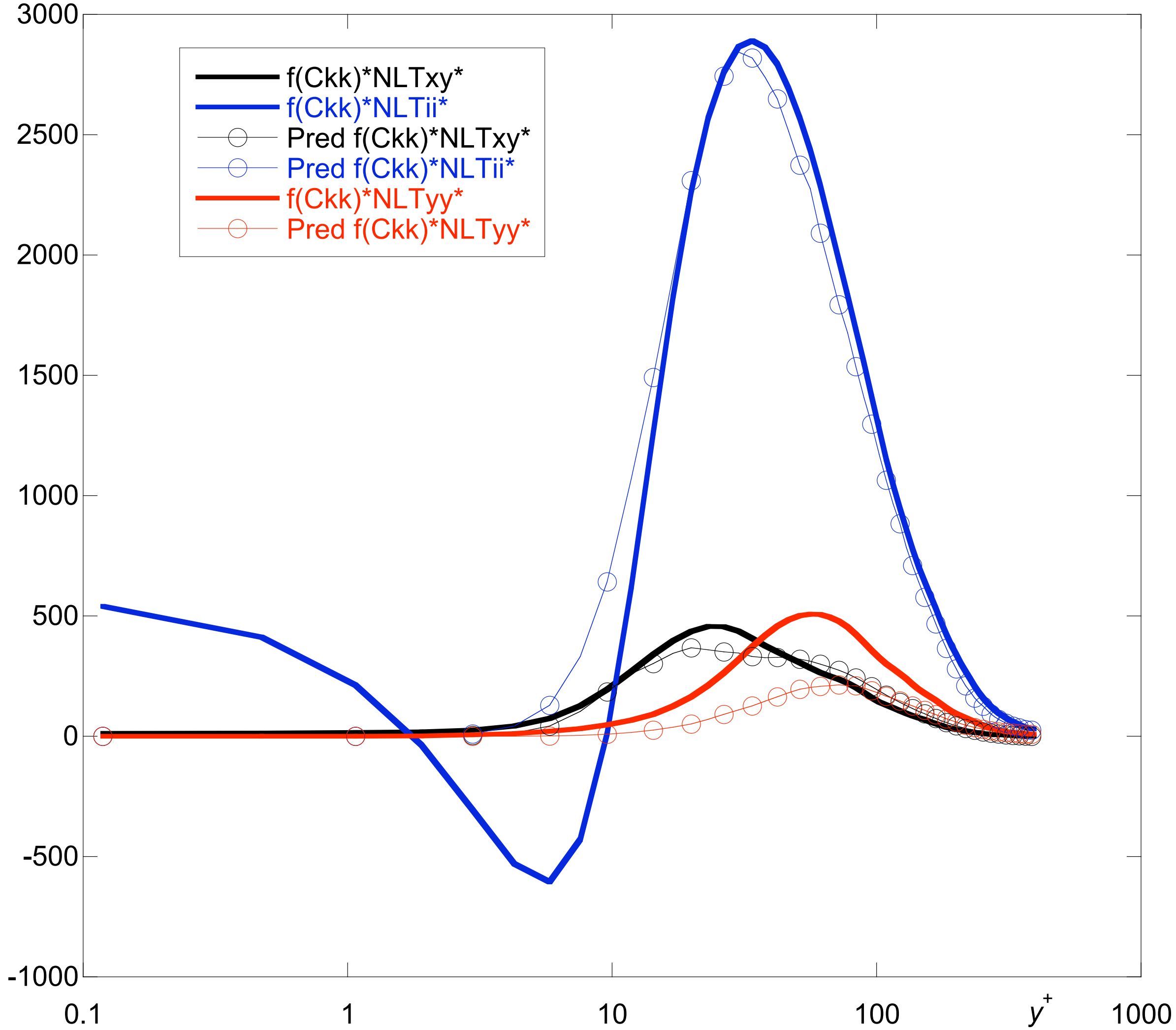
$$f\left(C_{mm}\right)\frac{NLT_{ij}}{\lambda} = function\left(S_{ij}, W_{ij}, C_{ij}, \varepsilon_{ij}^{N}, \frac{\partial \overline{u_i u_j}}{\partial x_k}, \frac{\partial C_{ij}}{\partial x_k}, \frac{\partial NLT_{ij}}{\partial x_n}, M_{ij}, \overline{u_i u_j}\right)$$

$$f\left(C_{mm}\right)\frac{NLT_{ij}}{\lambda} = f_{\mu_1}\left[\frac{C_{E_3} u_\tau^2}{\nu_0^2} C_{kk}\overline{u_i u_j} + \frac{C_{\alpha_{14}}}{\nu_0}\left(\overline{u_i u_k} W_{kn} C_{nj} + \overline{u_j u_k} W_{kn} C_{ni} + \overline{u_k u_i} W_{jn} C_{nk}\right)\right]$$

Model for NLT_{ij}

$C_{E_3} = 0.00035; C_{\alpha_{14}} = 0.00015$

$$f_{\mu_1} = \left(1 - \exp\left(-y^+/26.5\right)\right)^2$$



Viscoelastic turbulent transport: Q^V

$$Q^V \equiv \frac{\partial \overline{\tau'_{ik,p} u_i}}{\partial x_k} = \frac{\eta_p}{\lambda}\frac{\partial}{\partial x_k}\left[C_{ik}\overline{f\left(C_{mm}+c_{mm}\right)u_i} + \overline{c_{ik} f\left(C_{mm}+c_{mm}\right)u_i}\right]$$

$$C_{kk} > \sqrt{\overline{c'^2_{kk}}}$$

$$f\left(\hat{C}_{mm}\right) = \frac{L^2-3}{L^2-\left(C_{mm}+c_{mm}\right)}$$

Weak coupling between c_{kk} and c_{ij}, u_i

$$C_{ik}\overline{f\left(C_{mm}+c_{mm}\right)u_i} < \overline{c_{ik} f\left(C_{mm}+c_{mm}\right)u_i}$$

Neglect of this term is irrelevant because non-neglected term is modeled

$$Q^V = \frac{\eta_p}{\lambda}\frac{\partial}{\partial x_k}\left[f\left(C_{mm}\right)CU_{iik}\right]$$

