RANS/RACE TURBULENCE MODELS FOR VISCOELASTIC FLUIDS

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II Conferência Nacional de Métodos Numéricos em Mecânica de Fluidos e Termodinâmica

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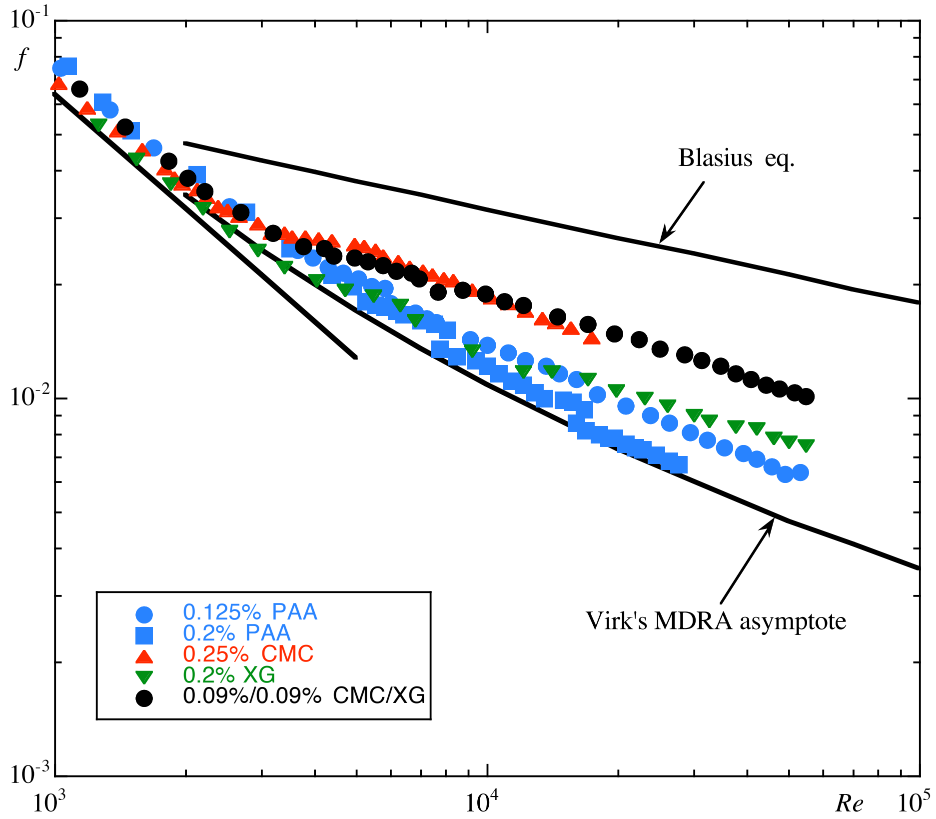
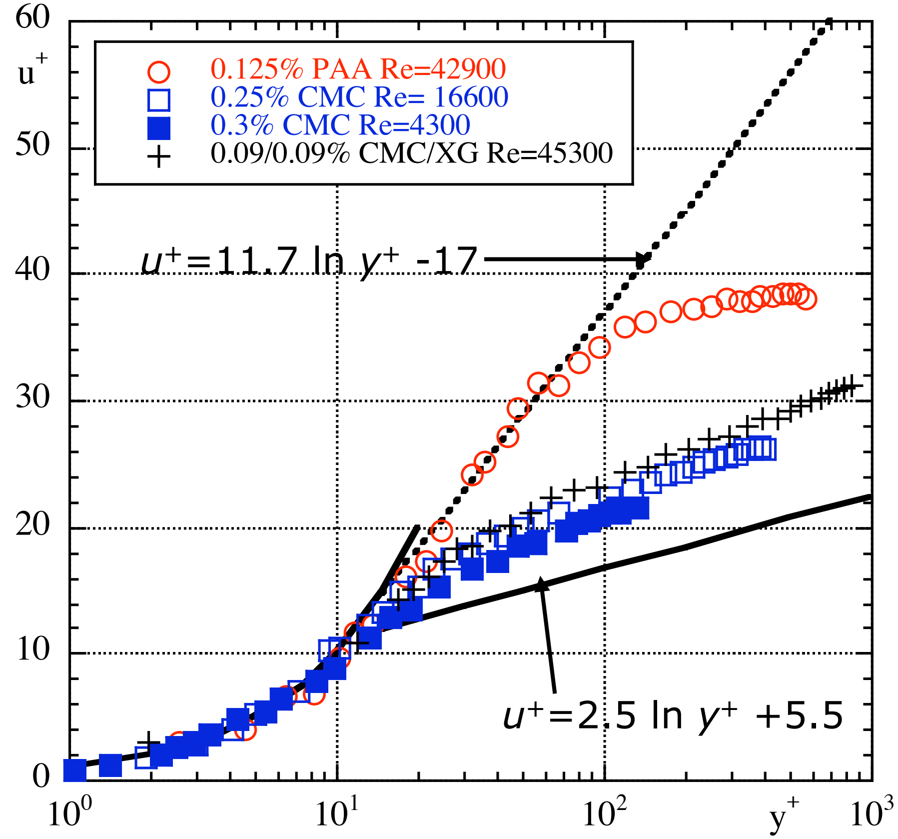
Universidade de Aveiro, Portugal

Drag reduction: some characteristics

Applications: drilling of oil an gas wells; district heating & cooling sys.





Escudier et al JNNFM (1999)

- Reduction of shear Reynolds stress (DR) → **Reynolds stress deficit**
- Increase of normal streamwise Reynolds stress
- Dampening of normal radial and tangential Reynolds stress

No universal law of the wall

Governing equations for non-Newtonian fluids

Reynolds decomposition $\hat{B} = B + b'$ where $\overline{b'} = 0$

^- instantaneous quantities
Overbar or upper-case letters- time-averaged quantities
' or lower-case letters- fluctuating quantities

Continuity (incompressible): $$\frac{\partial \hat{U}_i}{\partial x_i} = 0$$

Momentum: $$\rho \frac{\partial \hat{U}_i}{\partial t} + \rho \hat{U}_k \frac{\partial \hat{U}_i}{\partial x_k} = -\frac{\partial \hat{p}}{\partial x_i} + \frac{\partial \hat{\tau}_{ik,fluid}}{\partial x_k}$$

Rheological constitutive equation:

Power law fluid

Purely viscous: GNF $$\hat{\tau}_{ij} = 2\mu\left(I_{S_{ij}}, II_{S_{ij}}, III_{S_{ij}}\right)\hat{S}_{ij}$$

$$\mu = K\dot{\gamma}^{n-1} \qquad \dot{\gamma} = \sqrt{2S_{pq}S_{pq}}$$

Upper Convective Maxwell

Viscoelastic: $$\hat{\tau}_{ij,fluid} = 2\eta_s \hat{S}_{ij} + \hat{\tau}_{ij,p}$$

$$\hat{\tau}_{ij,p} + \lambda \overset{\nabla}{\hat{\tau}}_{ij,p} = 2\eta_p \hat{S}_{ij}$$

Existing models: 0 equations - Mizushina and Usui & Cruz

Mixing length with Van Driest damping (Stoke's 2nd problem)

$$\rho\frac{\partial U_i}{\partial t}+\rho U_k\frac{\partial U_i}{\partial x_k}=-\frac{\partial\bar{p}}{\partial x_i}+\frac{\partial}{\partial x_k}\left(\eta\frac{\partial U_i}{\partial x_k}\right)-\frac{\partial}{\partial x_k}\left(\rho\overline{u_i u_k}\right)$$

$$-\overline{uv}=\nu_T\frac{dU}{dy}$$

$$\nu_T=f_\mu(\kappa y)^2\left|\frac{dU}{dy}\right|$$

Mizushina & Usui (1977) PoF v20, S100

Turbulent relaxation time

$$f_\mu=1-\exp\left\{-\frac{y^+}{A^+}\left[-\alpha+\left(\alpha^2+1\right)^{1/2}\right]^{1/2}\right\}$$

$$\alpha=\frac{2\lambda_t}{\nu}\left(\frac{u_\tau}{A^+}\right)^2$$

$$\frac{\lambda_t}{\lambda}=3.76\times10^8\left(\frac{\lambda U}{D}\right)^{1.34}$$

$$\lambda=\frac{2}{5}\frac{\eta_s[\eta]^2 Mc}{kT}$$

Rouse relaxation time

Power law- purely viscous

Cruz et al (2000) HME v2, 1

$$f_\mu=\left[n\left(1-\left(1+\left|\frac{1-n}{1+n}\right|\frac{y^+}{A^+}\right)^{-\left|\frac{1+n}{1-n}\right|}\right)\right]^2$$

Existing models: 2 equations- Malin (inelastic fluids)

$$\rho\frac{\partial U_i}{\partial t}+\rho U_k\frac{\partial U_i}{\partial x_k}=-\frac{\partial \overline{p}}{\partial x_i}+\frac{\partial}{\partial x_k}\left(\eta\frac{\partial U_i}{\partial x_k}\right)-\frac{\partial}{\partial x_k}\left(\rho\overline{u_i u_k}\right)$$

Viscosity: Power law

No elasticity: No polymer stress

Malin: Lam-Bremhorst model (low Re *k*-ε)

Malin (1997) ICHMT v24,977

$$-\overline{u_i u_j}=2\nu_T S_{ij}-\frac{2}{3}k\delta_{ij}$$

$$\nu_T=C_\mu f_\mu\frac{k^2}{\varepsilon}$$

$$k:\quad \nabla.(Uk)=\nabla.\left[(\nu+\nu_T)\nabla k\right]+P_k-\varepsilon$$

$$\varepsilon:\quad \nabla.(U\varepsilon)=\nabla.\left[\left(\nu+\frac{\nu_T}{\sigma_\varepsilon}\right)\nabla\varepsilon\right]+\frac{\varepsilon}{k}\left(c_{1_\varepsilon}f_1P_k-c_{2_\varepsilon}f_2\varepsilon\right)$$

$$f_\mu=\left[1-\exp\left(-0.0165\,\mathrm{Re}/n^{1/4}\right)\right]^2\left(1+205/\mathrm{Re}_t\right)\qquad f_1=1+\left(0.05/f_\mu\right)^3$$

Existing models: Pinho et al (2003 to 2006)

