Flexibility Requirement when Tracking Renewable Power Fluctuation with Peer-to-Peer Energy Sharing

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***Abstract*—Flexible load at the demand-side has been regarded as an effective measure to cope with volatile distributed renewable generations. To unlock the demand-side flexibility, this paper proposes a peer-to-peer energy sharing mechanism that facilitates energy exchange among users while preserving privacy. We prove the existence and partial uniqueness of the energy sharing market equilibrium and provide a centralized optimization to compute the equilibrium. The centralized optimization is further linearized by a convex combination approach, turning into a multi-parametric linear program (MP-LP) with renewable power output deviations being the parameters. The flexibility requirement of individual users is calculated based on this MP-LP. To be specific, an adaptive vertex generation algorithm is proposed to construct a piecewise linear estimator of the optimal total cost subject to a given error tolerance. Critical regions and optimal strategies are retrieved from the obtained approximate cost function to evaluate the flexibility requirement. The proposed algorithm does not rely on exact characterization of optimal basis invariant sets and thus is not influenced by model degeneracy, a common difficulty faced by existing approaches. Case studies validate the theoretical results and show that the proposed method is scalable.**

***Index Terms*—critical region, distributed renewable energy, energy sharing, flexibility, multi-parametric program.**

NOMENCLATURE

A. Indices, Sets, and Functions

$\mathcal{I}$	Set of consumers.
$\mathcal{J}$	Set of prosumers.
$l \in \mathcal{L}$	Line l in set $\mathcal{L}$.
$f_k(.)$	Disutility function of user $k \in \mathcal{I} \cup \mathcal{J}$.

B. Parameters

I	Number of consumers.
J	Number of prosumers.
d_k	Contract demand of user $k \in \mathcal{I} \cup \mathcal{J}$.
w_k	Forecast renewable output of prosumer $k \in \mathcal{J}$.
Δw_k	Real-time renewable power output deviation of prosumer $k \in \mathcal{J}$.
τ	Parameter in user's objective function.
π_{kl}	Line flow distribution factors.
F_l	Power flow limit of line $l \in \mathcal{L}$.
$\alpha_k, \beta_k, \zeta_k$	Parameters of the disutility function $f_k(.)$.
$\underline{D}_k, \overline{D}_k$	Lower/Upper bound of demand adjustable range.

C. Decision Variables

Δd_k	Demand adjustment of user $k \in \mathcal{I} \cup \mathcal{J}$.
q_k^c	Actual sharing amount of user $k \in \mathcal{I} \cup \mathcal{J}$.
q_k	Expected sharing amount of user $k \in \mathcal{I} \cup \mathcal{J}$.
δ_k	The gap between actual and expected sharing amount of user $k \in \mathcal{I} \cup \mathcal{J}$.
r_k^d	Flexibility requirement of user $k \in \mathcal{I} \cup \mathcal{J}$.

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I. INTRODUCTION

EXPLOITING distributed renewable generation is an effective remedy to reduce the dependence on fossil fuel energy and achieve a sustainable society [1]. Meanwhile, how to tackle the volatile and intermittent energy supply caused by renewable generation has become a major concern. Existing literature concentrates on two issues: 1) how to balance the real-time power in an optimal way facing the uncertainties; 2) how to quantify the system's potential in accommodating uncertain renewable power.

For the first question, plenty of works developed effective scheduling methods for the bulk power system that runs in a centralized manner. Typical techniques are stochastic optimization (SO) [2], robust optimization (RO) [3], and distributionally robust optimization (DRO) [4]. In stochastic optimization, the uncertain factors are modeled by their empirical distributions, and then a stochastic programming or chance-constrained problem is built. Though historical data can provide some hints for the construction of empirical distributions, the exact distributions can hardly be obtained. This inaccuracy leads to a sub-optimal scheduling strategy. Robust optimization utilizes an uncertainty set that contains all possible scenarios of the renewable power output. The worst-case performance is optimized via Benders Decomposition [5] or Column & Constraint Generation (C&CG) [6] algorithms. Despite its convenience, the RO method can be too conservative since it treats all possible outputs with equal probability and neglects the fact that severe events rarely happen. Distributionally robust optimization is in-between SO and RO, where uncertainty is described by a family of inexact probability distributions restricted in an ambiguity set.