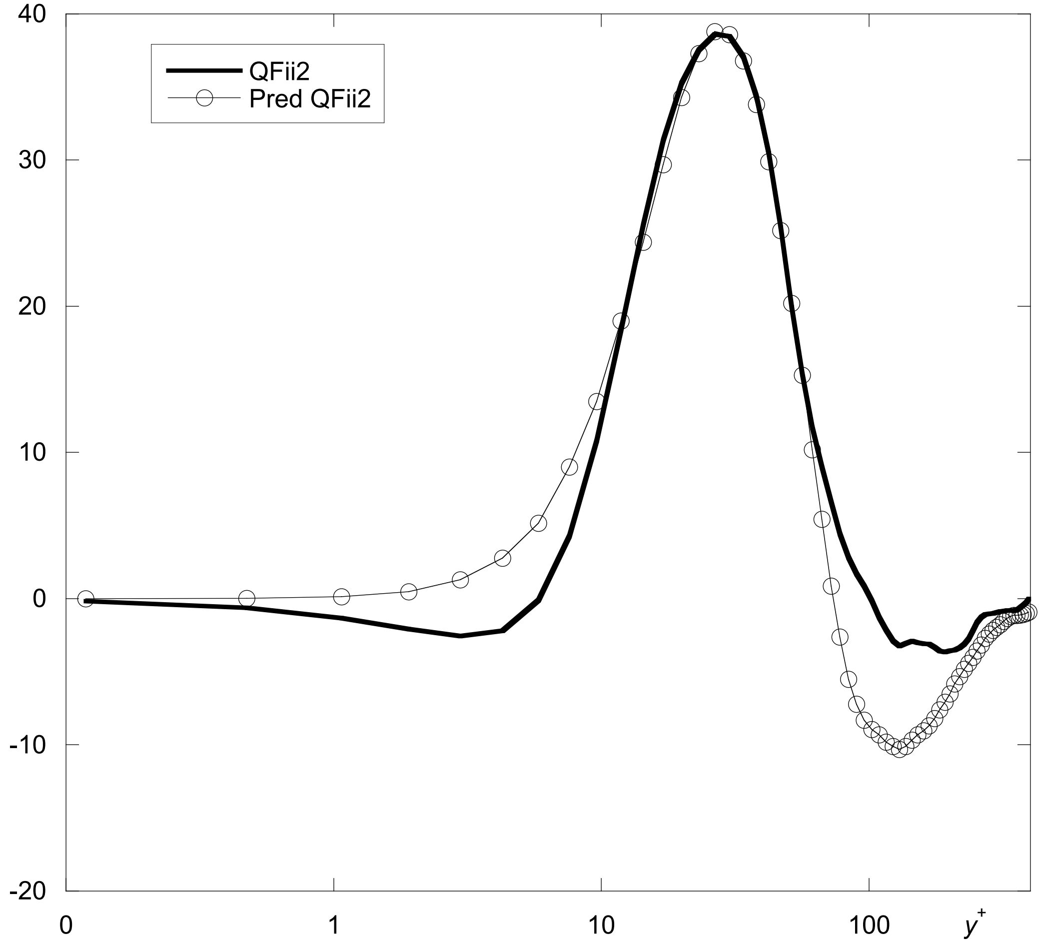
General case

Needs model $\left(CU_{ijk}\right)$

Model for CU_{ijk}

- Same modelling approach as with NLT_{ij}

$$\frac{f(C_{mm})CU_{ijk}}{\lambda} = f_{\mu_2}\left[-C_{\beta_1}\left(\overline{u_i u_m}\frac{\partial C_{kj}}{\partial x_m} + \overline{u_j u_m}\frac{\partial C_{ik}}{\partial x_m}\right) - \frac{C_{\beta_7}}{\lambda} f(C_{mm})\left[\pm\sqrt{\overline{u_j^2}}C_{ik} \pm \sqrt{\overline{u_i^2}}C_{jk}\right]\right]$$



$$C_{\beta_1} = 1.3; C_{\beta_7} = 0.37$$

$$f_{\mu_2} = 1 - \exp\left(-\frac{y^+}{26.5}\right)$$

Final equations: low *Re* *k*-ε type model for channel flow

Momentum:
$$\frac{d}{dy}\left[\eta_s \frac{dU}{dy} + \overline{\tau}_{p,xy} - \rho\overline{uv}\right] - \frac{d\overline{p}}{dx} = 0$$

$$\overline{\tau}_{xy,p} = \frac{\eta_p}{\lambda} f(C_{kk}) C_{xy}$$

$$f(C_{kk}) C_{xy} = \lambda C_{yy} \frac{dU}{dy} + \lambda NLT_{xy}$$

$$f(C_{kk}) C_{yy} = \lambda NLT_{yy} + 1$$

$$f(C_{kk}) C_{xx} = 2\lambda C_{xy} \frac{dU}{dy} + \lambda NLT_{xx} + 1$$

$$f(C_{kk}) C_{zz} = \lambda NLT_{zz} + 1$$

$$f(C_{kk}) = \frac{L^2 - 3}{L^2 - (C_{xx} + C_{yy} + C_{zz})}$$

Reynolds stress:
$$-\rho\overline{uv} = \rho\nu_T \frac{dU}{dy} \quad \text{with} \quad \nu_T = C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}^N}$$

k and *ε* transport equations: modified Nagano & Hishida

$$0=\frac{d}{dy}\left[\left(\eta_s+\frac{\rho\nu_T}{\sigma_k}\right)\frac{dk}{dy}\right]+P_k-\rho\tilde{\varepsilon}^N-\rho D+\eta_p\frac{d}{dy}\left[\frac{f(C_{mm})}{\lambda}\frac{CU_{nny}}{2}\right]-\eta_p\frac{f(C_{mm})}{\lambda}\frac{NLT_{nn}}{2}$$

$$\varepsilon^N=\tilde{\varepsilon}^N+D^N \qquad D^N=2\eta_s\left(\frac{d\sqrt{k}}{dy}\right)^2$$