Modified GNF: $\tau_{ij} = 2\mu S_{ij} \longrightarrow \mu = K_v\left[\dot{\gamma}^2\right]^{\frac{n-1}{2}} K_e\left[\dot{\varepsilon}^2\right]^{\frac{p-1}{2}}$

$$\rho\frac{\partial U_i}{\partial t} + \rho U_k\frac{\partial U_i}{\partial x_k} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_k}\left(\bar{\mu}\frac{\partial U_i}{\partial x_k}\right) + \frac{\partial}{\partial x_k}\left(\overline{\mu'\frac{\partial u_i}{\partial x_k}}\right) - \frac{\partial}{\partial x_k}\left(\rho\overline{u_i u_k}\right)$$

$$\bar{\mu} = f_v\bar{\mu}_h + (1-f_v)\eta_v \longrightarrow \eta_v = K_v\left[\dot{\gamma}^2\right]^{\frac{n-1}{2}}$$

$$\bar{\mu}_h = \left(C_\mu\rho\right)^{\frac{3m(m-1)A_2}{8+3m(m-1)A_2}} 2^{\frac{4m(m-1)A_2}{8+3m(m-1)A_2}} k^{\frac{6m(m-1)A_2}{8+3m(m-1)A_2}} \varepsilon^{\frac{[8-3(m-1)A_2]m}{8+3m(m-1)A_2}} B^{\frac{8}{8+3m(m-1)A_2}}$$

$$\overline{\mu'\frac{\partial u_i}{\partial x_j}} = \tilde{C}\frac{K_v K_e}{A_\varepsilon^{p-1}}\left[\frac{\rho C_\mu f_\mu k^2}{2\bar{\mu}\tilde{\varepsilon}}\left(2S_{mn}S_{mn}\right)\right]^{\frac{p+n-2}{2}}\sqrt{C_\mu f_\mu\frac{k^2}{\tilde{\varepsilon}}}\times\frac{1}{L_c}\times\frac{2S_{ij}}{\sqrt{2S_{pq}S_{pq}}}$$

Low Reynolds number anisotropic *k*-ε model

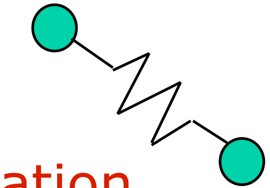
$$-\rho\overline{u_i u_j} = 2\nu_t S_{ij} - \frac{2}{3}k\delta_{ij} - f_{1n}\tilde{\beta}_2 k\left(S_{ik}^*S_{kj}^* - \frac{1}{6}S^{*2}\delta_{ij}\right) - f_{2n}f_{3n}\tilde{\beta}_3 k\left(W_{ik}^*S_{kj}^* - S_{ik}^*W_{kj}^*\right)$$

Pinho (2003), Cruz & Pinho (2003), Cruz et al (2004) **JNNFM**; Resende et al, 2006 Int. J. Heat Fluid Flow

Constitutive model for dilute polymer solutions: FENE-P

$$\hat{\tau}_{ij} = 2\eta_s \hat{S}_{ij} + \frac{\eta_p}{\lambda}\left[f\left(\hat{C}_{kk}\right)\hat{C}_{ij} - f(L)\delta_{ij}\right]$$

Molecular conformation



$$\beta = \frac{\eta_s}{\eta_s + \eta_p}$$

$$f\left(\hat{C}_{kk}\right)\hat{C}_{ij} + \lambda \overset{\nabla}{\hat{C}}_{ij} = \delta_{ij}$$

with

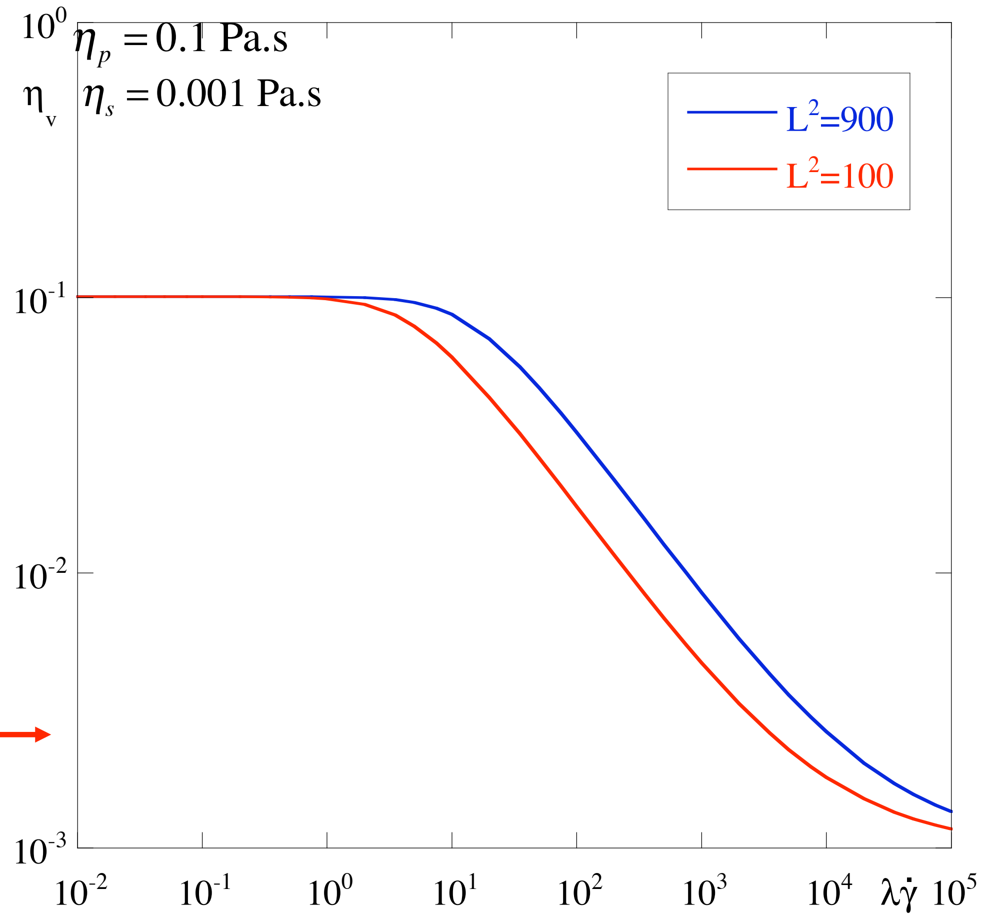
$$\overset{\nabla}{\hat{C}}_{ij} = \frac{\partial \hat{C}_{ij}}{\partial t} + U_k \frac{\partial \hat{C}_{ij}}{\partial x_k} - \hat{C}_{jk}\frac{\partial U_i}{\partial x_k} - \hat{C}_{ik}\frac{\partial U_j}{\partial x_k}$$

$$f\left(\hat{C}_{kk}\right) = \frac{L^2}{L^2 - \hat{C}_{kk}} \qquad f(L) = \frac{L^2}{L^2 - 3}$$

Couette flow

$$\eta(\dot{\gamma}) = \eta_p C_{22}(\dot{\gamma}) + \eta_s$$

$C_{22}(\dot{\gamma})$: analytical solution



Time-averaged governing equations: RANS and RACE

Continuity: $$\frac{\partial U_i}{\partial x_i}=0$$

Momentum balance:

$$\rho\frac{\partial U_i}{\partial t}+\rho U_k\frac{\partial U_i}{\partial x_k}=-\frac{\partial \bar{p}}{\partial x_i}+\eta_s\frac{\partial^2 U_i}{\partial x_k \partial x_k}-\frac{\partial}{\partial x_k}\left(\rho\overline{u_i u_k}\right)+\frac{\partial \bar{\tau}_{ik,p}}{\partial x_k}$$

$$\bar{\tau}_{ij}=2\eta_s S_{ij}+\bar{\tau}_{ij,p}$$

Rheological constitutive equation: **FENE-P**

$$\bar{\tau}_{ij,p}=\frac{\eta_p}{\lambda}\left[f\left(C_{kk}\right)C_{ij}-f\left(L\right)\delta_{ij}\right]+\frac{\eta_p}{\lambda}\overline{f\left(C_{kk}+c_{kk}\right)c_{ij}}$$

RACE →

$$\overset{\nabla}{C}_{ij}+\overline{u_k\frac{\partial c_{ij}}{\partial x_k}}-\left(\overline{c_{kj}\frac{\partial u_i}{\partial x_k}}+\overline{c_{ik}\frac{\partial u_j}{\partial x_k}}\right)=-\frac{\bar{\tau}_{ij,p}}{\eta_p}$$

M_{ij} $\quad CT_{ij}$ $\quad NLT_{ij}$

Closures required

Existing models for FENE-P: Li et al (2006)- 0 equation model

$$\rho\frac{\partial U_i}{\partial t}+\rho U_k\frac{\partial U_i}{\partial x_k}=-\frac{\partial \overline{p}}{\partial x_i}+\eta_s\frac{\partial^2 U_i}{\partial x_k \partial x_k}-\frac{\partial}{\partial x_k}\left(\rho\overline{u_i u_k}\right)+\frac{\partial \overline{\tau}_{ik,p}}{\partial x_k}$$

0 equation model
(shear stress only)

Reynolds stress

$$-\overline{uv}=\nu_{T,v}\frac{dU}{dy}$$

$$\nu_{T,v}=\phi\nu_{T,N}$$

$$\nu_{T,N}=\kappa u_\tau y$$

$$\phi=\left[a(DR)y+b(DR)\right]$$

$$\int_0^{\mathrm{Re}_\tau}\overline{\tau}_{xy,p}dy=\int_0^{\mathrm{Re}_\tau}M_{xy}dy+\int_0^{\mathrm{Re}_\tau}NLT_{xy}dy$$

$$I_{M_{xy}}=a'+b'DR+c'DR^2$$

$$I_{NLT_{xy}}=a''+b''DR+c''DR^2$$

$$DR=80\left\{1-\exp\left[-0.025\left(We_\tau-6.25\right)\left(\frac{\mathrm{Re}_\tau}{125}\right)^{-0.225}\right]\right\}\left[1-\exp\left(-0.0275L\right)\right]$$

Existing models for FENE-P: unpublished

FENE-P and based on DNS

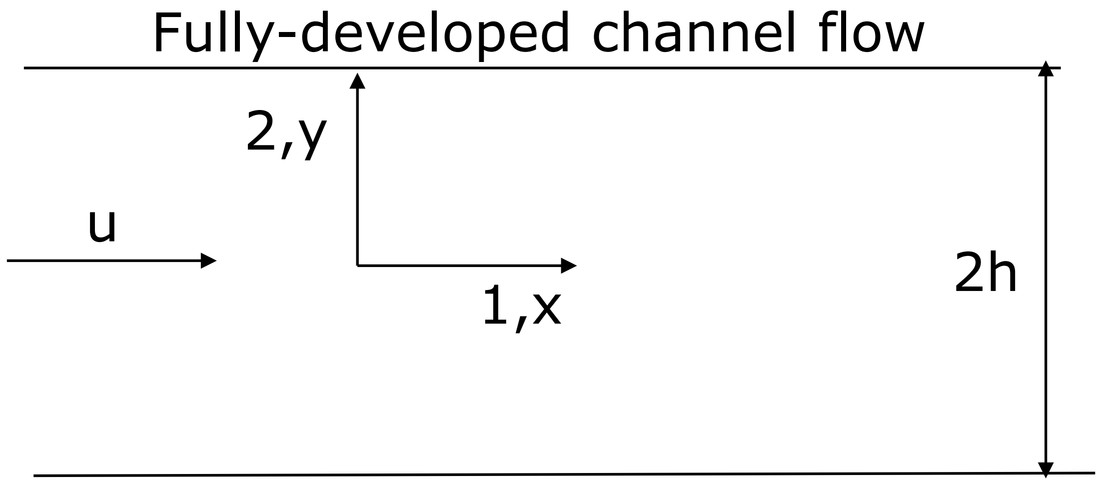
Leighton, Walker and Stephens (2002) APS meeting