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As the renewable energy generations change from large-capacity centralized units at the transmission level to small-capacity distributed units at the distribution level, various researchers seek to balance the real-time power with the help of peer-to-peer energy trading [7]. Existing energy sharing mechanisms can be categorized into cooperative-game-based ones, noncooperative-game-based ones, and optimization-based ones [8]. For the first category, allocation rules are designed so that all agents have the incentive to form coalitions and together acting towards the social optimum. A coalition formation game for peer-to-peer energy trading was established with proof of several nice properties [9]. Network constraint [10] and sustainable user participation [11] were further taken into account. Though high social efficiency can be achieved, private information is needed to design the allocation rules. For the second category, two typical models are Stackelberg games and (generalized) Nash games. In Stackelberg games, the operator moves first to determine the market prices and then the users follow as price-takers. The energy sharing among prosumers was modeled as a Stackelberg game [12] and the existence of a unique and stable equilibrium was proved [13]. A hybrid approach using stochastic optimization and Stackelberg game was developed for energy trading across the day-ahead and real-time periods [14], [15]. User's flexibility can be limited as they are followers under this setting. Moreover, it is hard to decide on an effective energy price especially with a large number of users, since private information may be needed and each user's capacity is too small to be observed. In (generalized) Nash games, the users first submit their bids and then the operator clears the market. The energy trading was modeled as a two-stage stochastic Nash game [16]. A generalized demand function based energy sharing mechanism was proposed [17], and a practical bidding process as well as its convergence condition was established [18]. The above models are for node-level sharing and how to incorporate network constraints remains to be investigated. For the third category, references [19] and [20] established two-stage energy sharing schemes based on alternating direction method of multipliers (ADMM) algorithms. They require the help of dual variables and the economic intuition behind is hard to explain. This paper aims to develop an energy sharing mechanism based on generalized Nash game, which can protect users' privacy, take into account network constraints, has a clear economic interpretation, and enlarge users' flexibility by letting them be bidders instead of followers. In this way, the flexibility of users is enhanced and it can facilitate the energy exchange among a wider area while network constraint is non-negligible.

For the second question, methods to quantify the flexibility of a system are based either on regions or metrics. For regions, the do-not-exceed limit (DNEL) region [21] and the dispatchable region [22] are two well-known concepts. The DNEL region is a hypercube of uncertain parameters that will not cause infeasibility of the scheduling problem, and its size is also optimized by the problem itself. Vast literature improves the DNEL region by considering the historical data [23], inexact probability distribution [24], and corrective topology control [25]. Projecting the feasible set of the scheduling problem on the uncertainty subspace, we can get the dispatchable region. In general, the DNEL region is an inner box approximation of the dispatchable region. For metrics, the flexibility envelope [26], the power/energy capacity [27], etc. were proposed. A method providing both region and metric was proposed [28]. The above works focused on centralized operation and cannot be directly applied to the energy sharing market. Reference [29] proposed the concept of absorbable region, but only region information is given and it viewed flexibility from a system level. This paper quantifies the flexibility requirement of individual users under peer-to-peer energy sharing by offering both region and metric information.

Quantifying the flexibility requirement of individual users involves solving a multi-parametric linear program (MP-LP) in this paper. In the majority of the current works, multi-parametric program algorithms rely on the computation of critical regions where degeneracy is a key challenge. To overcome this difficulty, reference [30] resorted to developing an approximate algorithm, which was then generalized to multi-parametric convex programs [31]. A prominent feature of the approximation methods is that the critical region is represented by simplices in the p-dimensional parameter space with $p+1$ extreme points. Such a partition may hide the impact of parameters on the optimal bases characterized by the exact critical regions which are not simplices in general. This paper proposes an alternative approximate algorithm for solving MP-LP. The basic idea is similar to [30] but no simplex is needed. The main contributions are two-fold:

1) **Energy Sharing Mechanism.** A peer-to-peer energy sharing mechanism that coordinates the energy exchange among users is presented. Each user bids the quantity it would like to share aiming to minimize its disutility and match the operator's sharing profile. The operator decides on the optimal sharing profile according to users' bids and the physical constraints. Compared to existing works, our mechanism is easy to implement without requiring private information; can incorporate the coupling network constraints; enlarges users' flexibility by letting them be bidders instead of followers; and have several nice provable properties. Under the proposed mechanism, all users play a generalized Nash game. We prove the existence and partial uniqueness of the generalized Nash equilibrium (GNE) theoretically. The equilibrium can be retrieved from a quadratic optimization problem parameterized in the renewable power output deviation, which is then turned into a multi-parametric linear program.

2) **Flexibility Characterization.** We characterize the flexibility under energy sharing based on the multi-parametric linear program derived above. To be specific, we develop an adaptive vertex generation (AVG) algorithm to partition the set of renewable generation deviations into critical regions, so that in each region, the demand adjustment of each user at equilibrium is shown to be a piecewise affine function in the deviations. Based on this unique feature, the flexibility requirement of users can be calculated. Compared to existing works, our method can provide both geometry information (critical region) and metric (flexibility requirement) about flexibility. The proposed AVG algorithm does not rely on the exact characterization of optimal basis invariant sets and thus is not influenced by model degeneracy, a common difficulty

encountered by solving multi-parametric linear programs.

The rest of this paper is organized as follows. Section II presents the energy sharing mechanism and the multi-parametric linear program to calculate the equilibrium. An adaptive vertex generation algorithm is developed in Section III to solve the multi-parametric program; based on the explicit demand adjustment policy, the flexibility requirement of individual users can be obtained. Case studies in Section IV validate the proposed models and methods. Conclusions are drawn in Section V.

II. Peer-to-Peer Energy Sharing Model

We consider a microgrid of I consumers (indexed by $k \in \mathcal{I} = \{1,...,I\}$) and J prosumers (indexed by $k \in \mathcal{J} = \{1,...,J\}$). All users $k \in \mathcal{I} \cup \mathcal{J}$ have demand while the prosumers are also equipped with renewable generators. We focus on the energy balancing at the real-time stage [32]. At the day-ahead stage, there are renewable power predictions $w_k, \forall k \in \mathcal{I}$ based on which the users decide on their contract demands $d_k, \forall k \in \mathcal{I} \cup \mathcal{J}$. Then at the real-time stage, the actual renewable power outputs may deviate from their predictions by $\Delta w_k, \forall k \in \mathcal{J}$. To maintain energy balancing, the users $k \in \mathcal{I} \cup \mathcal{J}$ can adjust their real-time demands by Δd_k and exchange q_k^c with each other via an energy sharing market. The disutility caused by demand adjustment can be represented as a quadratic function $f_k(\Delta d_k) := \alpha_k (\Delta d_k)^2 + \beta_k \Delta d_k + \zeta_k$, where α_k, β_k, and ζ_k are given parameters. This paper aims to quantify how much flexibility of individual users will be needed so that the real-time renewable power output deviations can be balanced.

Here, we assume the hour-ahead prediction is accurate and its deviation from the day-ahead prediction is accommodated by users' demand adjustments. To further consider the intra-hour forecasted errors, there are two possible approaches: 1) Implement the proposed energy sharing mechanism in a smaller time resolution, e.g. 15 min, where more precise predictions can be obtained. As shown in case studies, our mechanism is still applicable as it only takes 30 seconds to converge. 2) Settle the real-time unbalanced energy by buying (selling) from (to) the main grid, and introduce a penalty mechanism as in [12], [33] to settle the real-time losses according to each participant's contribution to the forecast error. Details can be found in Appendix A.