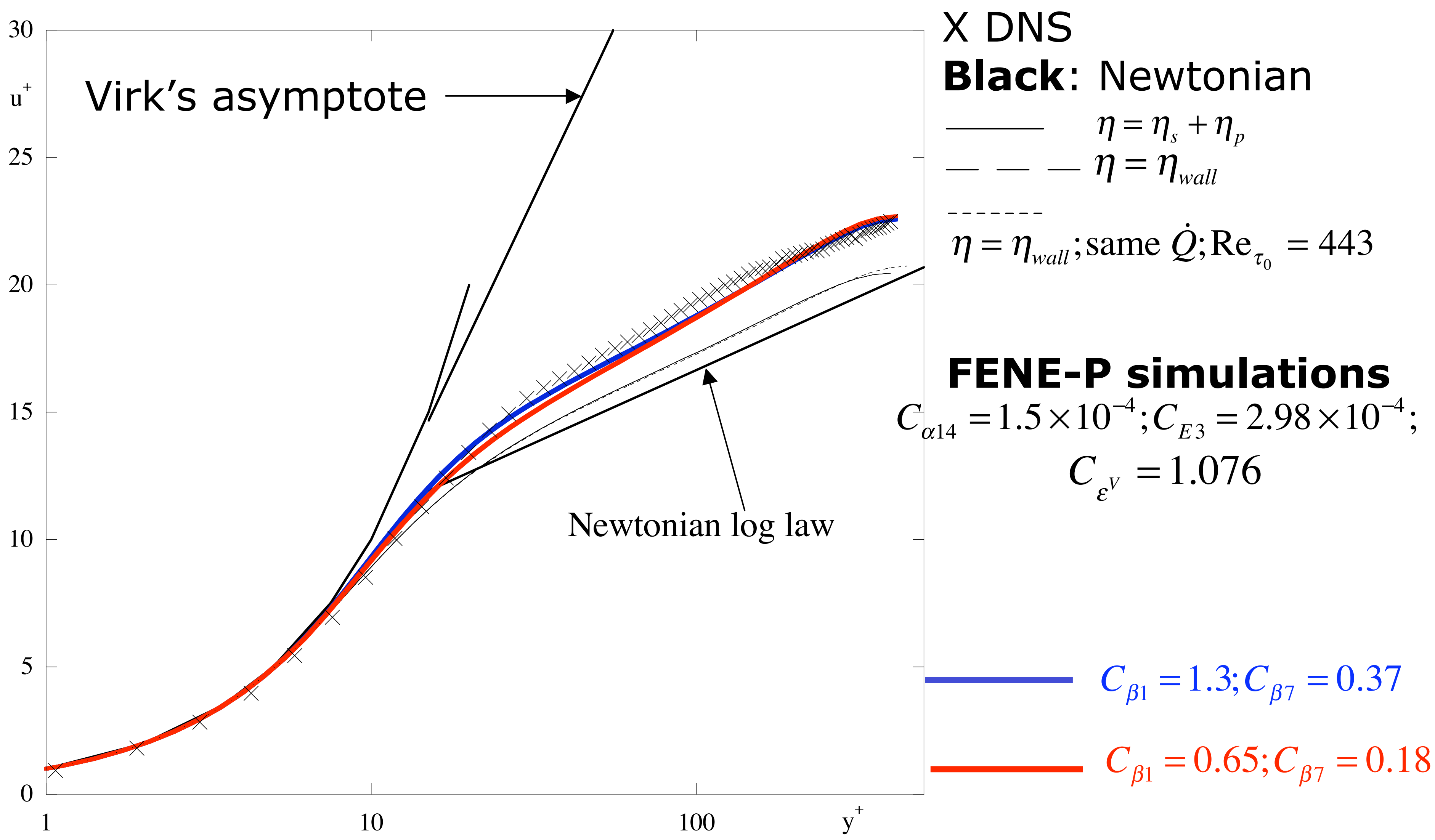
$$0=\frac{d}{dy}\left[\left(\eta_s+\frac{\rho\nu_T}{\sigma_\varepsilon}\right)\frac{d\tilde{\varepsilon}^N}{dy}\right]+\rho f_1 C_{\varepsilon_1}\frac{\tilde{\varepsilon}^N}{k}\frac{P_k}{\rho}-\rho f_2 C_{\varepsilon_2}\frac{{\varepsilon^N}^2}{k}+\rho E+E_{\tau_p}$$

$$E=\frac{\eta_s}{\rho}\nu_T\left(1-f_\mu\right)\left(\frac{d^2U}{dy^2}\right)^2$$

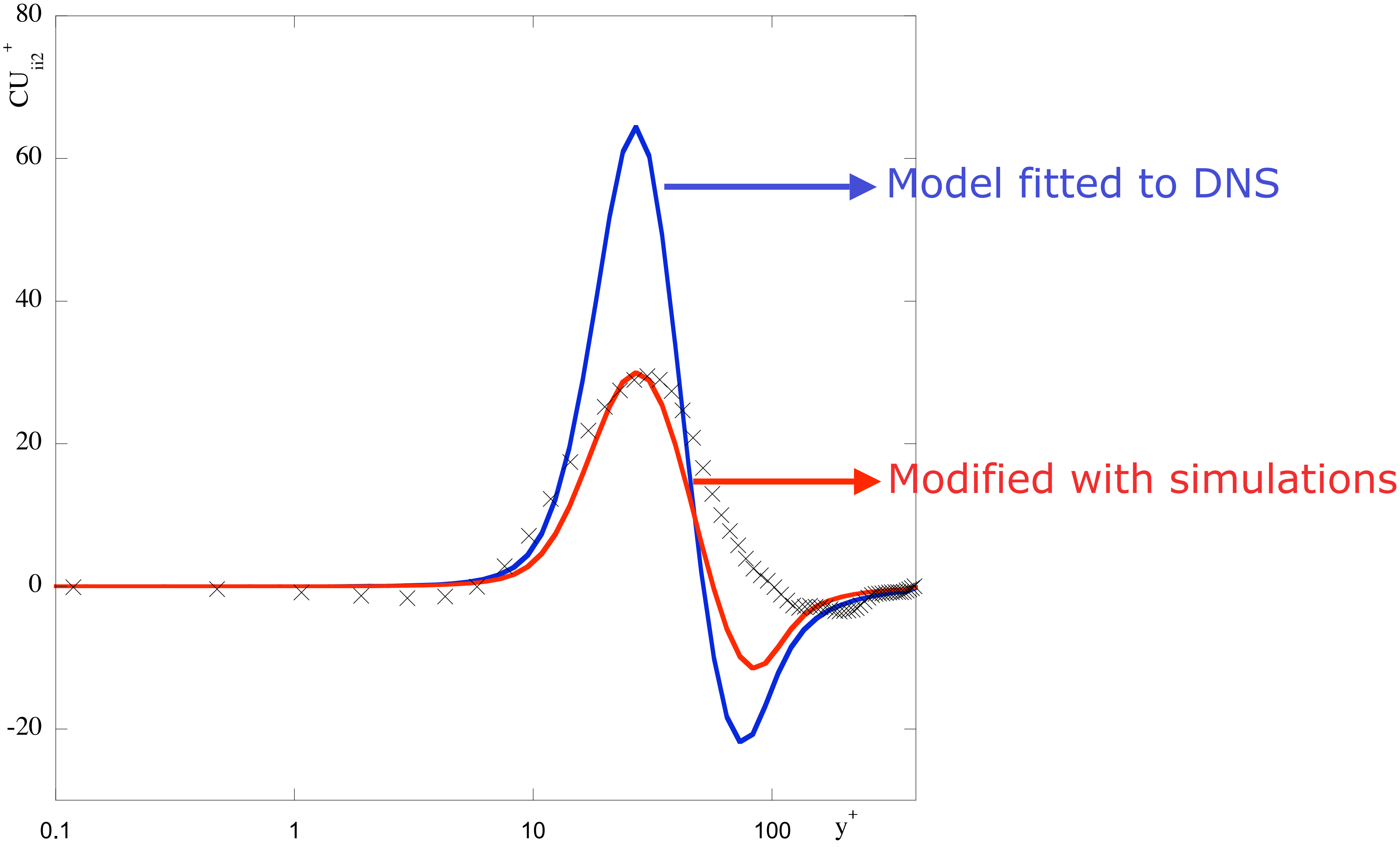
$$f_1=1 \qquad f_2=1-0.3\exp\left(-R_T^2\right) \qquad f_\mu=\left[1-\exp\left(\frac{-y^+}{26.5}\right)\right]^2$$

