

# Uncertainty Modeling for Participation of Electric Vehicles in Collaborative Energy Consumption

Naser Hashemipour, Jamshid Aghaei <sup>1</sup>, Senior Member, IEEE, Pedro Crespo Del Granado <sup>2</sup>,  
Abdollah Kavousi-Fard <sup>3</sup>, Member, IEEE, Taher Niknam <sup>4</sup>, Senior Member, IEEE,  
Miadreza Shafie-khah <sup>5</sup>, Senior Member, IEEE, and João P. S. Catalão <sup>6</sup>, Fellow, IEEE

**Abstract**—This paper proposes an accurate and efficient probabilistic method for modeling the nonlinear and complex uncertainty effects and mainly focuses on the Electric Vehicle (EV) uncertainty in Peer-to-Peer (P2P) trading. The proposed method captures the uncertainty of the input parameters with a low computational burden and regardless of the probability density function (PDF) shape. To this end, for each uncertain parameter, multitude of random vectors with the specification of corresponding uncertain parameters are generated and a fuzzy membership function is then assigned to each vector. Since the most probable samples occur repeatedly, they are recognized by the superposition of the generated fuzzy membership functions. The simulation results on various case studies indicate the high accuracy of the proposed method in comparison with Monte-Carlo simulation (MCs), Unscented Transformation (UT), and Point Estimate Method (PEM). It also scales down the computational burden compared to MCs. Also, a real-world case study is employed to examine the ability of the method in capturing the uncertainty of EVs' arrival and departure time. The studies on this case reveal that involving EVs in P2P trading augments the amount of energy traded within the prosumers.

**Index Terms**—EV uncertainty, P2P trading, uncertainty modeling, vehicle to home.

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Naser Hashemipour is with the Department of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz 71557-13876, Iran, and also with the Norwegian University of Science and Technology, 7491 Trondheim, Norway (e-mail: n.hashemipour@sutech.ac.ir).

Jamshid Aghaei is with the Department of Electrical Engineering, School of Energy Systems, Lappeenranta University of Technology, FI-53851 Lappeenranta, Finland, and also with the Department of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz 71557-13876, Iran (e-mail: jamshid.ghaei@lut.fi).

Pedro Crespo Del Granado is with the Norwegian University of Science and Technology Trondheim, 7491 Trondheim, Norway (e-mail: pedro@ntnu.no).

Abdollah Kavousi-Fard and Taher Niknam are with the Department of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz 71557-13876, Iran (e-mail: kavousi@sutech.ac.ir; niknam@sutech.ac.ir).

Miadreza Shafie-khah is with the School of Technology and Innovations, University of Vaasa, 65200 Vaasa, Finland (e-mail: miadreza@gmail.com).

João P. S. Catalão is with the Faculty of Engineering of the University of Porto and INESC TEC, 4200-465 Porto, Portugal (e-mail: catalao@fe.up.pt).

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## NOMENCLATURE

<i>Indexes</i>		
$h$	house.	29
$p$	peers.	30
$t$	time.	31
<i>Variables</i>		32
SoC	State of charge.	33
$S_{EV}^{(t,e)}$	State of the charge of the $e$ th EV at time $t$	34
$C_{EV}^{(t,e)}$	Charging of the $e$ th EV at time $t$	35
$D_{EV}^{(t,e)}$	Discharging of the $e$ th EV at time $t$	36
$\eta_{EV}^c$	EV charging efficiency.	37
$\eta_{EV}^d$	EV discharging efficiency.	38
$\alpha_{EV}$	EV charging rate.	39
$\beta_{EV}$	EV discharging rate.	40
$G^{(t,h)}$	Grid import of house $i$ at time $t$	41
$I_p^{(t,h \leftarrow p)}$	P2P energy import of house $h$ from $p$ at time $t$	42
$X_p^{(t,h \rightarrow p)}$	P2P energy export of house $h$ from $p$ at time $t$	43
$\psi_{p2p}$	loss factor.	44
$P_G^{(t)}$	Electricity price.	45
$d$	Traveled distance of the EV.	46
$C_{eff}$	Efficiency coefficient of the PEV during driving (km/kWh)	47
Cap	Capacity of the EV's battery.	48
<i>Parameters</i>		49
$t^{departure}$	Departure time of the EV.	50
$t^{arrival}$	Arrival time of the EV.	51
$SoC^{arrival}$	Arrival state of charge.	52
$a$	Random number between 0 and 1	53
$\sigma$	standard deviation.	54
$\mu$	mean.	55
$N$	Number of samples.	56
$X_{i'}^t$	$i^{th}$ sample	57
$X_{j'}^t$	$j$ th element of ascending sorted $X_{i'}$	58
$F_j$	$j$ th element of superposition vector $F$	59
$X_1$	uncertain parameter.	60
$X_2$	uncertain parameter.	61
$\alpha$	Scale parameter.	62
$\beta$	Shape parameter.	63
$\gamma_1$	skewness.	64
$K$	Kurtosis.	65

## I. INTRODUCTION

**E**LECTRIFICATION of the transportation systems is gaining a crucial role in solutions for environmental problems. Indeed, integrating the electric transportation assets to a grid supplied by renewable energy sources (RESs) will reduce the emission of greenhouse gasses and deal with the scarcity of non-renewable resources. The importance of this issue has been exhibited in many pieces of research, such as [1]–[7].

Ref. [1] reviews six EV charging strategies which are the optimization problem formulations in the vehicle to grid (V2G) or vehicle to home (V2H) programs. It also proposes an algorithm based on the logic of selling electricity back to the grid during peak hours. Paper [2] proposes a mixed-integer optimization that models the EVs as mobile storage. An optimal V2G model is presented by [3] which considers the battery aging of EVs along with requirements of driving patterns. A distributed control algorithm is then employed to implement the proposed strategy. Reference [4] proposes a long short-term memory (LSTM) based EV battery available capacity prediction that is going to help the frequency regulation in a micro-grid. Aggregation of the EVs in a fleet has been presented in [5]–[7] to support grid operation.

