Estimation of extreme response of floating bridges by Monte Carlo simulation

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ABSTRACT: This paper addresses the problem of estimating the extreme response statistics of floating bridges subjected to harsh weather conditions. The sea surface is modelled as a locally homogeneous stochastic field, while the wave actions are obtained using potential theory. The added mass and potential damping originating from the fluid-structure interaction is modelled using rational functions. The dynamic response is predicted in the time domain using Monte Carlo simulations and to extract the extreme value distribution from the simulated data, the average conditional exceedance rate method, is used. This is a non-parametric representation of the extreme value distribution inherent in the data. The method can be used to predict extreme response statistics of narrow-banded random response processes typical for floating bridges.

KEY WORDS: Floating bridge, extreme values, ACER, Monte Carlo simulations

1 INTRODUCTION

Even though the history of floating bridges may be traced back many thousands of years, it is only during the last three decades or so that floating bridges are being developed to the degree of sophistication so they can be applied as a critical part of modern infrastructure. In spite of this fact, compared with land-based bridges, including cable-stayed bridges, limited information [1] is currently available on floating bridges and even less on submerged tunnels for transportation. This concerns especially construction records, environmental conditions, durability, operations and performance. This is obvious from the fact that currently there are only about twenty long span floating bridges in the world.

A computational procedure to assess the dynamic response of floating bridges in time domain is outlined in this paper. The hydrodynamic actions have been modelled using first order potential theory, see for instance [2] or [3]. The sea surface is modelled as a homogeneous stochastic field [4, 5] and small amplitude waves (Airy waves) have been assumed. The motion induced forces have been approximated using rational functions. This is an approach strongly related to the state space models more commonly applied to replace the convolution integral in the Cummins equation that governs the equilibrium of a floating structure in time domain, see for instance [6-8].

The dynamic response of slender bridges is typically assessed using advanced finite element codes such as Abaqus or Ansys that are capable to model large deformations and advanced material behaviour. In this paper it is shown how the type of rational functions used in this paper can be used to develop a hydroelastic element that can be implemented in Abaqus using the user subroutine functionality.

Time series of the dynamic response of a floating bridge subjected to a sea state is of limited value, it is the extreme response that is needed in design. The extreme values are in this paper assessed using the Gumbel method of episodical extremes, see for instance [9], and the average conditional exceedance rates (ACER) method [10]. The latter is a non-parametric representation of the extreme value distribution inherent in the data, which implies that no assumption regarding the distribution of the extreme values needs to be made.

The Bergsøy sund Bridge displayed in Figure 1 is used as case study in this paper. The bridge is a pontoon bridge that crosses the Bergsøy sund strait in Norway. The bridge is 931 meters long and consists of a steel truss that is resting on 7 concrete pontoons. The bridge is constructed as an arch in the horizontal plane and there are no anchors that support the bridge between the abutments, which makes it one of the longest floating bridges in the world without side anchoring.

One of the most important components of the bridge is the steel rod displayed in Figure 2. The bridge is resting on rubber joints.
bearings in the vertical and horizontal direction at the abutments while it is only the steel rod that supports the bridge in the tangential direction of the arch. The extreme values of the axial forces in this rod will be studied in this paper.

2 HYDRODYNAMIC ACTIONS

2.1 Wave modelling

Wind generated sea waves are commonly approximated as a locally homogeneous random field for engineering purposes. The sea surface can then be expressed mathematically by the following Riemann-Stieltjes integral:

$$\eta(x,t) = \int e^{i(k_x x + k_y y)} dZ_\eta(k,\omega)$$ (1)

Here the surface elevation $\eta$ is a scalar quantity given as function of location $x = (x, y)$ in space (mean sea surface) and time $t$, $k = (k_x, k_y)$ is wave number vector, $\omega$ is frequency, and $Z_\eta$ is a spectral process with independent increments. For small amplitude water waves (Airy waves) the wave number and frequency is related through the dispersion relation given as:

$$2 \tanh(\frac{g}{h}) = \frac{\omega^2}{\kappa^2}$$ (2)

$$\cos \theta \sin \theta = k_x$$ (3)

where $\theta$ is the wave direction. The model outlined above reveals that the wave spectral density can be expressed as a function of frequency and direction

$$S_\eta^{(d)}(\omega, \theta) = S_\eta(\omega) D(\omega, \theta)$$ (4)