- Reynolds stress transport model
- Slow pressure-strain redistribution term is modified by polymer (limits energy redistribution)
- New term in RS equation: interaction of $\tau'_{p,ij}$ & turbulence
- New term in C_{ij} equation (NLT_{ij})
- Additional diffusive flux terms not modeled

Shaqfeh (2006) AIChE Conference

- k-ε v^2-f extension of Durbin's model (1995)
- Simplified model: $\overline{\tau}_{p,ij}$ proportional to mean strain (elongation)
- Coefficient has laminar and turbulent contribution
- Laminar part proportional to ∂U/∂y
- Turbulent part proportional to k;
- Modifies pressure strain (v2 equation)
- One transport equation for C_{kk};

DNS cases: channel flow



$$We_\tau = \frac{\lambda u_\tau^2}{\nu_0}$$

$$Re_\tau = \frac{h u_\tau}{\nu_0}$$

$$Re_\tau = 395, \beta = 0.9, L^2 = 900$$

Low Drag Reduction

$$We_\tau = 25, DR = 18\%$$

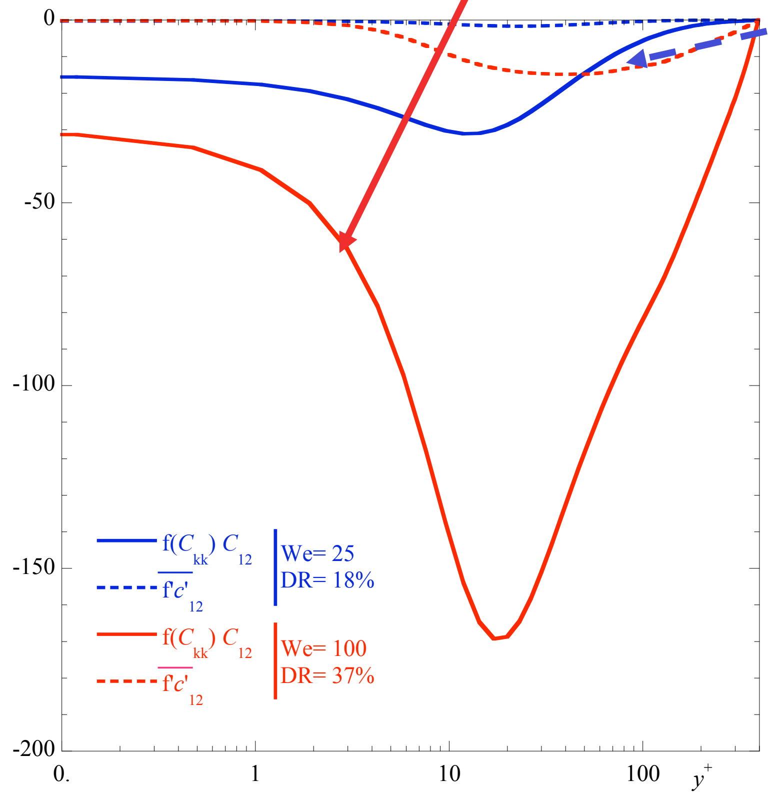
High Drag Reduction

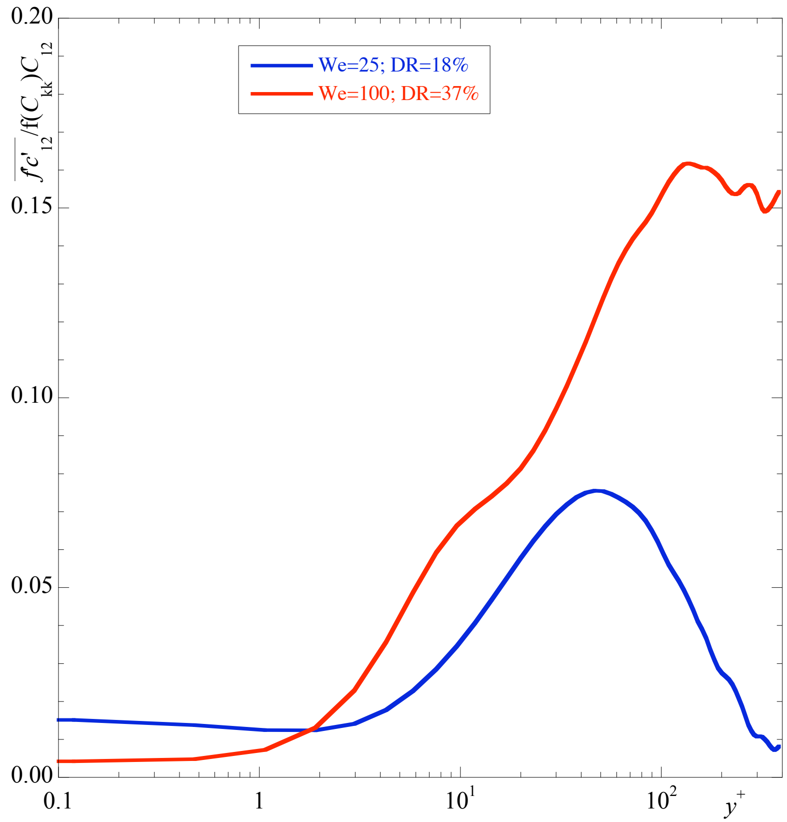
$$We_\tau = 100, DR = 37\%$$

- 2007 model**s** (Pinho et al JNNFM 2008 & unpublished) - Only **LDR**
- 2008 model (under develop.)- Recalculated DNS + **LDR** & **HDR**
- **Closures valid for 1st & higher order turbulence models**

Time- average polymer stress

$$\overline{\tau}_{ij,p} = \frac{\eta_p}{\lambda}\left[f(C_{kk})C_{ij} - f(L)\delta_{ij}\right] + \frac{\eta_p}{\lambda}\overline{f(C_{kk}+c_{kk})c_{ij}}$$



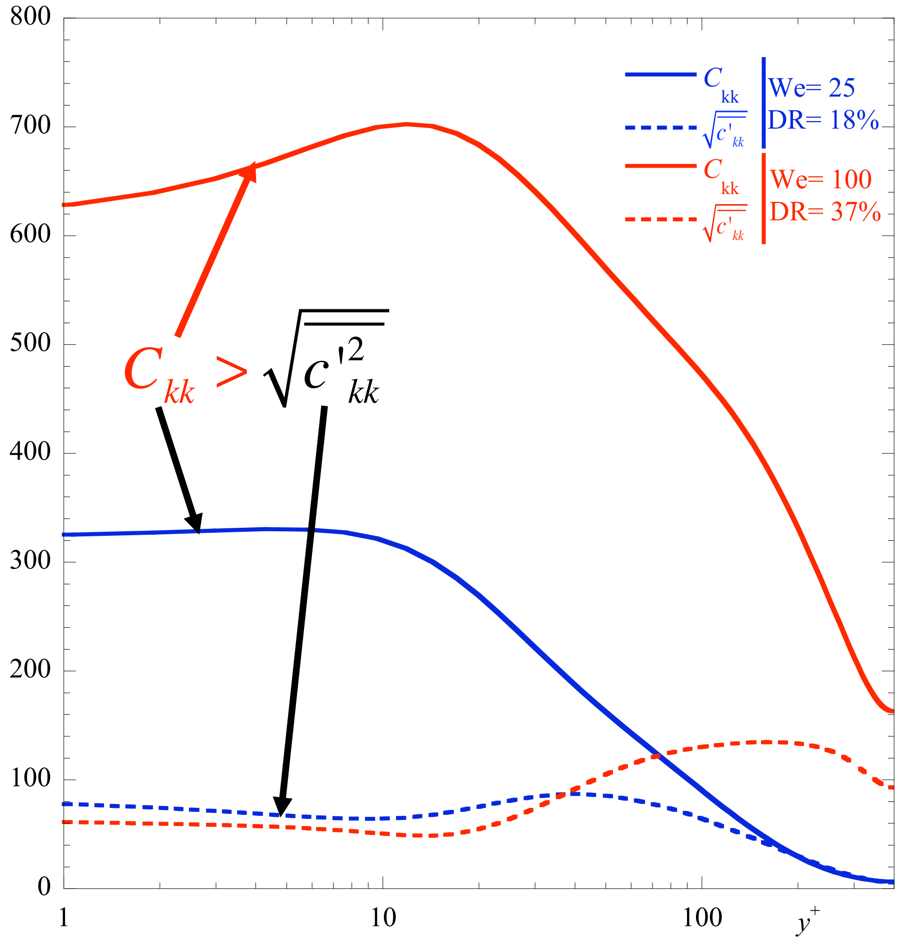


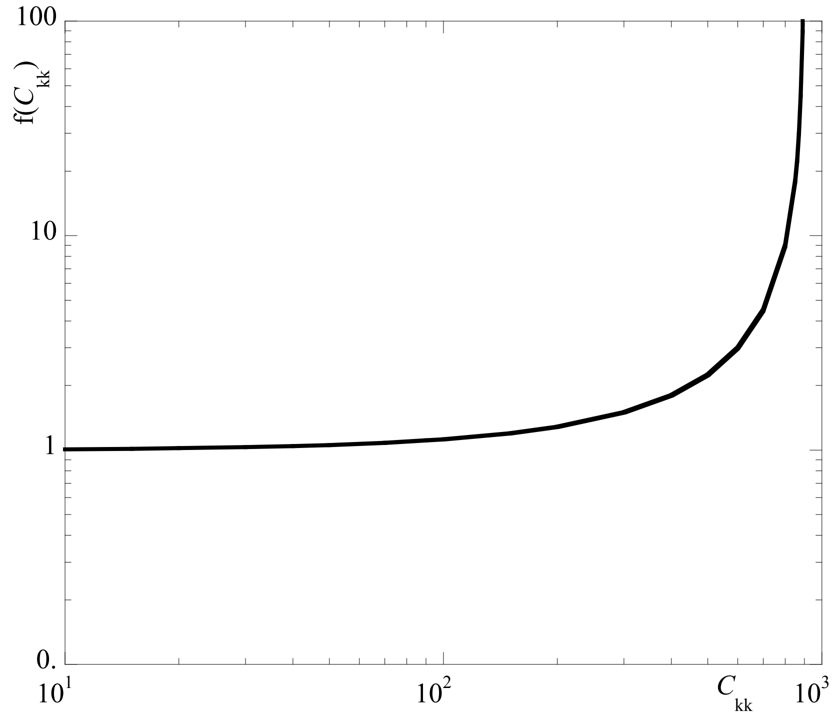
$$f(C_{kk})C_{12} >> \overline{f'c_{12}}$$

Can be neglected !