A transition from the centralized to decentralized or distributed structures of the grid operation is happening rapidly due to the higher potential for integration of the distributed resources [8]. In this situation, involving EVs, which make up a considerable part of the electric transportation systems in P2P trading programs, paves the way toward providing electrified transportation systems. The related researches to utilizing EVs in P2P trading of electricity are briefly reviewed in the following.

Ref. [9] proposes a collaborative energy consumption based on the renewable energy clusters by providing the best match of the EV, demand, and renewable resources. Reference [10] has presented a P2P trading system between local plug-in electric vehicles (PEVs) that trade. An aggregator collects the bids and offers and the demand data from EVs and determines the optimal P2P prices. Paper [11] has proposed P2P trading model to buy and sell electricity among local plug-in hybrid electric vehicles (PHEVs). This study satisfies the prerequisites such as security and privacy by consortium Blockchain. The presented approach has tried to issue security and privacy as well as mobility. It is to mention that, in [11], EVs are the only participants in the P2P market, and the other types of traders have not been considered. Proposing blockchain-based methods can reduce credit costs and enhance renewable energy integration. Also, Monte Carlo simulation has been used to show the uncertain nature of charging stations' charging demand. Another work [12] employs a private blockchain method to prove transaction records between EVs. This framework relies on a private blockchain-based P2P electricity trading approach to obtain secure electricity trading. Paper [13] has introduced a smart contract and blockchain-based energy trading system that directly provides conditions for direct interaction between providers and EV owners. This framework depends on utility companies for metering and billing to prevent significant infrastructure changes. Although blockchain can facilitate decentralization and security requirements, it does not

solve all problems of distributed structures, such as performance efficiency [14]. Hence, Trading strategies for inter vehicles (V2V) was analyzed in [14] which, addresses the problems of conventional blockchain. In [15], a P2P method for energy trading in the local electricity market was utilized. This method can help the PV owners to achieve more accuracy in the forecast. In another study [16], P2P trading through DSO, as the central party, has been presented. DSO keeps the overall data of all users and links prosumers and consumers. In [17], a novel approach for EVs' charging and discharging has been introduced. For the validation process, the presented method has been compared with standard consensus methods. Moreover, a new proof-of-Benefit (PoBen) consensus protocol was proposed that fills the gap of previous consensus methods. The experimental results demonstrated that PoBen method developed the security and sustainability of power fluctuations.

Generally, the uncertainty of a problem can be modeled by different methodologies, such as probabilistic methods [18], possibilistic approaches [19], hybrid methods [20], information gap decision theory [21], robust optimization [22], and interval analysis [23], depending on different factors. For example, probabilistic methods are applicable in cases that the PDF of the uncertain parameters are known. In contrast, possibilistic methods are not based on the PDF and use fuzzy functions to capture the uncertainty. Combination of these two methods are called hybrid models. However, information gap decision theory, robust optimization, and interval analysis, are based on the measurement error or estimation of the parameters, feasibility in the worst case, and uniform PDF, respectively. Since the proposed method can be considered a probabilistic method, more relevant papers are reviewed.

Owing to recent developments in the advanced technologies of electric vehicles, many sources of uncertainty have appeared in the energy systems. This shows the necessity of developing accurate and fast uncertainty modeling methods to enhance the decision-making of energy systems. Monte-Carlo simulation, which is well known for its high accuracy, is very time-consuming [6] and may not be practical in a wide range of applications. This drawback has been partially addressed by some techniques such as PEM [24], UT [25] and various scenario reduction methods. Importance sampling (IS) methods such as Cross-Entropy involve finding a distribution that estimates necessary samples of uncertain elements. In [26], the authors implemented a method using the cross-entropy function to minimize the distance between sampling distribution and the original one. In [27], the stratified sampling Monte Carlo method was employed to calculate the lightning performance of transmission and distribution systems. Non-Sequential Monte Carlo simulation was used in [28] to model statistically dependent time-varying quantities, including renewable energy sources. In order to reach a practical and logical computational burden, scenarios with high similarity or low probability were omitted from the scenario set in [29]. Compression of scenarios by scenario mapping technology was applied on the uncertain behavior of wind power in [30]. Authors in [31] provided a comparison on random sampling, importance sampling inspired method, distance-based method, and stratified scenario sampling

183 as scenario reduction methods. They concluded that the scenario  
184 reduction methods could effectively reduce the size of the larger  
185 models with a complete set of scenarios. However, for their case  
186 study, the distance-based method is the most accurate among the  
187 others.

188 Although these methodologies are accurate and more time  
189 efficient, they face some limitations in the complex analysis.  
190 For example, PEM is not capable of capturing the uncertainty in  
191 the correlated environment [24]. As for UT, it can be employed  
192 just when uncertain parameters have a symmetric distribution  
193 such as Normal [25].

194 Furthermore, in these methods, the computational burden  
195 is dependent on the number of uncertain parameters and can  
196 demand more computational effort than MCs in large-scale  
197 problems. This paper proposes a new uncertainty modeling  
198 method with high accuracy but a meager computational effort to  
199 deal with these problems. The main contributions of the paper  
200 are summarized as follows:

- 201 • Since the EV owners potentially could participate in the  
202 energy communities, this paper aims to analyze the impact  
203 of the uncertain EV owner behaviors on the energy trading  
204 in a community.
- 205 • The EVs' arrival and departure times usually follow known  
206 behaviors. However, these behaviors can be estimated by a  
207 combination of the different PDFs. So, the paper's second  
208 contribution is to propose a probabilistic uncertainty mod-  
209 eling approach that is not dependent on the features of the  
210 PDF, such as correlation, skewness, and so on.
- 211 • Analyzing the impact of local energy sharing on the distri-  
212 bution feeders under EV uncertainty is the third contribu-  
213 tion of this paper.

214 The remainder of this paper is organized as follows. The em-  
215 ployed P2P trading framework, as well as the uncertain behavior  
216 of the EVs, are explained in Section II. The proposed uncertainty  
217 modeling is then described in Section III. Section IV deals with  
218 evaluating the performance of the proposed method through four  
219 case studies covering a wide area and various situations. Finally,  
220 Section V wraps the paper up with a conclusion.