Here $S_\eta(\omega)$ is the one dimensional wave spectral density, while $D(\omega, \theta)$ symbolizes the directional distribution. The directional function for locally generated sea states is commonly approximated as independent of frequency. The one dimensional wave spectral density, applied in this study is the so-called ITTC spectrum [11] while the directional distribution applied is the so-called cos-2s distribution.

$$S_\eta(\omega) = 0.0081g^2\omega^{-5}\exp(-\frac{3.11}{\omega^5 H_s^2})$$

$$D(\theta) = \frac{\Gamma(s+1)}{2\sqrt{\pi}} \frac{\cos^s\left(\frac{\pi}{2} \theta\right)}{\Gamma(s+1/2)}$$ (5)

Here, $H_s$ represent the significant wave height while $s$ is a directional parameter.

2.2 Wave actions

The wave forces acting on our study bridge are generated by extending the simulation procedure outlined above. It is assumed that the pontoon behaves like a rigid body and the wave action can be expressed in terms of three force components and three moment components. These force components are in the frequency domain modelled in terms of transfer functions obtained using first order potential theory applying the software Wadam [12]. The real and imaginary parts of the transfer function for the vertical force on the pontoons are displayed in Figure 3 and Figure 4, respectively.

Time series of the forces acting on the pontoons can then be obtained by

$$P_n(x, y, t) = \sum M \sum \sqrt{2S(\omega, \theta)\Delta\omega\Delta\theta} \cos[k_i(x\cos(\theta) + y\cos(\theta)) - \omega t + \omega_i + \phi_i]$$ (6)

Here $P_n$, $n \in \{1..6\}$, symbolizes the force component, $F_n(\omega, \theta)$ the hydrodynamic transfer function while $\omega_i \in [0..2\pi]$ is a uniformly distributed random phase angle. Deep water conditions, $\tanh(\kappa h) = 1$, have been assumed.

Figure 3: Real part of the transfer function for the vertical force

Figure 4: Imaginary part of the transfer function for the vertical force

2.3 Added mass and potential damping

The hydrodynamic mass and damping of the study structure are frequency dependent which implies that the corresponding time domain properties will be time related. This can be visualized as a memory process in the fluid-structure system...
in the time domain representation. For single harmonic component, \(\exp(-i\omega t)\), with frequency \(\omega\), the motion induced part of the hydrodynamic force can be written as

\[
q(\omega, t) = m_{sd}(\omega) \ddot{u}(t) + c_{sd}(\omega) \dot{u}(t) + k_{sd} u(t)
\]  
\(7\)

Here, \(m_{sd}\) and \(c_{sd}\) are the frequency dependent hydrodynamic mass and potential damping matrices, \(k_{sd}\), is the hydrostatic restoring matrix (assumed herein as frequency independent which is judged valid for small amplitude linear waves), while \(u\) symbolizes the displacements of the structure. Eq. (7) is only valid for a single-frequency harmonic motion. However, by introducing the principle of superposition, Eq. (7) can be extended to any periodic or aperiodic motion by applying a Fourier integral representation. Then, the motion induced forces can be expressed as follows:

\[
G_{q}(\omega) = F_{sd}(\omega)G_{x}(\omega)
\]

\(8\)

\[
F_{sd}(\omega) = -\omega^2 m_{sd}(\omega) + i\omega c_{sd}(\omega) + k_{sd}
\]

\(9\)

Here, \(i\) is the imaginary unit, and \(G_{x}(\omega)\) is the Fourier transform of \(X(t)\), where \(X \in [u, q]\), and the matrix \(F(\omega)\) contains the hydrodynamic transfer functions defined in terms of the hydrodynamic mass and damping, and restoring matrices, which in this representation are treated as continuous functions of frequency.

The time domain representation of the motion induced forces can be obtained applying the inverse Fourier transform to Eq. (8). This results in the following equation:

\[
q(t) = \int_{-\infty}^{\infty} f_{sd}(t - \tau)u(\tau)d\tau
\]

\(10\)

Here, the matrix \(f\) contains the fluid-structure interaction impulse response functions defined in terms of the hydrodynamic mass, radiational damping and restoring matrices. In the cases when the hydrodynamic mass and damping matrices are obtained at discrete frequencies it is convenient to curve-fit the data using the following expression:

\[
F_{sd}(\omega) = a_1 + a_2i\omega + a_3(i\omega)^2 + \sum_{i=4}^{N} b_i i\omega \frac{i\omega}{i\omega + d_i}
\]

