RECENT DEVELOPMENTS AND CHALLENGES IN TURBULENCE MODELING FOR VISCOELASTIC FLUIDS

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Conference on "Complex Flows of Complex Fluids"

17th-19th March 2008

University of Liverpool, United Kingdom

Summary

- **Brief review of existing RANS models for FENE-P**

- **Governing equations for FENE-P in RANS/RACE form**
  (Reynolds decomposition)

- **Development of closures for RANS/RACE of FENE-P (2007 (LDR) and 2008 (LDR & HDR) closures)**

- **Some results (2007 models only)**
- **Conclusions and future prospects**

Governing equations: turbulent flow & FENE-P

Reynolds decomposition $\hat{B} = B + b'$ where $\overline{b'} = 0$

^- instantaneous quantities
Overbar or upper-case letters- time-averaged quantities
' or lower-case letters- fluctuating quantities

Continuity (incompressible): $\dfrac{\partial \hat{U}_i}{\partial x_i} = 0$

Momentum $\mathcal{M}\left(\hat{U}_{ij}\right)$: $\rho \dfrac{\partial \hat{U}_i}{\partial t} + \rho \hat{U}_k \dfrac{\partial \hat{U}_i}{\partial x_k} = -\dfrac{\partial \hat{p}}{\partial x_i} + \eta_s \dfrac{\partial^2 \hat{U}_i}{\partial x_k \partial x_k} + \dfrac{\partial \hat{\tau}_{ik,p}}{\partial x_k}$

Rheological constitutive equation: **FENE-P**

$$\hat{\tau}_{ij} = 2\eta_s \hat{S}_{ij} + \hat{\tau}_{ij,p}$$

$$\hat{\tau}_{ij,p} = \frac{\eta_p}{\lambda}\left[f\left(\hat{C}_{kk}\right)\hat{C}_{ij} - f(L)\delta_{ij}\right]$$

$$f\left(\hat{C}_{kk}\right) = \frac{L^2 - 3}{L^2 - \hat{C}_{kk}} \qquad f(L) = 1$$

$$\mathcal{L}\left(\hat{C}_{ij}\right) : \lambda\left(\frac{\partial \hat{C}_{ij}}{\partial t} + \hat{U}_k \frac{\partial \hat{C}_{ij}}{\partial x_k} - \hat{C}_{jk}\frac{\partial \hat{U}_i}{\partial x_k} - \hat{C}_{ik}\frac{\partial \hat{U}_j}{\partial x_k}\right) = -\left[f\left(\hat{C}_{kk}\right)\hat{C}_{ij} - f(L)\delta_{ij}\right]$$

Solution of governing equations

Direct **N**umerical **S**imul.

Sureshkumar, Beris, Handler (1997) PoF, v9, 743
Den Toonder, Hulsen, Kuiken, Nieuwstadt (1997) JFM 337, 193
Dimitropoulos, Sureshkumar, Beris (1998) JNNFM v79, 433
Dimitropoulos, Sureshkumar, Beris, Handler (2001) PoF v13, 1016
Angelis, Casciola, Piva (2002) Comput. Physics v31, 495
Ptasinski et al (2003) JFM v490, 251
Housiadas, Beris (2003) PoF v15, 2369
Zhou, Akhavan (2003) JNNFM v109, 115
Stone, Graham (2003) PoF v15, 1247
Yu, Kawaguchi (2003) IJHFF v24, 491
Vaithianathan, Collins (2003) J. Comput Physics v187, 1
Housiadas, Beris (2004) PoF v16, 1581
Dubief et al (2004) JFM v514, 271
Yu, Li, Kawaguchi (2004) IJHFF v25, 961
Dimitropoulos et al (2005) PoF v17, 011705
Li, Gupta, Sureshkumar, Khomami (2006) JNNFM v139, 177
Li, Sureshkumar, Khomami (2006) JNNFM v140, 23
Benzi, Angelis, L'vov, Procaccia, Tiberkevich (2006) JFM v551, 185
& others — see recent review
White, Mungal (2008) Ann. Rev Fluid Mech (2008) v40, 235

↓

Physical understanding

↓

Too costly for engineering calculations

↳ **Turbulence model development**

- **LES**
- **RANS/RACE** →

Time-averaged governing equations: RANS and RACE

Continuity: $\frac{\partial U_i}{\partial x_i} = 0$

Momentum balance:

$$\rho \frac{\partial U_i}{\partial t} + \rho U_k \frac{\partial U_i}{\partial x_k} = -\frac{\partial \bar{p}}{\partial x_i} + \eta_s \frac{\partial^2 U_i}{\partial x_k \partial x_k} - \frac{\partial}{\partial x_k}\left(\rho \overline{u_i u_k}\right) + \frac{\partial \bar{\tau}_{ik,p}}{\partial x_k}$$

$$\bar{\tau}_{ij} = 2\eta_s S_{ij} + \bar{\tau}_{ij,p}$$

Rheological constitutive equation: **FENE-P**

$$\bar{\tau}_{ij,p} = \frac{\eta_p}{\lambda}\left[f(C_{kk})C_{ij} - f(L)\delta_{ij}\right] + \frac{\eta_p}{\lambda}\overline{f(C_{kk}+c_{kk})c_{ij}}$$

RACE →

$$\overset{\nabla}{C}_{ij} + \overline{u_k \frac{\partial c_{ij}}{\partial x_k}} - \left(\overline{c_{kj}\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik}\frac{\partial u_j}{\partial x_k}}\right) = -\frac{\bar{\tau}_{ij,p}}{\eta_p}$$

M_{ij} $\quad CT_{ij}$ $\quad NLT_{ij}$

Closures required

Existing models for FENE-P: Li et al (2006)

$$\rho \frac{\partial U_i}{\partial t} + \rho U_k \frac{\partial U_i}{\partial x_k} = -\frac{\partial \overline{p}}{\partial x_i} + \eta_s \frac{\partial^2 U_i}{\partial x_k \partial x_k} - \frac{\partial}{\partial x_k}\left(\rho \overline{u_i u_k}\right) + \frac{\partial \overline{\tau}_{ik,p}}{\partial x_k}$$

0 equation model

(shear stress only)

Reynolds stress

$$-\overline{uv} = \nu_{T,v} \frac{dU}{dy}$$

$$\nu_{T,v} = \phi \nu_{T,N}$$

$$\nu_{T,N} = \kappa u_\tau y$$

$$\phi = \left[a(DR) y + b(DR)\right]$$

$$\int_0^{Re_\tau} \overline{\tau}_{xy,p} dy = \int_0^{Re_\tau} M_{xy} dy + \int_0^{Re_\tau} NLT_{xy} dy$$

$$I_{M_{xy}} = a' + b' DR + c' DR^2$$

$$I_{NLT_{xy}} = a'' + b'' DR + c'' DR^2$$

$$DR = 80\left\{1 - \exp\left[-0.025\left(We_\tau - 6.25\right)\left(\frac{Re_\tau}{125}\right)^{-0.225}\right]\right\}\left[1 - \exp\left(-0.0275 L\right)\right]$$

Existing models for FENE-P

FENE-P and based on DNS

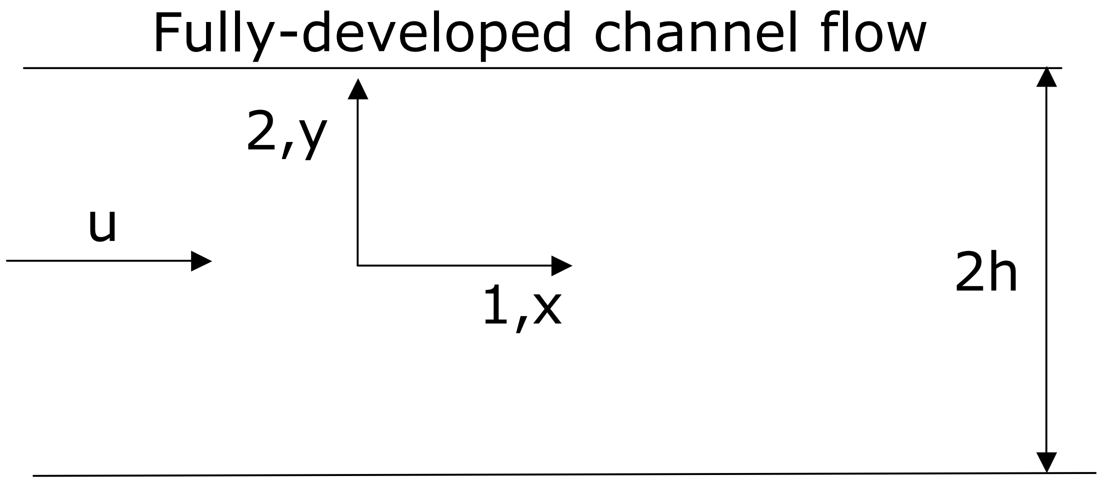
Leighton, Walker and Stephens (2002) APS meeting ?

- Reynolds stress transport model
- Slow pressure-strain redistribution term is modified by polymer (limits energy redistribution)
- New term in RS equation: interaction of $\tau'_{p,ij}$ & turbulence
- New term in C_{ij} equation (NLT_{ij})
- Additional diffusive flux terms not modeled

Shaqfeh (2006) AIChE Conference

- k-ε v^2-f extension model of Durbin (1995)
- Simplified model: $\overline{\tau}_{p,ij}$ proportional to mean strain (elongation)
- Coefficient has laminar and turbulent contribution
- Laminar part proportional to $\partial U/\partial y$
- Turbulent part proportional to k;
- Modifies pressure strain (v2 equation)
- One transport equation for C_{kk};

DNS cases: channel flow



$$We_\tau = \frac{\lambda u_\tau^2}{\nu_0}$$

$$Re_\tau = \frac{h u_\tau}{\nu_0}$$

$$Re_\tau = 395, \beta = 0.9, L^2 = 900$$

Low Drag Reduction

$$We_\tau = 25, DR = 18\%$$

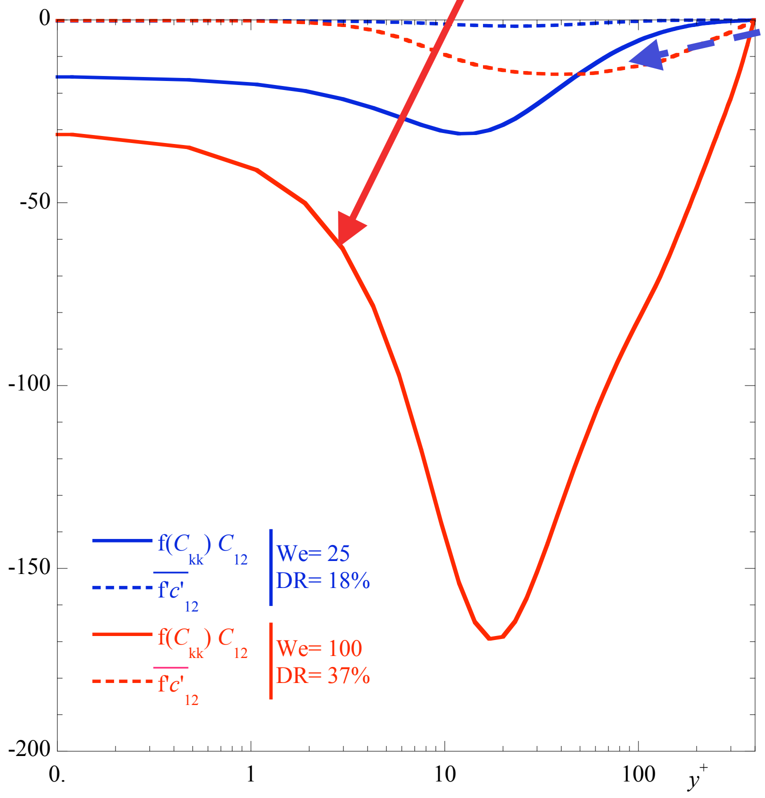
High Drag Reduction

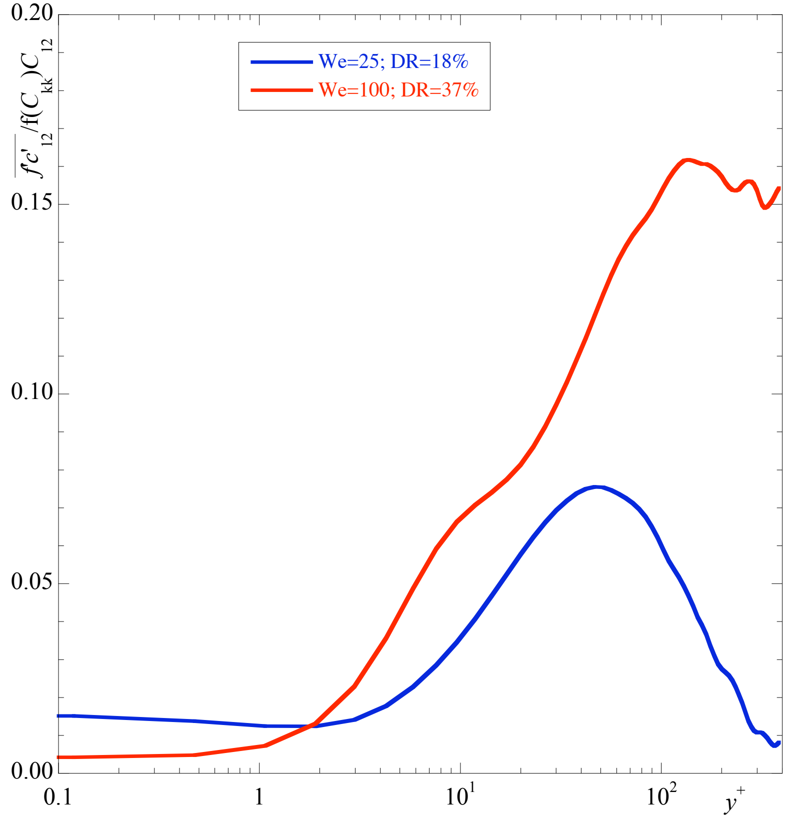
$$We_\tau = 100, DR = 37\%$$

- 2007 model**s** (Pinho et al JNNFM 2008 & unpublished) - Only **LDR**
- 2008 model (under develop.)- Recalculated DNS + **LDR** & **HDR**
- **Closures valid for 1st & higher order turbulence models**