Function $f\,(C_{kk})$

Function: $f\left(\hat{C}_{kk}\right)=\dfrac{L^2-3}{L^2-\left(C_{kk}+c'_{kk}\right)}$





$$\overline{f\left(\hat{C}_{kk}\right)b_{lm}d_i} \approx f\left(C_{kk}\right)\overline{b_{lm}d_i}$$

$$\overline{f\left(\hat{C}_{kk}\right)b_{ij}} \approx f\left(C_{kk}\right)\bar{b}_{ij}=0$$

This will be used frequently

Time-average evolution equation for the conformation: RACE

$$\lambda \overset{\nabla}{C}_{ij} + \lambda \left[\overline{u_k \frac{\partial c_{ij}}{\partial x_k}} - \left(\overline{c_{kj} \frac{\partial u_i}{\partial x_k}} + \overline{c_{ik} \frac{\partial u_j}{\partial x_k}} \right) \right] = -\left[f(C_{kk}) C_{ij} - f(L) \delta_{ij} \right]$$

$$\overset{\nabla}{C}_{ij} + \overline{u_k \frac{\partial c_{ij}}{\partial x_k}} - \left(\overline{c_{kj} \frac{\partial u_i}{\partial x_k}} + \overline{c_{ik} \frac{\partial u_j}{\partial x_k}} \right) + D_{ij} = -\frac{\overline{\tau}_{ij,p}}{\eta_p}$$

M_{ij} $\quad CT_{ij}$ $\quad NLT_{ij}$

Added for stability

Should be negligible

DNS: Housiadas et al (2005) Phys Fluids 17, 35106, Li et al (2006 a) JNNFM

M_{ij}: Oldroyd derivative
Mean flow distortion
Exact and large

NLT_{ij}: **turbulent distortion**
originates in distortion of Oldroyd derivative- not negligible
Must be modeled

CT_{ij}: originates in advective term, negligible
no need for modeling

Approximate equation for NLT_{ij}

$$
\overline{f\left(\hat{C}_{mm}\right) c_{kj} \frac{\partial u_i}{\partial x_k}} + \overline{f\left(\hat{C}_{mm}\right) c_{ik} \frac{\partial u_j}{\partial x_k}} + C_{kj}\,\overline{f\left(\hat{C}_{mm}\right)\frac{\partial u_i}{\partial x_k}} + C_{ik}\,\overline{f\left(\hat{C}_{mm}\right)\frac{\partial u_j}{\partial x_k}} + \lambda\left[\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial c_{kj}}{\partial t}} + \overline{\frac{\partial u_j}{\partial x_k}\frac{\partial c_{ik}}{\partial t}}\right] +
$$

$$
+\lambda\left[\frac{\partial C_{kj}}{\partial x_n}\overline{u_n\frac{\partial u_i}{\partial x_k}} + \frac{\partial C_{ik}}{\partial x_n}\overline{u_n\frac{\partial u_j}{\partial x_k}} + \overline{\frac{\partial\left(U_n c_{kj}\right)}{\partial x_n}\frac{\partial u_i}{\partial x_k}} + \overline{\frac{\partial\left(U_n c_{ik}\right)}{\partial x_n}\frac{\partial u_j}{\partial x_k}} + \overline{u_n\frac{\partial c_{kj}}{\partial x_n}\frac{\partial u_i}{\partial x_k}} + \overline{u_n\frac{\partial c_{ik}}{\partial x_n}\frac{\partial u_j}{\partial x_k}}\right] -
$$

$$
-\lambda\left[\frac{\partial U_k}{\partial x_n}\left(\overline{c_{jn}\frac{\partial u_i}{\partial x_k}} + \overline{c_{in}\frac{\partial u_j}{\partial x_k}}\right) + \frac{\partial U_j}{\partial x_n}\overline{c_{kn}\frac{\partial u_i}{\partial x_k}} + \frac{\partial U_i}{\partial x_n}\overline{c_{kn}\frac{\partial u_j}{\partial x_k}} + C_{kn}\left(\overline{\frac{\partial u_j}{\partial x_n}\frac{\partial u_i}{\partial x_k}} + \overline{\frac{\partial u_i}{\partial x_n}\frac{\partial u_j}{\partial x_k}}\right)\right] -
$$

$$
-\lambda\left[C_{jn}\overline{\frac{\partial u_k}{\partial x_n}\frac{\partial u_i}{\partial x_k}} + C_{in}\overline{\frac{\partial u_k}{\partial x_n}\frac{\partial u_j}{\partial x_k}} + \overline{c_{jn}\frac{\partial u_k}{\partial x_n}\frac{\partial u_i}{\partial x_k}} + \overline{c_{in}\frac{\partial u_k}{\partial x_n}\frac{\partial u_j}{\partial x_k}} + \overline{c_{kn}\frac{\partial u_j}{\partial x_n}\frac{\partial u_i}{\partial x_k}} + \overline{c_{kn}\frac{\partial u_i}{\partial x_n}\frac{\partial u_j}{\partial x_k}}\right] = 0
$$

First generation model for NLT_{ij} - 2007 1st model

First model

Exact equation is too complex.
Alternative model based on:
1) Identification of possible dependencies from inspection of exact equation
2) Simplicity, but capturing main features
3) *We*=25 (DR=18%)

$$f(C_{mm})\frac{NLT_{ij}}{\lambda} = function\left(S_{ij}, W_{ij}, C_{ij}, \varepsilon_{ij}^{N}, \frac{\partial \overline{u_i u_j}}{\partial x_k}, \frac{\partial C_{ij}}{\partial x_k}, \frac{\partial NLT_{ij}}{\partial x_n}, M_{ij}, \overline{u_i u_j} \right)$$

$$f(C_{mm})\frac{NLT_{ij}}{\lambda} = f_{\mu_1}\left[\frac{C_{E_3} u_\tau^2}{\nu_0^2} C_{kk}\overline{u_i u_j} + \frac{C_{\alpha_{14}}}{\nu_0}\left(\overline{u_i u_k} W_{kn} C_{nj} + \overline{u_j u_k} W_{kn} C_{ni} + \overline{u_k u_i} W_{jn} C_{nk} \right) \right]$$

2 coefficients

$$C_{E_3} = 0.00035; C_{\alpha_{14}} = 0.00015$$

1 damping function

$$f_{\mu_1} = \left(1 - \exp\left(-y^+/26.5\right)\right)^2$$

Pinho, Li, Younis, Sureshkumar (2008) JNNFM, in press

Second generation model for NLT_{ij} - 2007 2nd model

Based on:

1) Modeling of exact equation: **Ver Resende- 6ª feira, 11h15**
2) **HERE *We*=25 (DR=18%) ONLY**

$$f(C_{mm})\frac{NLT_{ij}}{\lambda} = \frac{f(C_{mm})}{\lambda}\left\{C_{N_1}\frac{u_\tau^2}{\beta\nu_s}C_{ij}f(C_{mm}) - C_{N_2}\left[C_{kj}\frac{\partial U_i}{\partial x_k} + C_{ik}\frac{\partial U_j}{\partial x_k}\right]\right\}$$

$$+C_{N_3}\frac{C_{kn}}{\nu_0\sqrt{2S_{pq}S_{pq}}}\left[\overline{u_i u_m}\frac{\partial U_j}{\partial x_k}\frac{\partial U_m}{\partial x_n} + \overline{u_j u_m}\frac{\partial U_i}{\partial x_k}\frac{\partial U_m}{\partial x_n}\right]$$

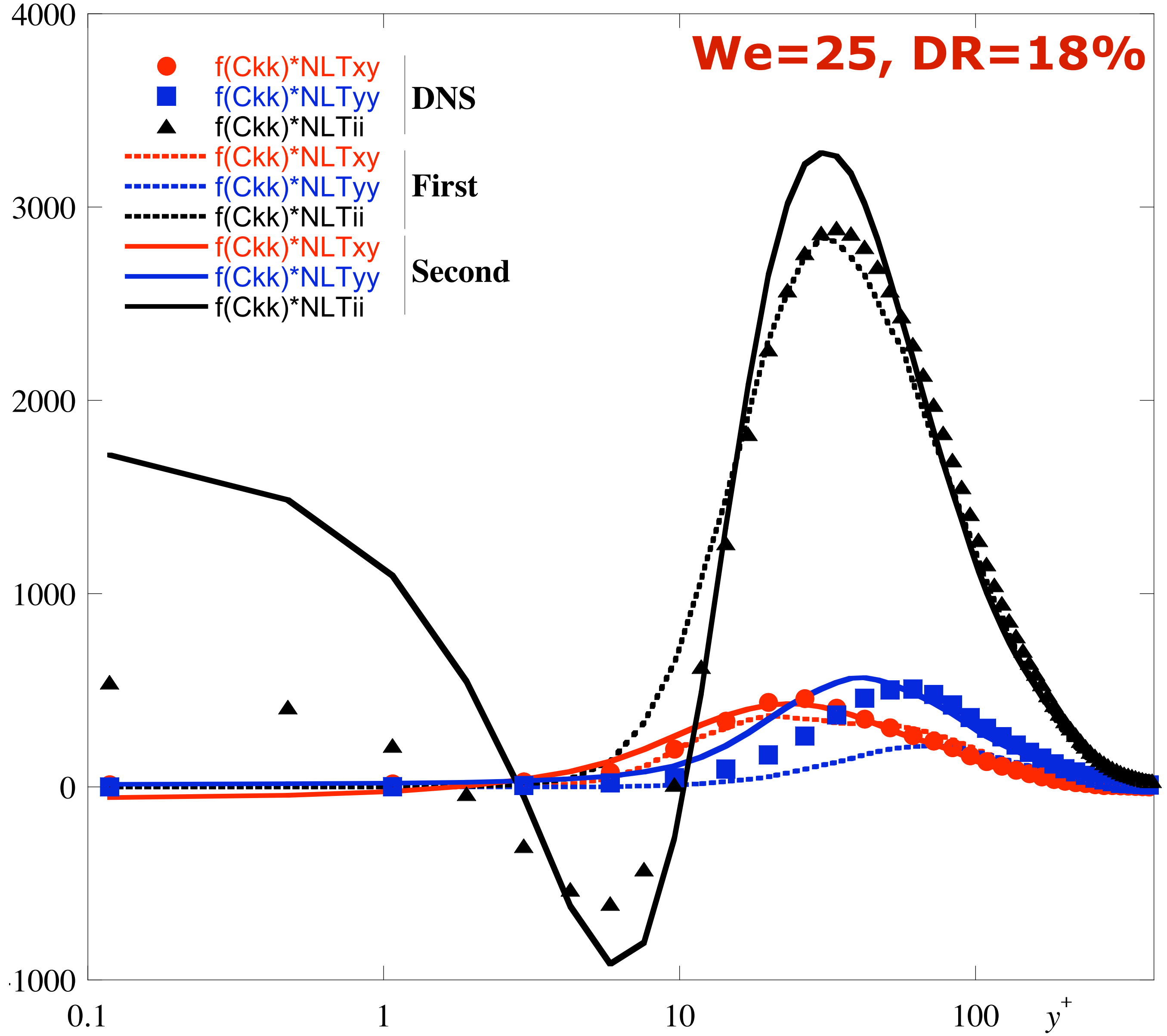
$$-f_{N_1}C_{N_4}\left[C_{jn}\frac{\partial U_k}{\partial x_n}\frac{\partial U_i}{\partial x_k} + C_{in}\frac{\partial U_k}{\partial x_n}\frac{\partial U_j}{\partial x_k} + C_{kn}\left(\frac{\partial U_j}{\partial x_n}\frac{\partial U_i}{\partial x_k} + \frac{\partial U_i}{\partial x_n}\frac{\partial U_j}{\partial x_k}\right)\right]$$

$$+C_{N_5}f_{N_2}\frac{4}{15}\frac{\varepsilon^N}{\beta\nu_s}C_{mm}\delta_{ij}$$

4 coefficients: $C_{N_1} = 0.0164; C_{N_2} = 0.32; C_{N_3} = 0.024; C_{N_4} = 1.11; C_{N_5} = 0.045$

2 damping functions: $f_{N_1} = \left[1 - 0.8\exp\left(-\frac{y^+}{30}\right)\right]^2$ $f_{N_2} = \left[1 - 0.8\exp\left(-\frac{y^+}{25}\right)\right]^4$

2007 models for NLT_{ij}: First versus second model



Modeling the Reynolds stress

Major issue: what model to use for Reynolds stresses?

