Fully probabilistic optimization of reinforced concrete elements using heuristic algorithm and neural network

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Introduction

Many mathematical programming applications in civil engineering, e.g.

• structural design
• water management
• reconstruction of transportation networks
• reliability related computations
Structural Design

Two ways of structural designing

• partial reliability factor method
  • all uncertainties are replaced with their characteristic values
  • calculation is simple
  • obtaining input data is easy

• fully probabilistic design
  • complex time-consuming computation
  • lack of input data
Design Optimization

Two ways to optimize structural design

• DBSO (Deterministic-Based Structural Optimization)
  • based on partial reliability factor method
  • provides suboptimal solution
  • quick

• RBSO (Reliability-Based Structural Optimization)
  • based on probabilistic design
  • „better“ solution
  • complex and time-consuming
DBSO Definition – Objective Function

Objective is to minimize beam’s price

\[
\min(C_C V_C(x) + C_S W_S(x))
\]

\(C_C, C_S\) – cost of concrete per m\(^3\), cost of steel per kg

\(V_C, W_S\) – volume of concrete [m\(^3\)], weight of steel [kg]

\(x\) – vector of design variables
DBSO Definition – Design Variables

The work focuses on beams

- e.g. simply supported beam with this scheme and cross section

- here then \( x = (b_1, b_2, b_3, h_1, h_2, h_3, A_{s1}, A_{s3}) \)
DBSO Definition – ULS

First constraint is ULS – ultimate limit state

• beam has to bear the prescribed load $\Leftrightarrow$ strains in concrete and steel cannot exceed prescribed values

\[
\varepsilon \geq \varepsilon_{c,\text{min}} \quad \text{in cross section vertices}
\]

\[
\varepsilon \leq \varepsilon_{s,\text{max}} \quad \text{in reinforcing steel positions}
\]

$\varepsilon$ – actual strain value

$\varepsilon_{c,\text{min}}$ – minimal allowed strain of concrete

$\varepsilon_{s,\text{max}}$ – maximal allowed strain of steel
DBSO Definition – SLS

Second constraint is SLS – serviceability limit state

• more possibilities, here it is limitation of deflection

• beam’s deflection cannot exceed prescribed maximum value

\[ w \leq w_{\text{max}} \]

\( w \) – beam’s deflection
DBSO Definition – FEM

• Mathematical model is rather complex
• To be able to assess ULS and SLS it is necessary to proceed with matrix calculations

\[ Ku = F \]

\( K \) – stiffness matrix; depends on \( x \)
\( F \) – vector of load; depends on load
\( u \) – vector of node deflections; strain \( \varepsilon \) and deflection \( w \) depend on it
RBSO Definition

• same as DBSO, except
  • some variables become random variables (such as load, material characteristics of concrete and steel, etc.)
  • ULS and SLS are redefined as probabilistic constraints
  • ULS
    \[
    P\left( (\varepsilon(\text{vertices}) \geq \varepsilon_{c,\text{min}}) \land (\varepsilon(\text{steel}) \leq \varepsilon_{s,\text{max}}) \right) \geq p_{ULS}
    \]
  • SLS
    \[
    P(w \leq w_{max}) \geq p_{SLS}
    \]
Solution

• the probability \( p_{ULS} \) should reach the level from \( 1-10^{-4} \) to \( 1-10^{-6} \) (depending on chosen reliability class), which is obviously troublesome to achieve

• large number of scenarios makes the problem insolvable in a reasonable time horizon

• therefore a different approach is proposed
  • this is based on algorithm divided into inner deterministic optimization (based on Reduced Gradient Method) and outer stochastic optimization (based on Regression Analysis)
Example

• shown on the mentioned simply supported beam
Example

- normal force $N$ and distributed load $q$ are random variables with gamma distribution
Solution – 1. Initialization

• The calculation is initialized by choosing 12 initialization scenarios

\[ \xi^{\text{init}}_k = (q^{\text{init}}_k, N^{\text{init}}_k), \quad k = 1, ..., 12 \]

• For these scenarios, the deterministic optimization is performed, giving design variables’ values \( x^{\text{init}}_k \) and values of objective function \( f^{\text{init}}_k \)

• Probabilities \( p^{\text{init}}_{U_k}, p^{\text{init}}_{S_k} \) that configuration \( x^{\text{init}}_k \) satisfies the ULS and SLS, are assessed using Neural Network on Monte Carlo Simulation
Solution – 2. Iteration

• Regression analysis is applied on known scenarios to approximate behavior of objective function values and probabilities with regards to values of $q$ and $N$

• Next scenario $\xi^\text{iter}_l$ is selected based on these approximations (so that it satisfies set probabilities and has minimal objective function value)
Solution – 2. Iteration
Solution – 2. Iteration
Solution – 2. Iteration
Solution – 2. Iteration

• The deterministic optimization is performed for the new scenario $\xi^\text{iter}_l$, giving design variables’ values $x^\text{iter}_l$ and values of objective function $f^\text{iter}_l$

• Also, probabilities $p^\text{iter}_U l$, $p^\text{iter}_S l$ that configuration $x^\text{iter}_l$ satisfies the ULS and SLS, are assessed using Neural Network on Monte Carlo Simulation

• If $p^\text{iter}_U l \geq p^\text{ULS}$ and $p^\text{iter}_S l \geq p^\text{SLS}$
  • than $x^\text{iter}_l$ is solution, end
  • else, $\xi^\text{iter}_l$ is added to regression and solution continues with next iteration
Solution – 2. Iteration
Solution – 2. Iteration
Solution – 2. Iteration
Results

• Results
  • desired probabilities $p_{ULS} = 99.99277 \%$, $p_{SLS} = 93.31928 \%$
  • obtained probabilities $p_U = 99.9928 \%$, $p_S = 99.9324 \%$
  • solution found for scenario $q = 34.4103 \text{ kN/m}$, $N = 5.3090 \text{ kN}$
The End

Thank you for your attention