Time- average polymer stress

$$\overline{\tau}_{ij,p} = \frac{\eta_p}{\lambda}\left[f(C_{kk})C_{ij} - f(L)\delta_{ij} \right] + \frac{\eta_p}{\lambda}\overline{f(C_{kk}+c_{kk})c_{ij}}$$

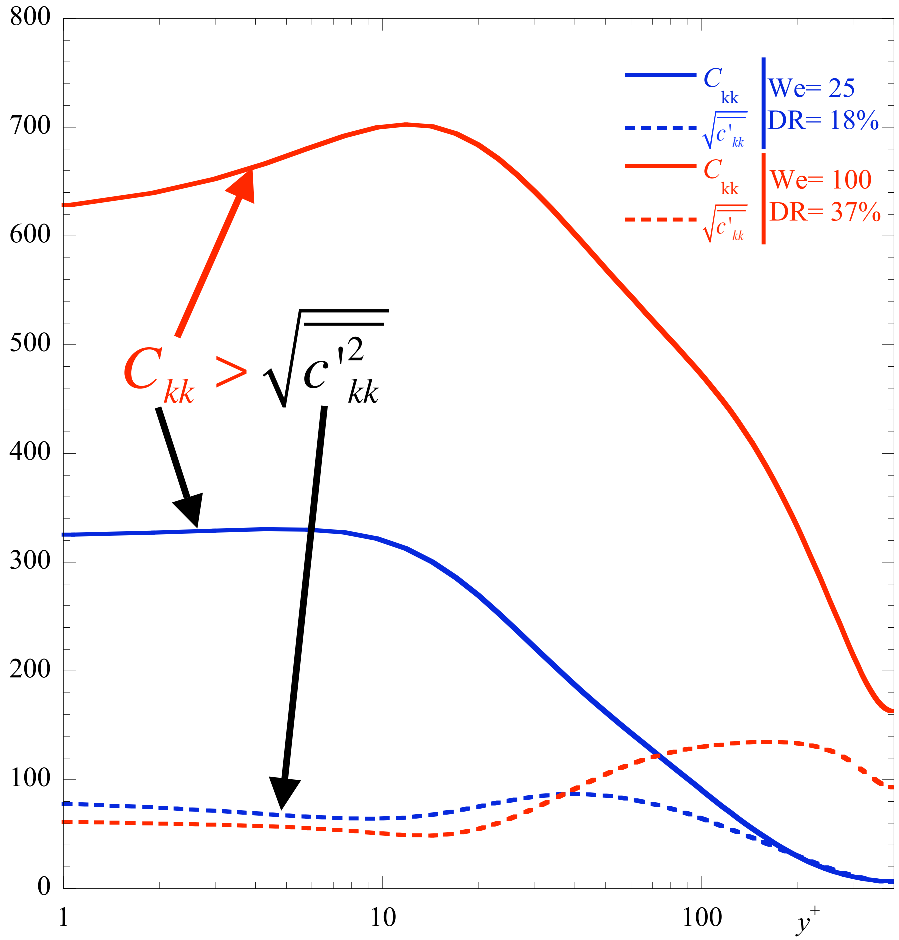


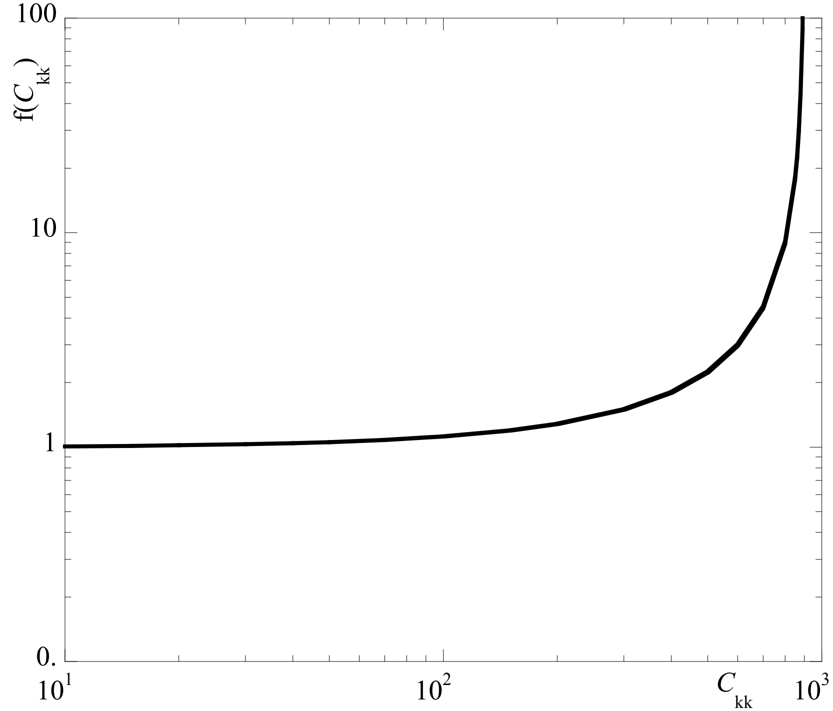


$$f(C_{kk})C_{12} \gg \overline{f'c_{12}}$$ → **Can be neglected !**

Function $f\,(C_{kk})$

Function: $f\left(\hat{C}_{kk}\right)=\dfrac{L^2-3}{L^2-\left(C_{kk}+c'_{kk}\right)}$





$$\overline{f\left(\hat{C}_{kk}\right)b_{lm}d_i}\approx f\left(C_{kk}\right)\overline{b_{lm}d_i}$$

$$\overline{f\left(\hat{C}_{kk}\right)b_{ij}}\approx f\left(C_{kk}\right)\overline{b}_{ij}=0$$

This will be used frequently

Time-average evolution equation for the conformation: RACE

$$\lambda \overset{\nabla}{C}_{ij} + \lambda\left[\overline{u_k \frac{\partial c_{ij}}{\partial x_k}} - \left(\overline{c_{kj}\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik}\frac{\partial u_j}{\partial x_k}}\right)\right] = -\left[f(C_{kk})C_{ij} - f(L)\delta_{ij}\right]$$

$$\overset{\nabla}{C}_{ij} + \overline{u_k \frac{\partial c_{ij}}{\partial x_k}} - \left(\overline{c_{kj}\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik}\frac{\partial u_j}{\partial x_k}}\right) + D_{ij} = -\frac{\bar{\tau}_{ij,p}}{\eta_p}$$

M_{ij} CT_{ij} NLT_{ij}

D_{ij}: **Added for stability**
Should be negligible

DNS: Housiadas et al (2005) Phys Fluids 17, 35106, Li et al (2006 a) JNNFM

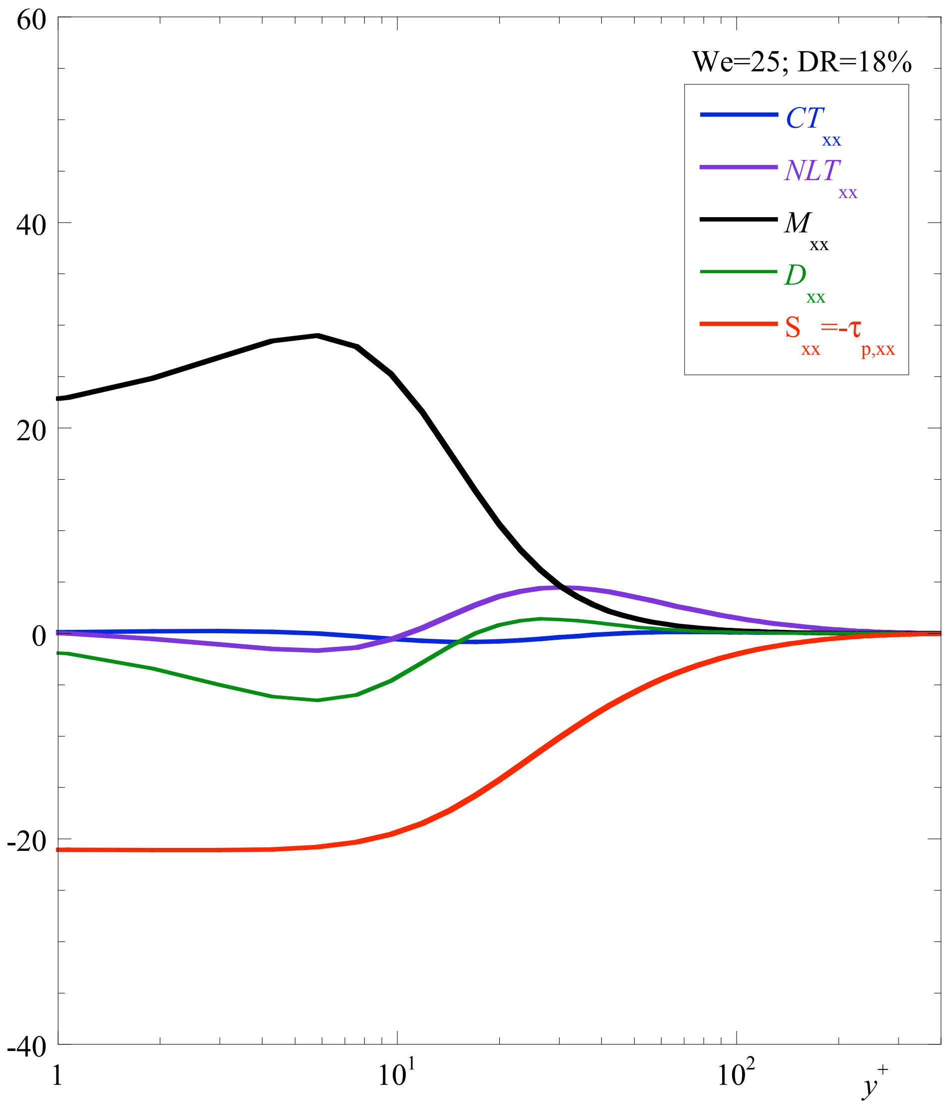
M_{ij}: Oldroyd derivative
Mean flow distortion
Exact and large

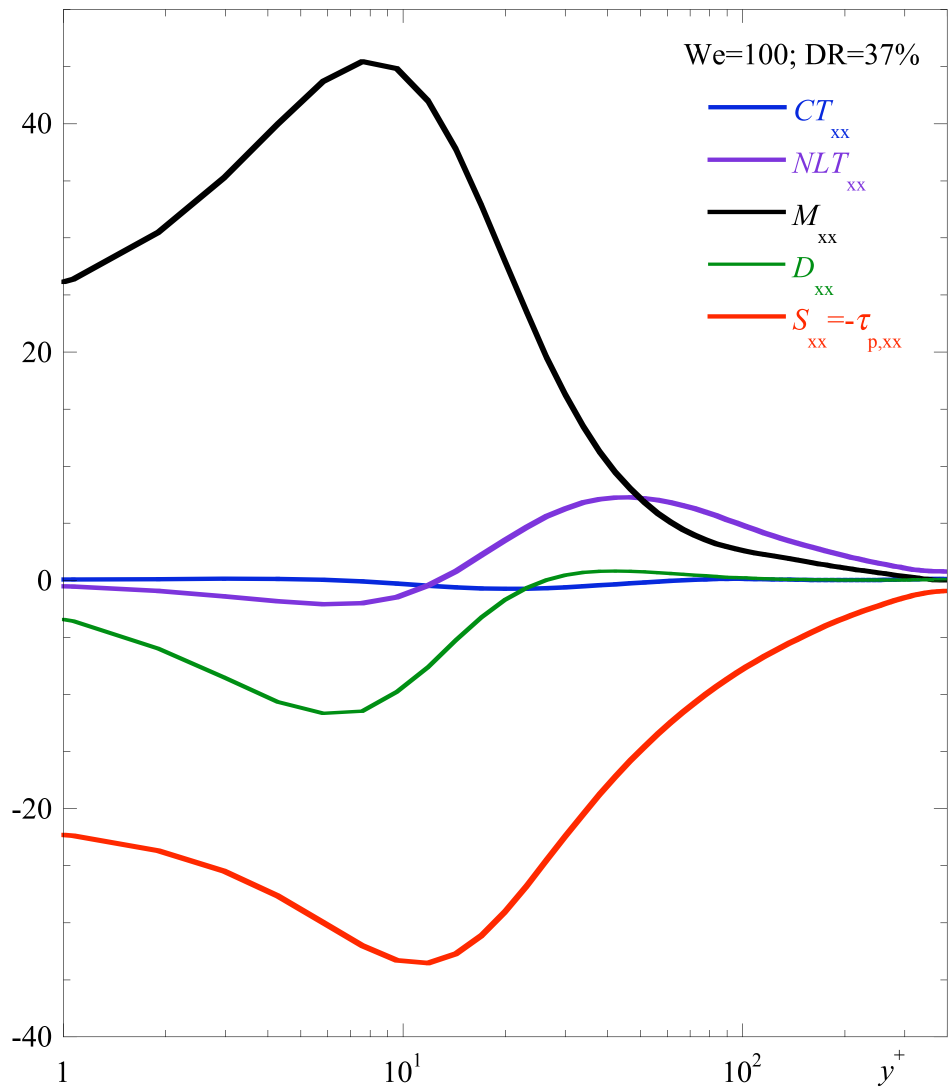
NLT_{ij}: **turbulent distortion**
originates in distortion of Oldroyd derivative- not negligible
Must be modeled