## 221 II. EV UNCERTAINTY IN P2P TRADING

222 In this section, we describe the community-based P2P trading,  
223 which has been proposed in [32]. This framework provides the  
224 opportunity for energy trading between various prosumers that  
225 build a community together. Indeed, the required energy of the  
226 prosumers and consumers is provided by the main grid, the  
227 renewable sources, as well as P2P trading with other members  
228 of the community shown by dashed arrows in Fig. 1. Since the  
229 houses in a neighborhood or the buildings in commercial or  
230 official centers can form a community, analysis in [3], [33], [34]  
231 are employed to model the uncertainties in arrival, sojourn, and  
232 departure times of the EVs which are charged near the owners'  
233 house or workplace.

234 Eqs. (1) to (6) describe the structure of the community-based  
235 P2P trading. The objective function (1) minimizes the cost of  
236 energy importing from the main grid. Equation (2) models the  
237 P2P energy trading between houses  $h$  and  $p$ , considering the  
238 loss factor  $\psi^{P2P}$ . Each house can trade with its peers at each

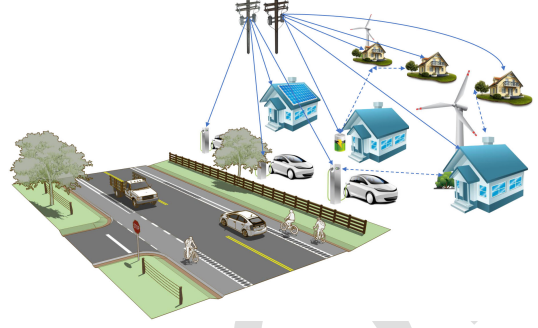


Fig. 1. Schematic illustration of a community made up of buildings-residential or office- as well as EV charging nodes.

time-step. So, equations (3) and (4) show the total Export and  
239 export of each house at each time-step, respectively. Since all  
240 trades happen within the community, the total amount of imports  
241 is proportional to the exports as it is shown in (5). 242

$$OF = \min \left\{ \sum_h \left( \sum_t \left[ p_G^{(t)} \cdot G^{(t,h)} \right] \right) \right\} \quad (1)$$

$$I_p^{(t,h \leftarrow p)} = \psi^{P2P} \cdot X_p^{(t,p \rightarrow h)} \quad \forall p \neq h, \quad (2)$$

$$X^{(t,h)} = \sum_{p \neq h} X_p^{(t,h \rightarrow p)} \quad (3)$$

$$I^{(t,h)} = \sum_{p \neq h} I_p^{(t,h \leftarrow p)} \quad (4)$$

$$\sum_h \psi^{P2P} \cdot X^{(t,h)} = \sum_h I^{(t,h)} \quad \forall t \in T. \quad (5)$$

Finally, (6) balances the input and output energy of each  
243 house at each time-step. In these equations,  $p_G^{(t)}$ ,  $G^{(t,h)}$ ,  $I_p^{(t,h \leftarrow p)}$ ,  
244  $X_p^{(t,p \rightarrow h)}$  are energy price, grid import of house  $i$ , energy import  
245 of house  $h$  from  $p$  and energy export of house  $p$  to  $h$ , all at  
246 time-step  $t$  respectively. 247

$$\overbrace{res^{(t,h)} + G^{(t,h)} + D_{ev}^{(t,h)} + I^{(t,h)}}^{\text{RES+Grid+EV disch. +P2P purchase}} \geq \overbrace{dem^{(t,h)} + C_{ev}^{(t,h)} + X^{(t,h)}}^{\text{Demand+EV charge+P2P sale}} \quad (6)$$

The equations related to battery or EV state of charge (SoC)  
248 must be included for the corresponding prosumers, as can be  
249 seen in equations (7) to (11). 250

$$S_{EV}^{(t,e)} = S_{EV}^{(t-1,e)} + \eta_{EV}^c \cdot C_{EV}^{(t,e)} - (1/\eta_{EV}^d) \cdot D_{EV}^{(t,e)} \quad (7)$$

$S_{EV}^{(t,e)}$ ,  $C_{EV}^{(t,e)}$ ,  $D_{EV}^{(t,e)}$ ,  $\eta_{EV}^c$ , and  $\eta_{EV}^d$  are state of charge, charg-  
251 ing, discharging, charging efficiency, and discharging efficiency,  
252 respectively. (8) and (9) specify the EVs' state of charge at their  
253 arrival and departure times, respectively. 254

$$S_{EV}^{(t,e)} = S_{EV}^{Arrival(e)} + \eta_{EV}^c \cdot C_{EV}^{(t,e)} - (1/\eta_{EV}^d) \cdot D_{EV}^{(t,e)} \quad t = Arrival \quad (8)$$

$$S_{EV}^{(t,e)} = S_{EV}^{Departure(e)} \quad t = Departure \quad (9)$$



TABLE I  
PROBABILITY DENSITY FUNCTION OF SOJOURN TIME FOR “CHARGE NEAR WORK,” “CHARGE NEAR HOME” CLUSTERS FOR THE FIRST 24 HOURS [33]

	PDF	Location	Scale	[min max] hours
Charge near work	Logistic	0.27	0.06	[5.00 18.52]
Charge near home	Logistic	0.56	0.08	[0.02 23.99]

255 Finally, Eqs. (10) and (11) define the charging and discharging  
256 rate of the EVs.

$$0 \leq C_{EV}^{(t,e)} \leq \alpha_{EV} \quad (10)$$

$$0 \leq D_{EV}^{(t,e)} \leq \beta_{EV} \quad (11)$$

257 As can be seen, the arrival and departure times are required  
258 to model the EVs’ participation in the community. According to  
259 (12), the departure time can be calculated based on arrival and  
260 sojourn times.

$$t^{departure} = t^{arrival} + t^{sojourn} \quad (12)$$

261 The authors in [33], [34] have analyzed real-world data sets and  
262 classified the EVs into three clusters named “Charge near work  
263 (CNW),” “Charge near home (CNH),” and “Park to charge”  
264 clusters, considering the influence of weekends and seasonal  
265 changes. Then, they have fitted distributions to the sojourn time  
266 of EVs belong to each cluster. Table I shows the probability den-  
267 sity functions of sojourn time for the first and second behavioral  
268 clusters.<sup>1</sup> It must be noted that the presented PDFs are based  
269 on normalized sojourn time concerning the last column ([min  
270 max]) of Table I.

