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Optimal and distributed energy management in interconnected energy hubs

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HIGHLIGHTS

- Developing a robust coordination model for Integrated Energy Hubs.
- Proposing a novel lambda-based iterative algorithm for more realistic applications.
- Addressing the transfer loss in the proposed model within a closed-loop approach.
- Elaborating robust scheduling as a single-level optimization framework.
- Forming the optimization model as a computationally efficient problem.

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ABSTRACT

Recently, multi-carrier energy systems (MCESs) have been rapidly developed as flexible multi-generation systems aiming to satisfy load demands by purchasing, converting, and storing different energy carriers. This study specifically focuses on the optimal and robust large-scale coordination of interconnected energy hubs (IEHs) in an iterative consensus-based procedure considering distribution network losses. Furthermore, a new robust-based hybrid IGDT/consensus algorithm is introduced to achieve risk-averse optimal energy management in IEHs under uncertainty. The fast convergence, needless to collect the total information from all hubs, minimal computational burden, and more robust communication system are the most important features of the proposed distributed consensus algorithm in this study. The effectiveness of the proposed consensus algorithm is verified by simulation results considering various energy trading structures in IEHs at different scales. The obtained results highlight the scalability capability of the proposed method. Regarding an IEHS of 30 energy hubs, the computation burden is lightened by 0.53 (s) and 0.1917 (s), respectively with and without uncertainty. Considering distribution network losses, the total purchasing costs can be increased by 8%. The simulation results also reveal an increase of 11% in the total power trading under the uncertainty.

1. Introduction

1.1. Motivation

Future electricity delivery system moves toward a reliable, flexible, and in a word, smarter grid. Looking forward, multi-carrier energy system (MCES) plays a key role in smart power systems. An energy hub (EH) is recognized as an interface framework for MCESs integrating multiple supply resources with various load demands. The ability of an EH to store, convert, and supply various energy demands plays a key role in improving flexibility [1]. By interconnecting MCESs, the concept of large-scale interconnected energy hub systems (LIEHSs) is formed in which a network of numerous energy hubs collaborates together. LIEHS opens a new critical pathway in future power systems for improving flexibility, especially with the rapid growth of RESs. The flexibility of MCESs in meeting multiple energy demands has a key role in optimal cooperative energy trading. Considering the strong energy (electricity-heating) coupling in a such large-scale coalition (LIEHS), the coordination of MCESs is more complicated. Concerning the wide integration of RES into a such complex large-scale coalition, the coordination can be quickly realized by adopting a fast distributed method.

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Nomenclature

Abbreviations

AC	Absorption Chiller
CHP	Combined Heat and Power
CM	Centralized Method
DM	Distributed Method
EC	Electrical Chiller
EDP	Economic Dispatch Problem
IEHs	Interconnected Energy Hubs
LIEHS	large-scale i=Interconnected Energy Hub Systems
IESs	Integrated Energy Systems
IGDT	Information Gap Decision Theory
EHs	Energy Hubs
GB	Gas Boiler
HE	Heat Exchanger
MCESs	Multi-Carrier Energy Systems
PV	Photovoltaic System
RESs	Renewable Energy Sources
ST	Solar Thermal
WT	Wind Turbine

Sets

e, h, g	Indices of electricity, heat, natural gas
i	Index of energy hub
k	Index of iteration number in the inner loop
t	Index of time
m	Index of RESs (e.g., WT, PV, and ST)
j	Index of iteration number in outer loop

Parameters

$\eta_{trans,i}$	Efficiency of transformer i
$P_{grid,t}^{max}$	Capacity of power grid (MW)
$l_{i,t}^{e/h/c}$	Electrical/heating/cooling demand (MW)
$\eta_{h,CHP,i}$	Heating efficiency of i-th CHP
$\eta_{e,CHP,i}$	Electrical efficiency of i-th CHP
$C_{EC,i}$	Efficiency of i-th EC
$C_{AC,i}$	Efficiency of i-th AC
$\eta_{GB,i}$	Efficiency of i-th GB
$\eta_{HE,i}$	Efficiency of i-th HE
$P_{CHP/GB,t}^{max}$	Capacity of CHP/GB (MW)
p_{ij}	Element (i, j) of matrix P
$\overline{D}_{e/m,i,t}$	Predicted electrical demand/RES's power generation (MW)

Variables

$P_{i,t}^{e}$	Purchased electricity (MW)
$P_{i,t}^{gas}$	Purchased natural gas (MW)
$P_{i,t}^{g,CHP/GB}$	Consumed natural gas by CHP/GB (MW)
$P_{i,EC/AC,t}^{e}$	Consumed electricity/heating by i-th EC/AC (MW)
$C_{E,i,t}$	Purchase cost of electrical power ($)
$C_{CHP/GB,i,t}$	Purchase cost of natural gas consumed by CHP/ GB ($)
$\lambda_{e/h,i,t}$	Electrical/heating increasing costs
$\Delta P_{i,t}$	Total supply-demand imbalance (MW)
$\Delta P_{e/h,i,t}$	Electrical power/heating imbalance (MW)
$\Delta D_{e/m,i,t}$	Uncertainty in electrical demand/RES's power generation (MW)
$D_{e/m,i,t}^{act}$	Actual electrical demand/RES's power generation (MW)

1.2. Literature review

Recently published studies have especially focused on optimal and cooperative energy trading in an interconnected energy hubs coalition. For example, a centralized optimization method was proposed in [2] to model transactive energy in an IEHS. Due to the non-linear generator power costs and non-linear NG network constraints, as well as considering the dispatch factor in the defined coupling matrix, the optimization problem is mainly non-convex, highly constrained, and multi-period in the presence of energy storage systems. In [3], the participation of the IEHs in DA energy and reserve markets has been modeled based on coordinated energy management. In [4], the economic scheduling of the electrical and thermal networks with flexi-renewable EHs is developed. In [5], a non-linear programming problem has been reformulated as mixed-integer linear programming using a piece-wise linear approximation. All these papers build the deterministic cooperative scheduling models. Considering the uncertainty, a scenario-driven stochastic scheduling model has been proposed in [6] for multi-energy hub systems minimizing the operation cost and total emissions of the system. A full AC non-linear power flow was used to accurately model the loss of distribution network. To handle the price uncertainty, an optimal robust-based load dispatch model has been presented in [7] for a community energy hub. A distributionally robust dispatching model has been established in [8] based on the energy sharing and profit allocation for a community energy hub. Relying on the stochastic-based uncertainty modeling, a linear coordinated power management scheme has been established in [9] considering uncertainties in loads, power prices, and RES generation power. A bi-level multi-objective model is developed in [10] for renewable networked microgrids. Wind uncertainty has been introduced in [11] for optimal scheduling of IEHSs via centralized stochastic optimization. Similarly, Ref. [12] presented a centralized stochastic framework for peer-to-peer energy trading in IEHSs. In [12], the authors have included both normal and resilient operation uncertainties.

In the present centralized optimization schemes in all the above-mentioned studies, information is exchanged between the central coordination unit and all participating hubs. Therefore, the significant computational burden compromises the centralized method's application in a large-scale IEHS. Alternatively, distributed optimization methods can be developed as a proper substitute for the central scheme. In this method, the optimal operation of EHs is individually solved in an isolated manner and, followed exchanged across adjacent hubs. Distributed optimization algorithms have been extensively used for multiple energy trading between prosumers in the context of P2P trading and local energy markets.

In this regard, the consensus algorithm is widely used owing to the fast convergence and global optimal solution. In consensus optimization, each agent has access to its local objective function and decision variables but lacks complete knowledge of the global problem. Extensive studies have investigated the state-of-the-art consensus algorithm to optimally schedule IEHs. There have been some attractive efforts in [13] to the optimal and deterministic operation of an IEHS using a distributed consensus algorithm. In [14], a consensus-based distributed algorithm is proposed for the optimal control of m-EHs. In [15], a new consensus-based decentralized transaction-based energy management is developed for an IEH system. [16] concerned with optimal energy management in an IEH System solved by an ADMM consensus-based algorithm. A fully distributed consensus-based ADMM approach is also established in [17] for fully distributed optimal cooperative scheduling in multi-energy hubs. Based on various types of energy hubs, [18] introduced a

Table 1
Taxonomy table covering some features for related references.

Ref.	CM	DM	Loss	Uncertainty	Stochastic optimization	robust optimization	Distributed IGTD-based method	Application Domain
[2]	✓	×	×	×	×	×	×	IEHs
[3]	✓	×	×	×	×	×	×	IEHs
[4]	✓	×	×	✓	✓	×	×	IEHs
[5]	✓	×	×	×	×	×	×	IEHs
[6]	✓	×	✓	✓	✓	×	×	IEHs
[7]	✓	×	×	✓	×	✓	×	IEHs
[8]	✓	×	×	✓	×	✓	×	IEHs
[9]	✓	×	✓	✓	✓	×	×	IEHs
[11]	✓	×	×	✓	✓	×	×	IEHs
[12]	✓	×	×	✓	✓	×	×	IEHs
[13]	×	✓	×	×	×	×	×	IEHs
[14]	×	✓	×	×	×	×	×	IEHs
[15]	×	✓	×	×	×	×	×	IEHs
[16]	×	✓	×	×	×	×	×	IEHs
[17]	×	✓	×	×	×	×	×	IEHs
[18]	×	✓	×	×	×	×	×	IEHs
[19]	×	✓	×	✓	✓	×	×	IEHs
[20]	×	✓	×	✓	✓	×	×	IEHs
[21]	×	✓	×	✓	✓	×	×	IEHs
[22]	×	✓	×	✓	✓	×	×	IEHs
[23]	×	✓	×	✓	✓	×	×	IEHs
[24]	×	✓	×	✓	✓	×	×	IEHs
[25]	×	✓	×	✓	×	✓	×	IEHs
[26]	×	✓	×	✓	×	✓	×	IEHs
[27]	×	✓	×	✓	×	✓	×	IEHs
[28]	×	✓	×	✓	×	✓	×	IEHs
Proposed framework	×	✓	✓	✓	×	×	✓	IEHs

fully distributed energy trading mechanism using an ADMM-based OPF model. In the previous papers [13–18], the inherent uncertainty of volatile renewable energy sources (RESs) and forecasted electrical demand were not considered.

To fulfill this research gap, appreciable efforts have specifically focused on stochastic and robust scheduling models of IEHs considering uncertainty while benefiting from the advantages of the distributed methods.

For example, the authors in [19] investigated a two-stage stochastic decentralized scheduling strategy for multi-region integrated energy systems (IESs) considering the uncertainty of RESs. Under the uncertainty, an advanced optimal dispatch model is proposed in [20] for multi-agent energy hubs relying on the distributed consensus algorithm. Moreover, the authors in [21] developed a scenario-driven stochastic optimization to model peer-to-peer multi-energy transactions in multi-energy hubs. An optimal stochastic power trading model is proposed in [22]. In that paper, a Bayesian game is used to solve the proposed model under incomplete information. A decentralized bi-level stochastic model is also developed in [23] for optimal operation of networked multi-energy microgrids. A bi-level stochastic optimization model has been proposed in [23] for the decentralized control framework in multi-agent microgrids. In stochastic optimization, several different uncertainty scenarios are defined based on historical information and data probability statistics.

Alternatively, a distributed robust optimization problem is also used in recent articles. A distributed robust synergies scheduling is proposed in [24] for multi-region IESs with uncertainties of wind turbine (WT) and electricity load. In [24], the provided max-min robust operation model is decentralized into multiple independent sub-problems using a consensus-based distributed ADMM. The authors in [25] proposed a fully distributed robust scheduling model for multi-area IESs. A distributed consensus-based ADMM cooperative scheduling model is established in [26] for IESs. In that study, a two-stage robust optimization is developed to handle the uncertainties. Pinpointing the distributed peer-to-peer heat and power trading, a two-stage robust optimization is developed in [27] considering different uncertainty budgets. A distributed robust operational optimization has been modeled in [28] to coordinate entities in different robustness levels. The previous investigations usually consider a max-min robust optimization to find the optimal feasible solution under the worst conditions. While the proposed algorithms have shown their superiority in convergence performance, the present algorithms can get slow for the complex large-scale IES.

1.3. Research gaps

The relevant research gaps can be divided into two categories. The first thread considers that the robust optimization should be applied to LIEHS. In reality, an IES is a network of numerous energy hubs that collaborate together. Considering the strong electricity-heating coupling, the cooperative scheduling is more complicated. In this regard, Information Gap Decision Theory (IGDT) can be regarded as a promising optimization approach with a slight computational burden in many coordination problems, especially for power systems such as economic dispatch, P2P energy trading, and real-time power dispatch. Nevertheless, in the proposed distributed models for the peer-to-peer multi-energy transactions in IEHs, IGDT models are not well addressed yet. To address this issue, an IGDT-based distributed robust method is developed in this paper to handle the uncertainty. One of the claimed advantages of the proposed method is its scalability for large-scale systems.