\(11\)

The model coefficients can be determined by fitting the expression, either to numerical or experimental data describing the frequency dependence of the fluid-structure system. In practice this can be done by using the following real expressions:

\[
\text{Im}(F_{sd}(\omega)) = a_3\omega^2 + \sum_{i=4}^{N} b_i \frac{d_i}{d_i^2 + \omega^2}
\]

\(12\)

\[
\text{Re}(F_{sd}(\omega)) = a_1 - a_0\omega^2 + \sum_{i=4}^{N} b_i \frac{\omega^2}{d_i^2 + \omega^2}
\]

\(13\)

The elements in the matrices, \(a_n\), \(n \in \{1 2 3 4\}\), and the constants \(d_i\) are determined by a least square fit to the data, i.e. the frequency dependent matrices of Eq.(8). The hydrodynamic impulse response functions can be obtained formally by taking the Fourier transform of Eq. (10). That gives the following expression:

\[
f_{sd}(\tau) = a_1\delta(\tau) + a_2\delta(\tau) + a_3\delta(\tau)
\]

\(14\)

\[
+ \sum_{i=4}^{N} a_i \left( \delta(\tau) - d_i e^{-d_i\tau} \right)
\]

\(15\)

Here, \(\delta\) represents the Dirac delta function, the dots indicate time derivative, and \(H\) is the Heaviside unit step function. Inserting the hydrodynamic impulse response function into Eq. (9) renders the following expression for the motion induced forces in time domain:

\[
q(t) = a_1 u(t) + a_2 \dot{u}(t) + a_3 \ddot{u}(t)
\]

\(16\)

\[
+ \sum_{i=4}^{N} a_i \left( u(t) - d_i \int_{-\infty}^{t} e^{-d_i(t-\tau)} u(\tau)d\tau \right)
\]

\(17\)

2.4 Finite element representation of motion induced forces

We consider the pontoon of the floating bridge as a rigid body attached to the flexible girders of the bridge system. Then the following equation of motion applies:

\[
m\ddot{u} + c\dot{u} + k\dot{u} + q(t) = P(t)
\]

\(18\)

Here \(m\), \(c\), and \(k\) represent the structural mass, damping and stiffness matrices respectively i.e. properties of the system without water (sometimes referred to as the dry system properties), while \(u\) symbolizes the displacements of the pontoon. The motion-induced hydrodynamic forces are denoted \(q\), contain the wet system additional mass, radiational damping, and restoring forces, while \(P\) represents the wave action. From Eq.(1.8) we get:

\[
q(t) = a_1 u(t) + a_2 \dot{u}(t) + a_3 \ddot{u}(t) + Z(t)
\]

\(19\)

\[
Z(t) = \sum_{i=4}^{N} a_i \left[ u(t) - d_i \int_{-\infty}^{t} e^{-d_i(t-\tau)} u(\tau)d\tau \right]
\]

\(20\)

\[
Z = QX
\]

\(21\)

\[
Q = [a_1 \ a_2 \ \cdots \ a_N]
\]

\(22\)

\[
X = \begin{bmatrix} x_1^T \ x_2^T \ \cdots \ x_{N-3}^T \end{bmatrix}^T
\]

\(23\)

Taking the derivative of the time history dependent term \(\dot{x}(t)\) gives us the following relation:

\[
\dot{x}_i = u - d_i \dot{x}_i
\]

\(24\)

The equation of motion can then be written as follows:
Proceedings of the 9th International Conference on Structural Dynamics, EURODYN 2014

\[
\begin{bmatrix}
(m + a_{i}) & 0 & \ddot{u} + (c + a_{z}) & 0 \\
0 & 0 & \dot{X} + E & B \\
-k + a_{i} & Q & u & 0
\end{bmatrix}
\begin{bmatrix}
\dot{X} \\
\dot{X} \\
\dot{X}
\end{bmatrix}
= \begin{bmatrix}
P \\
0 \\
0
\end{bmatrix}
\] (18)

\[
E = \begin{bmatrix}
I & I & I \\
I & I & I \\
I & I & I
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
I & I & I \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
d_{1}I & d_{2}I & \cdots & d_{N-3}I
\end{bmatrix}
\] (19)