CT_{ij}: originates in advective term, negligible
no need for modeling

Balance of RACE 1

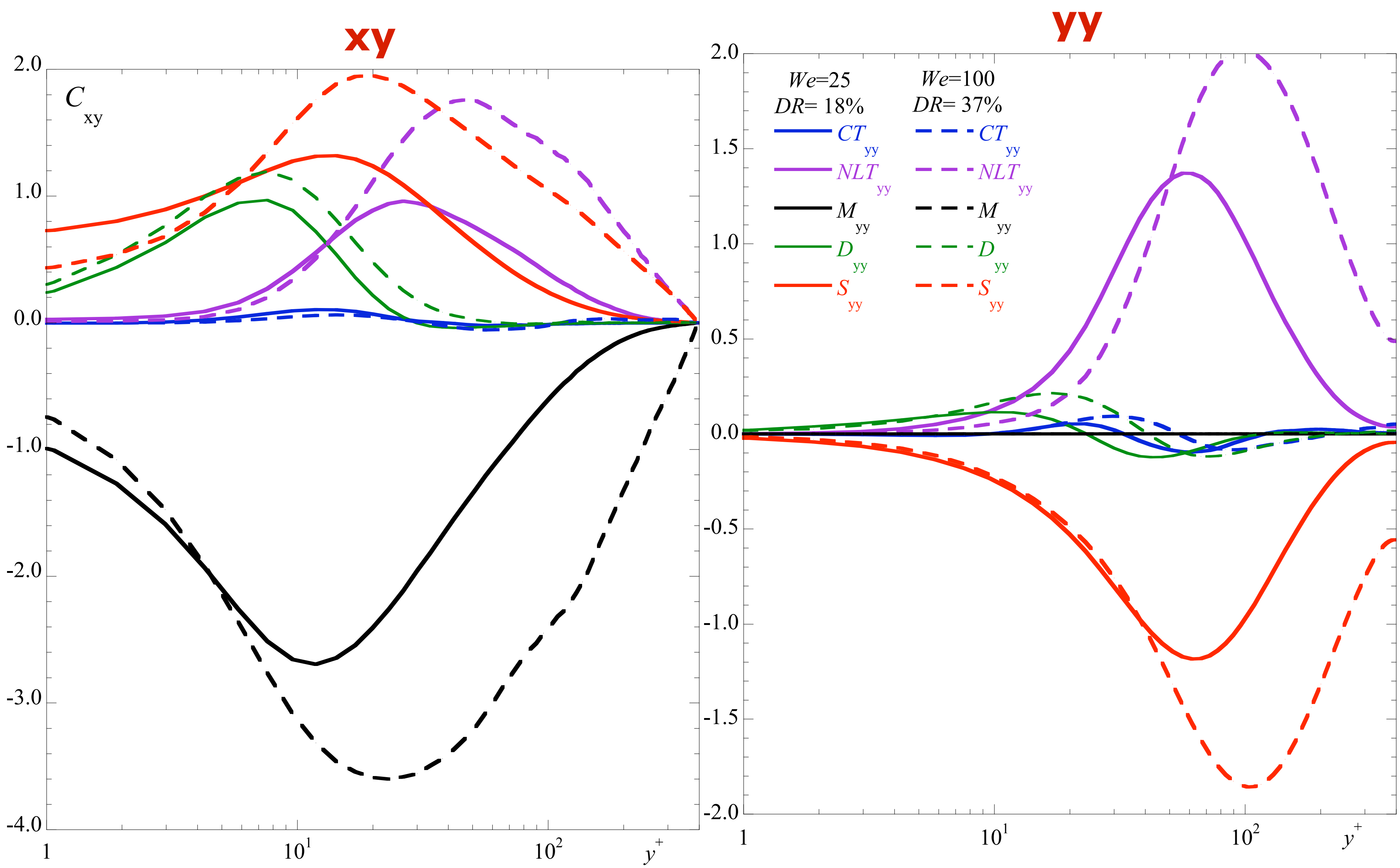
xx





DNS: Housiadas et al (2005), Li et al (2006) JNNFM

Balance of RACE 2



Approximate equation for NLT_{ij}

$$NLT_{ij} = \overline{c_{kj}\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik}\frac{\partial u_j}{\partial x_k}}$$

$$\overline{\mathcal{L}\left(\hat{C}_{kj}\right)\frac{\partial u_i}{\partial x_k}} + \overline{\mathcal{L}\left(\hat{C}_{ik}\right)\frac{\partial u_j}{\partial x_k}}$$

$$\boxed{\overline{f\left(\hat{C}_{mm}\right)c_{kj}\frac{\partial u_i}{\partial x_k}} + \overline{f\left(\hat{C}_{mm}\right)c_{ik}\frac{\partial u_j}{\partial x_k}}} + C_{kj}\overline{f\left(\hat{C}_{mm}\right)\frac{\partial u_i}{\partial x_k}} + C_{ik}\overline{f\left(\hat{C}_{mm}\right)\frac{\partial u_j}{\partial x_k}} + \lambda\left[\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial c_{kj}}{\partial t}} + \overline{\frac{\partial u_j}{\partial x_k}\frac{\partial c_{ik}}{\partial t}}\right] +$$

$$+\lambda\left[\frac{\partial C_{kj}}{\partial x_n}\overline{u_n\frac{\partial u_i}{\partial x_k}} + \frac{\partial C_{ik}}{\partial x_n}\overline{u_n\frac{\partial u_j}{\partial x_k}} + \overline{\frac{\partial\left(U_n c_{kj}\right)}{\partial x_n}\frac{\partial u_i}{\partial x_k}} + \overline{\frac{\partial\left(U_n c_{ik}\right)}{\partial x_n}\frac{\partial u_j}{\partial x_k}} + \overline{u_n\frac{\partial c_{kj}}{\partial x_n}\frac{\partial u_i}{\partial x_k}} + \overline{u_n\frac{\partial c_{ik}}{\partial x_n}\frac{\partial u_j}{\partial x_k}}\right] -$$

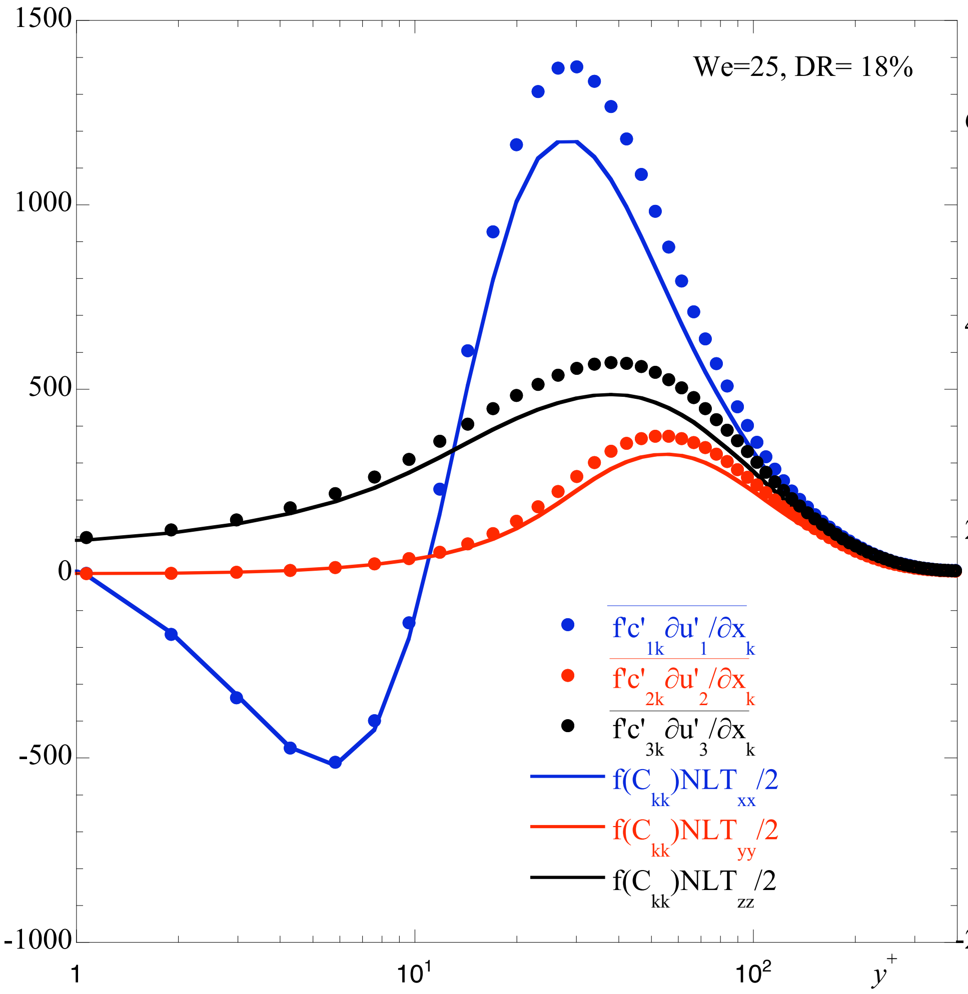
$$-\lambda\left[\frac{\partial U_k}{\partial x_n}\left(\overline{c_{jn}\frac{\partial u_i}{\partial x_k}} + \overline{c_{in}\frac{\partial u_j}{\partial x_k}}\right) + \frac{\partial U_j}{\partial x_n}\overline{c_{kn}\frac{\partial u_i}{\partial x_k}} + \frac{\partial U_i}{\partial x_n}\overline{c_{kn}\frac{\partial u_j}{\partial x_k}} + C_{kn}\left(\overline{\frac{\partial u_j}{\partial x_n}\frac{\partial u_i}{\partial x_k}} + \overline{\frac{\partial u_i}{\partial x_n}\frac{\partial u_j}{\partial x_k}}\right)\right] -$$

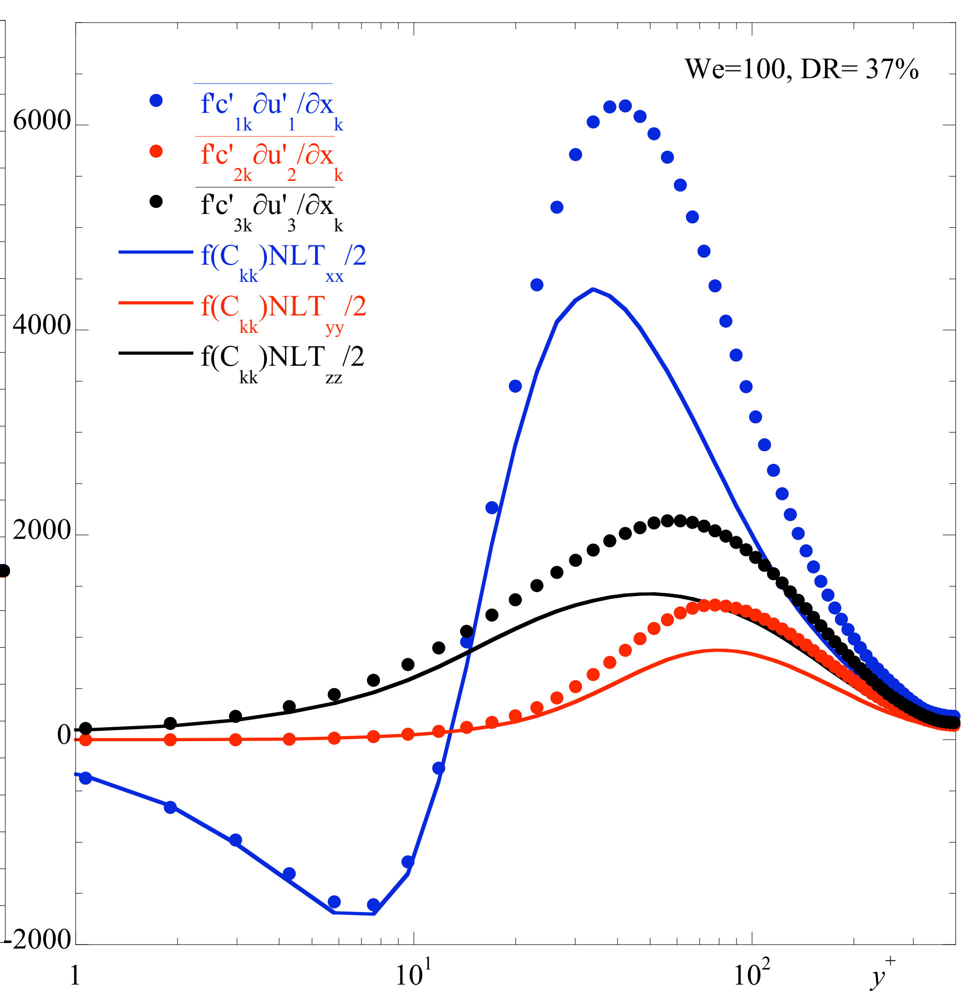
$$-\lambda\left[C_{jn}\overline{\frac{\partial u_k}{\partial x_n}\frac{\partial u_i}{\partial x_k}} + C_{in}\overline{\frac{\partial u_k}{\partial x_n}\frac{\partial u_j}{\partial x_k}} + \overline{c_{jn}\frac{\partial u_k}{\partial x_n}\frac{\partial u_i}{\partial x_k}} + \overline{c_{in}\frac{\partial u_k}{\partial x_n}\frac{\partial u_j}{\partial x_k}} + \overline{c_{kn}\frac{\partial u_j}{\partial x_n}\frac{\partial u_i}{\partial x_k}} + \overline{c_{kn}\frac{\partial u_i}{\partial x_n}\frac{\partial u_j}{\partial x_k}}\right] = 0$$

SIMPLER THAN EXACT EQUATION: $\overline{\frac{\mathcal{L}\left(\hat{C}_{kj}\right)}{f\left(\hat{C}_{mm}\right)}\frac{\partial u_i}{\partial x_k}} + \overline{\frac{\mathcal{L}\left(\hat{C}_{ik}\right)}{f\left(\hat{C}_{mm}\right)}\frac{\partial u_j}{\partial x_k}}$

Simplifications for modeling NLT_{ij}

$$\overline{f'c'_{jk}\frac{\partial u_i}{\partial x_k}+f'c'_{ik}\frac{\partial u_j}{\partial x_k}} \approx f\left(C_{mm}\right)\left(\overline{c_{kj}\frac{\partial u_i}{\partial x_k}}+\overline{c_{ik}\frac{\partial u_j}{\partial x_k}}\right)=f\left(C_{mm}\right)NLT_{ij}$$





Previous models for NLT_{ij} - 2007 1st model

First model

Exact equation is too complex.