based on Newtonian model of Nagano & Hishida (1984)

Predictions 1: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900

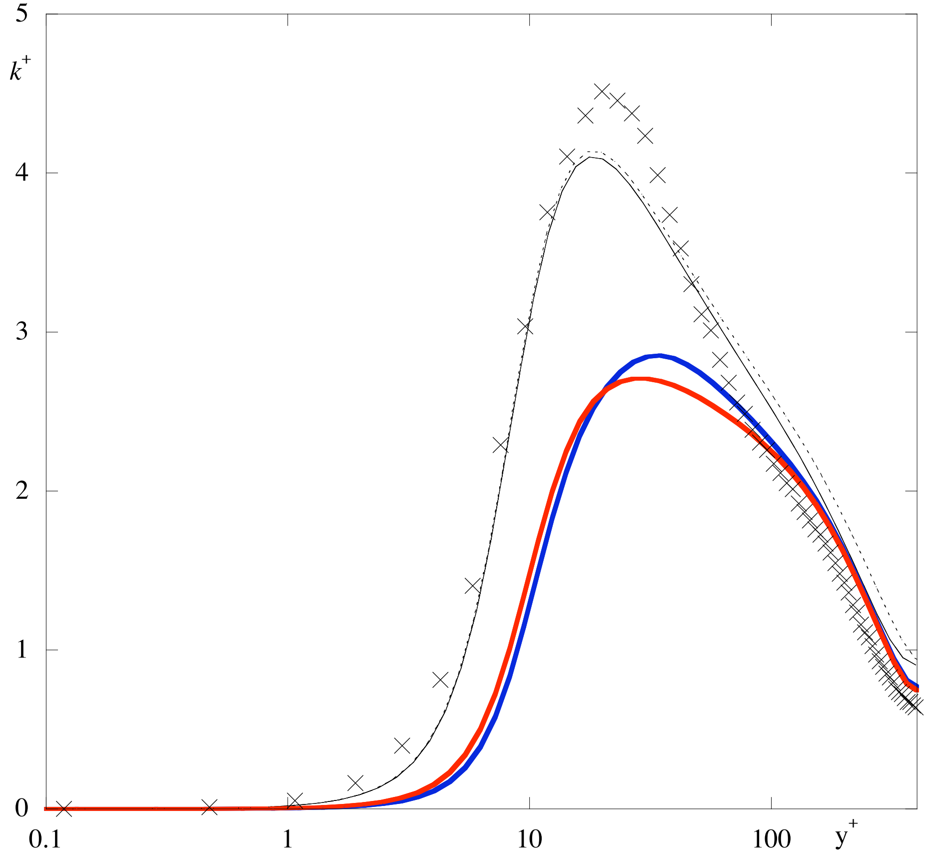


Predictions 2: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900

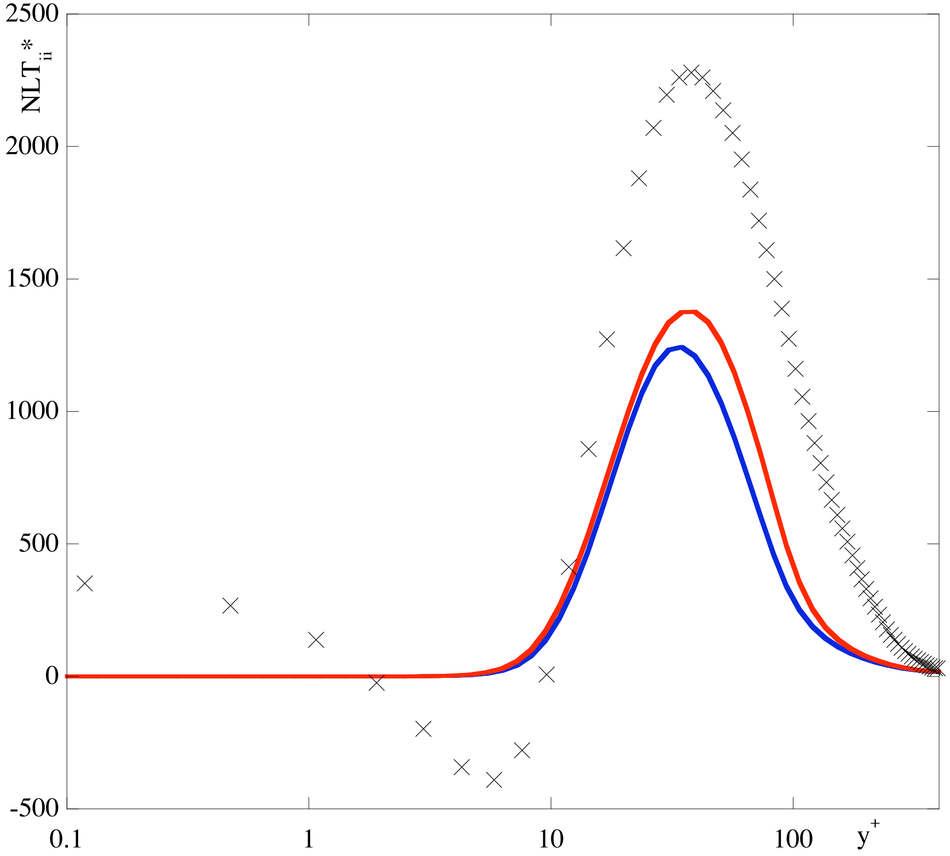