Secondly, the influence of energy loss is ignored in the above papers focused on the distributed optimization algorithms. The electricity and heating energy losses, which are approximately 10% and 5% in the distribution power system, significantly impact the optimal power trading. In relatively large-scale IESs, considering distribution network losses is deemed inevitable. It is worth noting that in [6,9], the energy loss of distribution network is integrated into the present model by nonlinear AC-OPF constraints. Nevertheless, the present scenario-driven stochastic model can't be implemented for a LIEHS since it has been

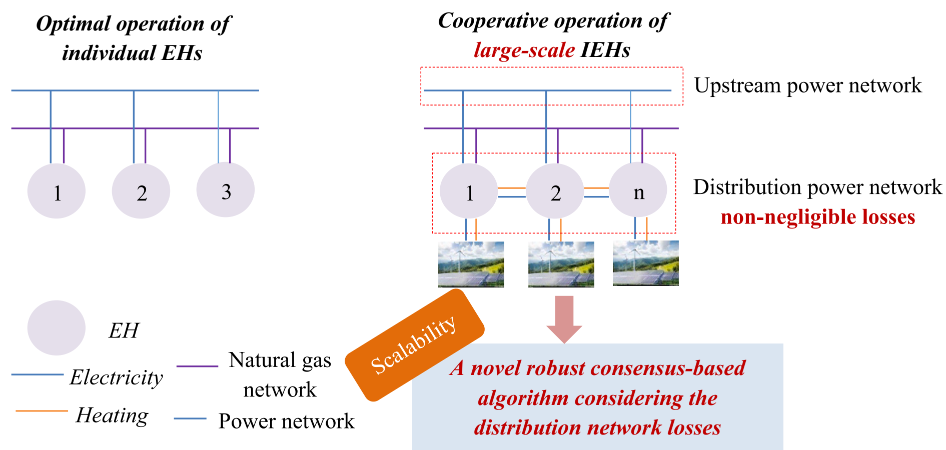

Fig. 1. Schematic of proposed framework

solved by a centralized method. Moreover, considering the nonlinear quadratic model of the energy loss makes the max-min robust optimization complex and non-convex needed to be reformulated using the convexification methods. To address this issue, a novel tractable iteratively distributed consensus-based algorithm is formulated in this paper. A useful taxonomy table covering some features of these references has been provided in Table 1. In order to discern how the contributions of this paper distinguish it from prior research, the reviewed works have been grouped in 4 different colors (blue, red, and purple) based on their similarities. Also, the last row indicates the features of this paper against existing ones. The proposed method is applicable in large-scale systems due to its scalability.

1.4. Contributions

In extension to previous research, this paper proposes the modeling and optimal operation of a cooperative LIEHS while taking distribution line losses into account. To achieve this goal, a novel heuristic iterative consensus-based algorithm is developed. Herein, a new robust consensus-based framework is also investigated to deal with uncertainty. Specifically, the main contributions of this study are summarized as follows:

- Previous appreciable efforts especially focused on distributed energy management of a small-scale IEHS using the consensus algorithm. While the power losses were overlooked in the related mentioned papers. Inspired by these studies, this paper proposes a new realistic lambda-based iterative algorithm for the optimal operation of a cooperative LIEHS considering losses. Due to the relatively large scale (MW) of the energy hubs, considering distribution network losses is deemed necessary. The proposed framework is executed in two different loops, the inner and outer loops. In the inner loop, the distributed consensus algorithm is performed for optimal P2P energy trading. In the outer loop, the loss factor is specially calculated based on the latest energy trading estimated by the inner loop, and the electricity demand is finally updated.
- The subject of the bi-level robust optimal energy management of IEH systems has attracted attention in previously published articles. To handle the uncertainty, this paper aims to focus on the distributed information gap decision theory (IGDT) optimization method for the risk-averse optimal scheduling of IEHs. The present algorithm can provide a robust scheduling decision for IEHS minimizing the total cost while maximizing the uncertainty. The proposed model is solved via a distributed consensus-based algorithm in a way that not only global optimal solution can be reached, but also the volume of computations can be lightened. In this way, the proposed consensus algorithm is expanded to a robust IGDT-based algorithm intended to assess the negative effects of uncertainty on optimal energy management. It is worth noting that the IGDT-based proposed method is generally known as bi-level. In this paper, the IGDT-based proposed model is reformulated as a single-level optimization. In the proposed scheduling problem, increasing the uncertainty has a negative impact on the scheduling cost. For example, if the uncertainty drops, the scheduling cost will decrease as well or vice versa if the uncertainty increases, the cost will certainly increase. In other words, the maximum cost of uncertainty is equivalent to the maximum uncertainty. Thus, the proposed IGDT model is broken into a single-level multi-objective problem. Due to its distributed implementation (consensus algorithm) and scalability (IGDT), the proposed method can be effectively utilized in a cooperative large-scale IES.

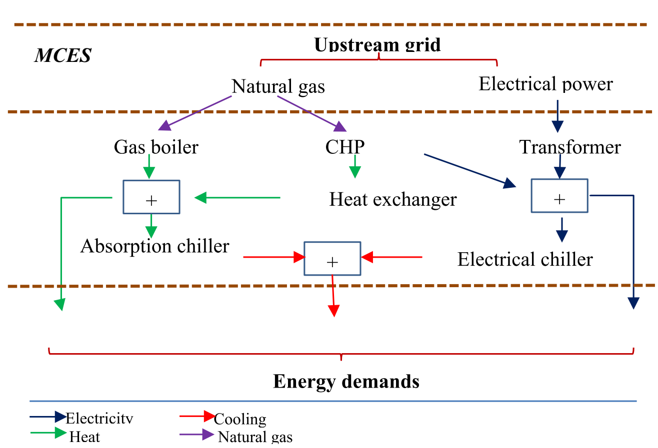

Fig. 2. The investigated MCES and its components

1.5. Paper organization

The rest of this paper is as follows: In Section 2, the mathematical model of an individual MCES is expressed. In Section 3, the optimal energy management strategy for an IEH system is formulated. The proposed iterative consensus-based framework, along with the mathematical formulation of the presented algorithm is discussed in Section 4. Considering the uncertainty, the proposed robust hybrid IGDT/consensus algorithm is also developed in Section 5. To assess the proposed method's effectiveness, case studies are simulated in Section 6. Finally, the conclusion is presented in Section 7.

2. General proposed framework

This paper aims to investigate a new robust-based distributed strategy for cooperative energy management of a large-scale IEH as indicated in Fig. 1. It is assumed that IES is a network of numerous energy hubs at the distribution level that collaborate together $(1,\ldots,n)$. Hence, the distribution network loss is also taken into consideration in the proposed framework. To deal with the loss, an iterative distributed consensus-based algorithm is introduced in Section 4. Load variations and uncertainties of RESs are also integrated into the presented model. The proposed robust-based model is discussed under uncertainty via a hybrid IGDT/consensus algorithm as detailed in Section 5. To describe the developed distributed energy management strategy in Section 3, it is necessary to provide preliminaries about MCESS modeling and their individual operation in the following section.

2.1. MCES modelling

As a foundational backbone of cooperative scheduling for IEHs, energy flow modeling and optimal operation analysis of an individual MCES are mathematically formulated in this Section. Fig. 2 describes the hierarchy energy hub schematic consisting of a transformer, combined heat and power (CHP) unit, gas boiler (GB), heat exchanger (HE), electrical chiller (EC), and absorption chiller (AC) to reliably deliver energy through coordination of optimal energy dispatch across multiple energy vectors. CHP is strongly used in MCESs to simultaneously produce electricity and heating energy. The interdependent relationship between $P_{i,t}^{e,CHP}$, $P_{i,t}^{h,CHP}$ and $P_{i,t}^{g,CHP}$ are respectively calculated in (1) and (2). A heat exchanger (HE) is also utilized for changing the temperature of generated heating by CHP based on (4).

$$P_{i,t}^{e,CHP} = P_{i,t}^{g,CHP} \times \eta_{e,CHP,i}; \forall i,t \tag{1}$$

$$P_{i,t}^{h,CHP} = P_{i,t}^{g,CHP} \times \eta_{h,CHP,i}; \forall i,t \tag{2}$$

$$P_{i,t}^{e,CHP} \le P_{CHP,t}^{max}; \forall i,t \tag{3}$$

$$P_{i,t}^{HE} = P_{i,t}^{h,CHP} \times \eta_{HE,i}; \forall i,t \tag{4}$$

Unlike CHP, GB is only able to produce heating energy according to (5). Similarly, the provided heating energy by GB should not exceed its capacity $\left(P_{GB,t}^{max}\right)$, according to (6).

$$P_{i,t}^{h,GB} = P_{i,t}^{g,GB} \times \eta_{GB}; \forall i,t \tag{5}$$

$$P_{i,t}^{h,GB} \le P_{GB,t}^{max}; \forall i,t \tag{6}$$

The required fuel for CHP and GB operating is natural gas $\left(P_{i,t}^{g,CHP}, P_{i,t}^{g,GB}\right)$. To balance energy production and consumption, natural gas $\left(P_{i,t}^{gas}\right)$ is necessarily purchased from the external energy grid according to (7). The maximum amount of natural gas that can be purchased is limited to the $P_{gas,t}^{max}$ in (8).

$$P_{i,t}^{gas} = P_{i,t}^{g,CHP} + P_{i,t}^{g,GB}; \forall i,t \tag{7}$$

$$P_{i,t}^{gas} \le P_{gas,t}^{max}; \forall i,t \tag{8}$$

In addition, electrical and absorption chillers are suitable options for converting energy (e.g., electricity and heating) to cooling. This conversion process is mathematically modeled as (9) and (10). Where $P_{i,EC,t}^{c}$ and $P_{i,AC,t}^{c}$ are limited by their predesigned capacity ($P_{EC,t}^{max}$ and $P_{AC,t}^{max}$) based on (11) and (12).

$$P_{i,EC,t}^{c} = C_{EC,i} \times P_{i,EC,t}^{e}; \forall i,t^{*} \tag{9}$$

$$P_{i,AC,t}^{c} = C_{AC,i} \times P_{i,AC,t}^{h}; \forall i,t \tag{10}$$

$$P_{i,EC,t}^{c} \le P_{i,EC,t}^{max} \ ; \forall i,t \tag{11}$$

$$P_{i,AC,t}^{c} \le P_{AC,t}^{max}; \forall i,t \tag{12}$$

Focusing on the energy component modeling presented above, the electrical, heating, and cooling balances are finally stated as (13)–(15).

$$P_{i,t}^{grid} + P_{i,t}^{e,CHP} = l_{i,t}^{e} + P_{i,EC,t}^{e}; \forall i,t \tag{13}$$

$$P_{i,t}^{HE} + P_{i,t}^{h,GB} = l_{i,t}^{h} + P_{i,AC,t}^{h}; \forall i,t \tag{14}$$

$$P_{i,EC,t}^{c} + P_{i,AC,t}^{c} = l_{i,t}^{c}; \forall i,t \tag{15}$$

Assuming the grid-connected MCES, the imported electrical power $\left(P_{i,t}^{grid}\right)$ can be calculated as (16). Additionally, (17) is used to restrict the input purchasing power according to the capacity of UG.

$$P_{i,t}^{grid} = P_{i,t}^{e} \times \eta_{trans,i}; \forall i,t \tag{16}$$

$$P_{i,t}^{grid} \le P_{grid,t}^{max}; \forall i,t \tag{17}$$

3. Cooperative optimal energy management for IEHs

Different from the individual MCES scheduling stated above, cooperative energy hubs can constitute an interdependent community minimizing the total cost via enabling their surplus energy trading. To do so, this paper especially focuses on modeling a system of interconnected multi-carrier systems in which both electrical and heating energy exchanges have been considered. The optimal energy management in this cooperative community can be modeled based on a constrained optimization formulation as follows:

$$min\ OF_t = \sum_{t=1}^{T=24} \sum_{i=1}^{N} \left[C_{E,i,t} + C_{CHP,i,t} + C_{GB,i,t}\right] \tag{18}$$