271 There are different approaches for estimation of a PDF for  
272 the EVs’ arrival times. In [3], a normal distribution has been  
273 assumed for the arrival time of the EVs to work or home. In  
274 another approach, according to [34] the arrival times can be  
275 uni/multi-modal or skewed in various situations. In ref [35], the  
276 arrival times are generated based on a normal distribution. How-  
277 ever, the average arrival time of the different EVs is generated  
278 by a Pearson distribution.

279 In this study, we assume a combination of two normal distri-  
280 butions for multi-modal situations and one normal PDF for the  
281 uni-modal cases as can be seen in Eq. (13) and (14), respectively.

$$f_{CNH}(x) = \begin{cases} \frac{1}{0.5\sqrt{2\pi}} \exp\left(-\frac{(x-33)^2}{2 \times 0.5^2}\right) & a < 0.5 \\ \frac{1}{1\sqrt{2\pi}} \exp\left(-\frac{(x-40)^2}{2 \times 1^2}\right) & \text{otherwise} \end{cases} \quad (13)$$

$$f_{CNW}(x) = \frac{1}{\frac{4}{3}\sqrt{2\pi}} \exp\left(-\frac{(x-14)^2}{2 \times (\frac{4}{3})^2}\right) \quad (14)$$

282 Which  $a$  is a random number between 0 and 1 with uniform  
283 distribution. So, in 50 percent of situations, a normal distribution  
284 with  $\mu = 33$  and  $\sigma = 0.5$  represents the EV arrival for CNH  
285 cluster. In the rest, another normal PDF with  $\mu = 40$  and  $\sigma = 1$   
286 shows the behavior of the arrival time for the mentioned cluster.  
287 Also, a normal distribution with  $\mu = 14$  and  $\sigma = \frac{4}{3}$  is considered  
288 for the arrival times of CNW cluster. It is worth noting that the  
289 mentioned formulation of the community-based P2P trading is

<sup>1</sup>We assume the communities contain residential, office or commercial pro-  
sumers.

a day ahead schedule with 30 min time-steps (48 time-steps for  
290 24 hours). Therefore, mean and standard deviations in equations  
291 (13) and (14) refer to the time step and standard deviation  
292 of the EV arrivals in each cluster. It is worth noting that the  
293 departure time is a summation of two uncertain parameters  
294 with logistic and a combination of normal distributions. As we  
295 don’t have the exact data for the skewness of the PDFs when  
296 they are skewed, we just follow the assumption in [3]. This  
297 assumption will not affect the proposed method proficiency, as  
298 we will show it’s ability in modeling the correlated-uncorrelated  
299 and symmetric/non-symmetric uncertain parameters. Moreover,  
300 arrival SoC would be the last uncertain parameter related to the  
301 EV behavior. Indeed, the arrival state of charge depends on the  
302 storage capacity, efficiency coefficient of the EV during driving  
303 ([km/kwh]) as well as the traveled distance, and can be calculated  
304 according to (15) [36].  
305

$$SoC^{arrival} = 100 - \frac{d}{C_{eff} \times Cap} \quad (15)$$

In this equation, the traveled distance is uncertain and leads  
306 to uncertain EVs’ state of charge at their arrival. Paper [37]  
307 uses a generalized extreme value distribution with  $\mu = 17.27$ ,  
308  $\sigma = 0.84$ , and  $k = -0.06$  to model this uncertain parameter.  
309

### III. UNCERTAINTY MODELING 310

Monte-Carlo simulation is a powerful tool to map the uncer-  
311 tain behavior of the input parameters to output in a process.  
312 However, it needs a high number of iterations and can be  
313 time-consuming. So, scenario reduction methods that can reduce  
314 the required iterations gain importance. This section introduces a  
315 heuristic approach to capture the main behavior of the uncertain  
316 parameter by a few points. Assume  $X$  is an uncertain parameter  
317 with an arbitrary PDF, e.g. see the blue curve in Fig. 2(a)–(c). A  
318 considerable run-time reduction can happen if the most probable  
319 samples are selected and other samples that are not as important  
320 as previous ones are omitted. To highlight the most probable  
321 scenarios of uncertain parameter  $X$ ,  $N$  vectors  $X_i[1 \times n]^T$  are  
322 generated according to the specifications of the PDF of  $X$  by  
323 (16). Then, the mean value ( $\mu_i$ ) and standard deviation ( $\sigma_i$ ) of  
324 each vector are calculated using (17) and (18).  
325

$$X'_i = \text{random}(\text{PDF of } X', \text{ shape, scale, } [1, n]) \quad (16)$$

$$\mu_i = \frac{1}{n} \sum X'_i \quad \forall i = 1 : N \quad (17)$$

$$\sigma_i = \sqrt{\frac{1}{n} \sum |X'_i - \mu_i|^2} \quad \forall i = 1 : N \quad (18)$$

Because of probabilistic nature, and may be different in each  
326 generated vector in case of low number of samples. Each time,  
327 the period of  $C_i = [\mu_i - k\sigma_i, \mu_i + k\sigma_i]$  is selected and an arbi-  
328 trary fuzzy function is set on it. The main reason for producing a  
329 reasonably unbiased PDF is that the aggregated fuzzy function  
330 would peak around the critical samples. However, the amplitude  
331 of the peak may differ depending on the importance of the  
332 samples. So, the amplitude of each peak is considered as the  
333 selection criteria. For instance, a Gaussian membership function  
334

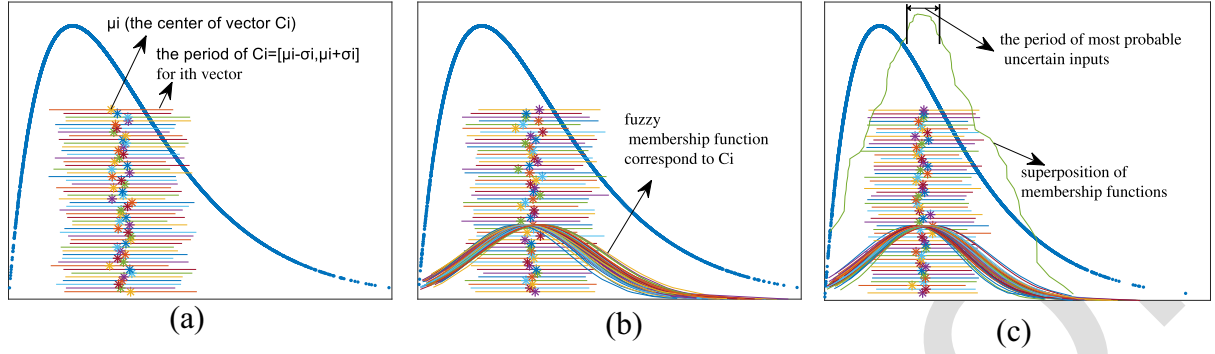


Fig. 2. (a) probability density function and confidence levels of uncertain input in various generations- (b) corresponding fuzzy membership function to each confidence level- (c) superposition of membership functions.