1) Reynolds stresses: Prandtl-Kolmogorov model (k-ε closure)

$$-\overline{u_i u_j} = 2\nu_T S_{ij} - \frac{2}{3} k \delta_{ij} \quad \text{with} \quad \nu_T = C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}^N + \varepsilon^V}$$

Next slide

2) Dissipation of turbulent kinetic energy: ε^N

$$2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\rho \frac{Du_i}{Dt}\right)} + 2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\rho u_k \frac{\partial U_i}{\partial x_k}\right)} + 2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\rho \frac{\partial u_i u_k}{\partial x_k}\right)}$$

$$+2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\frac{\partial p'}{\partial x_i}\right)} - 2\rho\nu_s^2 \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\frac{\partial^2 u_i}{\partial x_k^2}\right)} - 2\nu_s \overline{\frac{\partial u_i}{\partial x_m} \frac{\partial}{\partial x_m}\left(\frac{\partial \tau'_{ik,p}}{\partial x_k}\right)} = 0$$

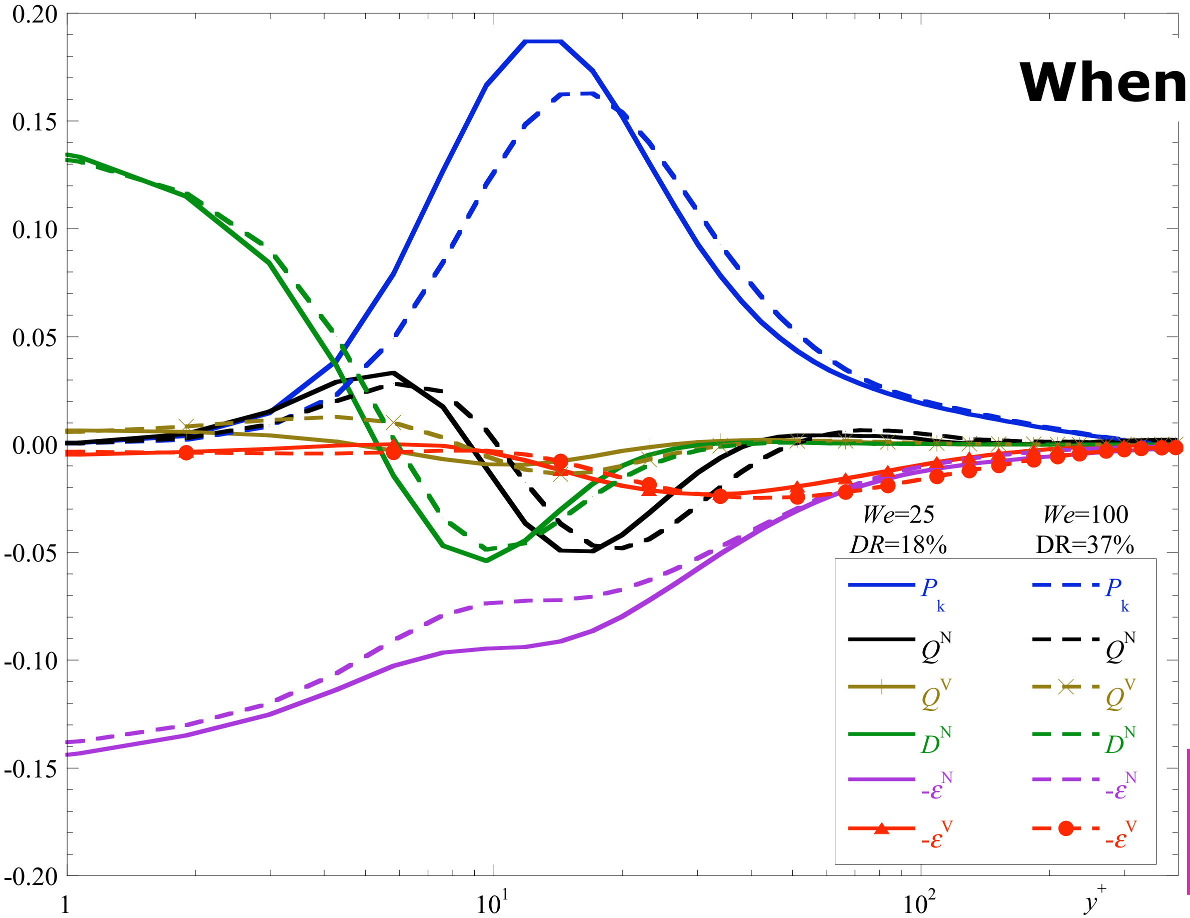
New term (will be considered in the future)

As for Newtonian fluids, most terms in ε^N are approximated

Transport equation for turbulent kinetic energy

$$\rho\frac{Dk}{Dt} = -\rho\overline{u_i u_k}\frac{\partial U_i}{\partial x_k} - \rho\overline{u_i\frac{\partial k'}{dx_i}} - \frac{\partial\overline{p'u_i}}{\partial x_i} + \eta_s\frac{\partial^2 k}{\partial x_i\partial x_i} - \eta_s\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_k}} + \frac{\partial\overline{\tau'_{ik,p}u_i}}{\partial x_k} - \overline{\tau'_{ik,p}\frac{\partial u_i}{\partial x_k}}$$

0 P_k Q^N D^N $-\varepsilon^N$ Q^V $-\varepsilon^V$



When *We* increases (DR↑)

P_k decreases

ε^N decreases

Q^V increases in buffer l., but remains small

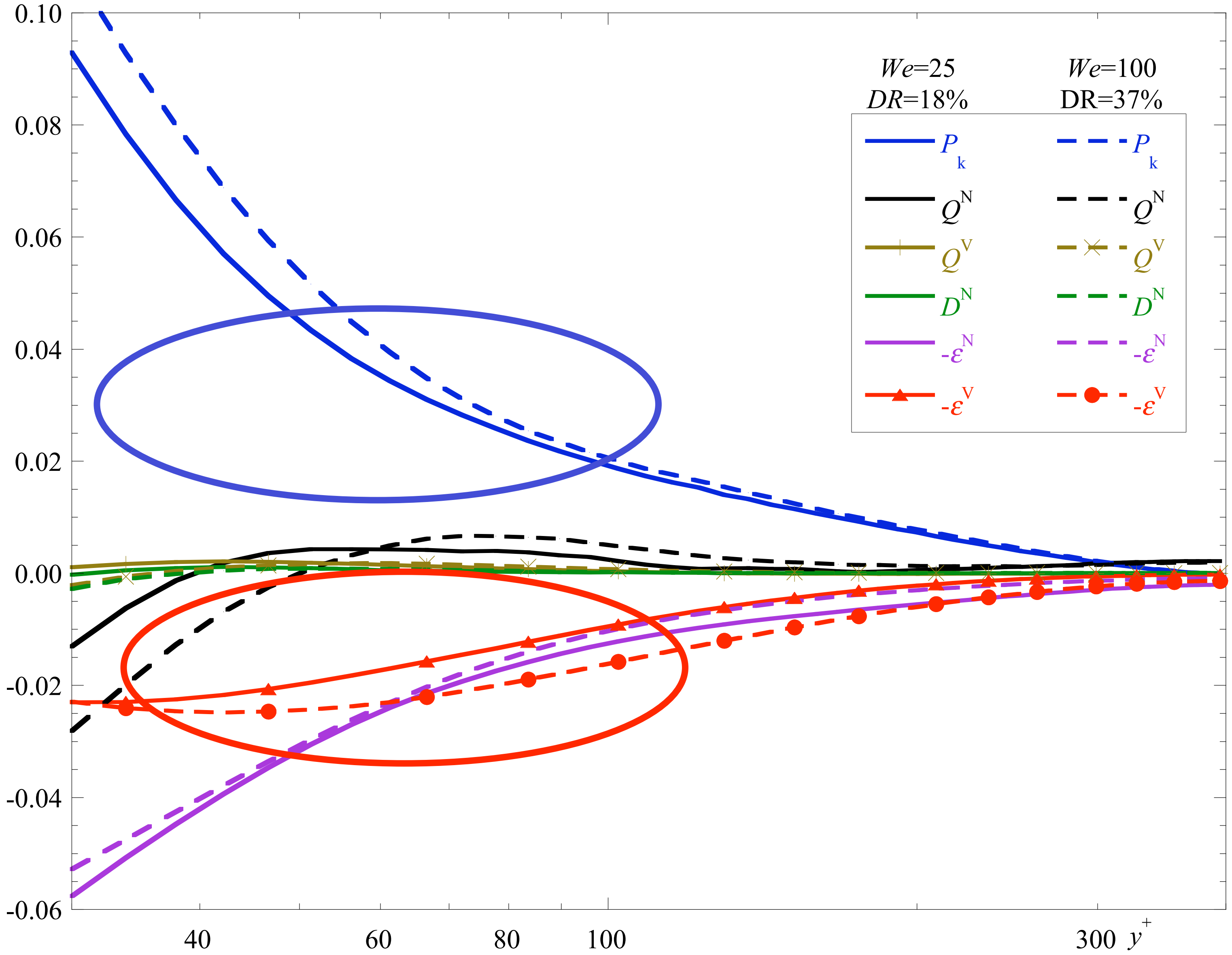
ε^V increases in inertial l.