Subject to (1)–(14), (19)–(21)

$$C_{E,i,t} = \alpha_i^e \left(P_{i,t}^{e}\right)^2 + \beta_i^e P_{i,t}^{e} + \gamma_i^e; \forall i,t \tag{19}$$

$$C_{CHP,i,t} = \alpha_i^{CHP} \left(P_{i,t}^{g,CHP}\right)^2 + \beta_i^{CHP} P_{i,t}^{g,CHP} + \gamma_i^{CHP}; \forall i,t \tag{20}$$

$$C_{GB,i,t} = \alpha_i^{GB} \left(P_{i,t}^{g,GB}\right)^2 + \beta_i^{GB} P_{i,t}^{g,GB} + \gamma_i^{GB}; \forall i,t \tag{21}$$

$$\sum_{i=1}^{N} \left[P_{i,t}^{grid} + P_{i,t}^{e,CHP} - P_{i,EC,t}^{e}\right] = \sum_{i=1}^{N} l_{i,t}^{e} + \sum_{j=1}^{n} P_{t,i}^{loss}; \forall t \tag{22}$$

$$\sum_{i=1}^{N} \left[P_{i,t}^{HE} + P_{i,t}^{h,GB} - P_{i,AC,t}^{h}\right] = \sum_{i=1}^{N} l_{i,t}^{h}; \forall t \tag{23}$$

$$P_{t,i}^{loss} = L_i \times \left(P_{i,t}^{exc}\right)^2; \forall i,t \tag{24}$$

This cooperative model aims to minimize the operation cost of all participants under the constraints of electrical and heating energy balances. Where the purchasing costs of input energy carriers ($P_{i,t}^{e}$, $P_{i,t}^{g,CHP}$, and $P_{i,t}^{g,GB}$) is quadratically defined for *i*-th energy hub according to (19)–(21). As each EH should be able to meet its demands via its own local energy supplies, (1)–(14) is also considered. This optimization problem is subject to the electrical and heating balance constraints as (22)–(23). Toward innovation, the power losses $\left(P_{t,i}^{loss}\right)$ is also included in (22). $P_{t,i}^{loss}$ can be calculated according to (24) where $P_{i,t}^{exc}$ is the exchanged

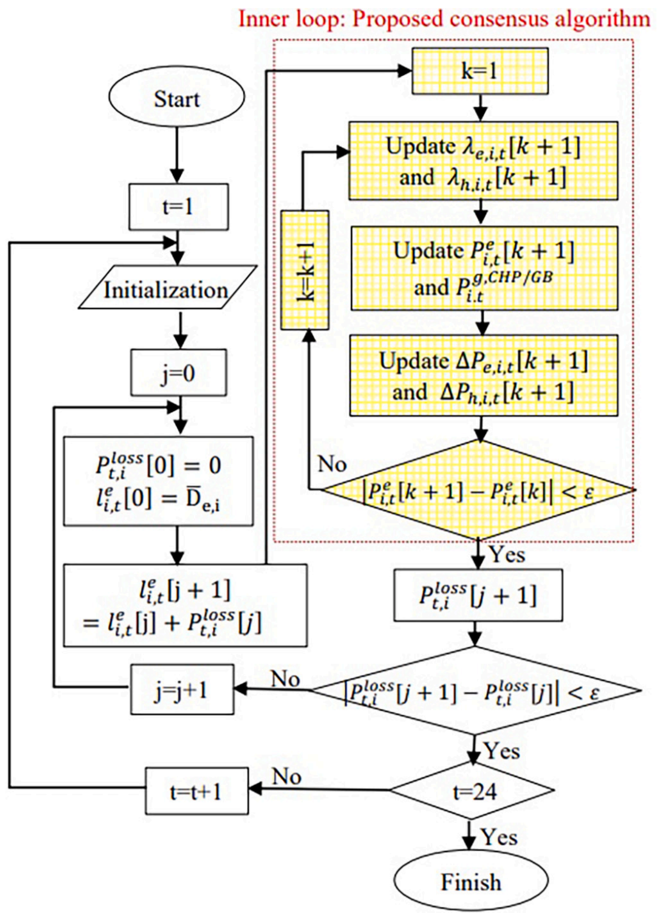

Fig. 3. Flowchart of the proposed iterative consensus-based framework considering the distribution network losses.

Table 2
Comparison of distributed method versus centralized approach.

Centralized	Distributed
Supervision of entire energy hubs by the central coordination unit	Supervision of each energy hub by its respective agent
Heavy calculation burden	Slight calculation burden
Achieve higher robustness	Achieve lower robustness
Application in a small-scale system in size	Application in a large-scale system in size

electrical power by the i-th energy hub [29]. In this work, the presented optimal operation model of IEHS is effectively solved through the proposed iterative framework considering loss. In this iterative framework, a distributed consensus algorithm is utilized in the inner loop.

4. Proposed iterative consensus-based heuristic framework considering energy loss

To solve the optimization problem in a cooperative mode considering the distribution network losses, a novel iterative consensus-based heuristic algorithm is proposed. The flowchart of the proposed iterative-based algorithm is indicated in Fig. 3.

The proposed consensus-based framework is executed in two different loops. In the inner loop, the distributed consensus algorithm is performed to obtain the optimal energy trading. It is worth noting that the loss factor is assumed to be fixed in the inner loop. In the outer loop, the loss is especially updated based on the latest energy trading estimated by the inner loop. In the following, the details of the inner and outer loops are briefly described.

4.1. Inner loop: distributed consensus algorithm

In the presented framework in Fig. 3, a novel distributed consensus algorithm is implemented in the inner loop for modeling optimal electrical and heating energy trading. Table 2 describes distinctive characteristics and differences between these two proposed methods. The distributed consensus algorithm is more robust and reliable to the single point of failure and the frequent changes in the power system topology since there is no need for the central coordination unit for the supervision of the entire IEHS. In this paper, the distributed consensus algorithm is used in the inner loop to solve the proposed optimization problem.

Herein, the consensus algorithm computes iteratively the optimal point using local communications (between agents and their adjacent on the network), needless to a supervision coordination unit. In this method, each agent has access to its local objective function and decision variables but lacks complete knowledge of the global problem. In this regard, all agents (energy hubs) coordinately work together to realize a shared consensus. The distributed consensus algorithm consists of three different steps as follows:

- Initially, the value of consensus variables ($\lambda_{e,i,t}$ and $\lambda_{h,i,t}$) is calculated in any iteration based on consensus equations in (25) and (26) after the initialization. It is worst-nothing that to achieve the optimal value, $\lambda_{e,i,t}$ and $\lambda_{h,i,t}$ are only communicated among the interconnected EHs after each iteration. Where, (25) and (26) reflect the distributed aspect of the proposed method.

$$\lambda_{e,i,t}[k+1] = \sum_{j=1}^{n} p_{ij}\lambda_{e,j,t}[k] + \varepsilon \Delta P_{i,t}[k]; \forall i,t \tag{25}$$

$$\lambda_{h,i,t}[k+1] = \sum_{j=1}^{n} p_{ij}\lambda_{h,j,t}[k] + \varepsilon \Delta P_{i,t}[k]; \forall i,t \tag{26}$$

Matrix P and Q indicate the network topologies in an IEH system which are row-stochastic and column-stochastic matrices, respectively. In the row-stochastic matrix, the sum of row entries is equal to 1. If m-th EH is connected to j-th EH, $P_{mj} > 0$ and $Q_{mj} > 0$, otherwise $P_{mj} = Q_{mj} = 0$. The weights of the entries in P and Q can be determined as $1/d_m^{+}$ and $1/d_m^{-}$, respectively.

- When the consensus variables are determined, the main decision variables (i.e.,$P_{i,t}^{e}$, $P_{i,t}^{e,CHP}$, $P_{i,t}^{g,GB}$) are individually calculated for each energy hub subsequently using (27)–(29). Where, $\Delta P_{i,t}[k]$ is calculated by the sum of the electrical and heating energy imbalances in i-th energy hub. Finally, the energy mismatch $\left(\Delta P_{i,t}[k]\right)$ is updated in k-th iteration by using (30) and (32).

$$P_{i,t}^{e}[k+1] = \frac{\eta_{trans,i}\,\lambda_{1,i,t}[k+1] - \beta_i^{e}}{2\alpha_i^{e}}; \forall i,t \tag{27}$$

$$P_{i,t}^{g,GB}[k+1] = \frac{\eta_{GB,i}\,\lambda_{2,i,t}[k+1] - \beta_i^{GB}}{2\alpha_i^{GB}}; \forall i,t \tag{28}$$

$$P_{i,t}^{g,CHP}[k+1] = \frac{\lambda_{1,i,t}[k+1] \times \eta_{e,CHP,i} + \lambda_{2,i,t}[k+1] \times \eta_{h,CHP,i} \times \eta_{HE,i} - \beta_i^{CHP}}{2\alpha_i^{CHP}}; \forall i,t \tag{29}$$

$$\Delta P_{i,t}[k] = \Delta P_{e,i,t}[k] + \Delta P_{h,i,t}[k]; \forall i,t \tag{30}$$

$$\Delta P_{e,i,t} = P_{i,t}^{grid} + P_{i,t}^{e,CHP} - P_{i,EC,t}^{e} - l_{i,t}^{e} - P_{t,i}^{loss}; \forall i,t \tag{31}$$

$$\Delta P_{h,i,t} = P_{i,t}^{HE} + P_{i,t}^{h,GB} - P_{i,AC,t}^{h} - l_{i,t}^{h}; \forall i,t \tag{32}$$

- Until $\left|P^{e}_{i,t}[k+1]-P^{e}_{i,t}[k]\right|$ is non-zero, this process (aforesaid steps) will be repeated. Herein, k is the discrete index of iteration in the proposed consensus algorithm in the inner loop. This condition is satisfied if the energy mismatch is within the acceptable range.

Remark 1. In the proposed iterative flowchart in Fig. 3, the distributed consensus algorithm is implemented in any iteration to determine the optimal energy trading. To guarantee the convergence of the presented consensus method, the convergence study has been presented in the appendix.

Remark 2. In this subsection, the distributed consensus algorithm is executed in the inner loop while ignoring the uncertainty. The robust consensus formulation ((25)–(32)) will be developed In Section 5 considering the uncertainty.

4.2. Outer loop: losses updating

The outer loop consists of three different steps:

- Step 1: Optimal energy management in IEHs is conducted without considering the fixed line losses. In this paper, it is assumed that $P^{loss}_{t,i}[1]=0$. Therefore, the electrical load demand is $l^{e}_{i,t}$. In this step, the inner loop is executed using the presented consensus algorithm and finally, the optimal electrical and heating energy trading $\left(P^{exc}_{i,t}\right)$ is obtained.
- Step 2: Distribution network loss $\left(P^{loss}_{t,i}[j] \text{ for } j=1,2,\dots,n\right)$ is updated based on the optimal energy trading $\left(P^{exc}_{i,t}\right)$ and accordingly $P^{loss}_{t,i}$ is acquired according to (24). Updating the electrical load ($l^{e}_{i,t}[j+1]=l^{e}_{i,t}[j]+P^{loss}_{t,i}[j+1]$) is finally conducted in this step.
- Step 3: Steps 2 and 3 will be regularly repeated until $P^{loss}_{t,i}$converge to the acceptable value.

5. Proposed robust hybrid IGDT/consensus algorithm

Variations of renewable energy resources (RESs), such as wind turbine, photovoltaic, and solar thermal, and the uncertainty of electrical demand, are the main challenges in the optimal operation of IEHs. This subsection seeks to investigate the optimal and robust operation of IEHs using combined IGDT optimization and consensus algorithm.

Inspired by information gap decision theory (IGDT), uncertainty can be defined as an error between the real-time and forecasted uncertain parameters. In this paper, the generation power of WT, PV, and ST as well as electrical demand are assumed to be uncertain. The uncertainty set can be formulated as (33) and (34). Where α_m and $\alpha_D$are the uncertainty horizon parameters.