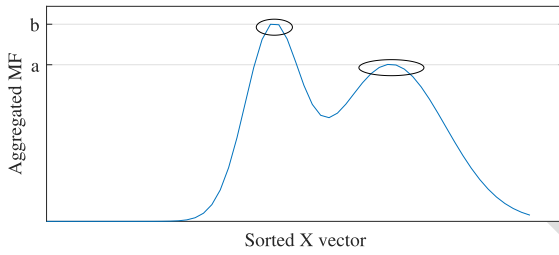


Fig. 3. Multi-modal aggregated fuzzy FM.

335 can be indicted by (19).

$$f_{ij}(X'_{ij}, \sigma_i, u_i) = e^{-\frac{(X'_{ij})^2}{2\sigma_i^2}} \quad \forall i = 1 : \text{length}(C_i) \quad (19)$$

336  $X'_{ij}$  is  $j$ th element of ascending sorted  $X'_i$ . This period and its  
 337 corresponded fuzzy membership function for all  $N$  vectors have  
 338 been shown in Fig. 2(a) and 2(b) respectively. As can be seen in  
 339 Fig. 2(c), the most probable values for uncertain parameter  $X$   
 340 can be detected by superposition of fuzzy functions. Equation  
 341 (20) shows the element-wise superposition of produced fuzzy  
 342 functions with (19).

$$F_j = \sum_{i=1}^N f_{ij}(X'_{ij}, \sigma_i, u_i) \quad (20)$$

343 where  $F_j$  is  $j$ th element of superposition vector  $F$ . In fact,  
 344 important samples are further repeated during sampling and  
 345 more number of fuzzy membership functions will be added  
 346 together. This leads to the fact that peak of vector  $F$  corresponds  
 347 to most probable sample.

348 In some situations, the superposition of the fuzzy MFs leads to  
 349 a multi-modal curve like Fig. 3. In such situations, although the  
 350 value of one local peak may be lower than the others, the peak  
 351 states that there are some important scenarios for the uncertain  
 352 parameter around that area. So, for selecting the most probable  
 353 samples all points around the local peaks of the  $F$  function must  
 354 be taken into account. Finally, the number of the samples around  
 355 each peak is proportional to the peak value of  $F$  in peak points  
 356 i.e.  $a$  and  $b$  in Fig. 3.

357 Table II provides pseudo code of the presented method.

TABLE II  
PROPOSED ALGORITHM

<b>Step 1</b>	<b>// Recognition of important samples</b> for $1 : N$ times // $N$ is the number of superposition Generate $1 \times n$ vector $X'_i$ with specification of $X$ Fit the fuzzy MF to ascending sorted $X'_i$ vector end add generated fuzzy functions Select the highest period as the most probable samples // Step 1 must be repeated for each uncertain parameter.
<b>Step 2</b>	<b>//Mapping of input uncertainty to output</b> Calculate the output function for selected inputs
<b>Step 3</b>	<b>//Calculation of output uncertain behavior</b> Compute the output parameters

#### IV. ASSUMPTIONS

This section briefly summarizes the assumptions made in this study.

- It is assumed that the arrival time of the CNW EVs, follows a normal distribution.
- It is assumed that the arrival time of the CNH EVs, follows a combination of two normal distribution as can be seen in (13). The histogram of the generated numbers by this equation has two modes and is not symmetric.
- It is assumed that the sojourn times of the CNH and CNW categories follow the distributions shown in Table I. The sojourn time is not.
- There is no feed-in tariff in the community model.
- It is assumed that the community manager operates the community independent of the grid operator.
- It is assumed that the residential loads have a constant power factor.

#### V. SIMULATION RESULT

In this paper, two case studies are employed to illustrate the impact of the EV uncertainty on collaborative energy consumption in a community. Since the community is operated by a community manager independent of the grid operator, the first case focuses on the community model and ignores the physical grid. The second case study deals with the propagation of the EV

TABLE III  
DESCRIPTION OF THE COMMUNITY ASSETS IN CASE IV

Method	Houses
2 [kW] PV	2, 7, 8, 9, 16, 20, 24, 25
4 [kW] PV	5, 15, 23
2.3 [kW] Wind	3, 15, 20, 25
4 [kWh] Storage	5, 15, 23

382 uncertainty on the grid parameters, mainly voltage magnitude.  
383 The cases, and results are explained in the following subsections.

#### 384 A. Case I: EV Uncertainties in Collaborative Energy 385 Consumption

386 A neighborhood consisted of 25 houses located in the U.K.  
387 has been employed to evaluate the performance of the proposed  
388 method. More information can be found in [9]. As can be seen  
389 in Table III, some buildings own assets like 4 kWh storage,  
390 2 kW and 4 kW PVs as well as 2.3 kW wind turbines. The  
391 real-world data used in [32] is employed for the mentioned  
392 assets. According to this data and configuration of the community,  
393 37% of the annual demand of the whole community is covered  
394 by renewable sources, approximately. Besides, EVs that belong  
395 to people who are living or working in the community, can  
396 participate in P2P trading program. It's worth noting that the  
397 simulations of this case study have been done for one month in  
398 spring, considering the seasonal impact on the EV availabil-  
399 ity patterns based on equations (13) and (14). It is assumed  
400 that three EVs owned by residents of the neighborhood (CNH  
401 cluster) join the community every day. Also, two EV owners  
402 who work near that area (CNW cluster) prefer to participate  
403 in P2P program of the community. Also, according to the model,  
404 EVs should be charged to a certain level before they leave. The  
405 demand profiles of the first case have been formed based on smart  
406 meter energy consumption data in London households, as part of  
407 Low Carbon London project [https://data.london.gov.uk/dataset/  
408 smartmeter-energy-use-data-in-london-households](https://data.london.gov.uk/dataset/smartmeter-energy-use-data-in-london-households). The infor-  
409 mation related to the wind and solar profiles, as well as  
410 the half-hourly energy prices are accessible through the fol-  
411 lowing GitHub page: [https://github.com/LocalEnergyMarkets/  
412 PCDGMModel-LocalCommunities](https://github.com/LocalEnergyMarkets/PCDGMModel-LocalCommunities)