In order to use traditional integration schemes, e.g. the Newmark’s β-methods, to solve the equation of motion, it is convenient to take the derivative of Eq. (15) two times instead of ones. This provides the following equation of motion for each hydroelastic element:

\[
\begin{bmatrix}
(m + a_{i}) & 0 & \ddot{u} + (c + a_{z}) & 0 \\
0 & 0 & \dot{X} + E & B \\
-k + a_{i} & Q & u & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u} \\
\dot{X} \\
\dot{u}
\end{bmatrix}
= \begin{bmatrix}
P \\
0 \\
0
\end{bmatrix}
\] (20)

Then applying a standard assembly procedure the global system equation can be created by adding the dry elemental matrices to the wet elemental matrices represented, in principle, by Eq. (20) above. The hydroelastic element is displayed in Figure 5.

In Figure 5. Hydroelastic element

2.5 **Finite element representation of motion induced forces**

The hydrodynamic element presented in the section above has been implemented as a user element in Abaqus. In Abaqus the HHT-α [15] algorithm is used to solve the dynamic equilibrium, but since α is assumed to be zero in the calculations performed in this work, the algorithm is equal to the well-known Newmark-β method [16], which simplifies the implementation of the element slightly. In the Newmark-β method the equilibrium is satisfied at time increments, and at \( t_{n+1} \) the equilibrium reads [17]

\[
\dot{\mathbf{M}} \ddot{\mathbf{u}}_{n+1} + \mathbf{g}(\mathbf{V}_{n+1}, \mathbf{V}_{n+1}) = \mathbf{P}_{n+1}
\] (21)

Here, \( \dot{\mathbf{M}} \ddot{\mathbf{u}}_{n+1} \) represents nodal forces related to the accelerations; \( \mathbf{g}(\mathbf{V}_{n+1}, \mathbf{V}_{n+1}) \) represents forces related to the displacements and velocities, while \( \mathbf{P}_{n+1} \) represents the dynamic actions. In Abaqus the solution is obtained by Newton iterations on the residual \( \mathbf{r} \), implying that the residual and its total derivative (Jacobian matrix) \( \mathbf{J} \) must be given in the subroutine.

\[
\mathbf{r} = \dot{\mathbf{P}}_{n+1} - \dot{\mathbf{M}} \ddot{\mathbf{u}}_{n+1} - \mathbf{g}(\mathbf{V}_{n+1}, \mathbf{V}_{n+1})
\]

\[
\mathbf{J} = \frac{\partial \mathbf{r}}{\partial \mathbf{V}} = -\frac{\partial \mathbf{r}}{\partial \mathbf{V}} - \frac{\partial \mathbf{r}}{\partial \mathbf{V}} - \frac{\partial \mathbf{r}}{\partial \mathbf{V}}
\] (22)

Since the applied hydrodynamic model is linear the residual and the Jacobian matrix for the system outlined in Eq. (22) can be defined as

\[
\mathbf{r} = \dot{\mathbf{P}}_{n+1} - \dot{\mathbf{M}} \ddot{\mathbf{u}}_{n+1} - \dot{\mathbf{C}} \dot{\mathbf{V}}_{n+1} - \dot{\mathbf{K}} \mathbf{V}_{n+1}
\]

\[
\mathbf{J} = \left( \frac{1}{\beta(\Delta t)} \right) \dot{\mathbf{M}} + \frac{\dot{\gamma}}{\beta(\Delta t)} \dot{\mathbf{C}} + \dot{\mathbf{K}}
\] (23)

Here β and γ are constants used to defined the approximation of the velocity and acceleration in the Newmark’s algorithm, see [16] for further details.

3 **DYNAMIC RESPONSE**

The finite element model used to calculate the wave induced dynamic response is displayed in Figure 6. As can be seen, the entire truss has been modelled using beam elements, and since the concrete pontoons have been assumed rigid they are included in the finite element model as a 6 by 6 mass matrix.

In Figure 6: Overview of the finite element model of the steel truss [18]
In order to verify the procedure outlined in section 0 the dynamic response obtained for a linear model in time domain is benchmarked with the dynamic response predicted using frequency domain power spectral density methods [19]. The resulting power spectral density of the vertical response of the pontoon at the middle of the bridge is displayed in Figure 7. The spectral density of the time series calculated in the time domain has been obtained using the Welch method. Seven time series of 2000 seconds was used to provide the estimate. As can be seen from the figure the time domain results corresponds to the frequency domain results in an excellent manner which confirms that the time domain formulation outlined above is able to model the hydroelastic effects in the time domain.