Need to model well ε^V

Q^V is small

Need to modify model of ε^N

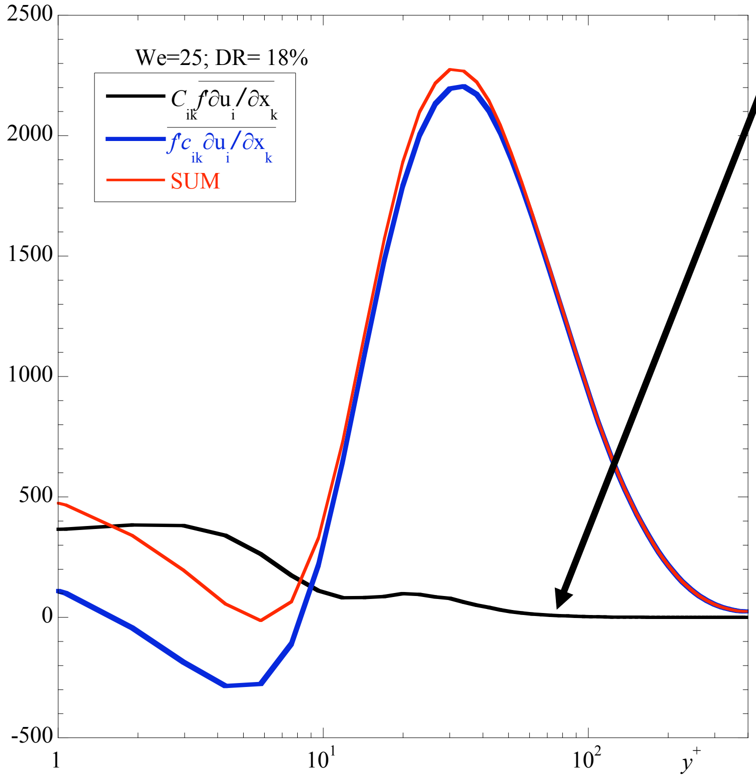
Zoom of balance of *k*: inertial sub-layer

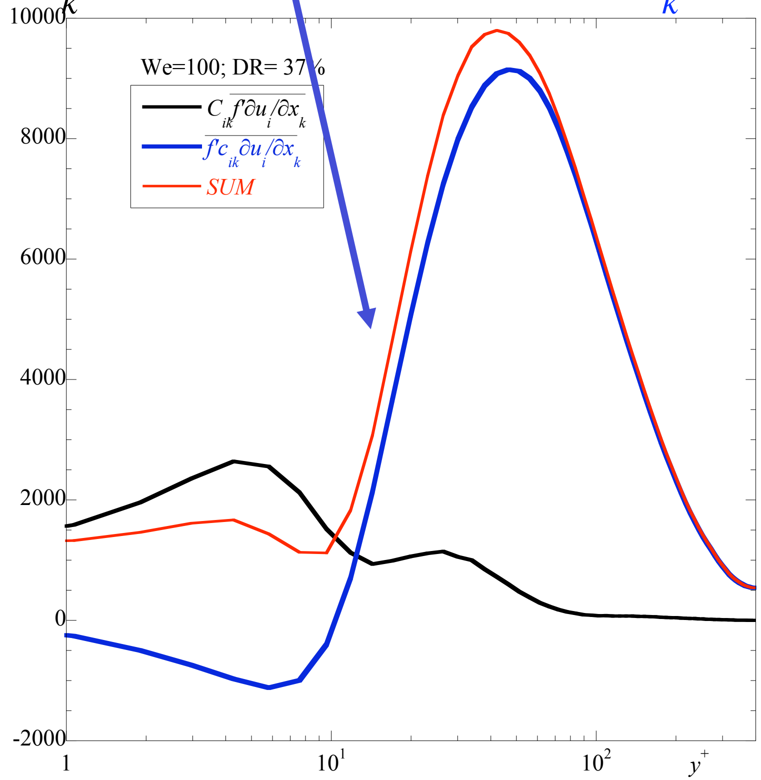


Assumptions for viscoelastic stress work: ε^V

$$\varepsilon^V \equiv \frac{1}{\rho}\overline{\tau'_{ik,p}\frac{\partial u_i}{\partial x_k}} = \frac{\eta_p}{\rho\lambda}\left[C_{ik}\overline{f(C_{mm}+c_{mm})\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik}f(C_{mm}+c_{mm})\frac{\partial u_i}{\partial x_k}}\right]$$

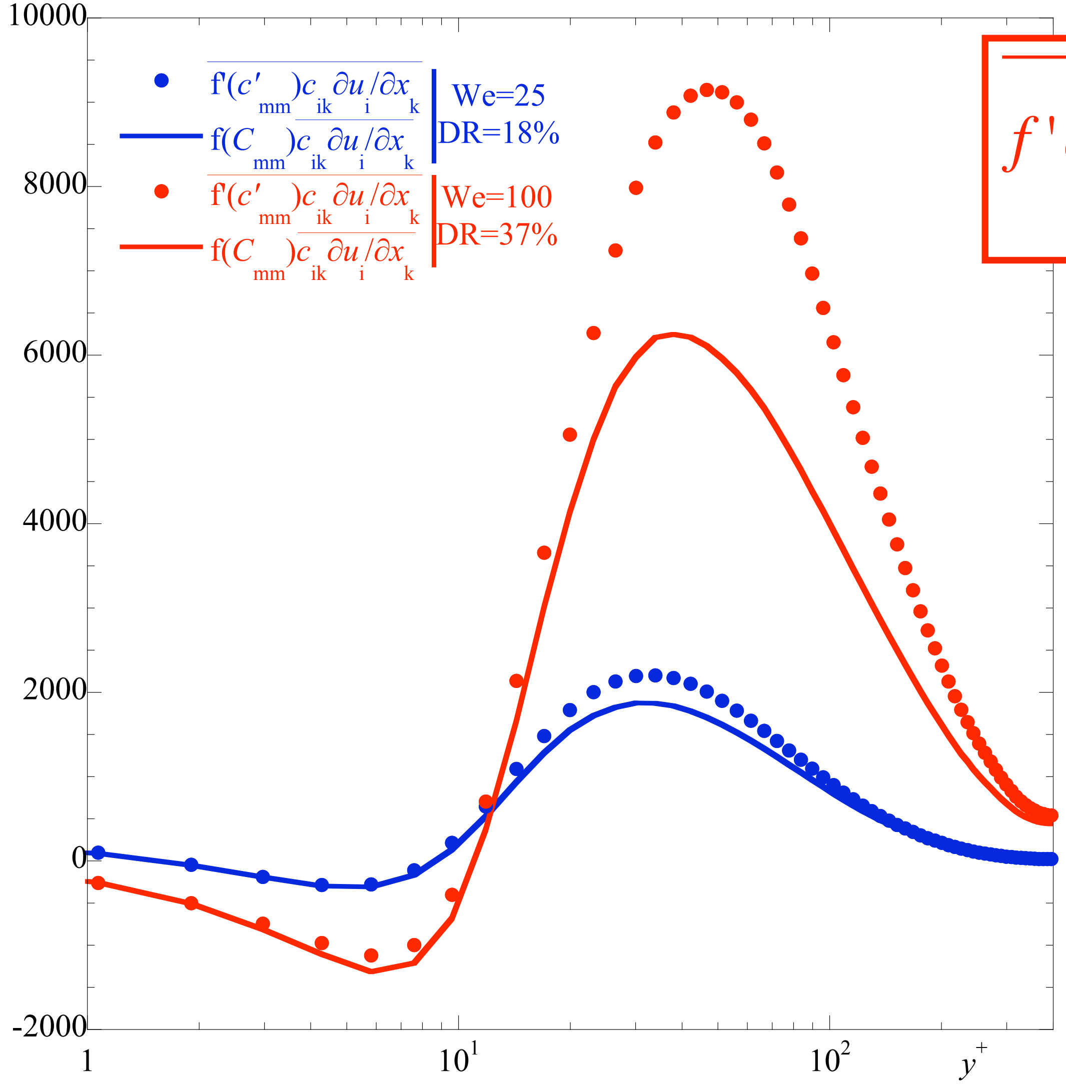
$$C_{ik}\overline{f(C_{mm}+c_{mm})\frac{\partial u_i}{\partial x_k}} \ll \overline{c_{ik}f(C_{mm}+c_{mm})\frac{\partial u_i}{\partial x_k}}$$





Except in viscous sublayer and buffer, but here ε^V is not important

Further assumptions for viscoelastic stress work: ε^V



$$\overline{f'c'_{ik}\frac{\partial u_i}{\partial x_k}} \approx C_{\varepsilon^V} \times f\left(C_{mm}\right)\overline{c_{ik}\frac{\partial u_i}{\partial x_k}}$$

$C_{\varepsilon^V} \sim O(1)$

at $We_{\tau_0} = 25$

but larger as DR increases

This is *NLT*$_{ii}$

Viscoelastic stress work model

$$\varepsilon^V \approx \frac{\eta_p}{\rho\lambda} C_{\varepsilon^V} f(C_{mm}) \overline{c_{ik}\frac{\partial u_i}{\partial x_k}} = C_{\varepsilon^V}\left(\frac{We_{\tau_0}}{25}\right)^{n-1} \frac{\eta_p}{\rho\lambda} f(C_{mm}) \frac{NLT_{ii}}{2}$$