413 In the following, we analyze the EV uncertainties as well as  
414 operation in two situations. At first, we assume the EVs have  
415 one-way charger and can import energy from the main grid or  
416 the other prosumers in the community. In the other situation,  
417 the EVs can actively participate in the P2P trading due to their  
418 bidirectional chargers.

419 The calculated confidence levels which are illustrated in Fig. 4  
420 cover the deterministic value of the objective function in (1)  
421 for the simulation period. It shows the ability of the proposed  
422 method in mapping the input uncertainty with various distribu-  
423 tions or a combination of PDFs like (13) to the output function.  
424 To calculate these confidence intervals, we have only selected  
425 20 samples of each uncertain parameter using the proposed  
426 method. Table IV compares the performance of the method  
427 in terms of confidence levels for two cases, i.e., 20 scenarios  
428 and 5 scenarios. Indeed, the number of scenarios refers to the  
429 number of the most probable samples that are recognized by

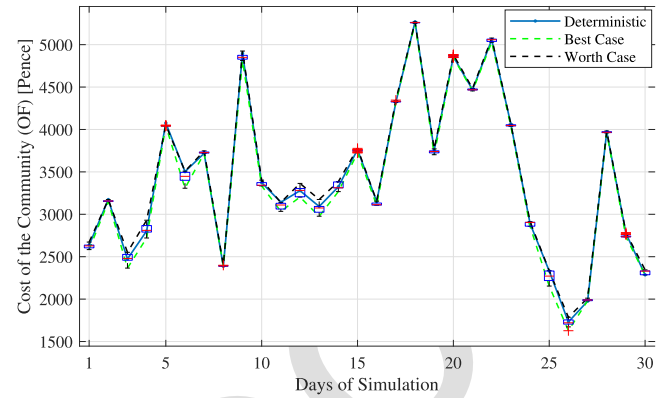


Fig. 4. Comparison of the deterministic and probabilistic community costs for 30 days.

TABLE IV  
COMPARISON OF THE RELATIVE CONFIDENCE LEVELS OF 20 SCENARIOS WITH 5 SCENARIOS

Number of Scenarios	Max Confidence [%]	Avg Confidence [%]
20	9.1433	2.7674
5	8.2390	2.0067

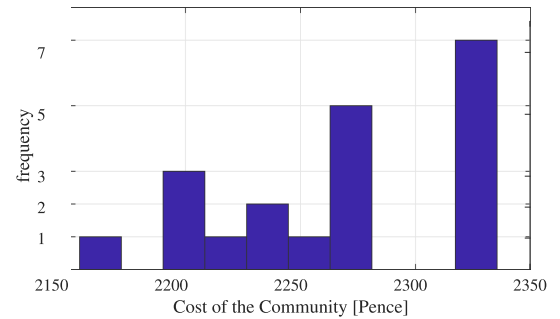


Fig. 5. Histogram of the community cost with 20 scenarios.

TABLE V  
IMPACT OF THE EVs IN P2P TRADING - ONE MONTH RESULT

Total trades [kWh]	No EV	One-way charger	Bidirectional charger
Import-all participants	1088	1154	1356
Export-all participants	1178	1249	1467
Import-EVs	-	139	161.4
Export-EVs	-	0	241.5

430 the proposed superposition method. In case of 20 scenarios,  
431 relatively higher confidence levels are achieved due to covering  
432 more scenarios than the other case shown in Table IV. To better  
433 analyze the performance of the proposed method, day 25 -with  
434 the biggest standard deviation as can be seen in Fig. 4- has been  
435 selected for more investigations. Fig. 5 illustrates the calculated  
436 histogram of the community cost based on 20 scenarios. Due to  
437 the low number of the selected samples, only the most probable  
438 scenarios are illustrated in this histogram. Scenarios with a  
439 cost around 2320–2340 [Pence] have repeated 7 times. The  
440 deterministic value of the community cost for the same day is  
441 2331 [Pence] which has been covered by the proposed method.  
442 Table V reveals the impact of the EVs in the amount of the  
443 total P2P trading in the community. As can be seen, involving  
444 EVs -even with unidirectional chargers- in P2P trading augments

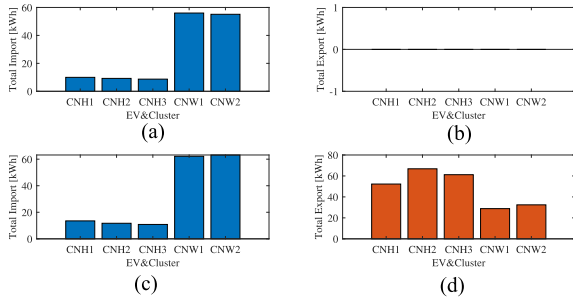


Fig. 6. Total P2P trading of EVs- (a) Import, Unidirectional charger - (b) Export, Unidirectional charger - (c) Import, Bidirectional charge - (d) Export, Bidirectional charger.

445 the volume of energy traded. This impact scales up when the  
 446 EVs can actively participate in the trades, using bidirectional  
 447 chargers. An interesting finding is the different tendency of the  
 448 mentioned clusters in P2P trades, as can be seen in Fig. 6. The  
 449 EVs belong to CNH cluster tend to export energy to their peers,  
 450 when is possible. It means that in case of using the bidirectional  
 451 chargers for charging EVs near the houses, the EVs behave like  
 452 the stationary batteries in their availability. On the other hand,  
 453 the EVs belong to CNW tend to import energy from the other  
 454 prosumers. It's due to the simultaneous availability times and  
 455 PV production periods.