Alternative model based on:

1) Identification of possible dependencies from inspection of exact equation
2) Simplicity, but capturing main features
3) *We*=25 (DR=18%)

$$f\left(C_{mm}\right)\frac{NLT_{ij}}{\lambda} = function\left(S_{ij}, W_{ij}, C_{ij}, \varepsilon_{ij}^{N}, \frac{\partial \overline{u_i u_j}}{\partial x_k}, \frac{\partial C_{ij}}{\partial x_k}, \frac{\partial NLT_{ij}}{\partial x_n}, M_{ij}, \overline{u_i u_j}\right)$$

$$f\left(C_{mm}\right)\frac{NLT_{ij}}{\lambda} = f_{\mu_1}\left[\frac{C_{E_3} u_\tau^2}{\nu_0^2} C_{kk}\overline{u_i u_j} + \frac{C_{\alpha_{14}}}{\nu_0}\left(\overline{u_i u_k} W_{kn} C_{nj} + \overline{u_j u_k} W_{kn} C_{ni} + \overline{u_k u_i} W_{jn} C_{nk}\right)\right]$$

2 coefficients

$$C_{E_3} = 0.00035; C_{\alpha_{14}} = 0.00015$$

1 damping function

$$f_{\mu_1} = \left(1 - \exp\left(-y^+/26.5\right)\right)^2$$

Pinho, Li, Younis, Sureshkumar (2008) JNNFM, in press

Modeling NLT_{ij} 1- 2008 model and 2nd 2007 model

1) $CT_{ij} = \overline{u_k \frac{\partial c_{ij}}{\partial x_k}} \approx 0$ suggests $\overline{u_n \frac{\partial c_{ij}}{\partial x_k}} \approx 0$

2) Homogeneous turbulence (negligible turbulent diffusion) $\overline{u_n \frac{\partial u_j}{\partial x_k}} \approx 0$

3) Invariance laws $U_n \left(\overline{\frac{\partial c_{kj}}{\partial x_n} \frac{\partial u_i}{\partial x_k}} + \overline{\frac{\partial c_{ik}}{\partial x_n} \frac{\partial u_j}{\partial x_k}} \right) = 0$

$$\overline{u_n \frac{\partial c_{kj}}{\partial x_n} \frac{\partial u_i}{\partial x_k}} + \overline{u_n \frac{\partial c_{ik}}{\partial x_n} \frac{\partial u_j}{\partial x_k}} \approx 0$$

4) Homogeneous isotropic turbulence

$$\overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_l}} = \frac{8}{3} \frac{k}{\lambda_f^2} \left[\delta_{ij}\delta_{kl} - \frac{1}{4}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} \right) \right]$$

Taylor's longitudinal micro-scale $\varepsilon = 20 \frac{\nu k}{\lambda_f^2}$

$$C_{kn}\left(\overline{\frac{\partial u_j}{\partial x_n} \frac{\partial u_i}{\partial x_k}} + \overline{\frac{\partial u_i}{\partial x_n} \frac{\partial u_j}{\partial x_k}} \right) + C_{jn} \overline{\frac{\partial u_k}{\partial x_n} \frac{\partial u_i}{\partial x_k}} + C_{in} \overline{\frac{\partial u_k}{\partial x_n} \frac{\partial u_j}{\partial x_k}} \approx C_{\varepsilon_N} f_{N_2} \frac{4}{15} \frac{\varepsilon}{\beta \times We_{\tau_0} \times \nu_T} C_{mm} \delta_{ij}$$

Hinze (1975); Mathieu & Scott (2000)

Modeling NLT_{ij} 2- 2008 model and 2nd 2007 model

5) *ad-hoc* + symmetry, invariance, permutation, realizability

$$\frac{\partial U_k}{\partial x_n}\left(\overline{c_{jn}\frac{\partial u_i}{\partial x_k}}+\overline{c_{in}\frac{\partial u_j}{\partial x_k}}\right)+\frac{\partial U_j}{\partial x_n}\overline{c_{kn}\frac{\partial u_i}{\partial x_k}}+\frac{\partial U_i}{\partial x_n}\overline{c_{kn}\frac{\partial u_j}{\partial x_k}}\approx$$

$$C_{N_3}\left[\left|\overline{\frac{\partial U_j}{\partial x_k}\frac{\partial U_m}{\partial x_n}}\right|C_{kn}\frac{\overline{u_i u_m}}{\nu_0\sqrt{2S_{pq}S_{pq}}}+\left|\overline{\frac{\partial U_i}{\partial x_k}\frac{\partial U_m}{\partial x_n}}\right|C_{kn}\frac{\overline{u_j u_m}}{\nu_0\sqrt{2S_{pq}S_{pq}}}\right]$$

6) $$C_{kj}\overline{f\left(\hat{C}_{mm}\right)\frac{\partial u_i}{\partial x_k}}+C_{ik}\overline{f\left(\hat{C}_{mm}\right)\frac{\partial u_j}{\partial x_k}}$$

$$\approx C_{kj}f\left(C_{mm}\right)\overline{\frac{\partial u_i}{\partial x_k}}+C_{ik}f\left(C_{mm}\right)\overline{\frac{\partial u_j}{\partial x_k}}=0$$

$$\approx C_{N_2}\left[C_{kj}f\left(C_{mm}\right)\frac{\partial U_i}{\partial x_k}+C_{ik}f\left(C_{mm}\right)\frac{\partial U_j}{\partial x_k}\right]$$

7) Decoupling 3rd order correlation

$$\overline{c_{jn}\frac{\partial u_k}{\partial x_n}\frac{\partial u_i}{\partial x_k}}+\overline{c_{in}\frac{\partial u_k}{\partial x_n}\frac{\partial u_j}{\partial x_k}}+\overline{c_{kn}\frac{\partial u_j}{\partial x_n}\frac{\partial u_i}{\partial x_k}}+\overline{c_{kn}\frac{\partial u_i}{\partial x_n}\frac{\partial u_j}{\partial x_k}}\approx$$

$$-C_{N_4}f_{N_1}\left[C_{jn}\frac{\partial U_k}{\partial x_n}\frac{\partial U_i}{\partial x_k}+C_{in}\frac{\partial U_k}{\partial x_n}\frac{\partial U_j}{\partial x_k}+C_{kn}\frac{\partial U_j}{\partial x_n}\frac{\partial U_i}{\partial x_k}+C_{kn}\frac{\partial U_i}{\partial x_n}\frac{\partial U_i}{\partial x_k}\right]$$

Closure for NLT_{ij}- 2008 model

We=25 (LDR) & 100 (HDR)

modified term & reprocessed DNS data

$$f(C_{mm})\frac{NLT_{ij}}{\lambda}=\frac{f(C_{mm})}{\lambda}\left\{C_{N_1}\frac{C_{ij}f(C_{mm})}{\lambda We_{\tau_0}}-C_{N_2}\left[C_{kj}\frac{\partial U_i}{\partial x_k}+C_{ik}\frac{\partial U_j}{\partial x_k}\right]\right\}$$

$$+C_{N_3}\frac{C_{kn}}{\nu_0\sqrt{2S_{pq}S_{pq}}}\left[\overline{u_i u_m}\left|\frac{\partial U_j}{\partial x_k}\frac{\partial U_m}{\partial x_n}\right|+\overline{u_j u_m}\left|\frac{\partial U_i}{\partial x_k}\frac{\partial U_m}{\partial x_n}\right|\right]$$

$$-C_{N_4}f_{N_1}\left[C_{jn}\frac{\partial U_k}{\partial x_n}\frac{\partial U_i}{\partial x_k}+C_{in}\frac{\partial U_k}{\partial x_n}\frac{\partial U_j}{\partial x_k}+C_{kn}\left(\frac{\partial U_j}{\partial x_n}\frac{\partial U_i}{\partial x_k}+\frac{\partial U_i}{\partial x_n}\frac{\partial U_j}{\partial x_k}\right)\right]$$

$$+C_{N_5}f_{N_2}\frac{4}{15}\frac{\varepsilon}{\beta\nu_s We_{\tau_0}}C_{mm}\delta_{ij}$$

$$f_{N_1}=\left[1-0.8\exp\left(-y^+/30\right)\right]^2$$

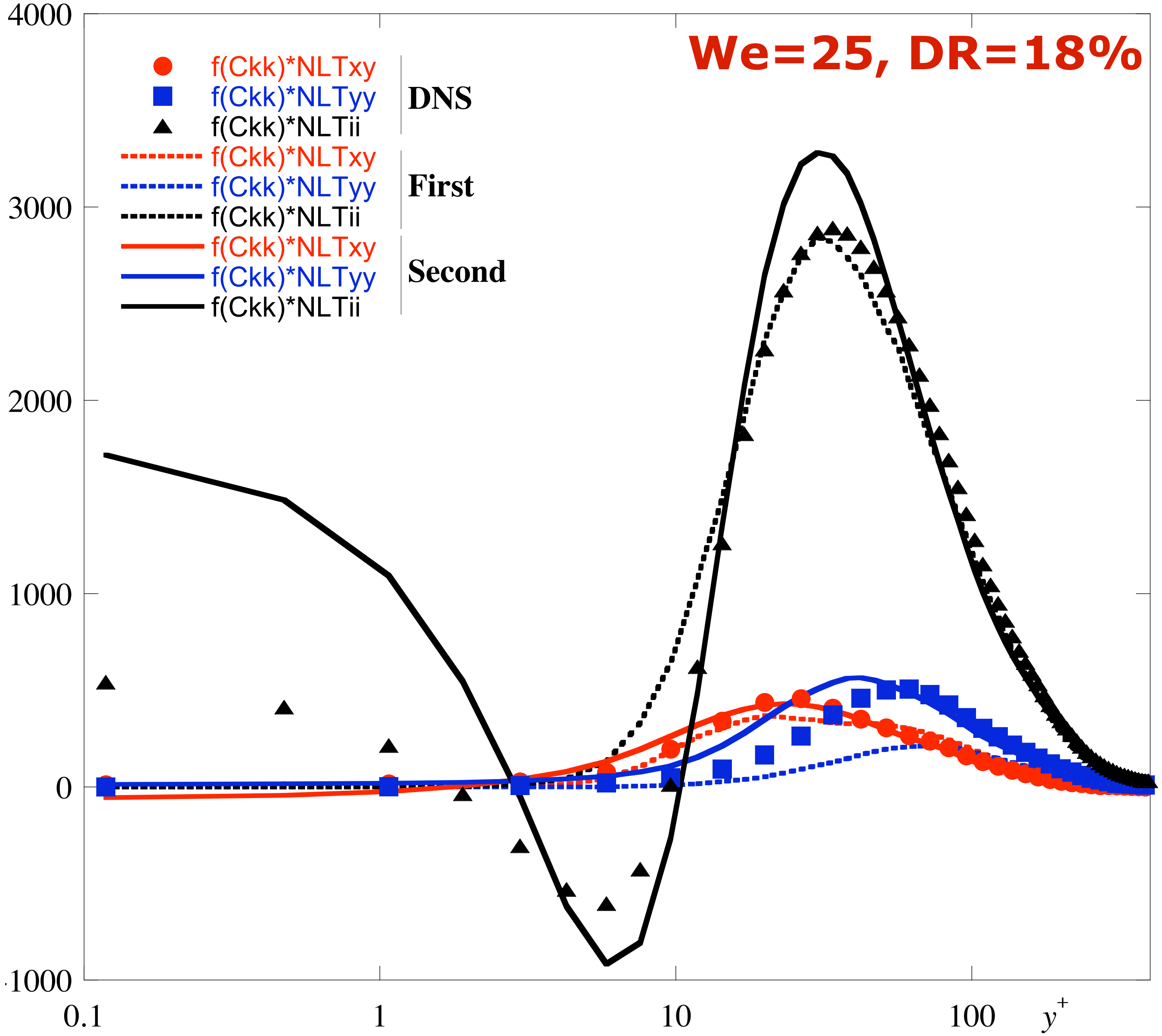
$$f_{N_2}=\left[1-\exp\left(-y^+/25\right)\right]^4$$