Predictions 3: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900

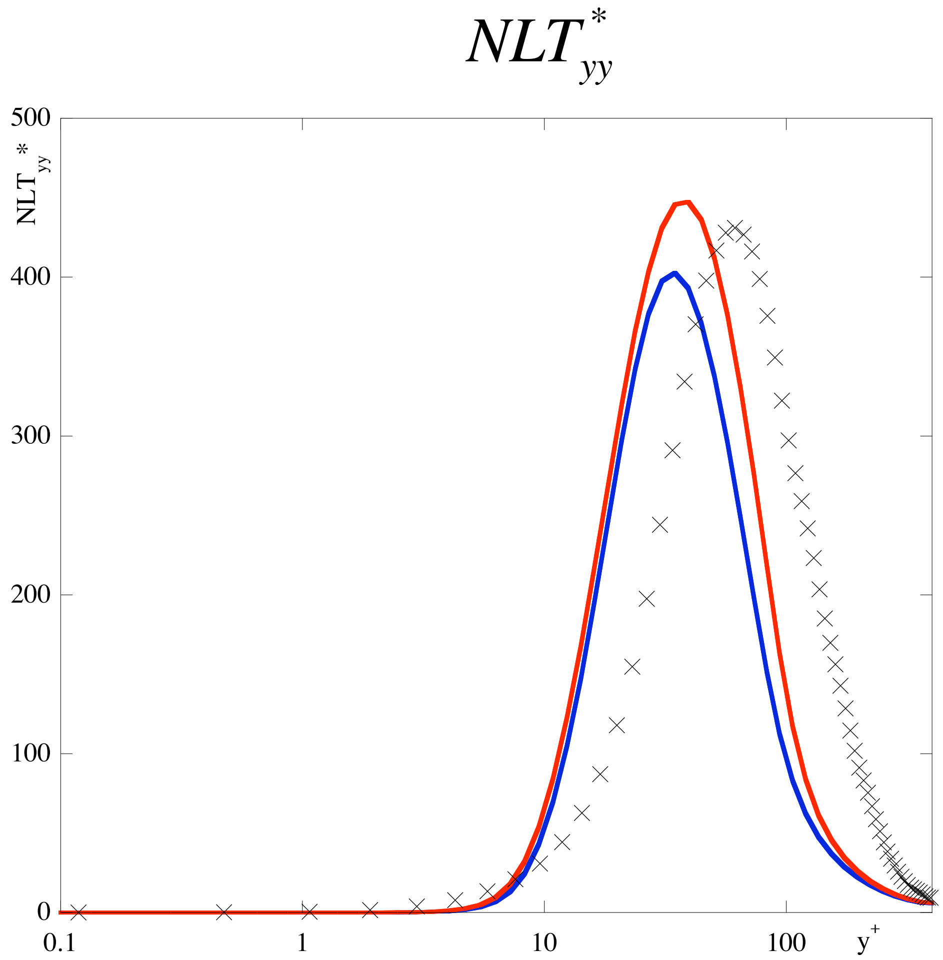
k^+

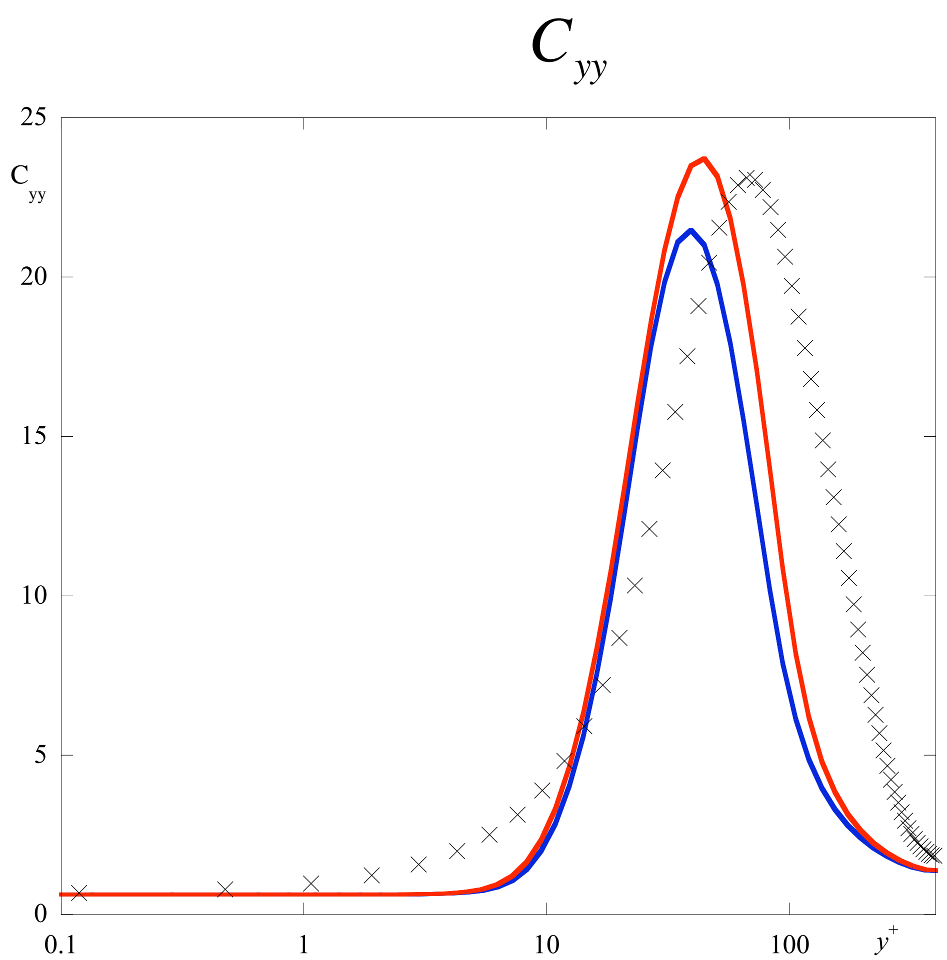


NLT_{ii}^*



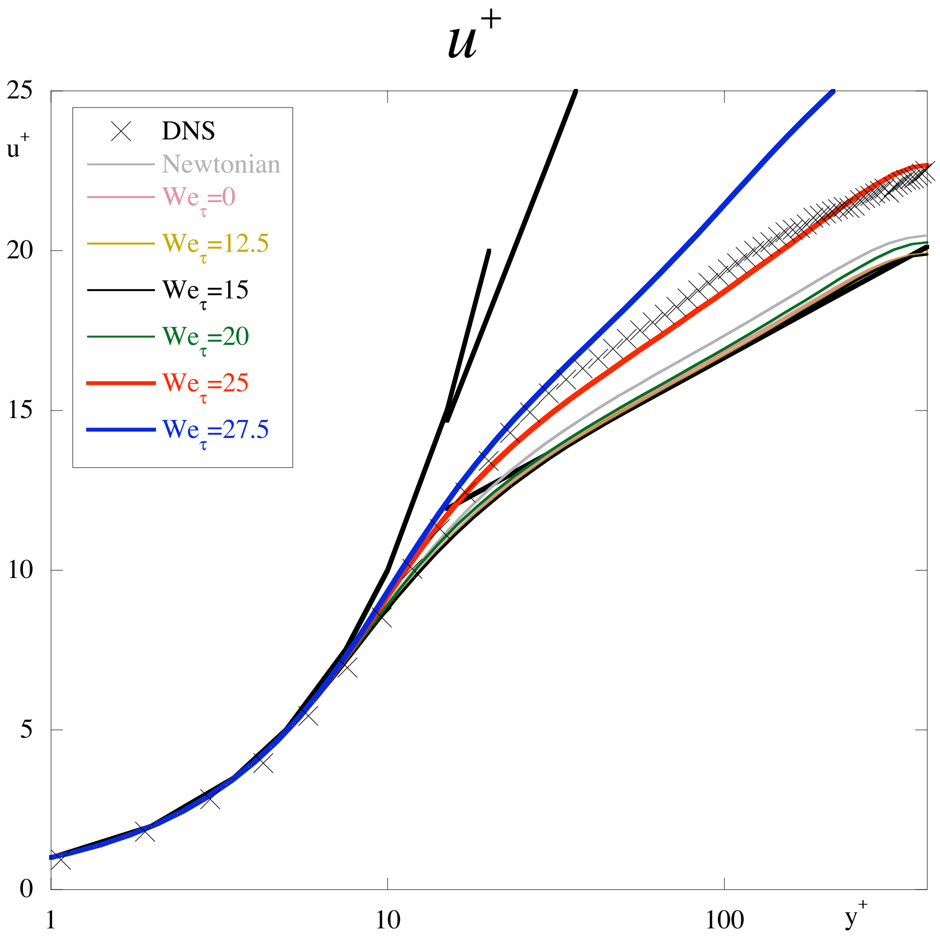
Predictions 4: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900