→ **Modeled**

$We_{\tau_0} = 25 \rightarrow 1.27$

$We_{\tau_0} = 100 \rightarrow 1.56$

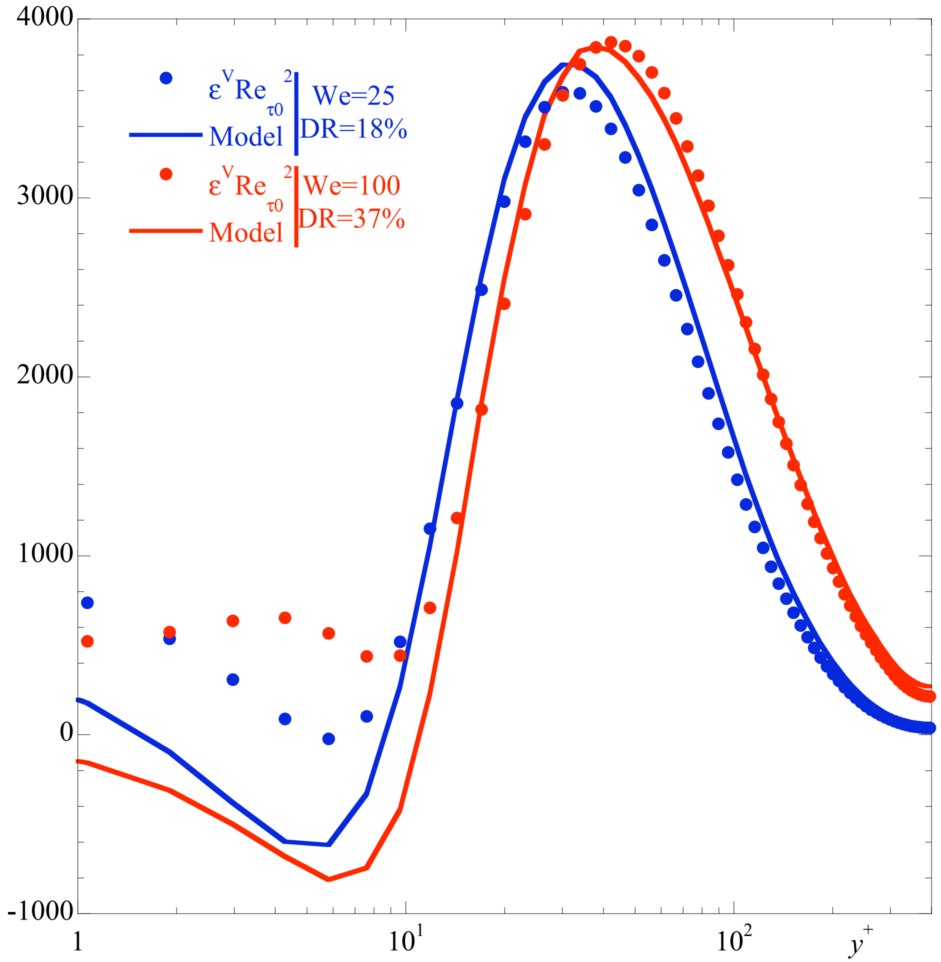
$C_{\varepsilon^V} = 1.27$

$n = 1.15$

$\varepsilon^{V+}\left(\mathrm{Re}_{\tau_0}\right)^2$

versus

$$C_{\varepsilon^V} \frac{\mathrm{Re}_{\tau_0}(1-\beta)}{We_{\tau_0}} f(C_{ii}) NLT^*_{jj}$$



Previous model: $C_{\varepsilon^V} = 1.076$
($We_{\tau 0}$= 25 only)

Pinho, Li, Younis, Sureshkumar (2008) JNNFM, in press

Viscoelastic turbulent transport: Q^V

$$Q^V \equiv \frac{\partial \overline{\tau'_{ik,p} u_i}}{\partial x_k} = \frac{\eta_p}{\lambda}\frac{\partial}{\partial x_k}\Big[\underbrace{C_{ik}\overline{f(C_{mm}+c_{mm})u_i}}_{CFU_{iik}} + \underbrace{\overline{c_{ik} f(C_{mm}+c_{mm})u_i}}_{CU_{iik}}\Big]$$

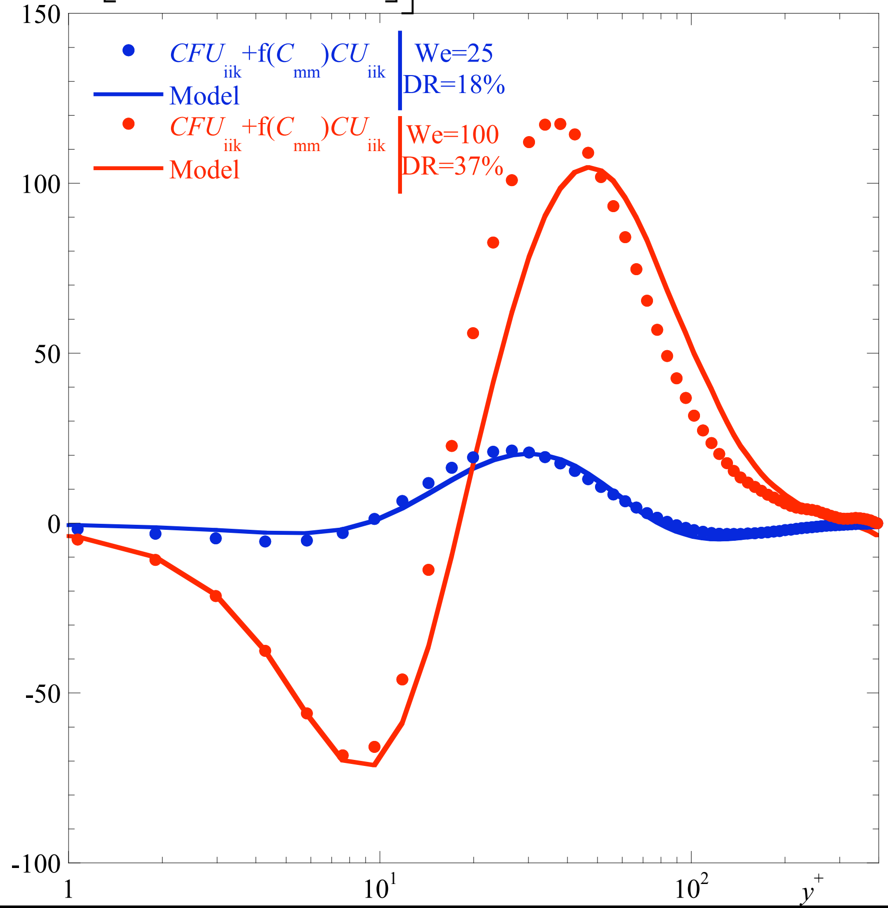
$$\frac{f(C_{mm})CU_{ijk}}{\lambda} = f_{\mu_2}\left(\frac{25}{We_{\tau_0}}\right)^{0.53}\left[-C_{\beta_1}\left(\overline{u_i u_m}\frac{\partial C_{kj}}{\partial x_m} + \overline{u_j u_m}\frac{\partial C_{ik}}{\partial x_m}\right) - \frac{C_{\beta_7}}{\lambda} f(C_{mm})\left[\pm\sqrt{\overline{u_j^2}}C_{ik} \pm \sqrt{\overline{u_i^2}}C_{jk}\right]\right]$$

$$f_{\mu_2} = 1-\exp\left(-y^+/26.5\right)$$

$$C_{\beta_1} = 1.1; C_{\beta_7} = 0.3$$

$$CFU_{iik} = C_{ik}\overline{f(C_{mm}+c_{mm})u_i}$$

$$\cong \frac{C_{FU}}{2}\frac{\lambda}{We_{\tau_0}} f(C_{mm})C_{kn}\frac{\partial \overline{u_i u_i}}{\partial x_n}$$



Closure development followed similar procedures as that for NLT_{ij}

k and ε transport equations: modified Nagano & Hishida

Based on Newtonian model of Nagano & Hishida (1984)

$$0=\frac{d}{dy}\left[\left(\eta_s+\frac{\rho f_T \nu_T}{\sigma_k}\right)\frac{dk}{dy}\right]+P_k-\rho\tilde{\varepsilon}^N-\rho D^N+\eta_p\frac{d}{dy}\left[\frac{f(C_{mm})}{\lambda}\frac{CU_{nny}}{2}\right]-\eta_p\frac{f(C_{mm})}{\lambda}\frac{NLT_{nn}}{2}$$

$$\sigma_k=1.1 \qquad \varepsilon^N=\tilde{\varepsilon}^N+D^N \qquad D^N=2\eta_s\left(\frac{d\sqrt{k}}{dy}\right)^2$$

$$f_T=1+3.5\exp\left[-\left(R_T/150\right)^2\right]$$

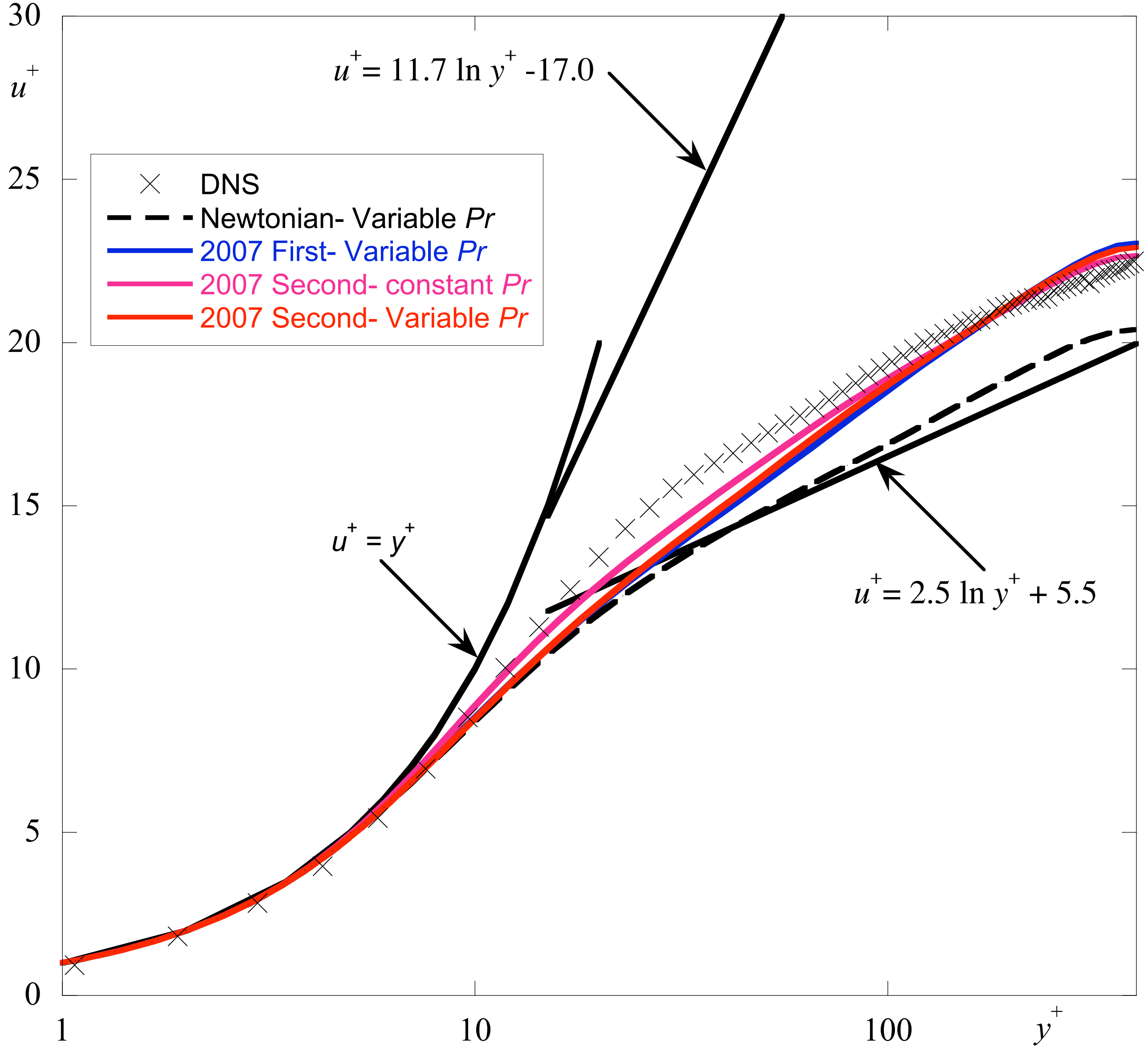
Variable Prandtl numbers: Nagano & Shimada (1993), Park and Sung (1995)

$$0=\frac{d}{dy}\left[\left(\eta_s+\frac{\rho f_T \nu_T}{\sigma_\varepsilon}\right)\frac{d\tilde{\varepsilon}^N}{dy}\right]+\rho f_1 C_{\varepsilon_1}\frac{\tilde{\varepsilon}^N}{k}\frac{P_k}{\rho}-\rho f_2 C_{\varepsilon_2}\frac{{\varepsilon^N}^2}{k}+\rho E+E_{\tau_p}$$

$$\sigma_\varepsilon=1.3 \qquad f_1=1 \qquad f_2=1-0.3\exp\left(-R_T^2\right)$$

$$C_{\varepsilon_1}=1.45 \qquad C_{\varepsilon_2}=1.90 \qquad E=\frac{\eta_s}{\rho}\nu_T\left(1-f_\mu\right)\left(\frac{d^2U}{dy^2}\right)^2 \qquad E_{\tau_p}=0$$