$\overline{P}_{m,i,t}$ and $P^{act}_{m,i,t}$ are the forecasted and actual generated electricity and heating energy by RESs, respectively. Moreover, $\overline{D}_{e,i,t}$ and $D^{act}_{e,i,t}$are respectively the forecasted and actual electrical demand. In the proposed robust scheduling model, the operator pursues a risk-averse strategy to guarantee the energy hub's robustness against the maximum uncertainty set. Using this Risk-averse strategy, the maximum uncertainty $[\widehat{\alpha}_m,\widehat{\alpha}_D]$ can be calculated as (37) considering the constrained cost. In (37), the maximum cost resulting from unfavorable deviations from the forecasted values should be restricted to the predefined upper-cost limit ($Z^{\text{limit\_}Cost}_{w}$). The higher $Z^{\text{limit\_}Cost}_{w}$ is, the maximum uncertainty will be obtained.

$$\forall\alpha_m\,\epsilon\,U\left(\overline{P}_{m,i,t},\alpha_m\right)=\left\{P^{act}_{m,i,t}:\left|\frac{P^{act}_{m,i,t}-\overline{P}_{m,i,t}}{\overline{P}_{m,i,t}}\right|\le\alpha_m\right\};\forall m\in\{WT;PV;ST\}\tag{33}$$

$$\forall\alpha_D\,\epsilon\,U\left(\overline{D}_{e,i,t},\alpha_D\right)=\left\{D^{act}_{e,i,t}:\left|\frac{D^{act}_{e,i,t}-\overline{D}_{e,i,t}}{\overline{D}_{e,i,t}}\right|\le\alpha_D\right\};\forall m\in\{WT;PV;ST\}\tag{34}$$

$$(1-\alpha_m)\overline{P}_{m,i,t}\le P^{act}_{m,i,t}\le(1+\alpha_m)\overline{P}_{m,i,t};\forall m\in\{WT;PV;ST\}\tag{35}$$

$$(1-\alpha_D)\overline{D}_{e,i,t}\le D^{act}_{e,i,t}\le(1+\alpha_D)\overline{D}_{e,i,t}\tag{36}$$

$$[\widehat{\alpha}_m,\widehat{\alpha}_D]=max\left\{\alpha_m,\alpha_D:\left(maxOF_t\le Z^{\text{limit\_}Cost}_{w}\right)\right\};\forall m\in\{WT;PV;ST\}\tag{37}$$

Bi-level-based IGDT is one of the main solutions for solving the provided optimization problem in (37). In the proposed scheduling problem, increasing the uncertainty has a negative impact on the scheduling cost. For example, if the uncertainty drops, the scheduling cost will decrease as well or vice versa if the uncertainty increases, the cost will certainly increase. In other words, the maximum cost of uncertainty is equivalent to the maximum uncertainty. Thus, the proposed IGDT model is broken into a single-level multi-objective problem. This alternative solution is also proposed in (38)–(43) in this paper. The proposed method aims to maximize the uncertainty and minimize the total operation cost of IES. Herein, the weight coefficient $w_{i,t}$ (uncertainty budget) can be calculated as (39) reflecting the $Z^{\text{limit\_}Cost}_{w}$ limitation under uncertainty. Suppose that Z^{Cost}_{w} is the total deterministic operation cost without uncertainty. With $w_{i,t}=0.1$, the uncertainty will be strongly maximized with a higher freedom degree with no cost consideration.

The worst-case scenario (maximum increase in electrical demand and maximum decrease in the output produced power of RESs) is considered in this paper. Aiming to maximize the uncertainty and minimize the total cost of IEHs, the proposed robust optimization framework is formulated as (38)–(43). Where the actual output power of RESs $\left(P^{act}_{m,i,t}\right)$ and actual electrical demand $\left(D^{act}_{e,i}\right)$ are determined using (40)–(41).

$$minOF^{robust}_{t}=\sum_{i=1}^{N}w_{i,t}\left[C_{E,i,t}+C_{CHP,i,t}+C_{GB,i,t}\right]-\left(1-w_{i,t}\right)\left[\Delta D^{2}_{e,i,t}+\Delta P^{2}_{WT,i,t}+\Delta P^{2}_{PV,i,t}+\Delta P^{2}_{ST,i,t}\right]\tag{38}$$

$$s.t.(1)-(14)\ and\ (24)$$

$$w_{i,t}=\frac{Z^{Cost}_{w}}{Z^{\text{limit\_}Cost}_{w}}\tag{39}$$

$$P^{act}_{m,i,t}=\overline{P}_{m,i,t}+\Delta P_{m,i,t};\forall i,t,m\in\{WT;PV;ST\}\tag{40}$$

$$D^{act}_{e,i,t}=\overline{D}_{e,i,t}+\Delta D_{e,i,t};\forall i,t\tag{41}$$

$$P^{act}_{WT,i,t}+P^{act}_{PV,i,t}\times\eta_{k,i}+P^{grid}_{i,t}+P^{e,CHP}_{i,t}-P^{e}_{i,EC,t}-D^{act}_{e,i,t}-P^{loss}_{t,i}=0;\forall i,t\tag{42}$$

$$P^{act}_{ST,i,t}+P^{HE}_{i,t}+P^{h,GB}_{i,t}-P^{h}_{i,AC,t}-l^{h}_{i,t}=0\,;\forall i,t\tag{43}$$

To solve the proposed robust framework, the robust consensus is implemented according to the present algorithm. The robust consensus algorithm is executed similarly to subsection 4.1. The optimal operation of EHs is individually solved in an isolated manner, $P^{e}_{i,t},P^{e,CHP}_{i,t},P^{g,GB}_{i,t},\lambda_{e,i,t}$, and $\lambda_{g,i,t}$ are determined for each hub and, followed exchanged across adjacent hubs. Herein, the uncertain variables are also updated in step 4 based on (44)-(46).

$$\Delta P_{WT,i,t} = \Delta D_{e,i,t} = \frac{\lambda_{e,i,t}}{2(1 - w_{i,t})}; \forall i, t \tag{44}$$

$$\Delta P_{PV,i,t} = \frac{\lambda_{e,i,t} \times \eta_{k,i}}{2(1 - w_{i,t})}; \forall i, t \tag{45}$$

$$\Delta P_{ST,i,t} = \frac{\lambda_{h,i,t}}{2(1 - w_{i,t})}; \forall i, t \tag{46}$$

Proposed robust consensus algorithm

1: initialization
2: Update $\lambda_{1,i,t}[k+1]$ and $\lambda_{2,i,t}[k+1]$
3: Update $P^{e}_{i,t}[k+1], P^{e,GT}_{i,t}$ and $P^{g,GB}_{i,t}[k+1]$
4: Update $\Delta D_{e,i,t}[k+1], \Delta P_{WT,i,t}[k+1], \Delta P_{PV,i,t}[k+1], \Delta P_{ST,i,t}[k+1]$
5: Update $\Delta P_{e,i,t}[k+1]$ and $\Delta P_{h,i,t}[k+1]$
6: Repeat step 2- step 5 until $\left|P^{e}_{i,t}[k+1] - P^{e}_{i,t}[k]\right| < \varepsilon$ where ε has the sufficiently low value.

Regarding distribution losses, the proposed optimization flowchart in Fig. 3 will be executed where the above algorithm is replaced with the proposed consensus algorithm in the inner loop due to taking the uncertainty into account.

6. Simulation results

In this section, the optimal and robust operation of an IEH system is simulated to illustrate the proposed consensus-based algorithm's performance. Herein, the electrical and heating efficiencies of CHP, transformer, GB, and EHP are assumed to be 0.3, 0.4, 0.98, 0.9, and 0.95, respectively. All of the other required parameters are presented in the nomenclature. To assess the proposed method, two different cases are examined (Case I and Case II) with and without uncertainty. In this study, uncertainty management is conducted under the worst-case scenario (maximum uncertainty) with no flexible resources. Considering ESs and DRPs in the energy hub structure provides an additional degree of freedom to handle the uncertainty and to decrease its negative security and economic impacts. Thus, to establish the worst-case scenario, no flexible resources are considered in the presented model.

Case I. Assessment of the proposed scheduling in IEHs without uncertainty.

Case II. Assessment of the proposed robust scheduling in IEHs with uncertainty.

6.1. Case I

In Case I, no uncertainty resources (RESs and uncertain load demands) are considered. Considering the power losses, the correctness of the distributed consensus framework is validated in this case. Case I consists of five different studies. In Study 1, the obtained results are firstly compared to the centralized method to analyze the accuracy and productivity of the distributed consensus-based framework. The numerical results are also more discussed in Study 1 compared to [18]. The major effects of EH's structure will be analyzed in Study 2 on optimal energy trading.

A detailed sensitive analysis is conducted in Study 3 varying the value of the most critical parameters. The main advantage of the proposed method is its scalability for large-scale systems. It is validated in systems at different scales in Study 4. Finally, the main effects of the power loss on the purchasing input carriers and energy trading framework are evaluated in Study 5 considering both small-scale (a network of 5 EHs) and large-scale IES (a network of 30 EHs).

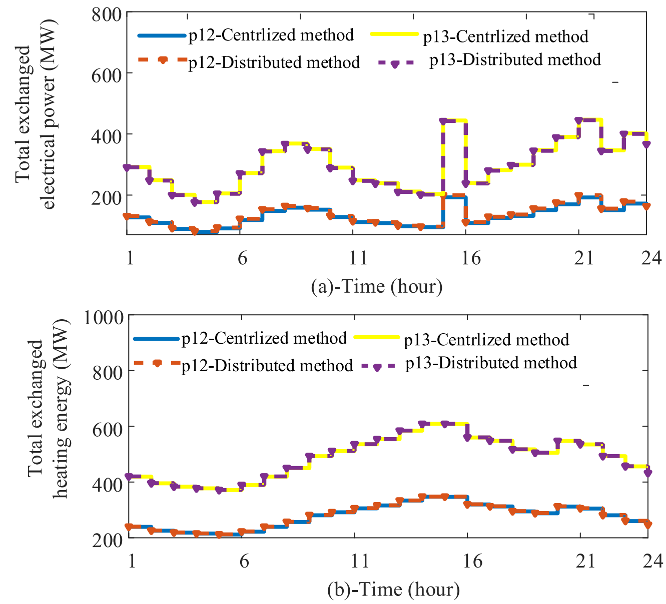

Fig. 4. Comparison results of the total traded energy compared to the centralized method in Case I

(a) electrical power (b) heating energy.

Table 3
Optimal power balance in the studied IEHs.

Optimal electrical energy flows (MW)	hub1	hub2	hub3
The produced electrical power	1131.56	396.7	241.74
Electrical demand	590	590	590
the exchanged electrical power	541.568	−193	−348

Table 4
Optimal heating balance in the studied IEHs.

Optimal heating energy flows (MW)	hub1	hub2	hub3
The produced heating energy	1350	450	270
heating demand	690	690	690
the exchanged heating energy	660	−240	−420

Study 1: Optimal cooperative scheduling of IEHs without uncertainty

To assess the effectiveness of the proposed framework, the daily optimal scheduling of IEHs is simulated in this subsection, ignoring the uncertainty. To do so, an IEH consisting of three MCESs is initially developed. The obtained numerical results are also compared to the centralized method (CM). Herein, HUB#1 is only assumed to be

Table 5
Comparison results of operation costs of energy hubs with and without considering the economic energy trading.

Cost (\$)($\times 10^6$)	Without interconnection			With interconnection		
	HUB#1	HUB #2	HUB #3	HUB #1	HUB #2	HUB #3
The electrical power purchasing	0.879	1.92	2.95	1.97	0.986	0.657
The input natural gas of CHP	0.383	0.801	1.21	1.44	0.481	0.288
The input natural gas of GB	0.247	0.906	1.56	1.97	0.658	0.394
Cost of MCESs	1.52	3.63	5.73	5.39	2.12	1.34
Total cost of IEHS	10.88			8.85		

Table 6
Results of the purchasing input energy carriers by energy hubs in IEHs compared to [18].

number	The proposed simulation results Without considering the emission cost in the objective function								The simulation results of [18] considering the emission cost in the objective function			
	$P^{e}_{i,t}$(MW)		$P^{g,CHP}_{i,t}$(MW)		$P^{g,GB}_{i,t}$(MW)		$P^{gas}_{t,i}$(MW)		$P^{e}_{i,t}$(MW)		$P^{gas}_{i,t}$(MW)	
	CM	DM	CM	DM	CM	DM	CM	DM	CM	DM	CM	DM
Hub 1	74.37	74.33	121.6	121	78.4	78.4	200	200	62.64	62.78	168.26	168.27
Hub 2	105.31	105.25	167.2	167	107.8	107.8	275	275	87.72	87.73	233.46	233.45
Hub 3	96.39	96.33	91.2	91	58.8	58.8	150	150	80.75	80.78	135.31	135.31
Hub 4	163.50	163.40	106.4	106	68.6	68.6	175	175	135.3	135.3	159.93	159.92
Hub 5	70.57	70.53	228	228	146.66	147	374.66	374.66	59.75	59.73	326.79	326.80
Cost ($)	10,856	10,847	7318.9	7300	4147.1	4151.2	11.466	11,451.2	8467.8	8467.8	12,021	12,021
Total cost	CM: 22322 DM: 22298.2								CM:20488.8 DM: 20488.8			

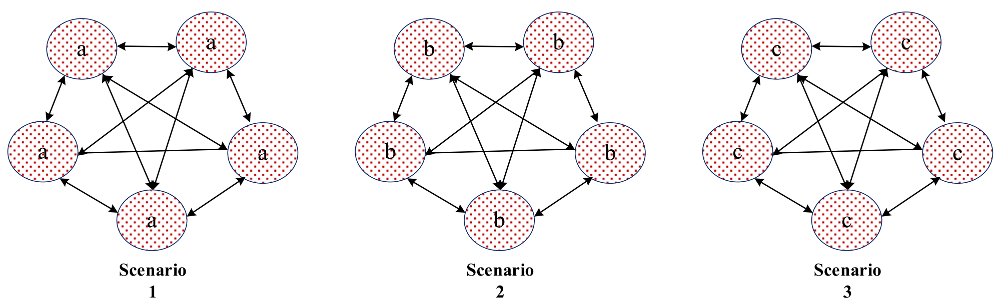

Fig. 5. Scenarios 1, 2, and 3 considered for the proposed energy trading in Study 2.A.

involved in the proposed energy trading (p23 = p32 = 0). Fig. 4 shows the optimal energy trading (electricity and heating energy) among MCESs using the proposed distributed method (DM) and centralized method (CM). In this figure, p12 and h12 are the electrical power and heating energy exchanged between HUB#1 and HUB#2, respectively. Furthermore, p13 and h13 are the exchanged electrical power and heating energy between HUB#1 and HUB#3. As a key result, the optimal solutions provided by the centralized and distributed approaches are similar.