Figure 7: Comparison of response spectra obtained in the time and frequency domain

To assess the extreme values of the axial force in the steel rod displayed in Figure 2, the dynamic response of the bridge have been calculated for 20 independent sea states of 3 hours duration. The significant wave height have been assumed \( H_s = 4 \) meters and the significant wave period taken as \( T_p = 10 \) seconds. The directional parameter, \( s = 20 \), was applied in Eq.(5). The calculations are performed using a Von Mises material model for the steel structure and geometric nonlinearities are also included. One of the obtained time series of the axial force is displayed in Figure 8, while the spectral density of the time series is displayed in Figure 9.

Figure 8: Time series of the axial force

Figure 9: Spectral density of the axial force

As can be seen from the figure, the axial force is fairly broadly banded with two distinct peaks.

4 EXTREME VALUE PREDICTION

4.1 Empirical estimation of average conditional exceedance rates

When estimating extreme value distributions it is often initially assumed that the up-crossings are independent. Under this assumption a good approximation of the CDF of the extreme value \( M(T) \) of the stochastic process \( F(t) \) during the time interval \( T \), is given by

\[
\text{Prob}(M(T) \leq \eta) = \exp\left[-v'(\eta)T\right] 
\]

(24)

Here \( v'(\eta) \) denotes the mean rate of up-crossings of the level \( \eta \) by the force process \( F(t) \). The question is then how to treat a situation where this assumption may be violated. Thus, we want to make use of a method which may be applied to any stochastic process represented as a sampled time series. To extract the extreme value distribution from the simulated data the average conditional exceedance rate method (ACER) may be used [10] This is a non-parametric representation of the extreme value distribution inherent in the data. The method can be used to predict extreme response statistics of narrow-banded random response processes typical for dynamic systems with low damping as investigated in the present paper.

As given in [16] we start by introducing the average conditional exceedance rate functions, \( \varepsilon_k(\eta) \) where \( k = 1, 2, ... \) \( k = 1 \) corresponds to independent data, meaning that all exceedances are counted for estimation of \( \varepsilon_1(\eta) \). For \( k = 2, 3, ... \) an exceedance of \( \eta \) is counted only if the immediately preceding \( k-1 \) data points are below \( \eta \). This gives the average conditional exceedance rate for the number of peaks or data points included in the time series, which again by sampling rate can be associated to a given time discretization.

The empirical estimation of the conditional average exceedance rate as described in detail in [10], is in practice given by

\[
\varepsilon(\eta) = \frac{1}{N-k+1} \sum_{j=1}^{N-k} \alpha_j(\eta), \quad k = 1, 2, ... 
\]

(25)

where

\[
\alpha_k(\eta) = \text{Prob}(X_{j+k} > \eta | X_{j+k-1} \leq \eta, ... , X_j \leq \eta)
\]

(26)
is the exceedance probability conditional on $k - 1$ preceding non-exceedances of the values of a time series sampled from a stochastic process given over a time interval $(0, T)$. $N$ is the total number of data processed and $\eta$ is the threshold value. The empirical estimation of the ACER function is found by replacing the ensemble mean by a corresponding time average where the sum equals the number of times $j$ when the event $\{X_j > \eta, X_{j-1} \leq \eta, \ldots, X_{j-k+1} \leq \eta\}$ occurs for $k = j \leq N$. That is the number of favourable incidents, i.e. exceedances combined with the requested number of preceding non-exceedance, for the total data ($N$) and then divided by $N - k + 1 = N$.

The confidence interval of the sample estimate of $\epsilon(\eta)$, may then be found by the assumption that there is available a suitable number $R$ of independent realizations. From an engineering point of view it is reasonable to choose $R$ no smaller than 15 to 20 realizations. The 95\% confidence interval for the mean value $\hat{\epsilon}(\eta)$ may then be approximated by

$$CI^+(\eta) = \bar{\epsilon}(\eta) \pm 1.96 \hat{s}_k / \sqrt{R}$$

where $\hat{s}_k$ is the sample standard deviation.