456 The energy community model has been implemented in Mat-  
 457 lab R2019b. Solving the community model for 25 houses and  
 458 5 EVs (on a laptop: RAM 32 GB, CPU intel core i 7) on average  
 459 takes 11 sec. This run-time includes building and solving the  
 460 model using the linprog solver. It should be noted that the sparse  
 461 implementation of the model can speed up the building process.  
 462 However, in this case, the time reported is based on the Matlab  
 463 optimization toolbox. The proposed method reduces the number  
 464 of runs (compared to MCs) by eliminating the unimportant  
 465 sample points.

### 466 B. Case II: The Impact of EV Uncertainties in Collaborative 467 Energy Consumption on the Grid Operation

468 This case study focuses on the impact of energy sharing on  
 469 the distribution feeder that supplies the end-users. In this case,  
 470 in addition to the demand profiles, prices, and solar profiles,  
 471 a low voltage distribution feeder is employed to evaluate the  
 472 energy sharing impact. This feeder (feeder lvgd-2388 connected  
 473 to the medium voltage grid called mvgd-2) is not a real but  
 474 a realistic synthetic low voltage grid generated by the Ding0  
 475 package<sup>2</sup> in python. The feeder has 84 buses and 83 branches  
 476 and supplies 27 residential end-users, and the nominal voltage  
 477 is 0.4 kV. The grid topology and the line characteristics are  
 478 shown in the appendix. Since this synthetic grid only provides  
 479 information about one snapshot of the demand and distributed  
 480 generation, various energy consumption, and generation profiles  
 481 are assigned to different grid nodes. In addition, there are 5 EVs  
 482 in the neighborhood. The battery capacity of the EVs in this  
 483 case is 50 kWh with a round-trip efficiency of 96% as the

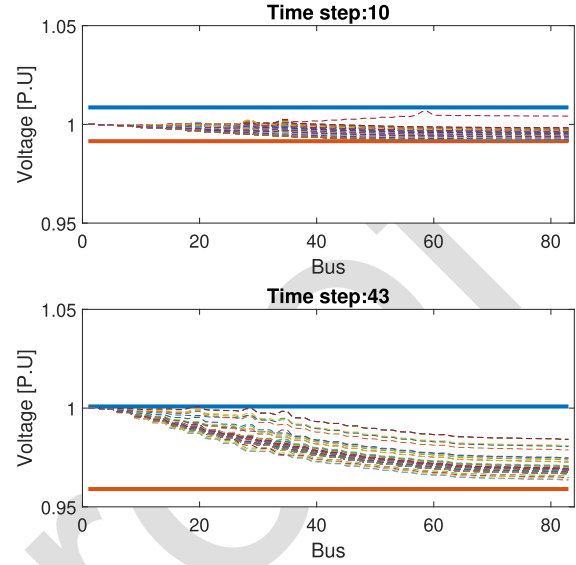


Fig. 7. Comparing the performance of the proposed method with MCs.

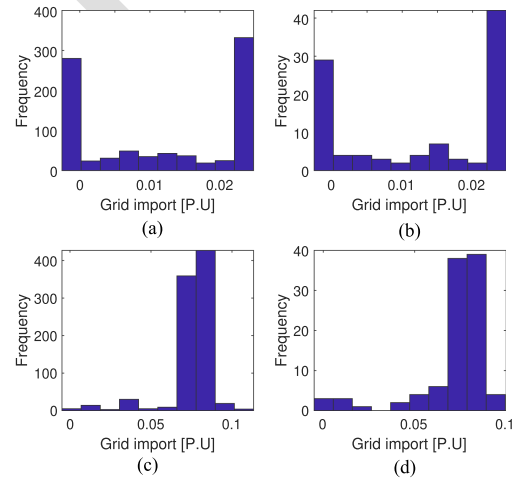


Fig. 8. Comparing the histograms of the grid import, obtained by the proposed method and MCs. (a) Time step:10, MCs. (b) Time step:10, Proposed method. (c) Time step:43, MCs. (d) Time step:43, Proposed method.

484 average of Nissan Leaf, Volkswagen e-Golf and Tesla S [9]. 484  
 485 The charging and discharging rate of the EVs are set to 7.3 kWh 485  
 486 per hour. It is worth noting that the profiles are based on the 486  
 487 same references as the previous case. To analyze the impact of 487  
 488 the local energy transactions on the grid, the community is first 488  
 489 analyzed under the uncertain behavior of the EVs, regardless 489  
 490 of the grid constraints. Then, the ex-post analysis is conducted 490  
 491 to understand the impact of the EV uncertainty on the grid. In 491  
 492 other words, the outcome of the energy community is converted 492  
 493 to the active power injections into different nodes of the grid. 493  
 494 The reactive power injections also are estimated based on the 494  
 495 loads' power factor. The Matpower toolbox<sup>3</sup> then is employed 495  
 496 to run the powerflow calculation on the grid. This assumption is 496  
 497 based on [38] and [39], that have separated the market and grid 497  
 498 layers. 498

<sup>2</sup>[Online]. Available: <https://dingo.readthedocs.io/en/dev/welcome.html>

<sup>3</sup>[Online]. Available: <https://matpower.org/>



Before dig in the result of this case, it is worth mentioning that the this case is based on 100 scenarios extracted from 1000 scenarios. Fig. 7 compares the voltage profiles obtained by each scenario with the minimum and maximum voltage magnitudes given by MCs. Indeed, the solid blue and red lines illustrate the maximum and minimum voltages over the feeder during the day calculated by MCs (850 scenarios). The dashed lines are the voltage profiles for extracted scenarios. Two time steps, representative of low load (10 - 05:00), afternoon (30), and high load (43 - 21:30) are exhibited in figure 7. As can be seen, the voltage profiles lay in the voltage range estimated by MCs. This figure also indicates that energy sharing does not jeopardize the grid in terms of over or under-voltage problems, even under heavy load. Obviously, this is not a general conclusion and is relevant for the case study. However, there might be some situations, especially in the future, that the end-users may experience overvoltage due to either EV participation in energy sharing programs or an increase in the share of renewable generation. Comparing the histograms of the energy imported from the grid for these two cases could be interesting. Fig. 8 shows the histograms of the energy imported from the main transformer at 05:00 and 21:30. Although the histograms do not represent a specific PDF, the proposed method provides similar histograms to the ones obtained by MCs.