A. Energy sharing mechanism

The structure of the proposed energy sharing mechanism is shown in Fig. 1. First, each user enters its private information into the smart meter it links to, including its disutility function $f_k(\Delta d_k)$, the contract demand d_k, the demand adjustable range $[\underline{D}_k, \overline{D}_k]$, and the predicted output of renewable generators w_k if it is a prosumer. Then the smart meter will determine the optimal bid q_k by solving (2) or (3) and submit the bid to the operator. Upon receiving all the bids, the operator decides on the optimal sharing schedule to meet users' expectation as much as possible while satisfying the physical constraints by solving (1). Then the obtained optimal schedule q_k^c and unfulfilled quantity $\delta_k := q_k^c - q_k$ are returned to the smart meter. The smart meter will adjust its bid and submit it to the operator again. This happens until the optimal schedule $q_k^c, \forall k \in \mathcal{I} \cup \mathcal{J}$ changes little between the last two iterations. Finally, the users will execute the sharing schedule $q_k^c, \forall k \in \mathcal{I} \cup \mathcal{J}$. If $q_k^c > 0$, user k is a buyer; if $q_k^c < 0$, user k is a seller; if $q_k^c = 0$, it does not take part in sharing. The key of the energy sharing mechanism design lies in how each user chooses the optimal bid q_k and how the operator decides on the optimal sharing schedule $q_k^c, \forall k \in \mathcal{I} \cup \mathcal{J}$.

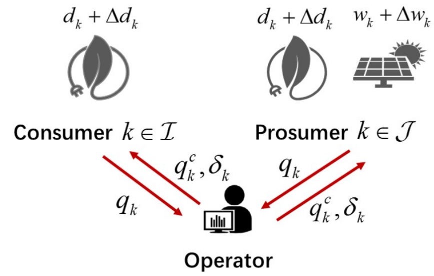

Fig. 1. Market structure of peer-to-peer energy sharing. Each consumer/prosumer offers a bid q_k to the operator; the operator clears the market and returns the sharing profile q_k^c with the gap δ_k. The users and operators are coordinated iteratively.

For the operator, given users' bids $q_k, \forall k \in \mathcal{I} \cup \mathcal{J}$, it solves problem (1) to set the optimal sharing schedule $q_k^c, \forall k \in \mathcal{I} \cup \mathcal{J}$:

$$\min_{q_k^c, \delta_k, \forall k \in \mathcal{I} \cup \mathcal{J}} \quad \sum\nolimits_{k \in \mathcal{I} \cup \mathcal{J}} \delta_k^2 \tag{1a}$$

$$\text{s.t. } \delta_k = q_k^c - q_k, \forall k \in \mathcal{I} \cup \mathcal{J} \tag{1b}$$

$$\sum\nolimits_{k \in \mathcal{I} \cup \mathcal{J}} q_k^c = 0 \tag{1c}$$

$$-F_l \le \pi_{kl} q_k^c \le F_l, \forall l \in \mathcal{L} \tag{1d}$$

The objective function (1a) minimizes the gap between the actual and expected sharing amount. Constraint (1c) means the energy sold equals the energy bought. The exchanged energy is transmitted via the power network limited by constraint (1d).

The smart meter decides on its optimal bid q_k by solving

For consumer $k \in \mathcal{I}$:

$$\min_{\Delta d_k, q_k} \quad f_k(\Delta d_k) + \frac{\tau}{2}(q_k^c - q_k)^2 \tag{2a}$$

$$\text{s.t. } q_k + \delta_k = d_k + \Delta d_k \tag{2b}$$

$$\underline{D}_k \le d_k + \Delta d_k \le \overline{D}_k \tag{2c}$$

For prosumer $k \in \mathcal{J}$:

$$\min_{\Delta d_k, q_k} \quad f_k(\Delta d_k) + \frac{\tau}{2}(q_k^c - q_k)^2 \tag{3a}$$

$$\text{s.t. } w_k + \Delta w_k + q_k + \delta_k = d_k + \Delta d_k \tag{3b}$$

$$\underline{D}_k \le d_k + \Delta d_k \le \overline{D}_k \tag{3c}$$

The objective of each user $k \in \mathcal{I} \cup \mathcal{J}$ consists of two parts: its disutility $f_k(\Delta d_k)$ and the gap between its bid q_k and operator's schedule q_k^c. Parameter τ indicates the trade-off between the above two parts and help evaluate the gap in monetary terms. Constraints include the power balance conditions for consumers (2b) and for prosumers (3b), and the demand adjustable range limits (2c) and (3c).