4.2 Prediction of extremes by optimal curve fitting to estimate ACER functions

Naess and Gaidai [10] argue that when one is using only sampled data as the basis for the extreme value estimation, the sub-asymptotic functional form of the ACER functions cannot easily be decided. However, by assuming that the extreme value distribution has an asymptotic tale similar to the Gumbel distribution they argue that it may be assumed that the behaviour in the tail is dominated by a function of the form $\exp[-a(\eta - b)c]$. Here $a$, $b$ and $c$ are suitable constants when the start value for $\eta$ is chosen as an appropriate tail marker, $\eta_0$, such that $\eta \geq \eta_0 \geq b$. Thus, for the estimation it may be assumed that the ACER functions in the tail are given by

$$\epsilon_i(\eta) = q_i(\eta) \exp\left\{-a_i \left(\eta - b_i\right)^c\right\}, \quad \eta \geq \eta_0 \quad (28)$$

where the function $q_i(\eta)$ is slowly varying compared with the exponential function and $a_i$, $b_i$ and $c_i$ are suitable constants that will be dependent on $k$. For practical applications it can be implemented in the following form

$$\epsilon_i(\eta) \approx q_i \exp\left\{-a_i \left(\eta - b_i\right)^c\right\}, \quad \eta \geq \eta_i \quad (29)$$

where $q_i$ is assumed constant and $\eta_i \geq \eta_0$. This function is then used to predict the extreme values. Due to the uncertainties in the empirical values of the ACER functions for high values of $\eta$ one should not include data points as the confidence band becomes large.

Optimal values of the four parameters may be obtained, as proposed by Naess et al. [20], by optimal fit carried out by minimizing the following mean square error function with respect to the four parameters in Eq. (29) at the log level. The objective function is then written as

$$F(a_i, b_i, c_i, q_i) = \sum_{i=1}^N w_i \left(\frac{\log \bar{\epsilon}_i(\eta_i) - \log q_i + a_i \left(\eta_i - b_i\right)^c}{\bar{s}_i}\right)^2$$

where $\bar{\epsilon}_i$ are the levels at which the ACER functions have been empirically estimated and $w_i$ denotes a weight factor that puts more emphasis on the more reliable data points. The choice of the weight factors is to some extent arbitrary. The following definition was suggested in [20]

$$w_i = \left[\frac{\log CI^-(\eta_i) - \log CI^+(\eta_i)}{\bar{s}_i}\right]^{\vartheta}$$

with $\vartheta = 1$ or 2, combined with a Levenberg-Marquardt least squares optimization method. This has proved to work well provided that a reasonable choice of the initial values for the parameters is made. Even though the choice of weight factor is to some extent arbitrary it may be deemed favourable to put stronger emphasis on the larger data, then the exponent $\vartheta$ should be set to 1. In this study $\vartheta = 1$ is adopted for the optimized fitting. Note that the definition adopted for $w_i$ puts some restriction on the use of the data. Usually, there is a level beyond which $w_i$ is no longer defined. Hence, the summation in the mean square error function given by Eq. (30) has to stop before that happens.

For a simple estimation of the 95\% confidence interval for the predicted values of $\epsilon(\eta)$ provided by the optimal curve, the empirical confidence band is re-anchored to the optimal curve. Further, the optimal curve fitting procedure is applied to the re-anchored confidence band boundaries. The fitted curves, extrapolated to the level of interest, will determine an optimized confidence interval of the estimated extreme value.

4.3 Numerical results

The average conditional exceedance rates of the twenty time series are displayed in Figure 10. The sampling interval used in the numerical integration of the equation of motion is $\Delta t = 0.1$ seconds, which implies that $k=100$ corresponds to a time period of 10 seconds.

![ACER plot](image-url)
When \( k=2 \) the ACER function corresponds to the definition of the mean up-crossing rate since it is required in Eq. (26) that one value is above and the previous value below the threshold \( \eta \). In order to use Eq.(24) to estimate the CDF of the extreme values it is crucial that the up-crossings are independent. This can be studied in Figure 10. For low threshold values the \( k \) values applied gives a slightly different ACER function which implies that the up-crossings are not independent. It is however the high threshold values that are of importance and as can be seen in Figure 10 all the ACER functions merge when \( \eta \) is larger than say 35 MN. This implies that there are low statistical dependencies in the tail region of the ACER function and that the Poisson assumption is fulfilled. The annual non-exceedance probability has to be selected in order to estimate the extreme value. Reasonable values are in the range 85-95% and we have used 85% in this work. Remembering that when \( k=2 \), the ACER function corresponds to the mean up-crossing rate the target value of the ACER function can be calculated using Eq.(24)

\[
\text{ACER}_2 = v'(\eta) - 0.1 \ln(0.85) / 10800 = 1.5 \times 10^{-6}
\]