## VI. CONCLUSION

This paper proposed a novel and accurate stochastic method for uncertainty modeling based on the superposition of uncertain input parameters. To this end, the most significant and probable samples of the input uncertain parameters are recognized through the superposition of various sampling vectors under fuzzy transformation. After forming some vectors containing samples of the uncertain parameter, an arbitrary fuzzy membership function is assigned to each vector. The simulation results show the appealing performance of the proposed method with high accuracy and a very low computational burden. As another significant feature of the proposed method, it can calculate the output histogram with just a few number of input samples. These findings reveal a promising role for the proposed method in modeling the uncertainty effects in the real practical power system problems. For example, EVs that are being utilized increasingly impose uncertainty on the grid or market operation. Besides, their arrival or sojourn times may have bi-modal and skewed or non-skewed PDFs that can not be modeled by the methods like PEM or UT accurately. But, the proposed method showed satisfactory performance in the described situation. Such a special feature can play an important role in addressing the big issues of computational burden, high complexity and low accuracy in the literature. Taken together, this paper has identified the tendency of the EVs to participate in local P2P tradings. Indeed, the EVs that are charged near the owners' houses tend to export energy to the other prosumers. Because they are connected to the charging stations in the evening and leave the house in the morning. So, case of using bidirectional chargers, their operation is similar to stationary batteries in the sojourn time. It means that active operation of the EVs belong to CNH cluster in P2P trading can increase the flexibility of the local markets. On the other hand,

the availability time of the CNW cluster is from the morning when the people go to their offices until they go back to their homes. So, they tend to import energy from the neighborhood buildings that have renewable productions.

To sum up, regarding the input data, the energy price, renewable profiles, and the demand of the houses are available and can be easily found. However, the arrival and departure times of the EVs for a certain period are not available to be considered as the basis of the comparisons. The results have been compared to the artificial EV data generated by the known PDFs.

## APPENDIX

*Low voltage synthetic grid data generated by Ding0 package:*  
The low voltage distribution feeder used in the second case study

TABLE A1  
BRANCH INFORMATION

From bus	To bus	R [P.U]	X [P.U]
1	2	0.00135625	0.00013296875
15	16	0.0165	0.004227896875
16	17	0.0007015625	0.0001325
18	19	0.009	0.002306125
18	21	0.0066625	0.00326725625
15	18	0.0066625	0.003265625
19	20	0.0007015625	0.0001325
21	22	0.0165	0.004227896875
21	24	0.0066625	0.003265625
22	23	0.0007015625	0.0001325
24	25	0.009	0.002306125
24	27	0.0066625	0.003265625
25	26	0.0007015625	0.0001325
27	28	0.0165	0.004227896875
27	30	0.0066625	0.003265625
28	29	0.0007015625	0.0001325
30	31	0.009	0.002306125
30	33	0.0066625	0.003265625
3	4	0.012628125	0.002390625
1	3	0.02510625	0.00980176875
31	32	0.0007015625	0.0001325
33	34	0.0165	0.004234375
33	36	0.0066625	0.00326725625
34	35	0.0007015625	0.0001325
36	37	0.009	0.0023125
36	39	0.0066625	0.00326725625
37	38	0.0007015625	0.0001325
39	40	0.0165	0.004234375
39	42	0.0066625	0.00326725625
40	41	0.0007015625	0.0001325
42	43	0.009	0.0023125
42	45	0.0066625	0.00326725625
43	44	0.0007015625	0.0001325
45	46	0.0165	0.004234375
45	48	0.0066625	0.00326725625
4	5	0.0007015625	0.0001325
46	47	0.0007015625	0.0001325
48	49	0.009	0.002306125
48	51	0.0066625	0.003265625



From bus	To bus	R [P.U]	X [P.U]
49	50	0.0007015625	0.0001325
51	52	0.0165	0.004227896875
51	54	0.0066625	0.003265625
52	53	0.0007015625	0.0001325
54	55	0.009	0.0023125
54	57	0.0066625	0.003265625
55	56	0.0007015625	0.0001325
57	58	0.0165	0.004227896875
57	60	0.0066625	0.003265625
58	59	0.0007015625	0.0001325
60	61	0.009	0.002306125
60	63	0.0066625	0.003265625
6	7	0.012628125	0.002385646875
6	9	0.0066625	0.00326725625
1	6	0.005078125	0.003234375
61	62	0.0007015625	0.0001325
63	64	0.0165	0.004227896875
63	66	0.0066625	0.003265625
64	65	0.0007015625	0.0001325
66	67	0.009	0.002306125
66	69	0.0066625	0.00326725625
67	68	0.0007015625	0.0001325
69	70	0.0165	0.004227896875
69	72	0.0066625	0.003265625
70	71	0.0007015625	0.0001325
72	73	0.009	0.002306125
72	75	0.0066625	0.003265625
73	74	0.0007015625	0.0001325
75	76	0.0165	0.004227896875
75	78	0.0066625	0.003265625
7	8	0.0007015625	0.0001325
76	77	0.0007015625	0.0001325
78	79	0.009	0.0023125
78	81	0.0066625	0.00326725625
79	80	0.0007015625	0.0001325
81	82	0.0165	0.004234375
82	83	0.0007015625	0.0001325
9	10	0.0231515625	0.0043736859375
9	12	0.005078125	0.003234375
10	11	0.0007015625	0.0001325
12	13	0.012628125	0.002390625
12	15	0.0066625	0.00326725625
13	14	0.0007015625	0.0001325

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567 has been generated by the Ding0 package which is a tool for  
568 generating synthetic medium and low voltage grids. The grid  
569 topology and the line impedance are presented in the table below.  
570 The base values for voltage and power are 0.4 kV and 0.25 MVA,  
571 respectively. This grid has 13 solar units connected to different  
572 nodes. All of them are models as PQ buses, as they do not have  
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574 The ID of this grid is lvgd-2388, connected to the medium  
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