IMPROVED PREDICTIONS OF LOW AND MODERATE DRAG REDUCTION IN TURBULENT CHANNEL FLOW OF FENE-P FLUIDS USING $k-\varepsilon$ MODEL

P. R. Resende  
*Centro de Estudos de Fenómenos de Transporte, Universidade do Porto, Portugal*

F. T. Pinho  
*Centro de Estudos de Fenómenos de Transporte, Universidade do Porto, Portugal*

K. Kim  
*Dep. Mechanical Engineering, Hanbat National University, Daejeon, South Korea*

R. Sureshkumar  
*Dep. Energy, Environmental and Chemical Engineering, Washington University of St. Louis, St Louis, MO, USA*

B. A. Younis  
*Dep. Civil and Environmental Engineering, University of California, Davis, USA*

**Vth Annual European Rheology Conference**  
15th-17th April 2009  
Cardiff, UK
Motivation and DNS cases: channel flow of FENE-P

**Objective: modifications to existing \( k-\varepsilon \) model + performance**

Pinho, Li, Younis & Sureshkumar (2008) JNNFM
A low Reynolds number turbulence closure for viscoelastic fluids

Turbulence modeling: Step 1) a priori DNS closure development
Step 2) building closures into the model

---

**Fully-developed channel flow**

$$\begin{align*}
W_{e_\tau} &= \frac{\lambda u_{\tau}^2}{\nu_0} \\
Re_{\tau} &= \frac{hu_{\tau}}{\nu_0}
\end{align*}$$

**DNS test/calibration cases (FENE-P model)**

$$Re_{\tau} = 395, \beta = 0.9, L^2 = 900$$

**Low Drag Reduction**

$$W_{e_\tau} = 25, DR = 18\%$$

**High Drag Reduction**

$$W_{e_\tau} = 100, DR = 37\%$$

Continuity: \[ \frac{\partial U_i}{\partial x_i} = 0 \]

Momentum balance:
\[ \rho \frac{\partial U_i}{\partial t} + \rho U_k \frac{\partial U_i}{\partial x_k} = - \frac{\partial \overline{p}}{\partial x_i} + \eta_s \frac{\partial^2 U_i}{\partial x_k \partial x_k} - \frac{\partial}{\partial x_k} \left( \overline{\rho u_i u_k} \right) + \frac{\partial \overline{\tau_{ik,p}}}{\partial x_k} \]

Rheological constitutive equation: **FENE-P**
\[ \overline{\tau}_{ij,p} = \frac{\eta_p}{\lambda} \left[ f(C_{kk}) C_{ij} - f(L) \delta_{ij} \right] + \frac{\eta_p}{\lambda} f(C_{kk} + c_{kk}) c_{ij} \]

\[ \nabla C_{ij} + u_k \frac{\partial C_{ij}}{\partial x_k} - \left( \frac{c_{kj}}{\partial x_k} + \frac{c_{ik}}{\partial x_k} \right) = - \frac{\overline{\tau}_{ij,p}}{\eta_p} \]

Improved predictions of low and high DR

**Closures required**

Resende, Pinho, Kim, Sureshkumar and Younis

CEFT-FEUP Centro de Estudos de Fenómenos de Transporte

AERC 2009, Cardiff, UK
Conformation (RACE) equation

\[ \lambda \frac{\partial C_{ij}}{\partial t} + \lambda M_{ij} \nabla \left[ u_k \frac{\partial u_i}{\partial x_k} - \left( c_{kj} \frac{\partial u_i}{\partial x_k} + c_{ik} \frac{\partial u_j}{\partial x_k} \right) \right] = -\left[ f(C_{kk})C_{ij} - f(L)\delta_{ij} \right] - \frac{f(C_{kk} + c_{kk})c_{ij}}{NLT_{ij}} \]


\[ f(C_{mm}) \frac{NLT_{ij}}{\lambda} = f_{\mu_i} \left[ \frac{C_{E3} u_i u_j}{v_{0}^2} \right] + \frac{C_{\alpha_{14}}}{v_{0}} \left( u_i u_k W_{kn} C_{nj} + u_j u_k W_{kn} C_{ni} + u_k u_i W_{jn} C_{nk} \right) \]