Table 3 and Table 4 show the power and heating energy balance in the IEHS. As a result, it can be found that a significant part of load demands in HUB#2 and HUB#3 is provided by higher purchasing energy carriers in HUB#1 (i.e., 1131.56 MW for electricity and 1350 MW for heating). This emphasizes supplying energy shortage in HUB#2 (−193 and − 240 MW) and HUB#3 (−348 and − 420) by the exchanged energy carriers (i.e., 541.568 MW for electricity and 660 MW for heating).

Table 5 provides the details on the total costs in the defined cooperative IEHS. It can be found that cooperative energy trading reduces the total cost of HUB#2 and HUB#3 in IEHS by 41% and 76%, respectively. Furthermore, the total cost of HUB#1 declined by up by 71% due to the higher purchasing input energy carriers. Finally, the simulation results verify the proposed cost-effective operation which decreases the total IEHS costs in cooperative mode.

In the other comparison, the effective performance of the proposed distributed method (DM) has also been verified (see Table 6) by comparing the simulation results to the centralized and decentralized methods (CM and DM) in Ref. [18]. In this regard, the IEHS is assumed to be a network of 5 EHs according to [18]. Table 6 shows that the purchased energy carriers (electricity and natural gas) have increased concerning [18]. This increase is due to neglecting the emission cost in the defined objective function.

Study 2: Analysis of IEH's structures

This subsection consists of two different studies. Herein, the structures of IEHS and its building elements (EHs) will be analyzed on optimal energy trading respectively in Study 2. B and 2. A. It is worth-noting that the electrical and heating load demands are initialized in this Study at 150 MW and 140 MW.

-Study 2. A: In Study 2. A, the task is simplified by assuming that the cooperative coalition (i.e., IEHS) consists of five MCESs that collaborate together according to Fig. 5 (Scenarios 1, 2, and 3). As IEHs building elements, we denote three interconnected EHs where all three constituent EHs are assumed to be with structures (a), (b), or (c) as indicated in

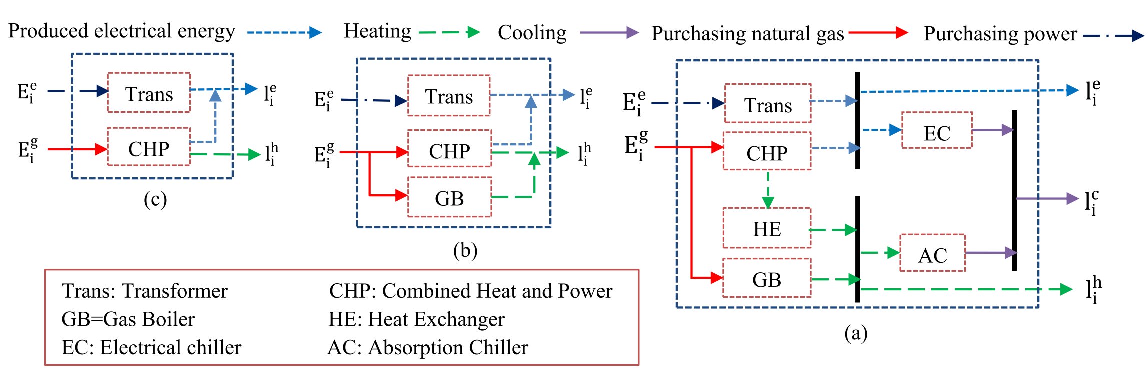

Fig. 6. The various understudied energy hub structures considered in Study 2.A.

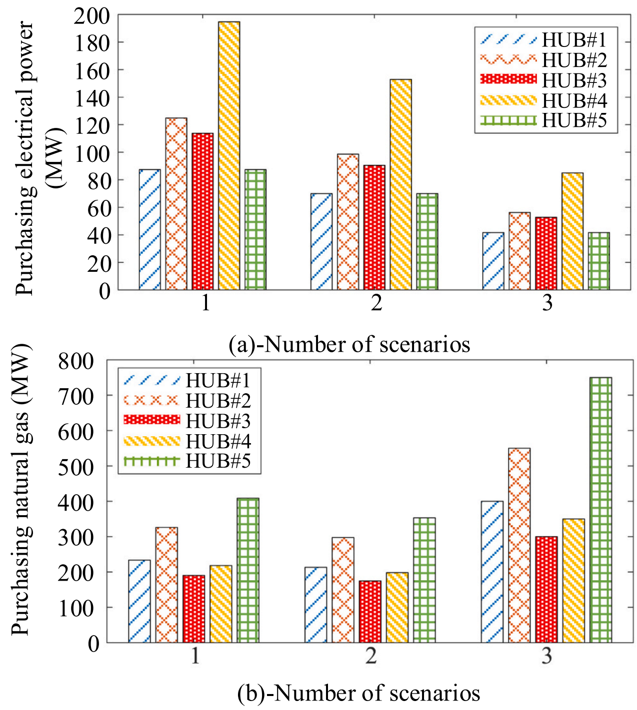

Fig. 7. Comparison of the effects of different structures of EHs on the purchasing input energy in Study 2.A. (a) Electrical power (b) Natural gas.

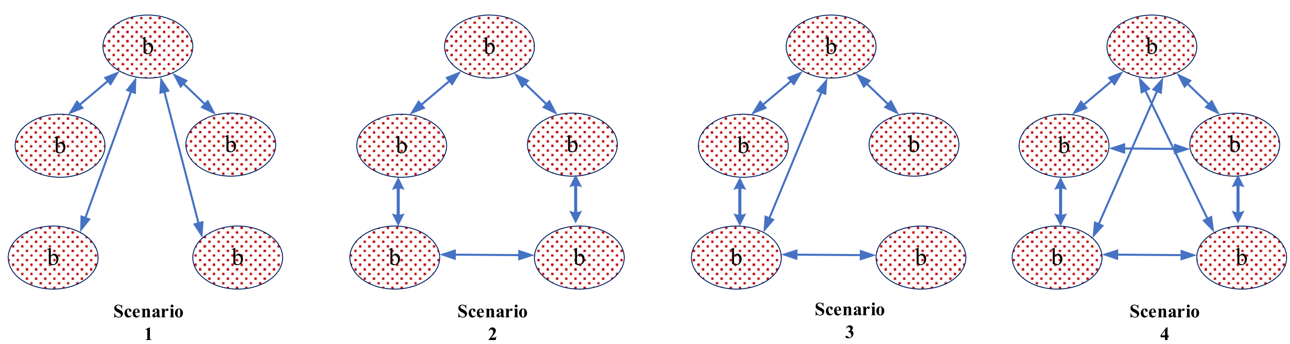

Fig. 8. Scenarios 1, 2, 3 and 4 considered for the proposed energy trading in Study 2.B.

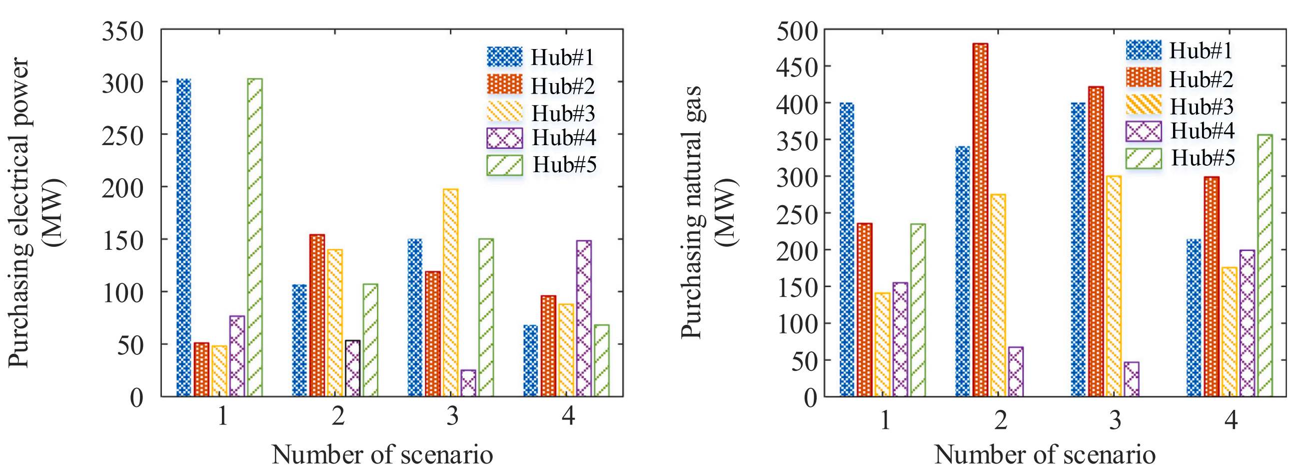

Fig. 9. Comparison of the effects of different structures of energy trading in IEHs on the purchasing input energy in Study 2.B (a) Electrical Power (b) Natural Gas

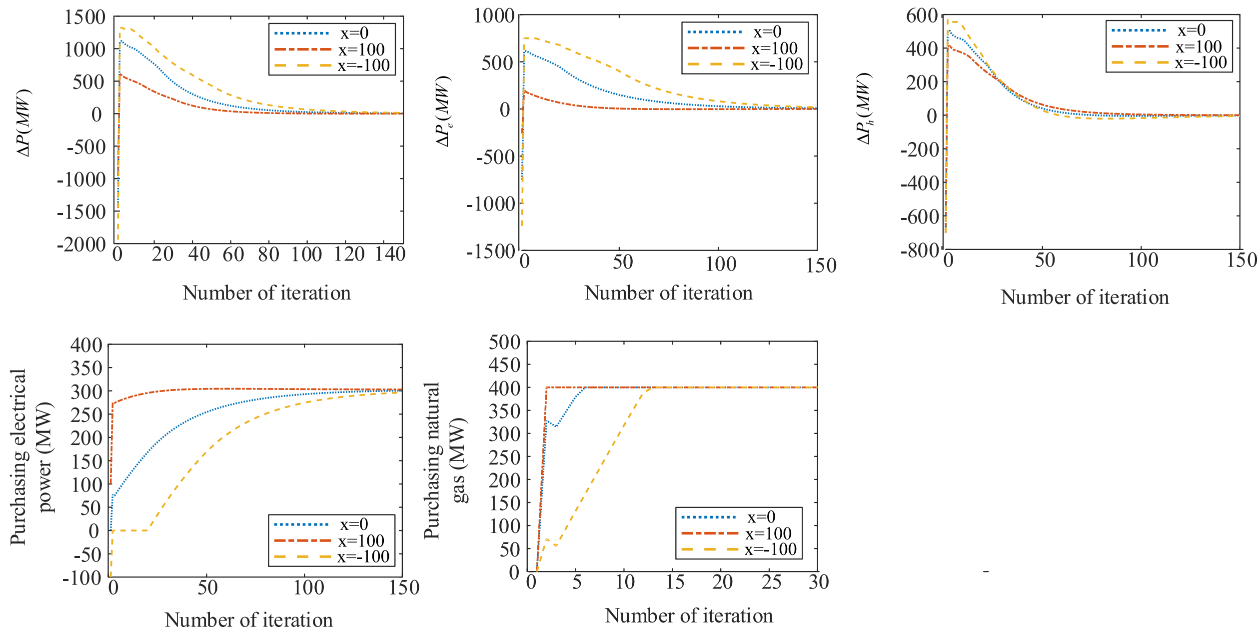

Fig. 10. Sensitive analysis of proposed distributed framework with different initial points (a) Total energy mismatch (b) Power mismatch (c) Heating mismatch (d) Purchasing power (e) Purchasing heating energy.