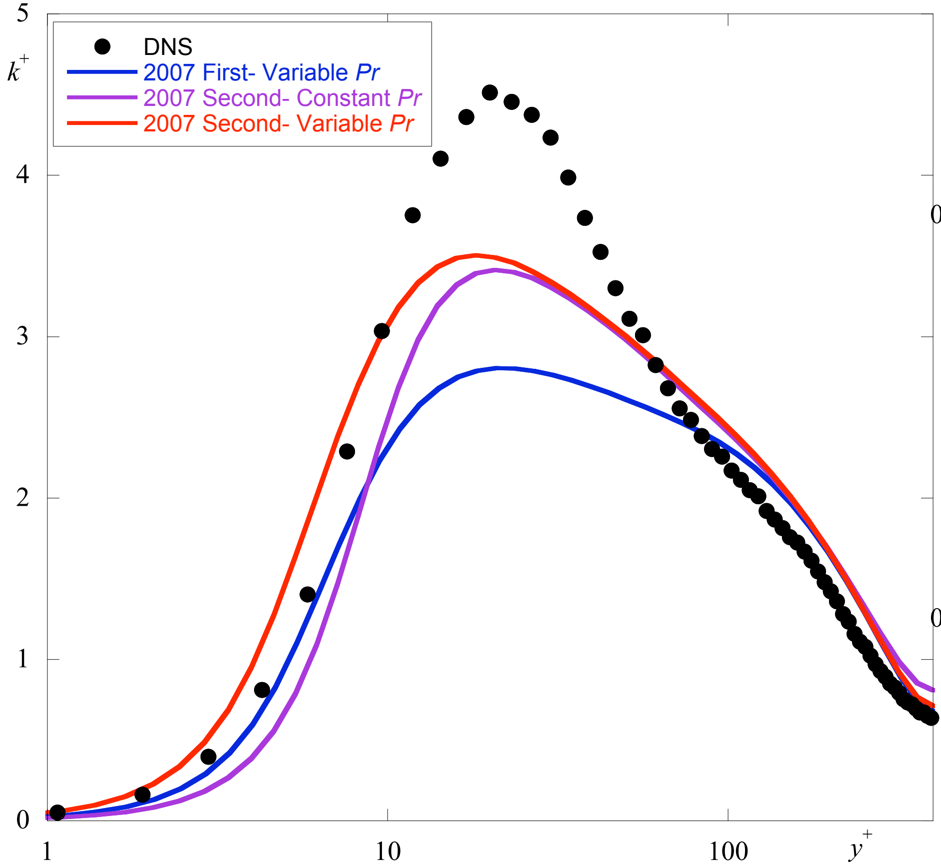
Predictions U^+: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900

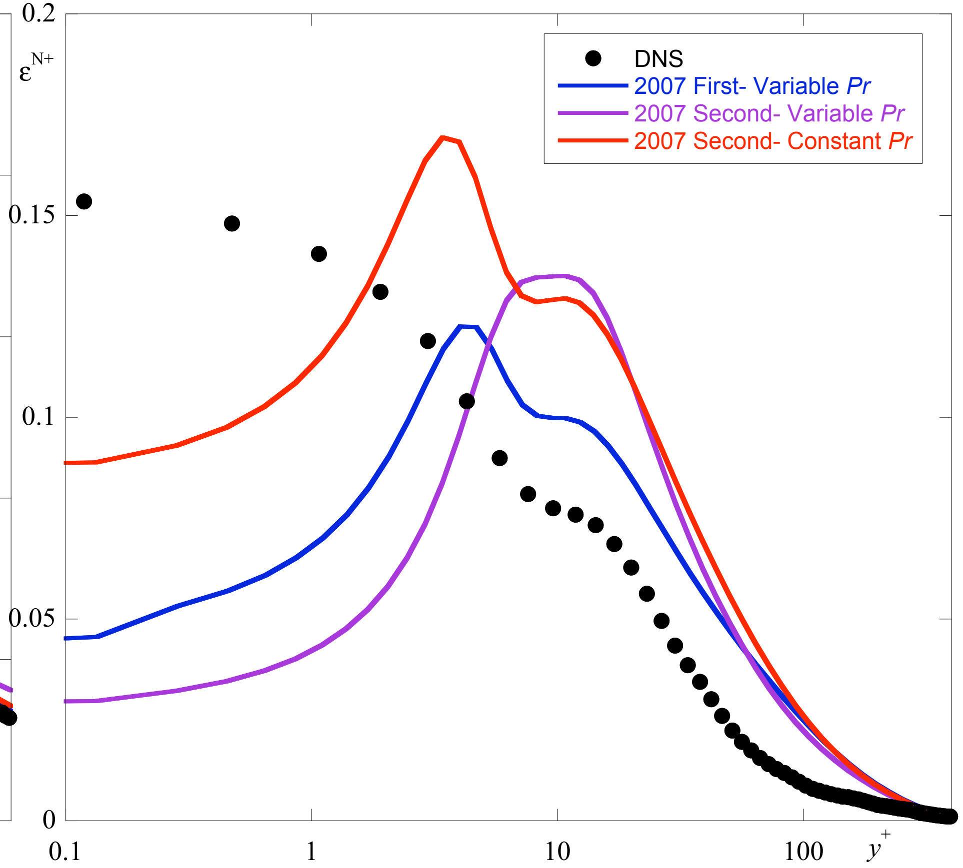


2007 1st model
2007 2nd model
2007 2nd model

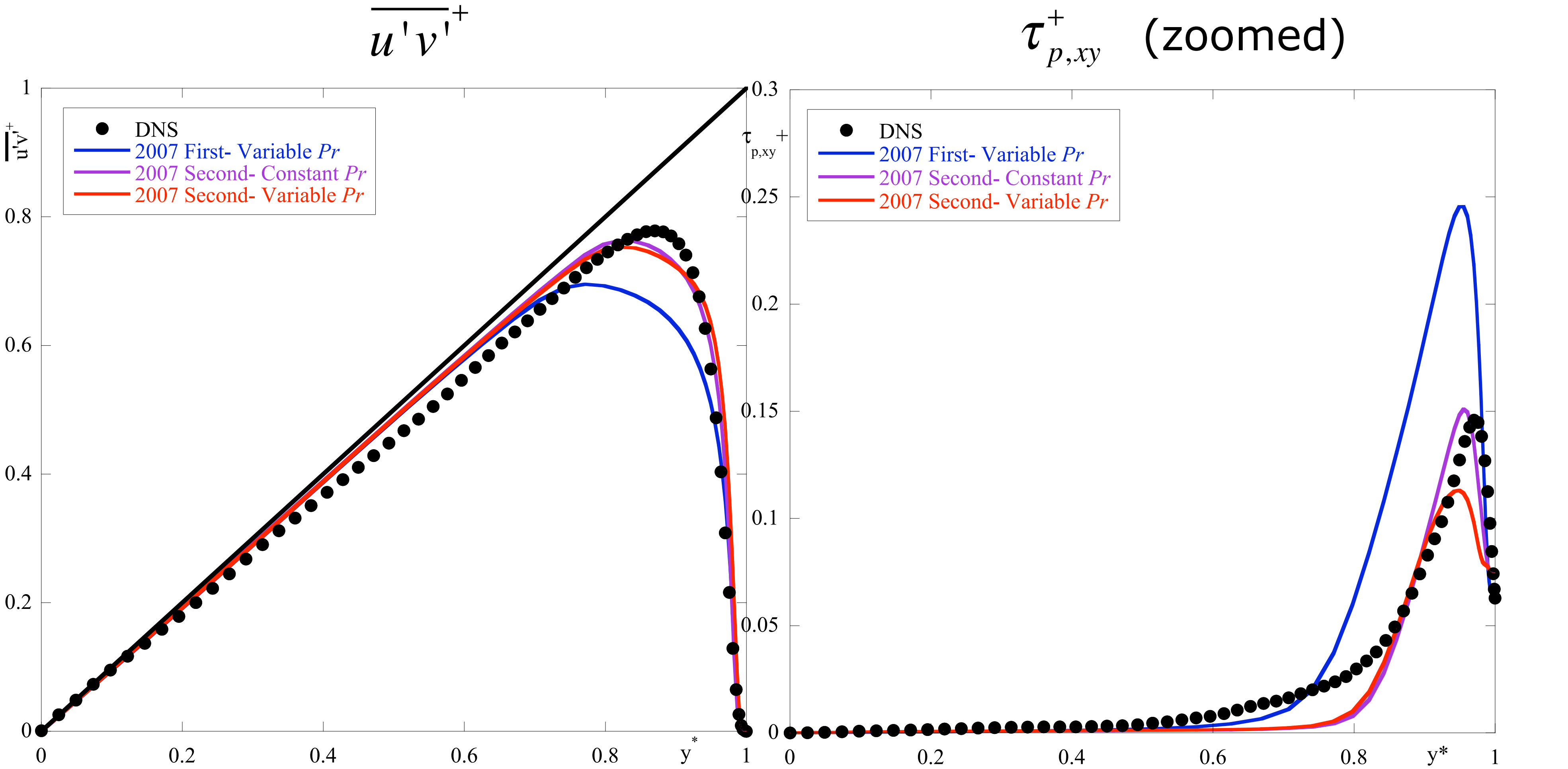
Predictions k & ε^N: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900

k^+ ε^{N^+}





Predictions *u'v'* & $\tau_{p,xy}$: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900



Conclusions and Acknowledgments

- Closures for Low DR and High DR **(Resende, 6ª feira, 11h15)**
- Closures for NLT_{ij}, ε^V and Q^V (in fact for $\varepsilon_{ij}{}^V$ and $Q_{ij}{}^V$)
- Developed simple low Reynolds *k*-ε model works reasonably well
- Need to incorporate with better Reynolds stress closures:

  k-ω, modified *k*-ε or *k*-ω, Menter's SST or Durbin's v2-f

  or RS transport (deficiencies in base model are imp.)
- Need to extend models to Maximum DR, & β & L^2
- DNS in other canonical flows required for extension of turb. models

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