New model (a priori DNS at BSR 2008 meeting)

\[ f(C_{mm}) \frac{NLT_{ij}}{\lambda} = f(C_{mm}) \frac{f(C_{mm})}{\lambda} \left( f_{N_1} C_{ij} f(C_{mm}) - f_{N_2} \left[ c_{kj} \frac{\partial U_i}{\partial x_k} + c_{ik} \frac{\partial U_j}{\partial x_k} \right] \right) \]

\[ + f_{N_3} \left[ \frac{C_{km}}{v_{0} \sqrt{2 S_p q_p}} \left( u_i u_m \frac{\partial U_j}{\partial x_k} \frac{\partial U_m}{\partial x_n} + u_j u_m \frac{\partial U_i}{\partial x_k} \frac{\partial U_m}{\partial x_n} + u_k u_m \frac{\partial U_j}{\partial x_k} \frac{\partial U_i}{\partial x_n} + u_k u_m \frac{\partial U_j}{\partial x_k} \frac{\partial U_i}{\partial x_n} \right) \right] \]

\[ - f_{N_4} \left[ c_{km} \frac{\partial U_k}{\partial x_n} \frac{\partial U_i}{\partial x_k} + c_{jn} \frac{\partial U_k}{\partial x_n} \frac{\partial U_j}{\partial x_k} + \frac{C_{kn}}{v_{0} \sqrt{2 S_p q_p}} \left( \frac{\partial U_k}{\partial x_n} \frac{\partial U_i}{\partial x_k} \frac{\partial U_j}{\partial x_n} + \frac{\partial U_k}{\partial x_n} \frac{\partial U_i}{\partial x_k} \frac{\partial U_j}{\partial x_n} \right) \right] + f_{N_5} \frac{4}{15} \frac{\varepsilon}{\beta v_{s}} C_{mm} \delta_{ij} \]

\[ f_{N_1} = f(W_{E_{\tau_0}}, y^+) \]
Reynolds stress model: eddy viscosity model

Prandtl-Kolmogorov model ($k$-$\varepsilon$ closure)

$$-u_i u_j = 2v_T S_{ij} - \frac{2}{3} k \delta_{ij}$$


$$v_T = C_\mu f_\mu \frac{k^2}{\langle \varepsilon \rangle^N + \varepsilon^V}$$

Dissipation of $k$ by solvent

Viscoelastic stress work

New model

$$v_T = v_T^N - v_T^P$$

$$v_T^N = C_\mu f_\mu \frac{k^2}{\langle \varepsilon \rangle^N}$$

$$\frac{k^2}{\langle \varepsilon \rangle^N} \propto l u'$$

$$v_T^P = C_\mu^P f_\mu^P C_{kk} \frac{k^2}{\langle \varepsilon \rangle^N}$$
Transport equation for $k$ and viscoelastic stress work ($\varepsilon^V$)

$$\rho \frac{Dk}{Dt} = -\rho u_i u_k \frac{\partial U_i}{\partial x_k} - \rho u_i \frac{\partial k'}{\partial x_i} - \frac{\partial p'u_i}{\partial x_i} + \eta_s \frac{\partial^2 k}{\partial x_i \partial x_i} - \eta_s \frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_k} + \frac{\partial \tau'_{ik,p} u_i}{\partial x_i} - \tau'_{ik,p} \frac{\partial u_i}{\partial x_k}$$

0 $P_k$ $Q^N$ $D^N$ $-\varepsilon^N$ $Q^V$ $-\varepsilon^V$

exact unchanged exact

\[ \varepsilon^V \equiv \frac{1}{\rho} \tau'_{ik,p} \frac{\partial u_i}{\partial x_k} \approx \frac{\eta_p}{\rho \lambda} \left[ c_{ik} f \left( C_{mm} + c_{mm} \right) \frac{\partial u_i}{\partial x_k} \right] \]

\[ f' c'_{ik} \frac{\partial u_i}{\partial x_k} \approx f_{e^V} \times f \left( C_{mm} \right) c_{ik} \frac{\partial u_i}{\partial x_k} \]

Model is the same, except for $NLT_{ii}$ and corrective $f_{e^V}$

\[ 0 = \frac{d}{dy} \left[ \left( \eta_s + \frac{\rho f_T v_T}{\sigma_k} \right) \frac{dk}{dy} \right] + P_k - \rho \varepsilon^N - \rho D^N + \frac{\eta_p}{\lambda} \frac{d}{dy} \left[ f \left( C_{mm} \right) C_{nk} \left( FU \right)_n + CU_{mm} \right] - \eta_p \frac{f \left( C_{mm} \right) NLT_{nn}}{2} \]
Transport equation for $\varepsilon^N$

Exact equation: not used, this equation is completely modeled

\[
2v \frac{\partial u_i}{\partial x} \frac{\partial}{\partial x_i} \left( \rho \frac{Du_i}{Dt} \right) + 2v \frac{\partial u_i}{\partial x} \frac{\partial}{\partial x_i} \left( \rho u_i \frac{\partial U}{\partial x_i} \right) + 2v \frac{\partial u_i}{\partial x} \frac{\partial}{\partial x_i} \left( \rho \frac{\partial u_i}{\partial x_i} \right) + 2\nu \frac{\partial u_i}{\partial x} \frac{\partial}{\partial x_i} \left( \frac{\partial p'}{\partial x_i} \right) - 2\rho \nu \frac{\partial u_i}{\partial x} \frac{\partial}{\partial x_i} \left( \frac{\partial \varepsilon'_i}{\partial x_i} \right) - 2\nu \frac{\partial u_i}{\partial x} \frac{\partial}{\partial x_i} \left( \frac{\partial \tau'_i}{\partial x_i} \right) = 0
\]