$C_{N_1}=12.7$ $\quad C_{N_4}=1.11$

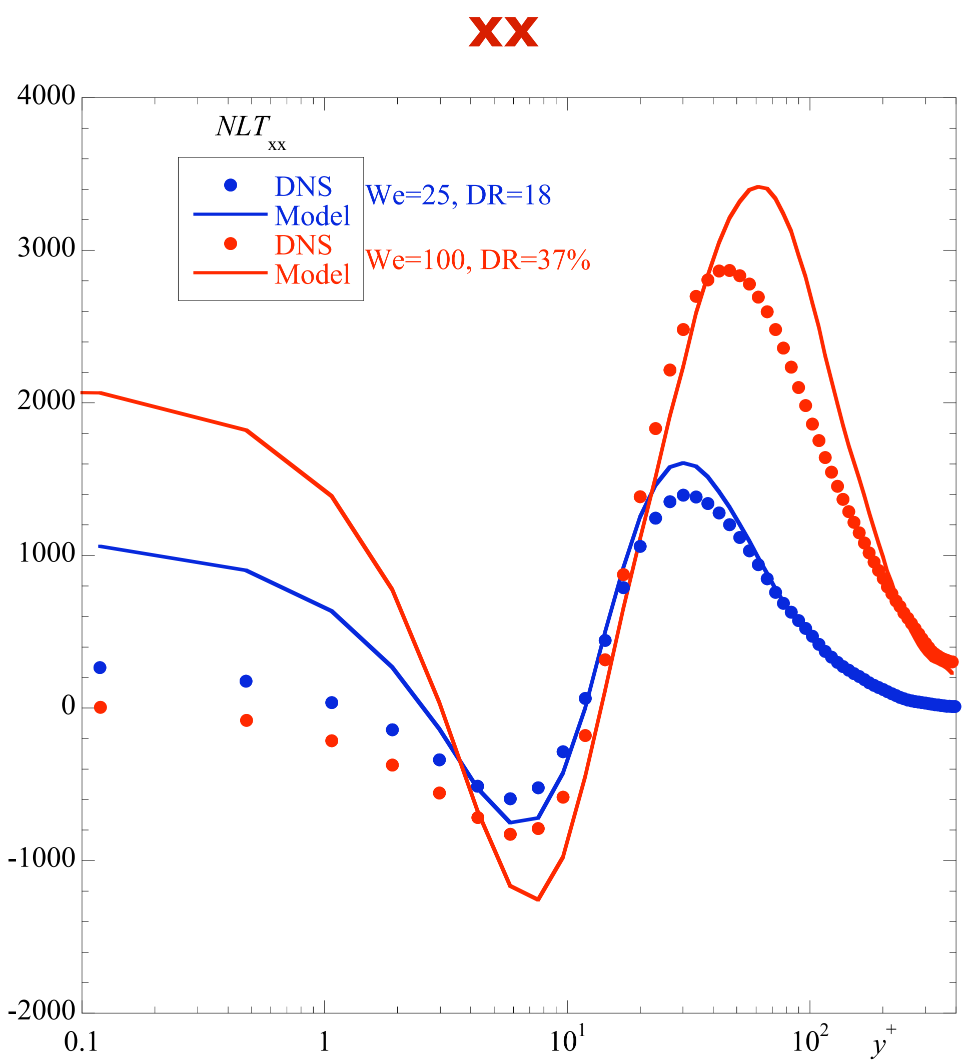
$C_{N_2}=0.32$ $\quad C_{N_5}=1.13$

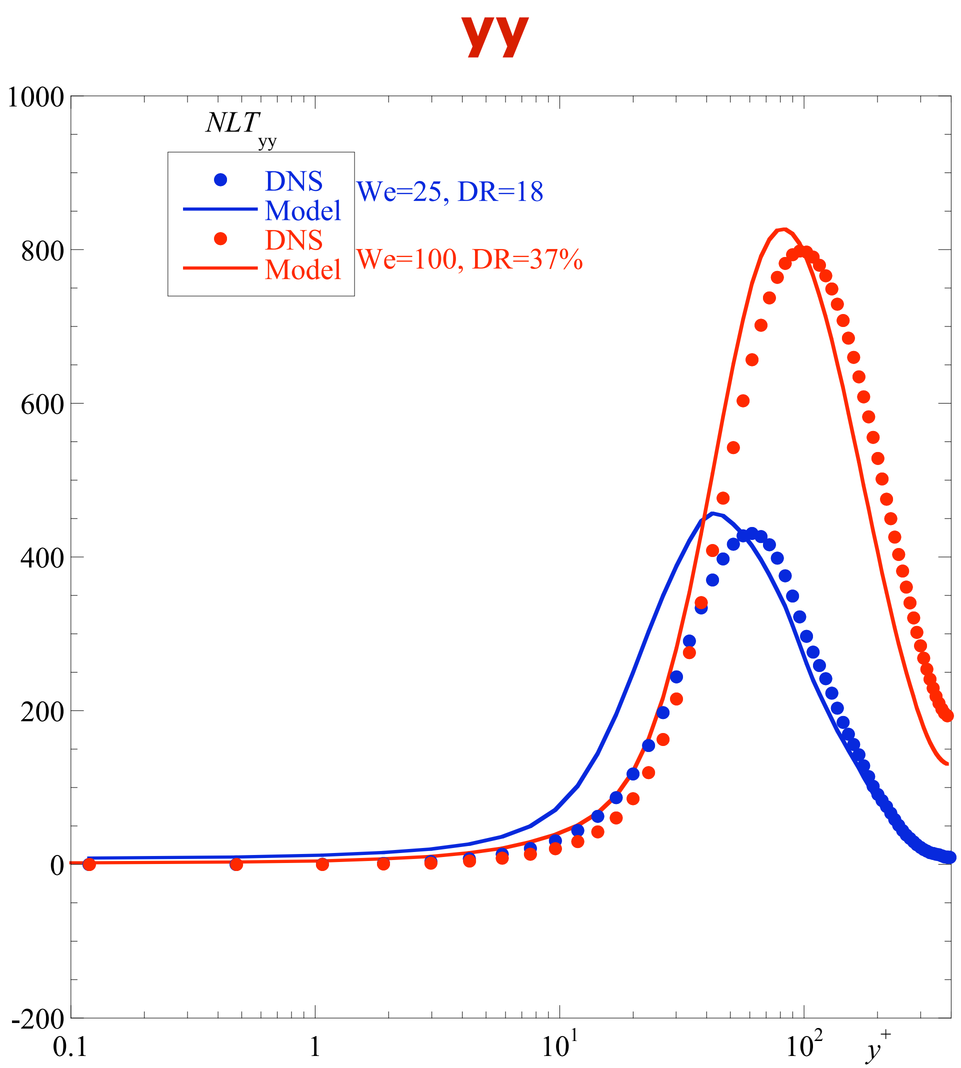
$C_{N_3}=0.024$

2007 models for NLT_{ij}: First versus second model

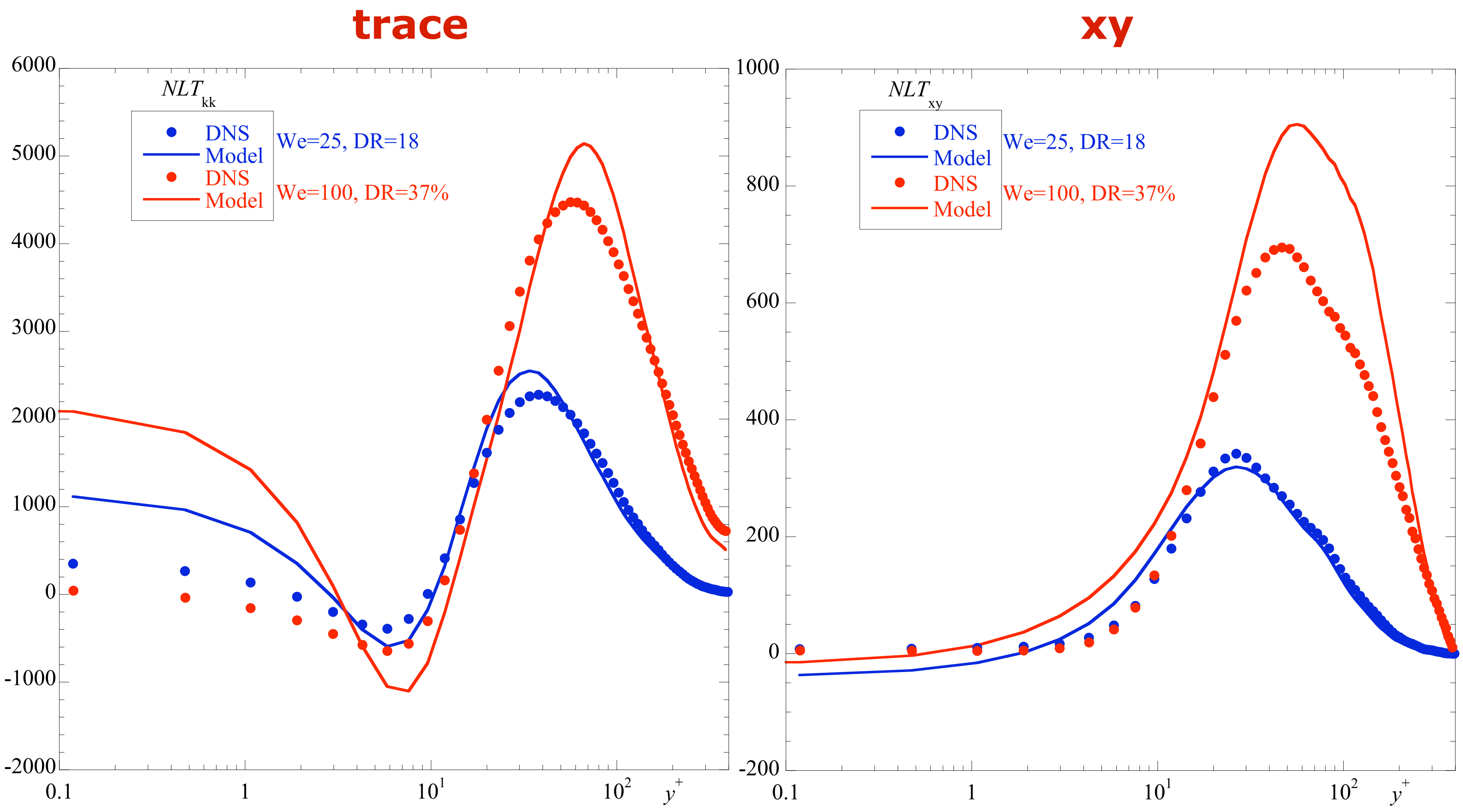


Performance of the 2008 NLT_{ij} closure: xx and yy





Performance of the 2008 NLT_{ij} closure: trace and xy



Modeling the Reynolds stress

Major issue: what model to use for Reynolds stresses?

1) Reynolds stresses: Prandtl-Kolmogorov model (k-ε closure)

$$-\overline{u_i u_j} = 2\nu_T S_{ij} - \frac{2}{3} k \delta_{ij} \quad \text{with} \quad \nu_T = C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}^N + \varepsilon^V}$$

Next slide

2) Dissipation of turbulent kinetic energy: ε^N

$$2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\rho \frac{Du_i}{Dt}\right)} + 2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\rho u_k \frac{\partial U_i}{\partial x_k}\right)} + 2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\rho \frac{\partial u_i u_k}{\partial x_k}\right)}$$

$$+2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\frac{\partial p'}{\partial x_i}\right)} - 2\rho \nu_s^2 \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\frac{\partial^2 u_i}{\partial x_k^2}\right)} - 2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\frac{\partial \tau'_{ik,p}}{\partial x_k}\right)} = 0$$

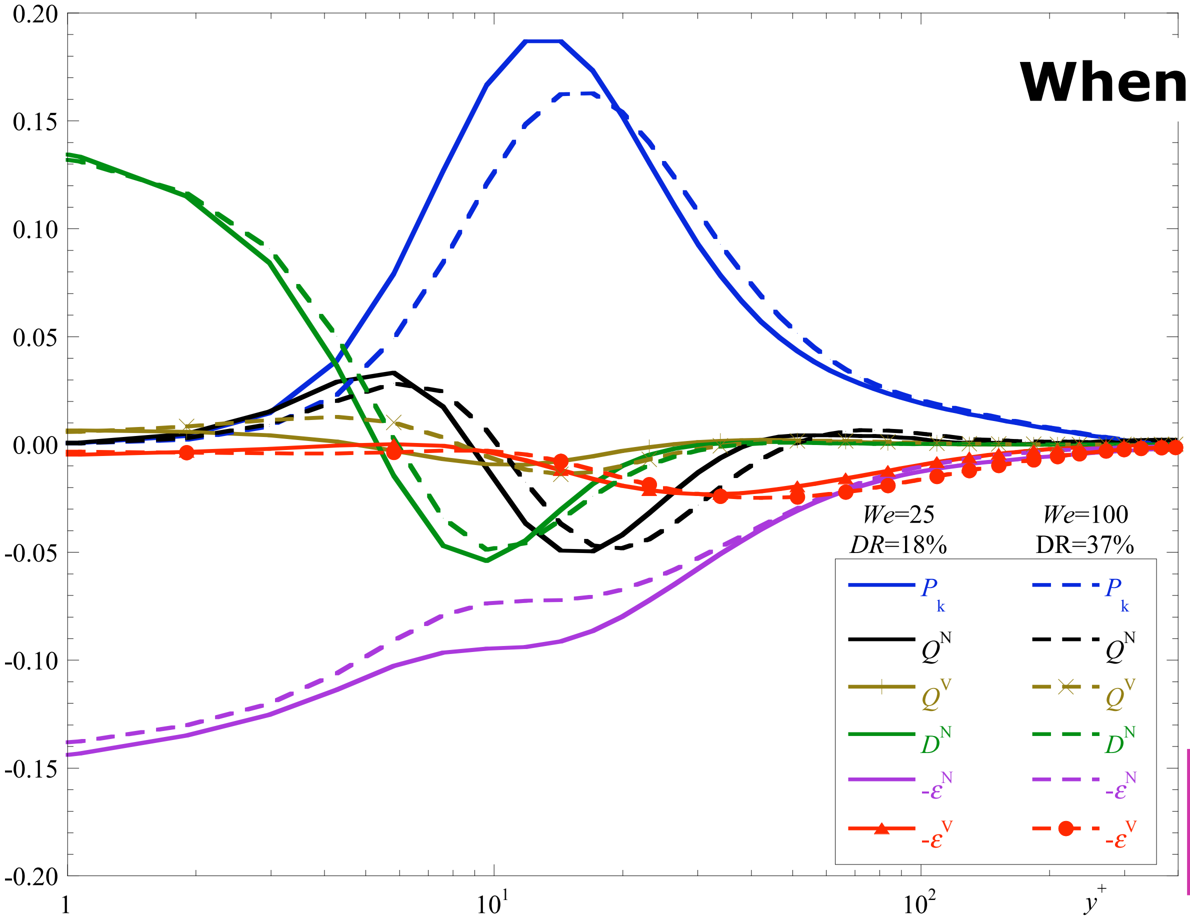
New term (will be considered in the future)

As for Newtonian fluids, most terms in ε^N are approximated

Transport equation for turbulent kinetic energy

$$\rho \frac{Dk}{Dt} = -\rho \overline{u_i u_k} \frac{\partial U_i}{\partial x_k} - \rho \overline{u_i \frac{\partial k'}{dx_i}} - \frac{\partial \overline{p' u_i}}{\partial x_i} + \eta_s \frac{\partial^2 k}{\partial x_i \partial x_i} - \eta_s \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_k}} + \frac{\partial \overline{\tau'_{ik,p} u_i}}{\partial x_k} - \overline{\tau'_{ik,p} \frac{\partial u_i}{\partial x_k}}$$

$0 \qquad P_k \qquad Q^N \qquad D^N \qquad -\varepsilon^N \qquad Q^V \qquad -\varepsilon^V$



When *We* increases (DR↑)