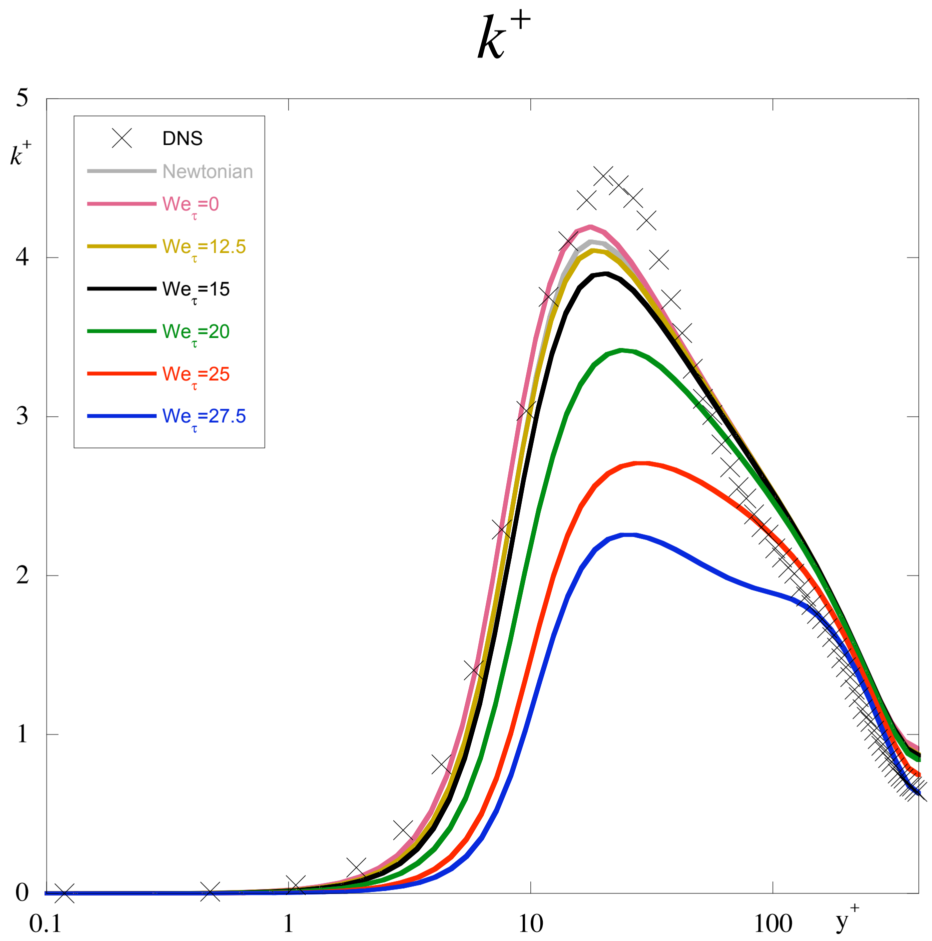




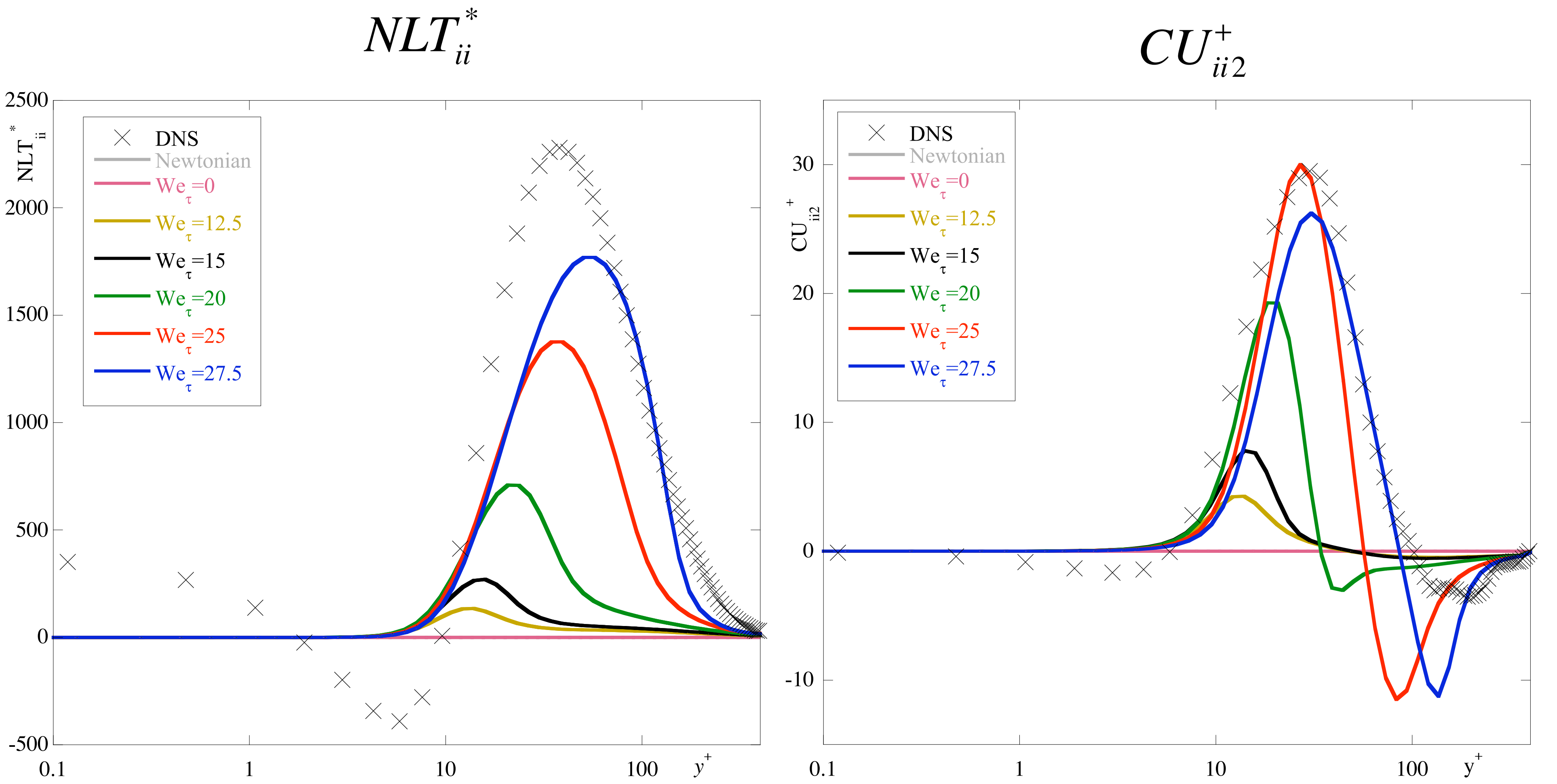
Effect of Weissenberg 1: $Re_{\tau 0}$= 395; β=0.9, L^2=900

$$Re_\tau = 395; C_{\beta_1} = 0.65; C_{\beta_7} = 0.18; C_{\alpha_{14}} = 1.5\times 10^{-4}; C_{E3} = 2.98\times 10^{-4}$$