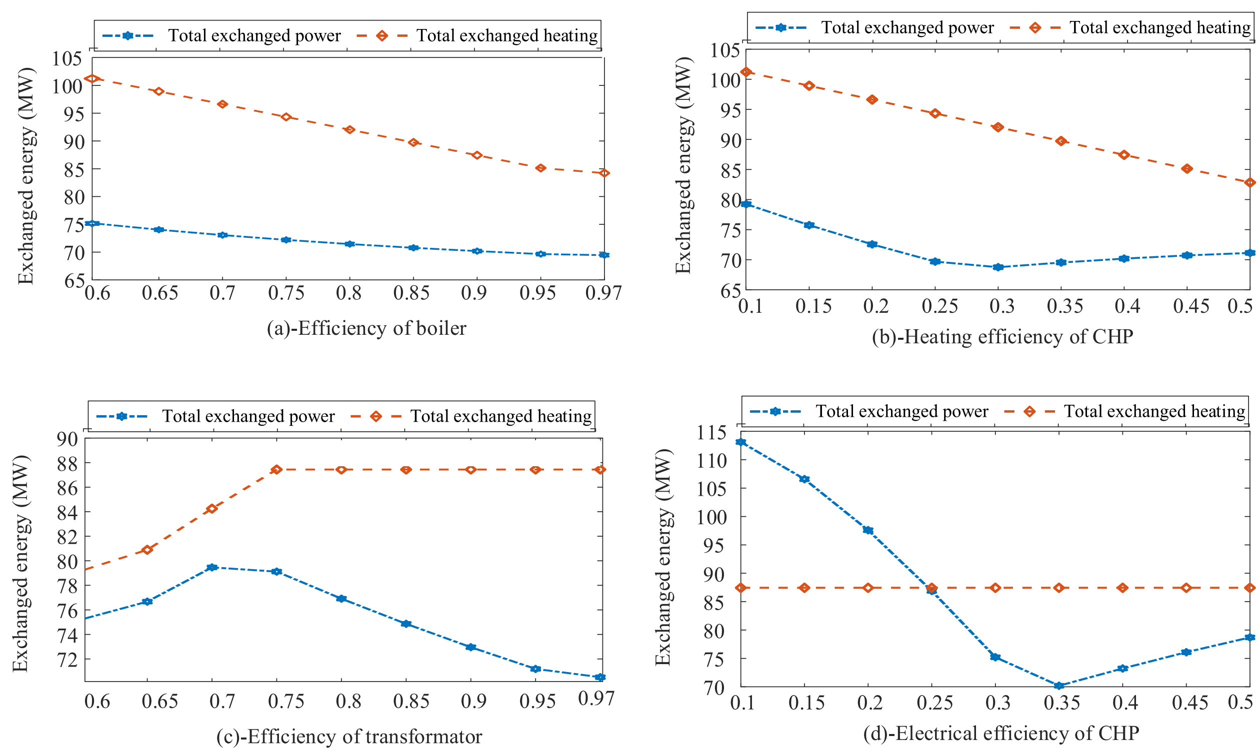

Fig. 11. Sensitive analysis of proposed distributed framework in Study 3 with different (a) electrical efficiency of CHP (b) heating efficiency of CHP (c) efficiency of transformer(d) efficiency of gas boiler.

Fig. 6. Figs. 7. (a) and 7. (b) shows the input purchasing energy (i.e., electrical and heating) for Scenarios 1, 2, and 3. In this study, all three defined MCESs are assumed to be involved in the proposed energy trading. The obtained results show the highest purchased power and natural gas, respectively in Scenarios 1 and 3. The exchanged electrical power is decreased in Scenario 2 compared to Scenario 1 although their heating trading is almost the same. The total purchasing electrical power is 607.8915, 481.7864, and 277.405 MW, respectively in Scenarios 1 to 3. Moreover, 1378, 1237.3, and 2350 MW of natural gas are totally purchased from the upstream network. As a result, the total cost is decreased by 14,053$ in Scenario 2 compared to Scenario 1 (17904$), and Scenario 3 (27767$).

-Study 2. B: In this study, a total of 5 trading scenarios were considered for IES's structures according to Fig. 8. Herein, IES is a network of five energy hubs cooperatively trading electricity and heating energy. Fig .9 (a) and Fig. 9 (b) indicate the electrical power and

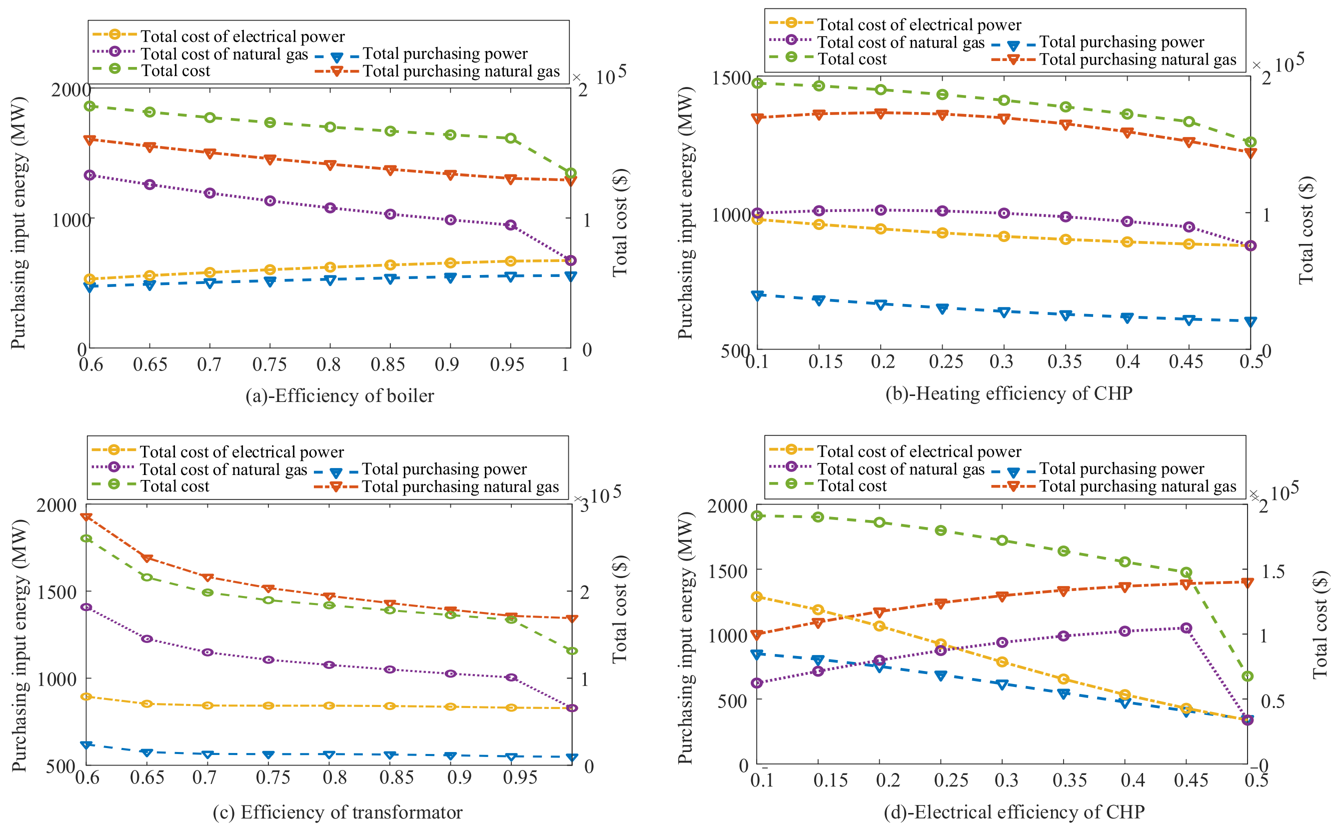

Fig. 12. Sensitive analysis of proposed distributed framework in Study 3 with different (a) electrical efficiency of CHP (b) heating efficiency of CHP(c) efficiency of transformer(d) efficiency of gas boiler.

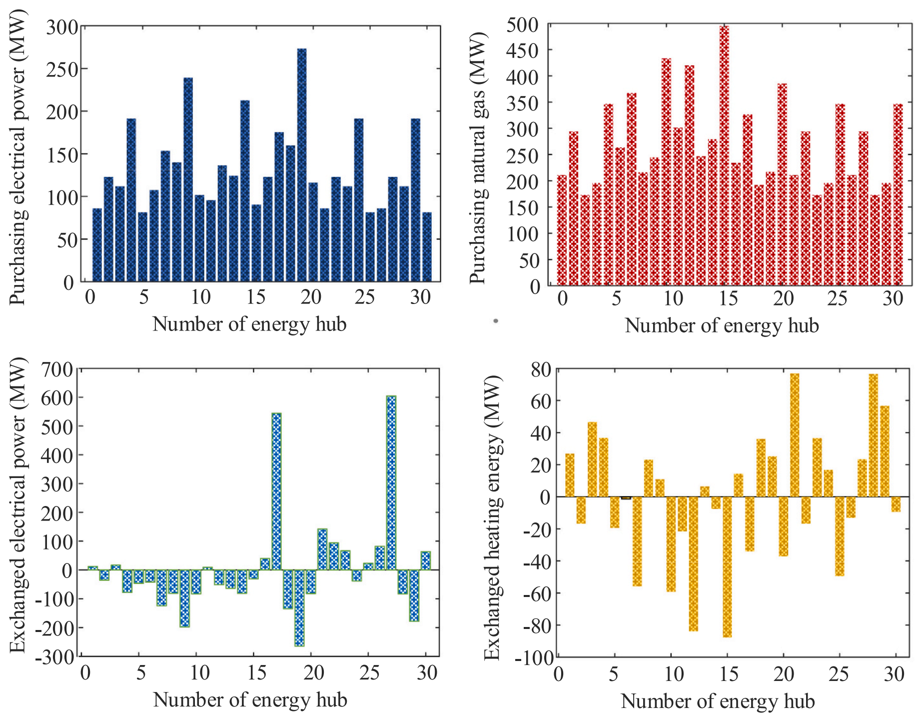

Fig. 13. The scalability evaluation of economic power trading in Study 4 (a) Total purchasing power (b) Total purchasing natural gas (c) Exchanged electricity power (d) Exchanged heating energy.

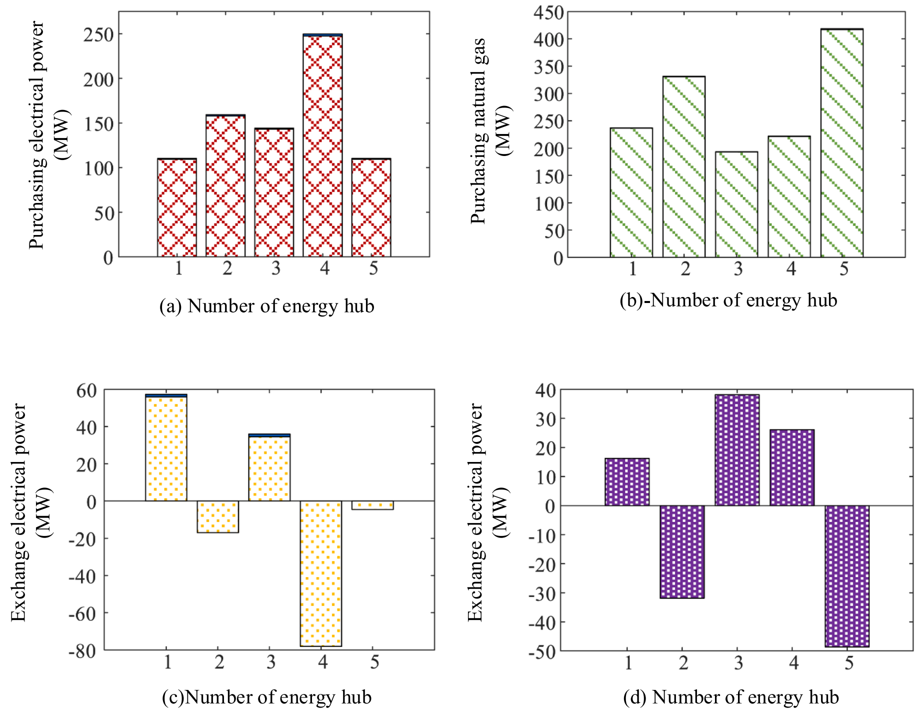

Fig. 14. The evaluation of economic power trading considering distribution losses in Study 5. A (a) Total purchasing power (b) Total purchasing natural gas (c) Exchanged electricity power (d) Exchanged heating energy.

natural gas purchased by participating hubs in each scenario. The total purchasing electrical power is obtained as 526.1250, 494.9209, 519.0869, and 478.3303 MW, respectively in Scenarios 1 to 4. It is worth noting that 232.30, 180.90, 241.85, and 58.6684 MW of the total exchanged power are respectively achieved.

The obtained results show the highest power trading respectively in Scenarios 3 and 1. In Scenario 1, HUB#1 is only involved in energy trading. In this scenario, HUB#2, HUB#3, and HUB#4 are highly dependent on HUB#1 to provide their required demand (higher energy trading). Inversely in Scenario 4, all five EHs participate in energy trading. Nevertheless, the total exchanged electricity decreased by 58.66 MW in Scenario 4. As a key result, the input purchasing carriers (both electricity and natural gas) are almost evenly distributed among all five energy hubs in this Scenario. This is also true for heating energy trading. Despite 1160.8 MW of purchasing natural gas, the lower heating trading is achieved in Scenario 1. Herein, 103.94, 226.1012, 254.4423, and 80.5032 MW of heating energy are totally exchanged among EHs, respectively in Scenarios 1 to 4.