P_k decreases

ε^N decreases

Q^V increases in buffer l., but remains small

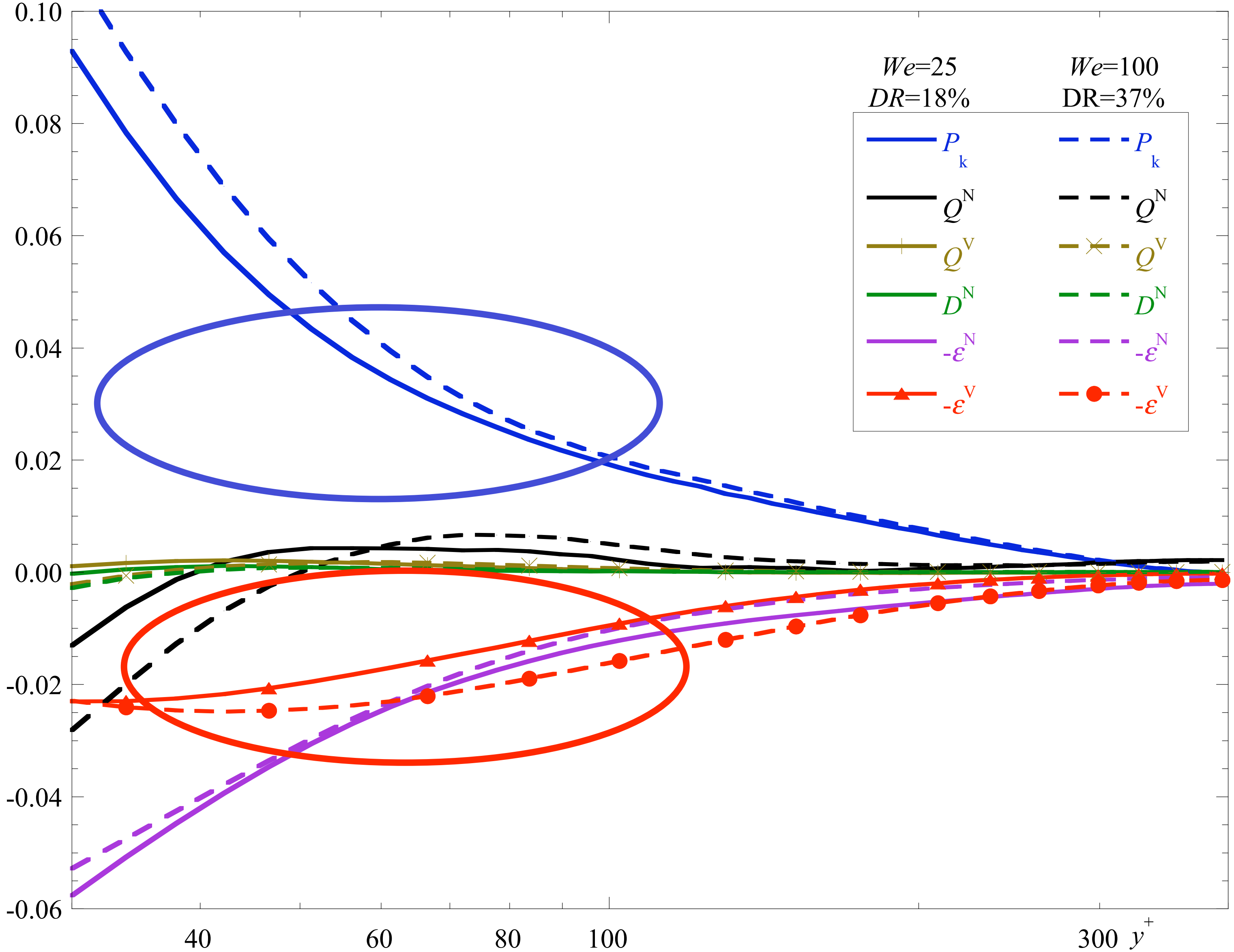
ε^V increases in inertial l.

Need to model well ε^V

Q^V is small

Need to modify model of ε^N

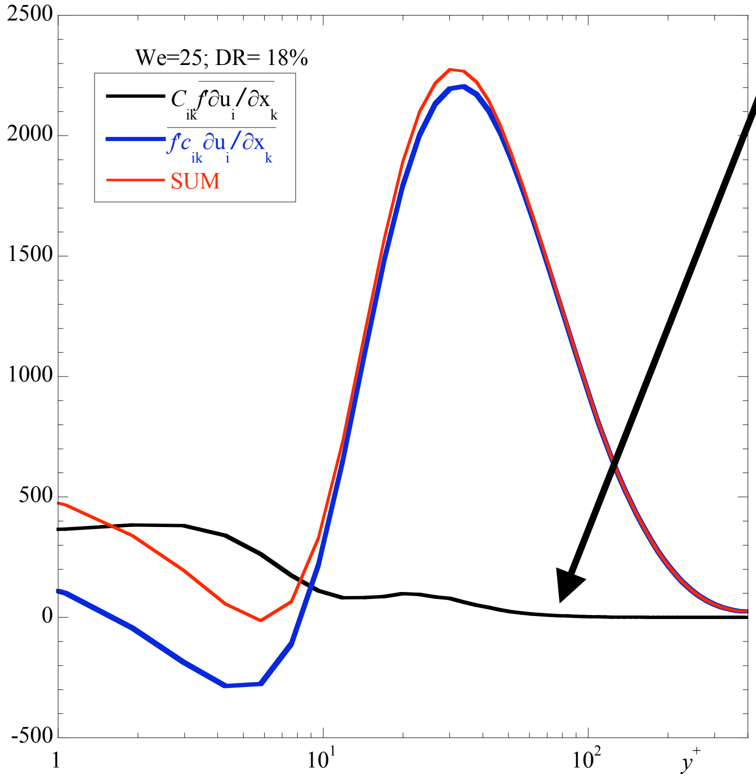
Zoom of balance of *k*: inertial sub-layer

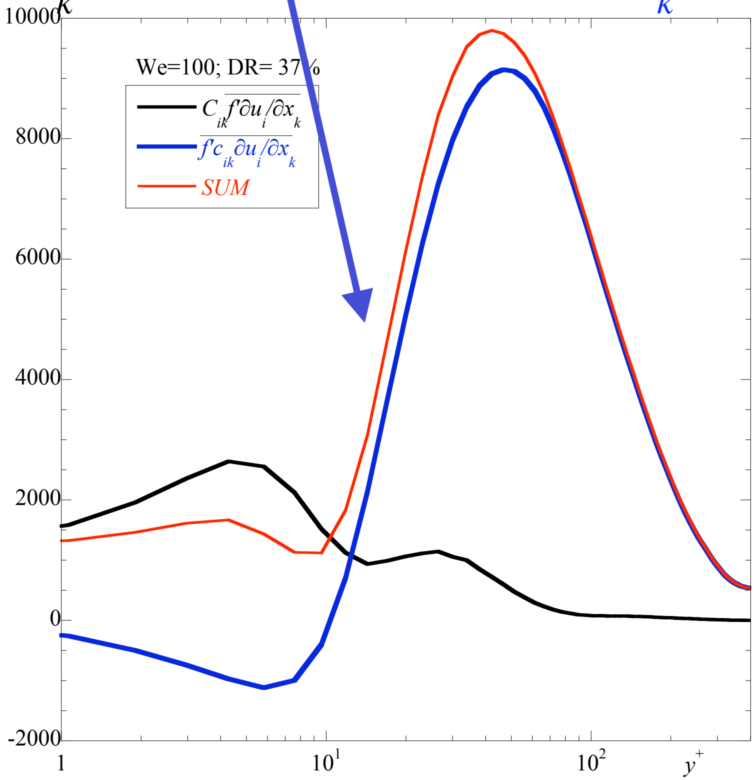


Assumptions for viscoelastic stress work: ε^V

$$\varepsilon^V \equiv \frac{1}{\rho}\overline{\tau'_{ik,p}\frac{\partial u_i}{\partial x_k}} = \frac{\eta_p}{\rho\lambda}\left[C_{ik}\overline{f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik}f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}}\right]$$

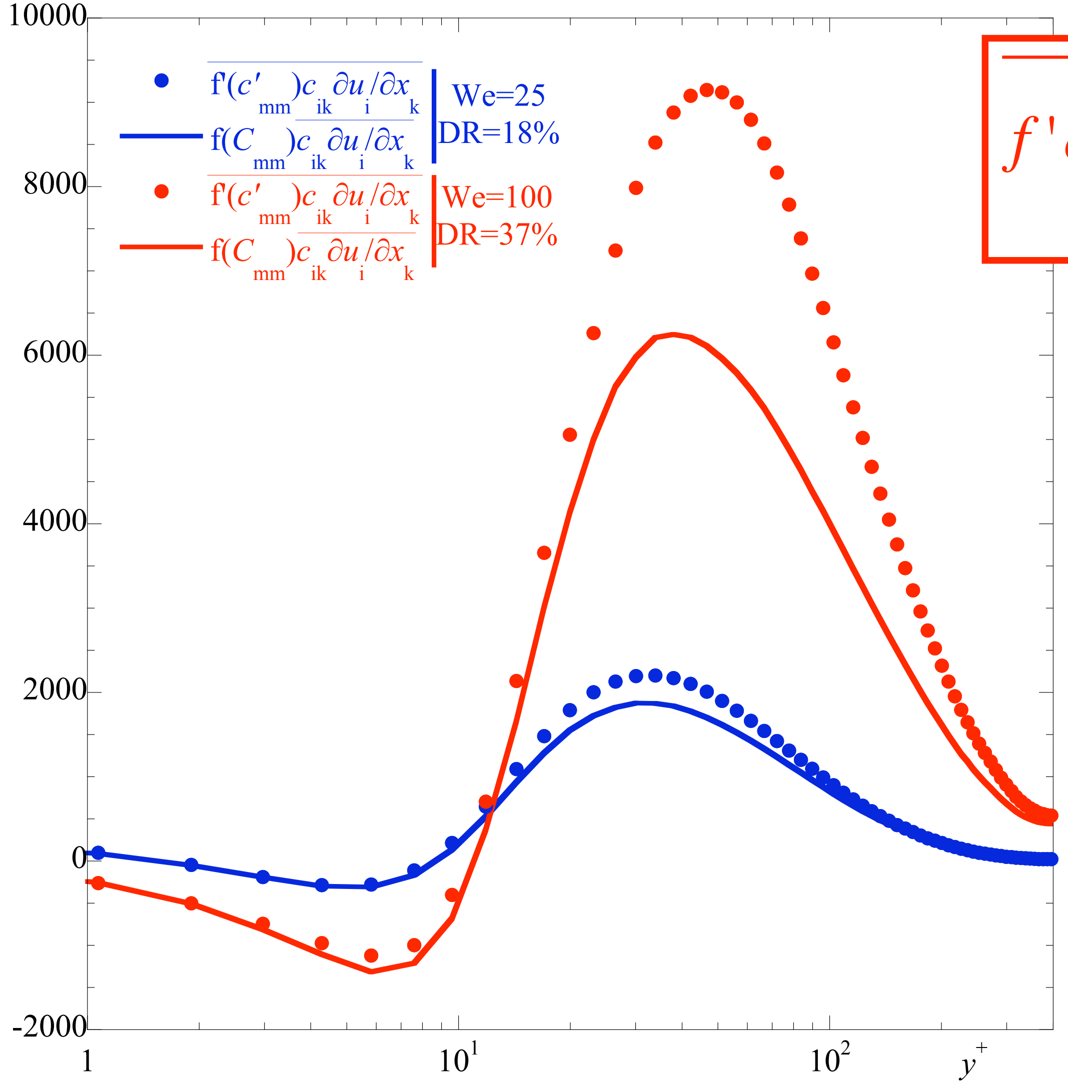
$$\overline{C_{ik}f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}} << \overline{c_{ik}f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}}$$





Except in viscous sublayer and buffer, but here ε^V is not important

Further assumptions for viscoelastic stress work: ε^V



$$\overline{f'c'_{ik}\frac{\partial u_i}{\partial x_k}} \approx C_{\varepsilon^V} \times f\left(C_{mm}\right)\overline{c_{ik}\frac{\partial u_i}{\partial x_k}}$$

$C_{\varepsilon^V} \sim O(1)$

at $We_{\tau_0} = 25$

but larger as DR increases

This is *NLT*$_{ii}$

Viscoelastic stress work model

$$\varepsilon^V \approx \frac{\eta_p}{\rho\lambda} C_{\varepsilon^V} f\left(C_{mm}\right) \overline{c_{ik}\frac{\partial u_i}{\partial x_k}} = C_{\varepsilon^V}\left(\frac{We_{\tau_0}}{25}\right)^{n-1}\frac{\eta_p}{\rho\lambda} f\left(C_{mm}\right)\frac{NLT_{ii}}{2}$$

Modeled

$We_{\tau_0} = 25 \rightarrow 1.27$

$We_{\tau_0} = 100 \rightarrow 1.56$

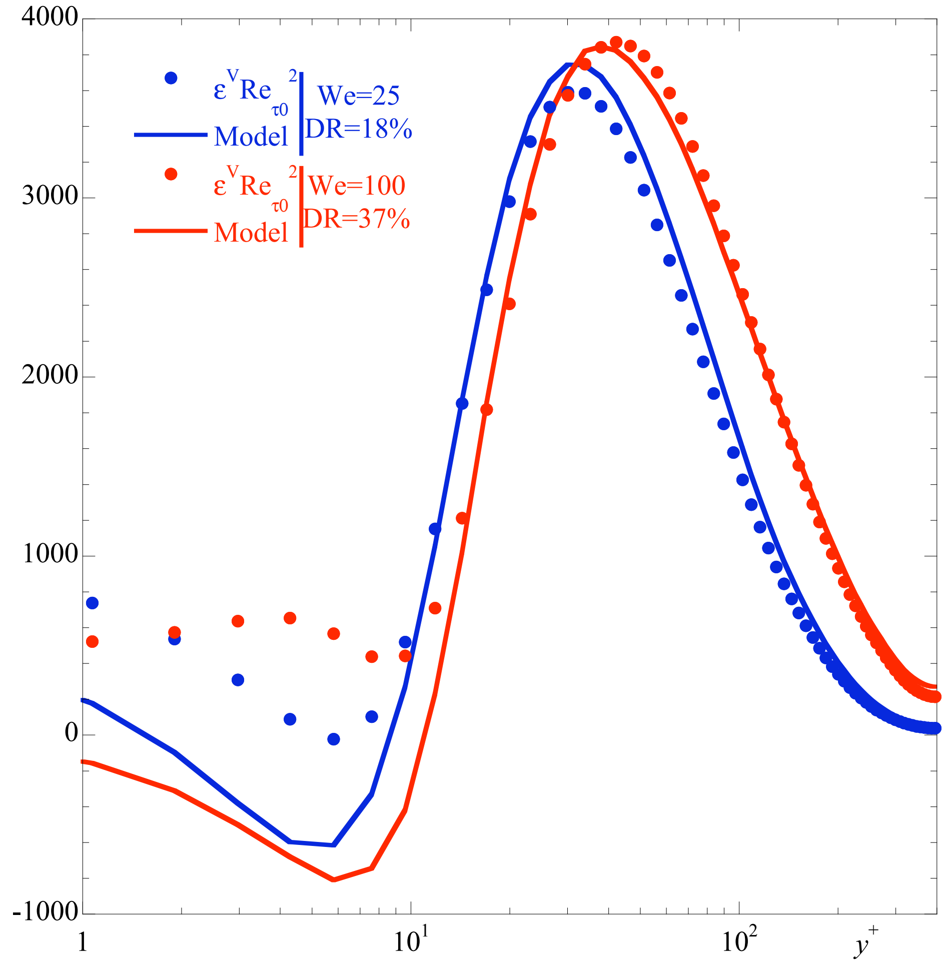
$C_{\varepsilon^V} = 1.27$

$n = 1.15$

$$\varepsilon^{V+}\left(\mathrm{Re}_{\tau_0}\right)^2$$

versus

$$C_{\varepsilon^V}\frac{\mathrm{Re}_{\tau_0}\left(1-\beta\right)}{We_{\tau_0}} f\left(C_{ii}\right) NLT_{jj}^*$$