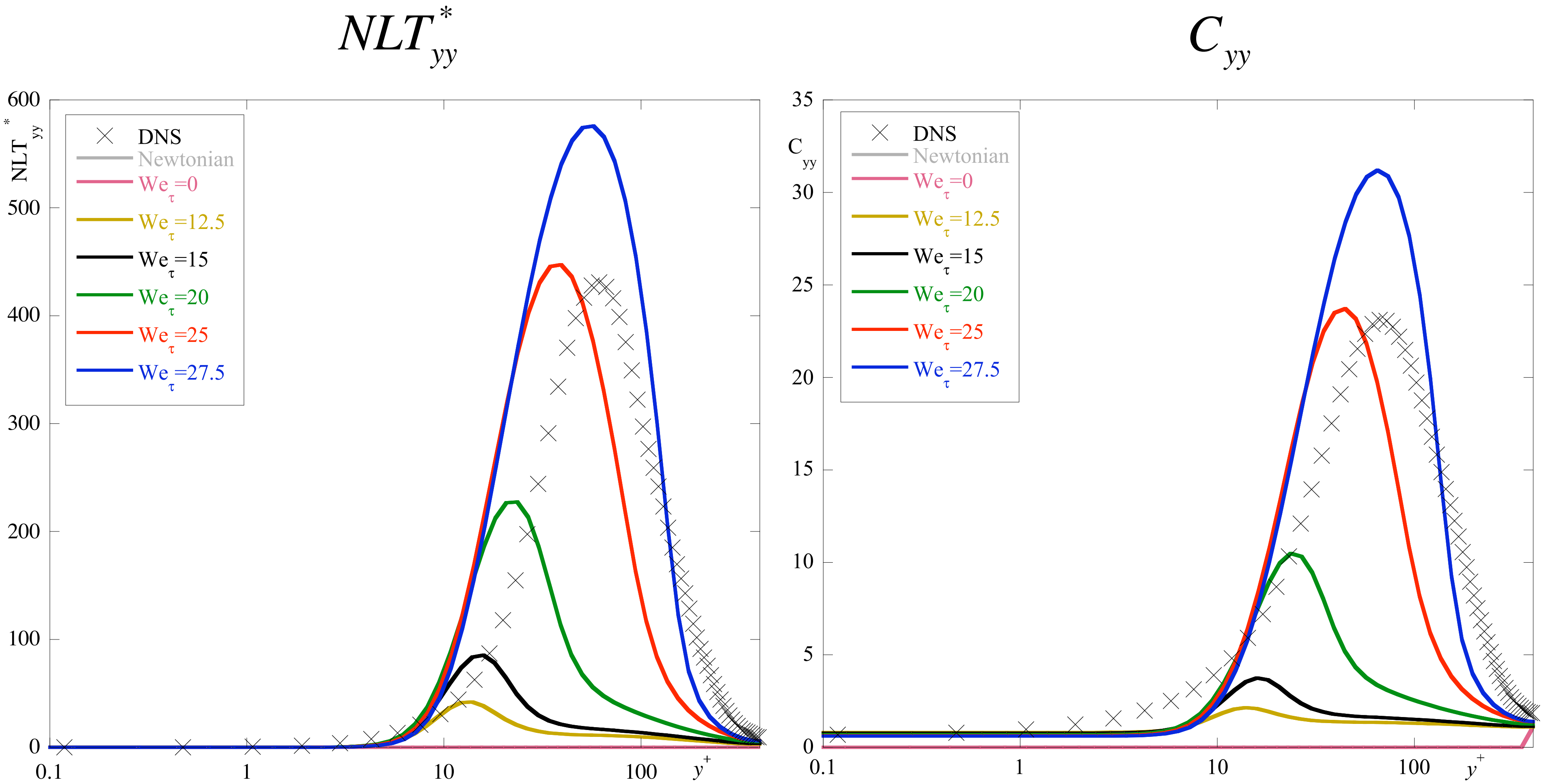




Effect of Weissenberg 2: $Re_{\tau 0}$= 395; β=0.9, L^2=900

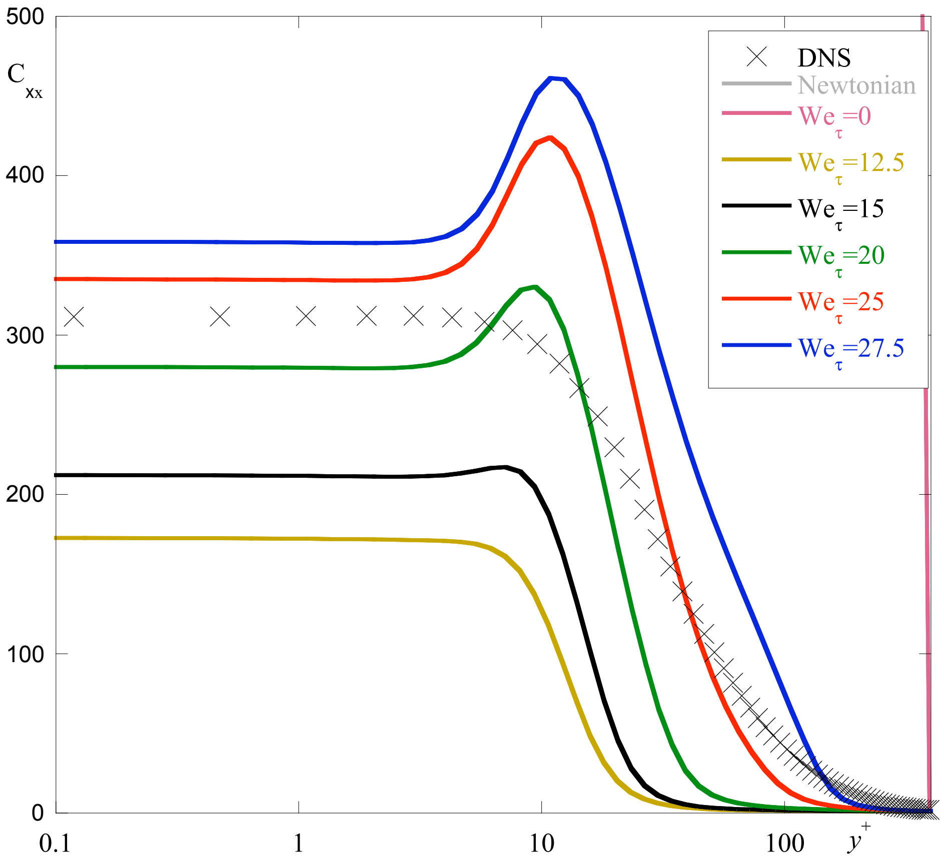


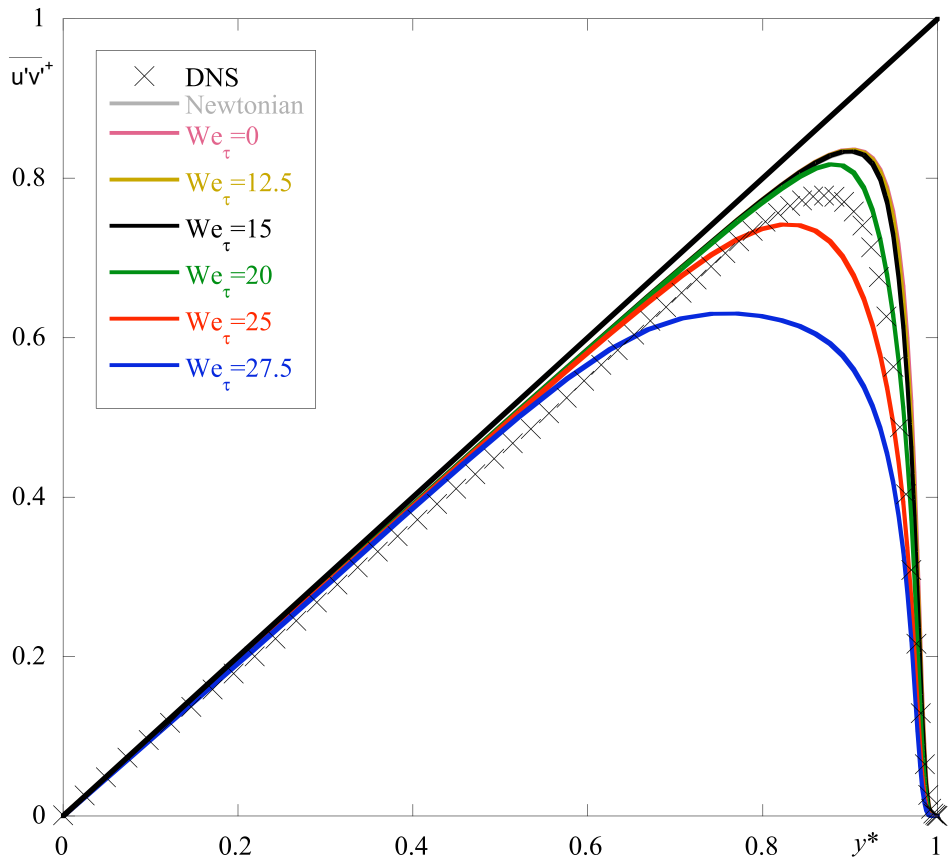
Effect of Weissenberg 3: $Re_{\tau 0}$= 395; β=0.9, L^2=900



Effect of Weissenberg 4: $Re_{\tau 0}$= 395; β=0.9, L^2=900

C_{xx}





Conclusions

- Developed simplified k-ε model: code and closures are working
- Viscoelastic turbulent transport not very important at DR= 18%
- Viscoelastic turbulent transport is reasonably well modeled

- Viscoelastic stress power well modeled by NLT_{ij}

- NLT_{ij} required for C_{ij} and ε_{ij}^{V}
- NLT_{ij} closure needs significant improvement
- Need to model viscoelastic turbulence production close to wall

- Deficiencies also related to Newtonian part of model
- Isotropic turbulence does not allow a good model
- Need to consider anisotropic turbulence: anisotr. k-ε and RSM