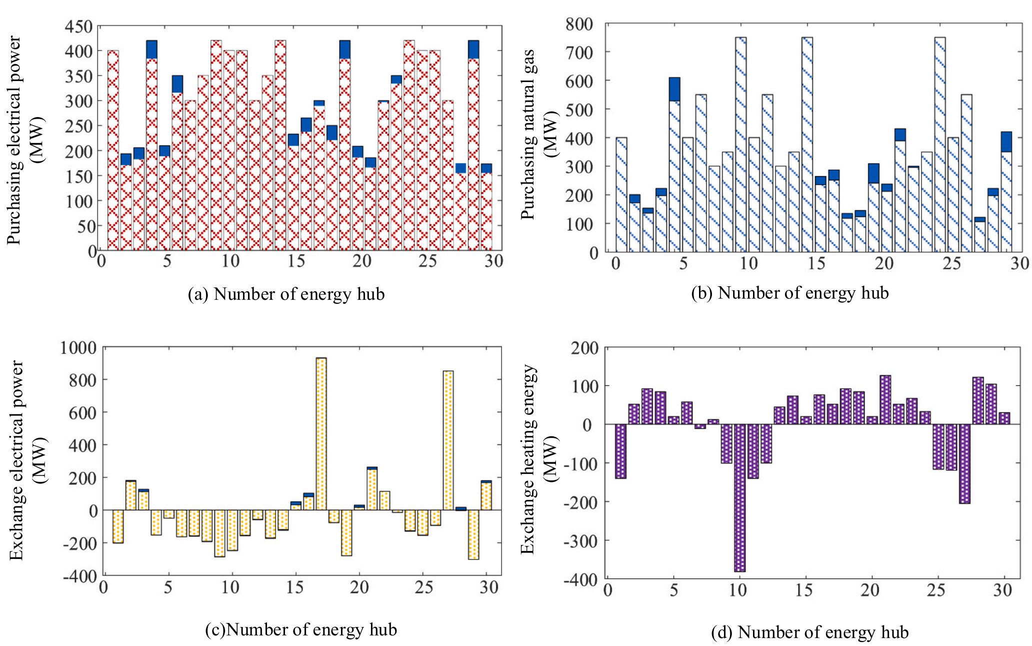

Fig. 15. The evaluation of economic power trading considering distribution network loss in Study 5. B (a) Total purchasing power (b) Total purchasing natural gas (c) Exchanged electricity power (d) Exchanged heating energy.

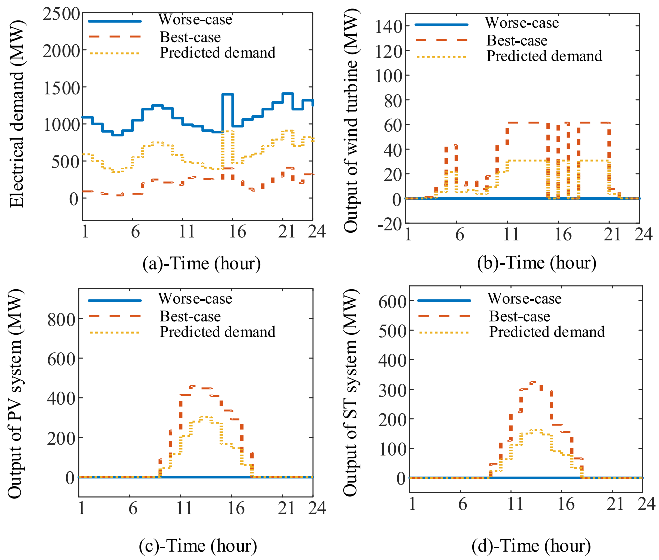

Fig. 16. Results of the proposed robust consensus algorithm
(a) Actual electrical demand (b) Actual power generation of WT (c) Actual power generation of PV (d) Actual heating energy generation of ST

Study 3: Sensitivity analysis.

In this study, a sensitivity analysis is first conducted varying the initial points. To do so, the main decision variables were initialized at three different scenarios ($x = 0.x = 100$ and $x = -100$). Herein, the power and heating load demands are assumed to be equal to 150 MW and 140 MW in EHs. The obtained mismatch and energy inputs are shown in Fig. 10.

After approximately 100 iterations, it can be found that the algorithm and its main decision variables converge to a common value. Moreover, both electricity and heating mismatches converge to zero. The obtained results also show the independence of algorithm performance on the initial parameters selection.

In the following, the parameter sensitivity analysis is performed. To do so, the IEH is assumed to be a network of 5 similar MCESs with structure (b). We Study how the converter's efficiency affects the proposed energy trading as indicated in Fig. 11. Fig. 12 also shows the effect of these parameters on the purchasing input carriers (electricity and natural gas) and total obtained costs with various efficiencies. It is clear that the total traded power and heating dose not affected by the efficiency of the gas boiler and electrical efficiency of CHP, respectively (see Fig. 11 (a) and Fig. 11 (d)). The exchanged heating and power energy is highly decreased by 83 MW and 70 MW as the heating efficiency of CHP increases. As a key result, the range of the heating and electrical efficiencies of CHP significantly affects the traded heating and power, respectively. In this regard, an efficiency of 0.35 is determined as a critical point for CHP in energy trading. Furthermore, the purchasing electrical power and total operation cost are highly decreased in Fig. 12 (d) as the electrical efficiency of CHP increases. Nevertheless, more natural gas is purchased from the upstream grid (an increase in the total cost of natural gas). In this regard, the impact of heating efficiency is ignorable (see Fig. 12 (b)). According to Fig. 12 (a), the gas boiler strongly affects the purchasing input energy carriers (both power and natural gas). As an important result, while the efficiency of the gas boiler increases the purchasing power, the total cost has decreased.

Study 4: Scalability analysis

One of the main advantages of the proposed method is its scalability for large-scale systems. In this study, it has been validated in systems at different scales (5, 20, and 30 EHs). As a key result, the proposed distributed algorithm is able to handle large-scale IES in a way that not only global optimal solution can be reached, but also the volume of computations can be lightened. It is notable that the proposed distributed algorithm can be completely executed at 0.0815 (s), 0.1318 (s), and 0.1917 (s) or IESs with respectively 5, 20, and 30 MCESs. In this study, a network of 30 EHs in which all MCESs are involved in energy trading is only considered for more detailed discussion. The obtained simulation results are indicated in Fig. 13. Herein, the exchange energy of i-th MCES reflects the total power and heating energy traded between i-th MCES with the others and is supplied/consumed by i-th MCES. (See Fig. 14.)

Study 5: Distributed consensus algorithm considering the power loss

This subsection consists of two different studies, Study 5. A, Study 5. B. In study 5. A, the distribution network consists of five MCESs with structure (b). To handle the scalability, a large-scale IES (a network of 30 MCESs) is also considered in Study 5.B. Fig. 15 shows the effect of losses on the energy flows in IEHs. The total loss is estimated as 5.6917 MW and 1175.4 MW, respectively for studies 5. A and 5. B. It can be found that the electricity and heating energy losses, significantly effect on the purchasing input energy in Study 5. B. For instance, the purchasing natural gas is increased by 600 MW in 5-th MCES. In this regard, the purchasing input energy is averagely increased by 7.8% and 8.53%, respectively for natural gas and electrical power considering the distribution losses. Thus, considering distribution network loss is deemed inevitable in relatively large-scale IESs compared to Study 5. B. Considering line losses, the total power traded among multi-energy hubs is shown in Fig. 15 (c) and (d). It is worst nothing that the total costs of purchasing power and natural gas are respectively increased by 1.199×10^7($) and 6.12450×10^6 ($) in Study 5. B.

6.2. Case II

In Case II, it is assumed that multiple wind turbines, PV systems, and solar thermals were installed in different energy hubs. Moreover, the

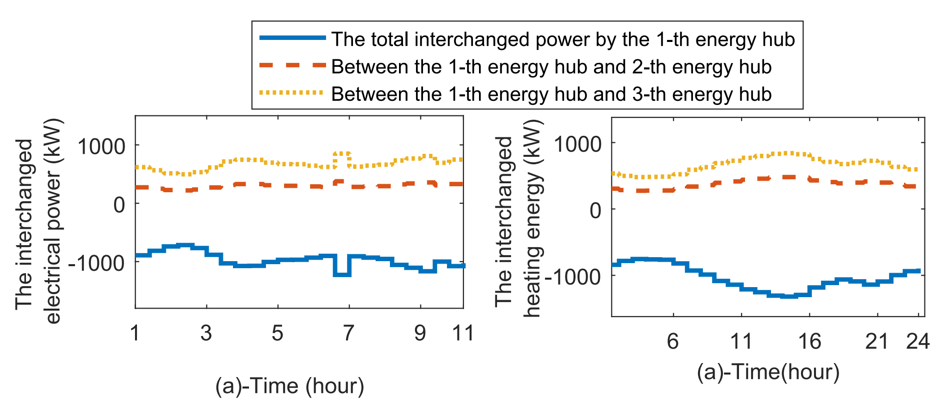

Fig. 17. Comparison results of the optimal and cooperative robust energy management of IEHs (a) electricity (b) heating.

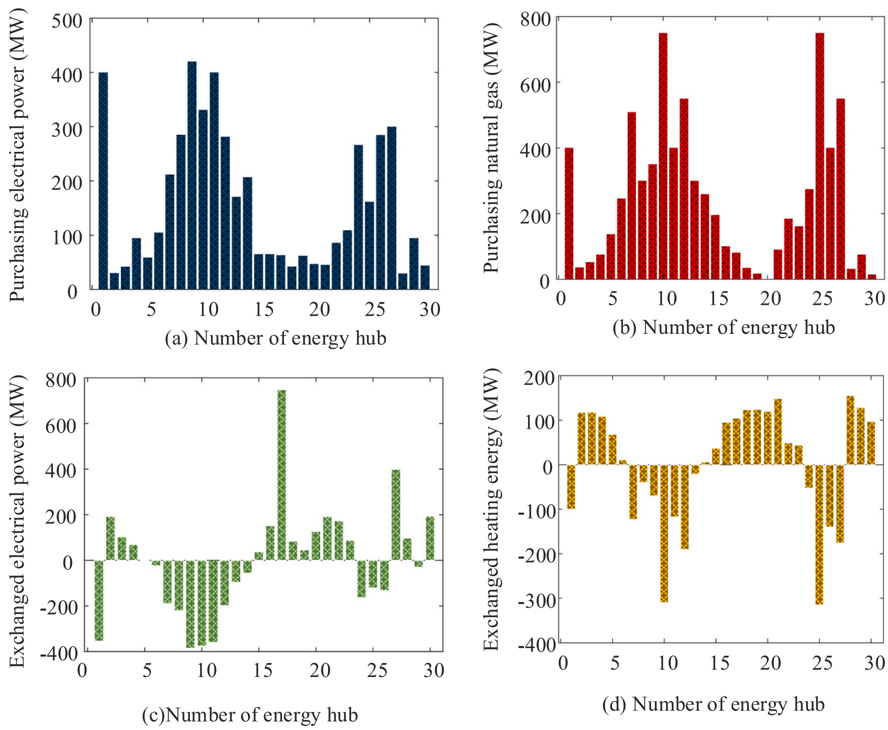

Fig. 18. The scalability evaluation of economic power trading in Study 2 in case II considering the worst-case uncertainty (a) Total purchasing power (b) Total purchasing natural gas (c) Exchanged electricity power (d) Exchanged heating energy.

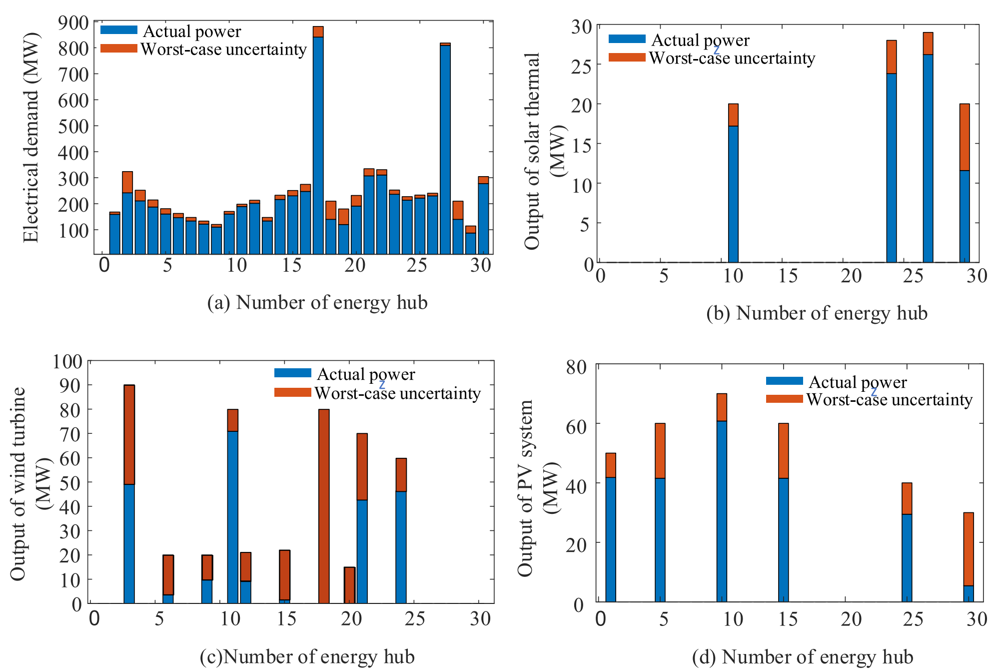

Fig. 19. The obtained maximum uncertainty in Study 2 in case II (a) electrical demand (b) heating energy generation of ST (c) power generation of WT (d) power generation of PV.

electrical demand is also assumed to be uncertain. The efficiency of the proposed robust consensus-based algorithm is evaluated in this case compared with the deterministic assessment. Case II consists of two different studies, Study 1 and Study 2. To assess the effectiveness of the robust proposed framework, the daily optimal scheduling of IEHs (consisting of three MCESs) is simulated in Study 1 considering the worst-case uncertainty. Considering a large-scale IES consisting of (30 MCESs), the scalability evaluation is discussed in Study 2.