Previous model: $C_{\varepsilon^V} = 1.076$
($We_{\tau 0}$= 25 only)

Pinho, Li, Younis, Sureshkumar (2008) JNNFM, in press

Viscoelastic turbulent transport: Q^V

$$Q^V \equiv \frac{\partial \overline{\tau'_{ik,p} u_i}}{\partial x_k} = \frac{\eta_p}{\lambda}\frac{\partial}{\partial x_k}\left[\underbrace{C_{ik}\overline{f\left(C_{mm}+c_{mm}\right)u_i}}_{CFU_{iik}} + \underbrace{\overline{c_{ik} f\left(C_{mm}+c_{mm}\right)u_i}}_{CU_{iik}}\right]$$

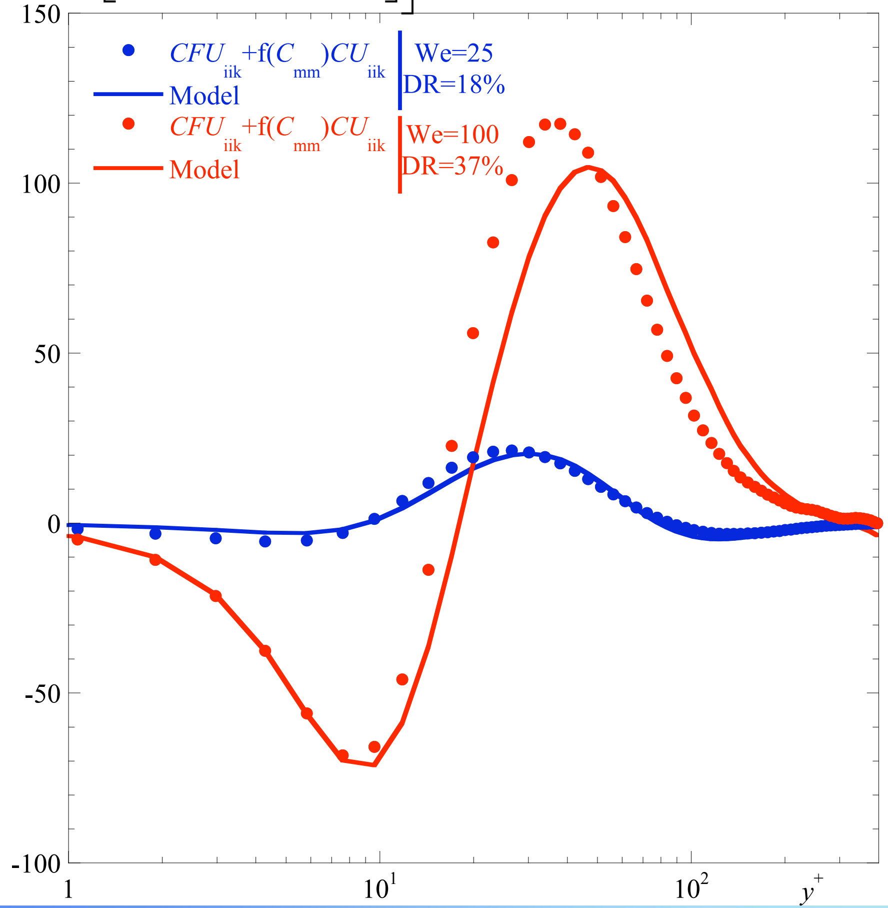
$$\frac{f(C_{mm})CU_{ijk}}{\lambda} = f_{\mu_2}\left(\frac{25}{We_{\tau_0}}\right)^{0.53}\left[-C_{\beta_1}\left(\overline{u_i u_m}\frac{\partial C_{kj}}{\partial x_m} + \overline{u_j u_m}\frac{\partial C_{ik}}{\partial x_m}\right) - \frac{C_{\beta_7}}{\lambda} f(C_{mm})\left[\pm\sqrt{\overline{u_j^2}}C_{ik} \pm \sqrt{\overline{u_i^2}}C_{jk}\right]\right]$$

$$f_{\mu_2} = 1 - \exp\left(-y^+/26.5\right)$$

$$C_{\beta_1} = 1.1; C_{\beta_7} = 0.3$$

$$CFU_{iik} = C_{ik}\overline{f\left(C_{mm}+c_{mm}\right)u_i}$$

$$\cong \frac{C_{FU}}{2}\frac{\lambda}{We_{\tau_0}} f\left(C_{mm}\right) C_{kn} \frac{\partial \overline{u_i u_i}}{\partial x_n}$$



Closure development followed similar procedures as that for NLT_{ij}

Final equations for channel flow: RANS and RACE

Momentum:

$$\frac{d}{dy}\left[\eta_s \frac{dU}{dy} + \overline{\tau}_{p,xy} - \rho\overline{uv}\right] - \frac{d\overline{p}}{dx} = 0$$

$$\overline{\tau}_{xy,p} = \frac{\eta_p}{\lambda} f(C_{kk}) C_{xy}$$

$$f(C_{kk}) C_{xy} = \lambda C_{yy} \frac{dU}{dy} + \lambda NLT_{xy}$$

$$f(C_{kk}) C_{yy} = \lambda NLT_{yy} + 1$$

$$f(C_{kk}) C_{xx} = 2\lambda C_{xy} \frac{dU}{dy} + \lambda NLT_{xx} + 1$$

$$f(C_{kk}) C_{zz} = \lambda NLT_{zz} + 1$$

$$f(C_{kk}) = \frac{L^2 - 3}{L^2 - (C_{xx} + C_{yy} + C_{zz})}$$

Reynolds stress:

$$-\rho\overline{uv} = \rho \nu_T \frac{dU}{dy}$$ with $$\nu_T = C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}^N + \varepsilon^V}$$ and $$f_\mu = \left[1 - \exp\left(\frac{-y^+}{26.5}\right)\right]^2$$

k and ε transport equations: modified Nagano & Hishida

Based on Newtonian model of Nagano & Hishida (1984)

$$0=\frac{d}{dy}\left[\left(\eta_s+\frac{\rho f_T \nu_T}{\sigma_k}\right)\frac{dk}{dy}\right]+P_k-\rho\tilde{\varepsilon}^N-\rho D^N+\eta_p\frac{d}{dy}\left[\frac{f(C_{mm})}{\lambda}\frac{CU_{nny}}{2}\right]-\eta_p\frac{f(C_{mm})}{\lambda}\frac{NLT_{nn}}{2}$$

$$\sigma_k=1.1$$

$$\varepsilon^N=\tilde{\varepsilon}^N+D^N$$

$$D^N=2\eta_s\left(\frac{d\sqrt{k}}{dy}\right)^2$$

$$f_T=1+3.5\exp\left[-\left(R_T/150\right)^2\right]$$

Variable Prandtl numbers: Nagano & Shimada (1993), Park and Sung (1995)

$$0=\frac{d}{dy}\left[\left(\eta_s+\frac{\rho f_T \nu_T}{\sigma_\varepsilon}\right)\frac{d\tilde{\varepsilon}^N}{dy}\right]+\rho f_1 C_{\varepsilon_1}\frac{\tilde{\varepsilon}^N}{k}\frac{P_k}{\rho}-\rho f_2 C_{\varepsilon_2}\frac{{\varepsilon^N}^2}{k}+\rho E+E_{\tau_p}$$

$$\sigma_\varepsilon=1.3$$

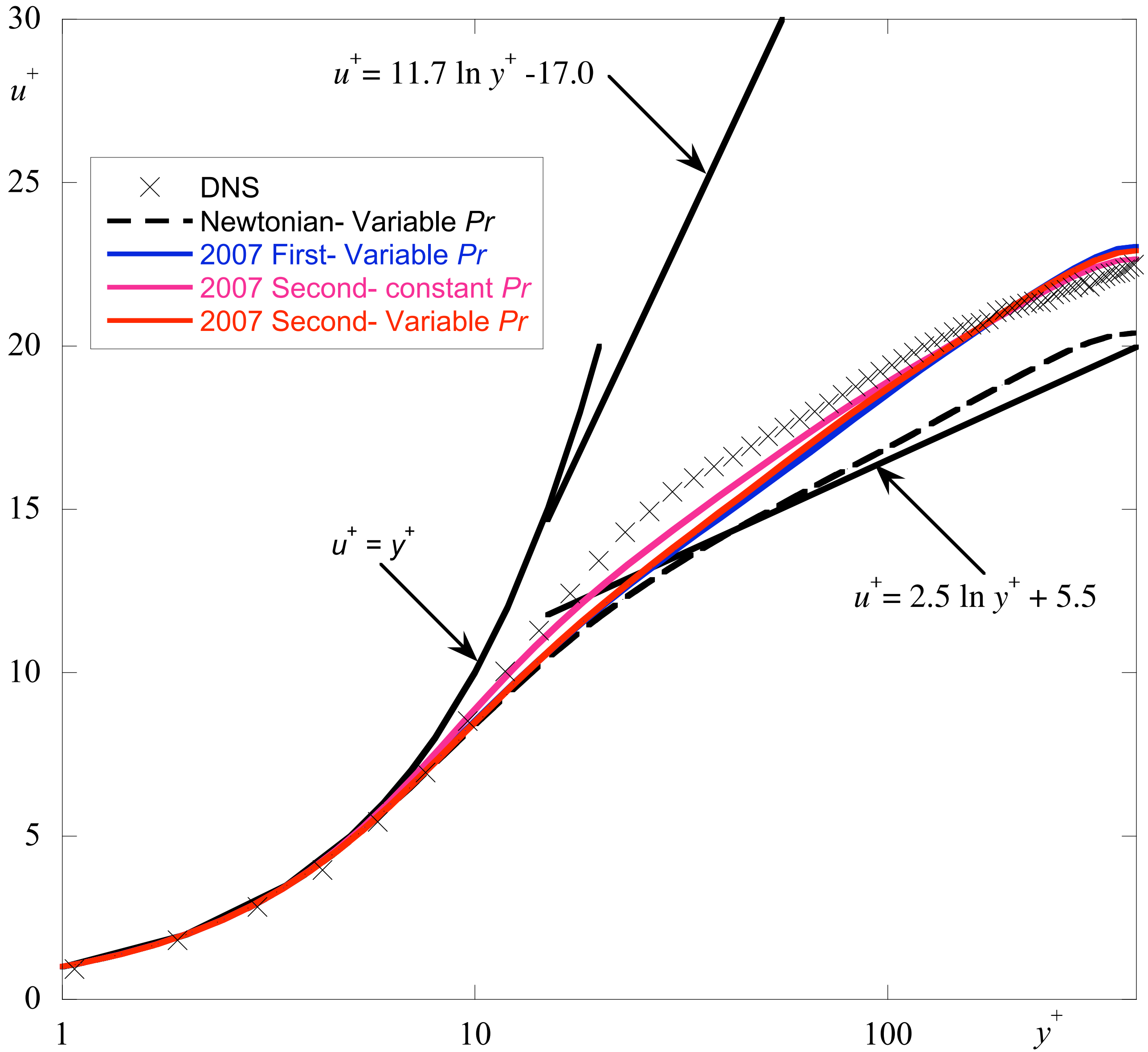
$$f_1=1 \qquad f_2=1-0.3\exp\left(-R_T^2\right)$$

$$E=\frac{\eta_s}{\rho}\nu_T\left(1-f_\mu\right)\left(\frac{d^2U}{dy^2}\right)^2$$