Polymer modifies $\varepsilon^N$ equation (neglected by Pinho et al (2008) but considered in new model)

- Need to reduce $\varepsilon^N$ as polymer drag reduction increases
- Extra term should be a destruction term, like to the classical destruction term, i.e., proportional to $\varepsilon^2/k$
Transport model equation for $\varepsilon^N$


$$0 = \frac{d}{dy} \left[ \left( \eta_s + \frac{\rho f_T v_T}{\sigma_\varepsilon} \right) d\tilde{\varepsilon}^N \right] + \rho f_1 C_{\varepsilon_1} \frac{\tilde{\varepsilon}^N}{\rho} \frac{P_k}{k} - \rho f_2 C_{\varepsilon_2} \frac{\varepsilon^{N^2}}{k} + \rho E + E_{\tau_p}$$

with $E_{\tau_p} = 0$

Destruction term

Low Re correction
(Nagano & Hishida, 1984)

New model

$E_{\tau_p} \neq 0$

Modeled as an extra destruction related to polymer extension and viscoelastic stress work

$$E_{\tau_p} = - \left\{ f_{E_{\tau p1}} (\varepsilon^V) + f_{E_{\tau p2}} \left[ C_{nn} f (C_{ii}) \right]^2 \varepsilon^N \right\} \frac{\tilde{\varepsilon}^N}{k} \propto \frac{\varepsilon \times \varepsilon^N}{k} \sim \frac{u^{14}}{l^2}$$

$$f_{E_{\tau p1}}, f_{E_{\tau p2}} = f_{E_{\tau p}} (We, y^+)$$
Reference cases 1: $Re_{x_0} = 395; \beta = 0.9, L^2 = 900$

**Mean velocity**

\[ u' = 11.7 \ln y' - 17.0 \]
\[ u' = 2.5 \ln y' + 5.5 \]

**$k^+$**

- DNS - Mansour ($We = 0$)
- DNS - $We = 25$
- DNS - $We = 100$
- $We = 25$ - Old (Pr = variable)
- $We = 0$ - New
- $We = 25$ - New
- $We = 100$ - New

Improved predictions of low and high DR

CEFT-FEUP Centro de Estudos de Fenómenos de Transporte

Resende, Pinho, Kim, Sureshkumar and Younis

AERC 2009, Cardiff, UK
Reference cases 2: $Re_{\tau_0} = 395; \beta=0.9, L^2=900$

**$\epsilon^{N^+}$**

- DNS- Mansour (We= 0)
- DNS- We= 25
- DNS- We= 100
- We= 25- Old (Pr= variable)
- We= 0- New
- We= 25- New
- We= 100- New

**$NLT^*_ii$**

- DNS- We= 25
- DNS- We= 100
- We= 25- Old (Pr= variable)
- We= 0- New
- We= 25- New
- We= 100- New
Reference cases 3: $Re_0 = 395; \beta = 0.9, L^2 = 900$

Improved predictions of low and high DR

Resende, Pinho, Kim, Sureshkumar and Younis

CEFT-FEUP Centro de Estudos de Fenómenos de Transporte

AERC 2009, Cardiff, UK
Parametric study 1: $Re_{\tau_0} = 395; \beta=0.9, L^2=900$

**Mean velocity**

- $u^* = 11.7 \ln y^* - 17.0$
- $u^* = 2.5 \ln y^* + 5.5$

**$k^+$**

- $We_{\tau_0}$, DR [%]
  - 0
  - 14 9.3
  - 19.2 15.6
  - 25.0 19.7
  - 42.4 27.6
  - 63.5 33.5
  - 100 39
  - 153 43.8

**$We_{\tau},DR$**

$We_{\tau} = We_{\tau_0} - DR$
Parametric study 2: $Re_{\tau_0} = 395; \beta=0.9, L^2=900$

Improved predictions of low and high DR

Resende, Pinho, Kim, Sureshkumar and Younis

AERC 2009, Cardiff, UK
Parametric study 3: $Re_{\tau_0} = 395; \beta = 0.9, L^2 = 900$

$\overline{u'n'}^+$

$\overline{\tau^+_{xy,p}}$

$We_{\tau, DR}$

$We_{\tau, DR}$
Conclusions, Future Work and Acknowledgments

- Closure for elastic contribution to $\nu_T$
- Closure for elastic term $(E_{tp})$ in $\varepsilon^N$ equation: destruction of $\varepsilon^N$
- $k-\varepsilon$ model works well at Low DR and High DR (45%)

- Need to extend models to Maximum DR, & $\beta$ & $L^2$
- To solve the existing deficiencies:
  
  improve $k$, improve polymer and Reynolds stress predictions

Acknowledgments - Funding

Fundação para a Ciência e Tecnologia
FCT Scholarship SFRH/BD/18475/2004