Study 1: daily optimal scheduling of IEHs under the worst-case uncertainty

In this section, the optimal and robust energy trading in IEHs is simulated according to the present algorithm in Section 5 considering the worst-case uncertainty of RESs and electrical demand. It is worst noting that $w_{i,t}$ is assumed to be 0.5 in this study for each energy hub.

The optimal simulation results are expressed in Fig. 16 and Fig. 17. Herein, the maximum increase in the electrical demand combined with the maximum decrease in the output power of RESs has been taken as the worst-case scenario in the simulation. The obtained maximum electrical demand has been indicated in Fig. 16 (a). To supply maximum uncertainty, higher power is exchanged among studied MCESs compared to the deterministic case. Since solar thermal is the only uncertain resource, the exchanged heating energy doesn't sufficiently change in the worst-case scenario.

Study 2: Scalability analysis

In Study 2, IEH is considered as a network of 30 EHs. Herein, it is assumed to that the wind turbines were highly installed in HUB#11, HUB#3, HUB#6, HUB#9, HUB#12, HUB#15, HUB#18, HUB#21, HUB#24 and HUB#20, the PV systems were installed in HUB#1, HUB#5, HUB#10, HUB#15, HUB#20, HUB#25 and HUB#30, and the solar thermals were installed in HUB#24, HUB#27, HUB#30, and HUB#11. Moreover, the electrical demand is also assumed to be uncertain. Fig .18 (a) and Fig. 18 (b) indicate the purchasing of electrical power and natural gas in Study 2. It is worst nothing that the worst-case uncertainty is also taken into account in this study ($w_{i,t} = 0.1$).

While the total purchasing input energy is expected to be decreased in Study 2 (due to RESs), the numerical results show an increase in the input energies compared to case I with no RESs (Fig. 13). Compared to the deterministic case, the total purchasing power is respectively increased by 6.5 while the input natural gas does not strongly be affected by the uncertainty. In other words, the proposed algorithm can compensate for the obtained worst-case uncertainty by an increase (11%) in the total exchanged electrical power compared to the deterministic case. As a key result, the proposed distributed algorithm is able to handle large-scale IES with a lightened volume of computational burden. It is worst-noting that the proposed robust consensus-based algorithm has been completely executed at 0.20 (s) and 0.53 (s) respectively for the defined small (5 MCESs) and large-scale IESs (30 MCESs). In this regard, maximum uncertainty is achieved for RES power generation and electrical load according to Fig. 19.

7. Conclusion

In this study, a novel hybrid distributed IGDT/consensus algorithm has been proposed for the risk-averse cooperative optimal operation of a large-scale IEHS. Considering the strong energy coupling, the optimal energy trading has been modeled as a distributed consensus algorithm to improve the system's overall efficiency. Considering distribution network losses, this paper also proposes a new realistic lambda-based iterative algorithm for the optimal coordination of IEHs.

By taking the proposed consensus algorithm, the total operation cost is decreased by 18.65% compared to the individual scheduling of EHs neglecting the interconnection. The simulation results have been compared to the centralized approach. As a key characteristic of the proposed method, scalability has been validated in IEHs at different scales in two different cases, with and without uncertainty. The running time can be lightened by 0.53 (s) and 0.1917 (s) in a large-scale IEHs with 30 energy hubs. The sensitive analysis also reveals how the distributed algorithm performance is affected by the initial parameter selection. The different structures of EHs have also been analyzed on the optimal energy trading mechanism. Purchasing input energy can be averagely increased by 8% affected by distribution network losses in large-scale IES. Nevertheless, the impact of network losses is ignorable in an IEHS with five energy hubs. As a key result, the worst-case uncertainty led to an increase in electrical power trading by 11% compared to the deterministic case. Considering a high uncertainty budget, the total operation cost can be even higher than case I with no RESs. In future works, the CHP's feasible region can be integrated into the proposed distributed model. How the proposed distributed method is susceptible to cyber-attacks will also be investigated in future works.

CRediT authorship contribution statement

Maryam Azimi: Visualization, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Abolfazl Salami:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Investigation. **Mohammad S. Javadi:** Writing – review & editing, Visualization, Validation, Resources, Methodology, Conceptualization. **João P.S. Catalão:** Writing – review & editing, Supervision, Resources, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A

To evaluate the convergence of the studied proposed consensus algorithm, the main decision variables in k + 1-th iteration can be rewritten in the general form as (A.1) and (A.2)

$$E^g(k+1) = B^{1,g}\lambda_1(k+1) + B^{2,g}\lambda_2(k+1) + \alpha^g \tag{A.1}$$

$$E^e(k+1) = B^e\lambda_1(k+1) + \alpha^e \tag{A.2}$$

Where, λ_1, λ_2, α^g and α^e are the column stack vectors of $\lambda_{1,i}$, $\lambda_{2,i}$, $-\beta_i^e/2\alpha_i^e$ and $-\beta_i^e/\alpha_i^e$. Also, B^e, $B^{1,g}$ and $B^{2,g}$ are calculated as (A.3)-(A.5).

$$B^e(k+1) = diag\left[\frac{\eta_{trans,1}}{2\alpha_1^e}, \frac{\eta_{trans,2}}{2\alpha_2^e}, \ldots, \frac{\eta_{trans,n}}{2\alpha_n^e}\right] \tag{A.3}$$

$$B^{1,g} = daig\left(\left[\frac{\eta_{GB,1}}{2\alpha_1^g}, \frac{\eta_{GB,2}}{2\alpha_2^g}, \cdots, \frac{\eta_{GB,n}}{2\alpha_n^g}\right]\right) \tag{A.4}$$

$$B^{2,g} = daig\left(\left[\frac{r_1}{2\alpha_1^g}, \frac{r_2}{2\alpha_2^g}, \cdots, \frac{r_n}{2\alpha_n^g}\right]\right) \tag{A.5}$$

$$r_i = \eta_{GB,i} + \eta_{h,GT,i} \times \eta_{HE,i} \tag{A.6}$$

The consensus variables (λ_1, λ_2) is updated in any iteration according to (A.7) and (A.8).

$$\lambda_1(k+1) = P\,\lambda_1(k) + \varepsilon\Delta P_e(k) + \varepsilon\Delta P_g(k) \tag{A.7}$$

$$\lambda_2(k+1) = P\,\lambda_2(k) + \varepsilon\Delta P_e(k) + \varepsilon\Delta P_g(k) \tag{A.8}$$

The electrical and heating imbalances are expressed as follows:

$$\Delta P_e(k+1) = Q\Delta P_e(k) - (E_e(k+1) - E_e(k)) \tag{A.9}$$

$$\Delta P_g(k+1) = Q\Delta P_g(k) - \left(E_g(k+1) - E_g(k)\right) \tag{A.10}$$

If $x(k-1) = \begin{bmatrix} \lambda_g(k-1) \\ \Delta P_g(k-1) \\ \Delta P_e(k-1) \end{bmatrix}$, $x(k)$ can be calculated as the following matrix equation in (A.11) by combining (A.1)- (A.10) together, where, $W(k)$ is defined as a matrix that relies on the cost coefficient and P and Q matrices. Thus, $W(k)$ is independently calculated in any iteration as (A.13).

$$x(k) = W(k) \times x(k-1) \tag{A.11}$$

$$x(k) = \begin{bmatrix} \lambda_g(k) \\ \Delta P_g(k) \\ \Delta P_e(k) \end{bmatrix}, x(k-1) = \begin{bmatrix} \lambda_g(k-1) \\ \Delta P_g(k-1) \\ \Delta P_e(k-1) \end{bmatrix} \tag{A.12}$$

$$W = \begin{bmatrix} P & \varepsilon I & \varepsilon I \\ B^g(I-P) & Q - \varepsilon B^g & -\varepsilon B^g \\ B^e(I-P) & -\varepsilon B^e & Q - \varepsilon B^g \end{bmatrix} \tag{A.13}$$

Theorem 1. *The convergence of the proposed consensus algorithm will be guaranteed if $\lim_{k\to\infty} x(k)$ exists for $\forall x(0) < \infty$.*

$$\lim_{k\to\infty} x(k) = \lim_{k\to\infty} w^k x(0) = x^* \tag{A.14}$$

Lemma1: The consensus algorithm converges to an optimal point if the absolute value of the largest eigenvalue of W is smaller than 1.

Proof: Considering $w = U\Lambda U^{-1}$, the $\lim_{k\to\infty} x(k)$ in (A.14) can be rewritten as (A.15).

$$\lim_{k\to\infty} w^k x(0) = \lim_{k\to\infty} \left(U\Lambda U^{-1}\right)^k x(0) \tag{A.15}$$

Where, U and Λ are matrices of eigenvectors and eigenvalues of W, respectively. Thus, considering Λ^k as (A.16), $\lim_{k\to\infty} \Lambda^k$ should be finite to guarantee $\lim_{k\to\infty} x(k) = x^*$ according to the (A.14). $\lim_{k\to\infty} \Lambda^k < \infty$ means that the absolute value of the largest eigenvalue is smaller than 1.

$$\Lambda = \begin{bmatrix} \Lambda_1^k & 0 & 0 & 0 \\ 0 & \Lambda_2^k & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \Lambda_n^k \end{bmatrix} \tag{A.16}$$

Win (A.13) can be decomposed to the M and N matrices as follows:

$$M = \begin{bmatrix} P & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ B^e(I-P) & 0 & Q - \varepsilon B^e & 0 \\ B^{1,g}(I-P) & B^{2,g}(I-P) & -\varepsilon B^{1,g} - \varepsilon B^{2,g} & Q - \varepsilon B^{1,g} - \varepsilon B^{2,g} \end{bmatrix}$$

$$N = \begin{bmatrix} 0 & 0 & \varepsilon I & \varepsilon I \\ 0 & 0 & \varepsilon I & \varepsilon I \\ 0 & 0 & 0 & -\varepsilon B \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{A.17}$$

The eigenvalues of M are the union of the eigenvalues of P, Q, $Q - \varepsilon B^e$and $Q - \varepsilon B^{1,g} - \varepsilon B^{2,g}$. The eigenvalues of P and Q are $\Lambda_1 = \Lambda_2 = 1$ and the rest eigenvalues lie in the open unit disk on the complex plane. The largest eigenvalues of $Q - \varepsilon B^e$and $Q - \varepsilon B^{1,g} - \varepsilon B^{2,g}$ are smaller than 1acccording to Lemma 2.

Lemma 2. *According to Weyl's inequality, the maximum eigenvalue of $Q - \varepsilon B^e$ is equal to the sum of eigenvalues of Q and $-\varepsilon B^e$. This fact can be also used for determining the maximum eigenvalue of $Q - \varepsilon B^{1,g} - \varepsilon B^{2,g}$.*

$$max[\Lambda(Q - \varepsilon B^e)] = \Lambda(Q) + \Lambda(-\varepsilon B^e) \tag{A.18}$$

The eigenvalue of Q is equal to 1. since $\Lambda(Q) + \Lambda(-\varepsilon B^e) \leq 1$ and $\Lambda(Q) + \Lambda\left(-\varepsilon B^{1,g} - \varepsilon B^{2,g}\right) \leq 1$ then the largest eigenvalues of $Q - \varepsilon B^e$and $Q - \varepsilon B^{1,g} - \varepsilon B^{2,g}$ will be smaller than 1. So, it can be concluded that $\lim_{k\to\infty} x(k) = x^*$ due to $\lim_{k\to\infty} \Lambda^k < \infty$ based on lemma l and lemma 2.

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