$$E_{\tau_p}=0$$

$$C_{\varepsilon_1}=1.45 \qquad C_{\varepsilon_2}=1.90$$

Predictions U^+: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900



2007 1st model

2007 2nd model

2007 2nd model

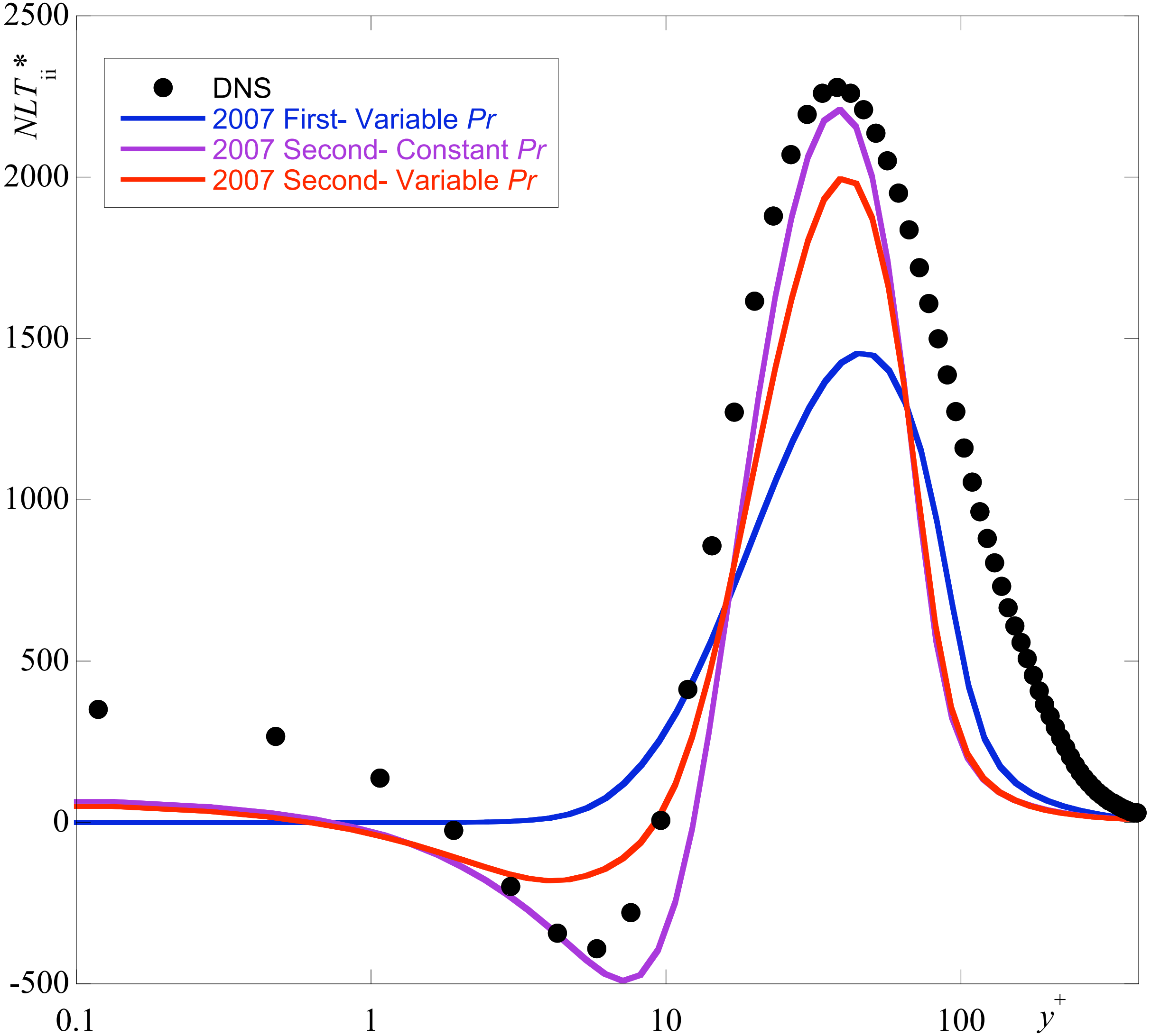
Models for

NLT_{ij} & CU_{ijk}

changed

Predictions NLT_{ii}: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900

NLT^*_{ii}



Models fitted to DNS

Code diverges

Models modified with simulations

Model 2

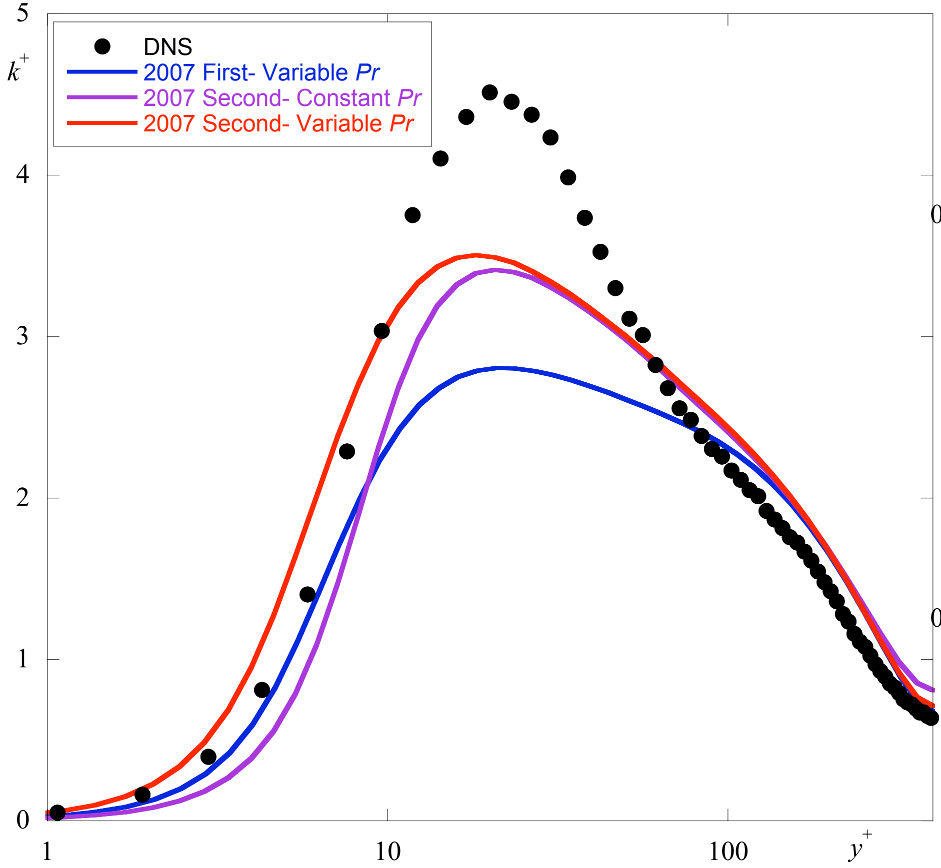
Coefficients C_{N_1} & C_{N_2} reduced by 4

Coefficient C_{N_3} increased 60%

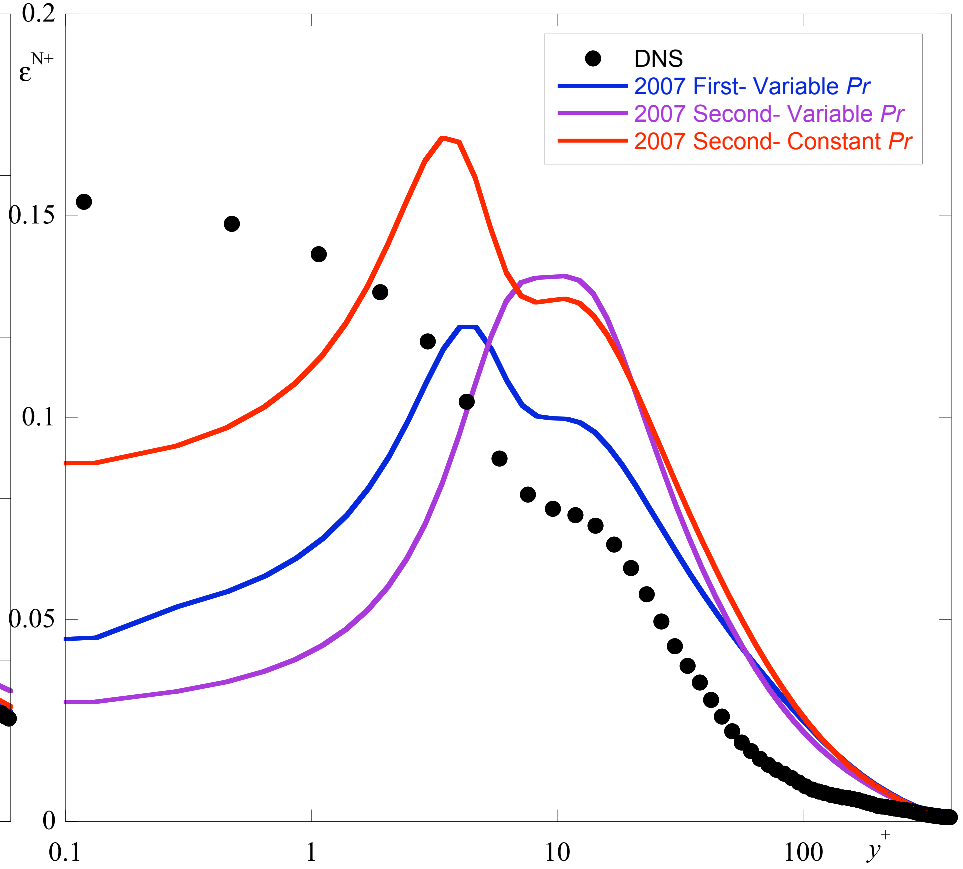
Coefficient C_{N_5} reduced 30%

Predictions k & ε^N: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900

k^+

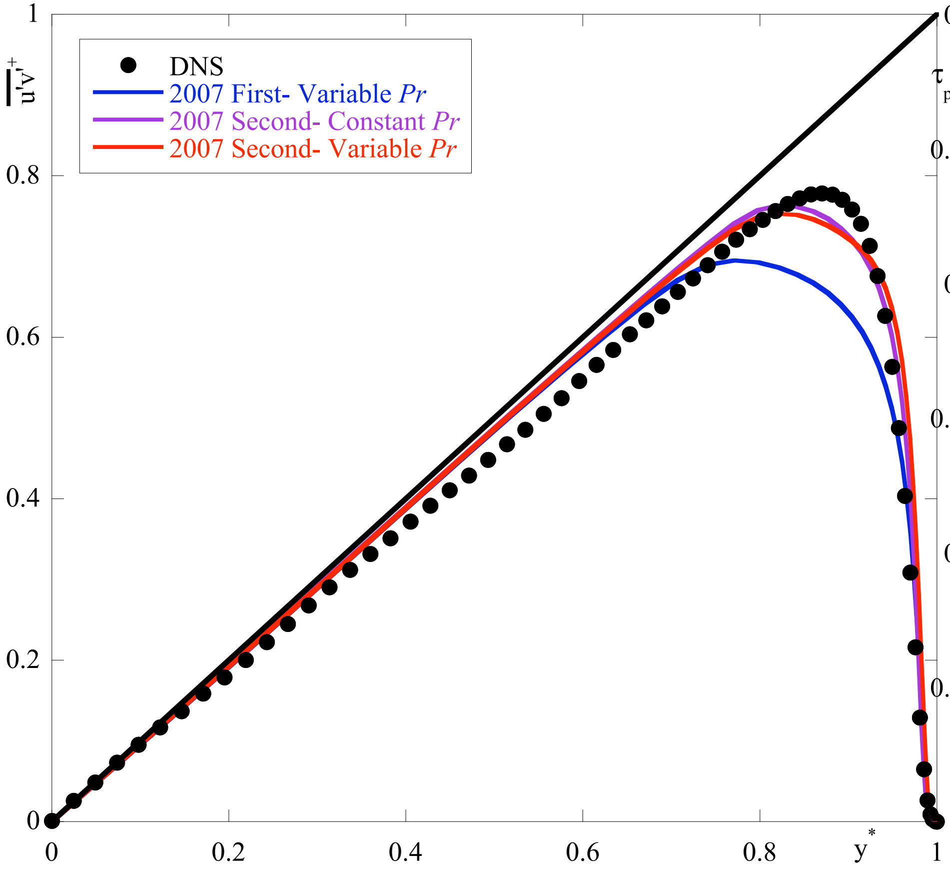


ε^{N+}

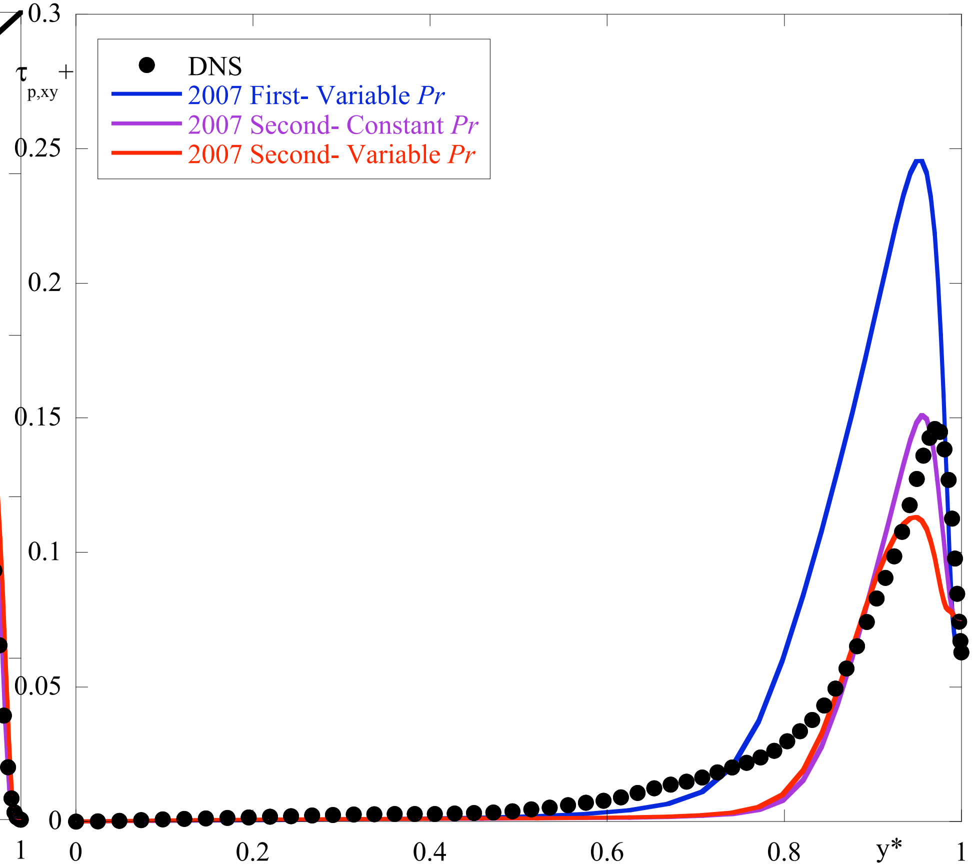


Predictions $u'v'$ & $\tau_{p,xy}$: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900

$\overline{u'v'}^+$



$\tau^+_{p,xy}$ (zoomed)



Conclusions and Acknowledgments

- Closures for Low DR and High DR
- Closures for NLT_{ij}, ε^V and Q^V (in fact for ε_{ij}^V and Q_{ij}^V)
- Developed simple low Reynolds k-ε model works reasonably well
- Need to incorporate with better Reynolds stress closures:

  k-ω, modified k-ε or k-ω, Menter's SST or Durbin's v2-f

  or RS transport (deficiencies in base model are imp.)
- Need to extend models to Maximum DR, & β & L^2
- DNS in other canonical flows required for extension of turb. models

Acknowledgments - Funding
Fundação Calouste Gulbenkian: Project 72259
Fundação para a Ciência e Tecnologia
Projects SFRH/BSAB/507/2005 ; SFRH/BD/18475/2004
Project "Turbopol" POCI/EQU/56342/2004