The predicted extreme value and the confidence interval are also presented in Table 1

![Extreme Values](image)

In order to investigate the performance of the ACER method it is interesting to benchmark the results with the Gumbel method of episodical extremes. The maximum values of each time series are then assumed to be Gumbel distributed and plotted in the Gumbel probability plot displayed in Figure 12. In a Gumbel probability plot the observed extremes are plotted versus \(-\log(-\log(m/(N+1)))\) for \( m=1,...,20 \). The fitted straight line in the represents the fitted Gumbel distribution based on the moments estimation method. The cumulative Gumbel distribution is defined by

\[
\text{Prob}(M(T) \leq \eta) = \exp \left[ - \exp \left( \frac{\eta - \alpha}{\beta} \right) \right] \quad (33)
\]

The parameters \( \alpha \) and \( \beta \) are related to the mean value \( m_M \) and the standard deviation \( \sigma_M \) of \( M(T) \) as follows

\[
\alpha = m_M - 0.57722 \beta \quad \text{and} \quad \beta = \sigma_M / 1.28255
\]

The estimates of \( m_M \) and \( \sigma_M \) obtained from the available sample therefore provide estimates of \( \alpha \) and \( \beta \) which gives the fitted Gumbel distribution by the moments method.

The 85% percentile value of the axial force of the fitted Gumbel distribution is indicated by the black horizontal line in Figure 12 and is also given in Table 1. Since the Gumbel method does not provide confidence intervals these are estimated using the parametric bootstrapping method [9, 21]. Here it is assumed that the fitted Gumbel distribution in Figure 12 is the true distribution of the extreme values. Since the distribution is assumed known it is possible to draw several samples of size 20 from this distribution. The empirical PDF of 100,000 bootstrapped samples is displayed in Figure 13 together with the estimated 95% confidence intervals. The confidence intervals are also presented in Table 1.

![Gumbel Plot](image)

The ACER and the Gumbel method provided estimates of the extreme value of the axial force that is very similar. The confidence intervals are however a bit different. As can be seen in Table 1 the confidence intervals predicted using the
parametric bootstrapping technique is significantly wider than the one predicted using the ACER method. This implies that more reliable estimates can be obtained using the ACER method in addition to that it is possible to verify that the upcrossings are independent such that the poisson assumption is fulfilled.

Table 1. Predicted extreme values and confidence intervals

<table>
<thead>
<tr>
<th>Method</th>
<th>Extreme value (MN)</th>
<th>95% Confidence intervals (MN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gumbel</td>
<td>50.3</td>
<td>45.5…54.4</td>
</tr>
<tr>
<td>ACER</td>
<td>50.7</td>
<td>48.6…52.9</td>
</tr>
</tbody>
</table>

5 CONCLUDING REMARKS

The wave induced dynamic response of the Bergsøysund Bridge has been assessed in time domain in this paper. The hydrodynamic actions have been modelled using first order potential theory. A hydrodynamic finite element has been developed to model the motion induced forces. In this element the convolution integral are replaced by first order differential equations. The dependent variables in these differential equations are termed hydrodynamic degrees of freedom and are solved together with the displacement degrees of freedom of the FE model applying the well-known Newmark integration scheme. The results predicted using the hydrodynamic element have been compared to results obtained applying traditional power spectral density methods and the results corresponds to each other in an excellent manner.

The extreme values of the axial force in a crucial component of the bridge have been carefully studied applying the average conditional exceedance rates (ACER) method and the Gumbel method of epistodical extremes. Applying the ACER method it is possible to evaluate whether the upcrossings are independent and no assumption regarding the distribution of the extreme values needs to be made. The results predicted using both methods were very similar, but the ACER method provided a confidence interval that was significantly narrower. This implies that an estimate with less uncertainty was obtained using the ACER method for the simulated data in our case study.

ACKNOWLEDGMENTS

This research was carried out with financial support from the Norwegian Public Roads administrations. The authors greatly acknowledge this support. We would like to thank Sindre Hermstad for his work on the Abaqus model used. This has supported the following work significantly. Thanks are also given to Professor Bernt Leira, and Abdillah Suyuthi as well as Sindre Hermstad for their work on the linear potential theory calculations performed in WADAM.

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