The proposed energy sharing mechanism well fits the practice in that the users do not need to know the network constraint (1d) available only to the operator, and the operator does not require local constraints (2c) and (3c) private to each user. Compared to centralized dispatch, our mechanism has two advantages: 1) **Regarding privacy.** Centralized dispatch needs to gather information from all users to make the central decision. However, the users may be unwilling to provide this sort of private information. In the proposed peer-to-peer energy sharing, each user only needs to submit a bid to the operator with its private data input only to its own smart meter, so privacy can be preserved. 2) **Less communication burden.** Under peer-to-peer energy sharing, the only information exchanged will be the bid, which is a scalar. However, under centralized dispatch, the operator needs to collect information such as cost coefficients, demands, and demand adjustable boundaries, from all users, which incurs a high communication burden overhead. Meanwhile, this mechanism can achieve a social optimal outcome (proved in Proposition 1 later).

Remark 1: Our proposed energy sharing mechanism focuses on the real-time market [32], which will be conducted hourly. In real-time, when the uncertainty is realized, we resort to a mechanism similar to the economic dispatch [34] (happens every several mins or an hour) to balance the system. Specially, instead of balancing the real-time power via centralized dispatch, we develop an energy sharing mechanism that allows the users to adjust their elastic demands and exchange energy with each other. Automatic Generation Control (AGC) that happens at a smaller time resolution and usually at the transmission level is not considered in this paper as most of the economic dispatch works [4], [34].

Remark 2: In this paper, deciding on the optimal bids and finally the sharing quantities involve solving optimization problems (2) or (3) and two-way communication with the operator. With the current technology, advanced smart meters have certain data processing abilities and enable two-way communication between the meter and the central system [35]. Therefore, it is reasonable to assume that with the assist of a smart meter, the user would be able to decide on the optimal bids. When taking into account various kinds of appliances and facilities, the decision-making may become more complex. In that case, we may need a more sophisticated energy management system to handle all the devices.

B. Market equilibrium

The above peer-to-peer energy sharing market can be regarded as a generalized Nash game [36] with its elements summarized in TABLE I. For simplicity, we use $\mathcal{G}=\{\mathcal{K},\mathcal{X},\Pi\}$ to denote the sharing game in an abstract form, where $\mathcal{K}$ is the set of players, $\mathcal{X}$ the action set, and Π the collection of cost functions. It's worth noting that, the action sets of users depend on the strategies of the operator (the $q_k^c, \delta_k, \forall k \in \mathcal{I}\cup\mathcal{J}$); the action sets of the operator depends on the strategies of the users (the $\Delta d_k, q_k, \forall k \in \mathcal{I}\cup\mathcal{J}$). Therefore, it constitutes a generalized Nash game, whose equilibrium is harder to analyze than the standard Nash game due to the complex coupling [37]. The equilibrium of the generalized Nash game, from which every player has no incentive to deviate, is defined below.

Definition 1. (Generalized Nash Equilibrium) A profile $(\Delta d^*, q^*, q^{c*}, \delta^*) \in \mathcal{X}$ is a *generalized Nash equilibrium* (GNE) of the energy sharing game $\mathcal{G}=\{\mathcal{K},\mathcal{X},\Pi\}$ if $\forall k=1,...,(I+J)$

$$\begin{aligned}(\Delta d_k^*, q_k^*) = \ & \text{argmin}_{\Delta d_k, q_k} \Pi_k(q_k^{c*}, \delta_k^*) \\ & \text{s.t.} \ \ (\Delta d_k, q_k) \in \mathcal{X}_k(q_k^{c*}, \delta_k^*)\end{aligned} \tag{4}$$

and

$$\begin{aligned}(q^{c*}, \delta^*) = \ & \text{argmin}_{q^c, \delta} \Pi_{I+J+1} \\ & \text{s.t.} \ \ (q^c, \delta) \in \mathcal{X}_{I+J+1}(\Delta d^*, q^*)\end{aligned} \tag{5}$$

In the following proposition, we prove the existence of the equilibrium and offer a centralized optimization problem to compute it, which casts down to multi-parametric program parameterized in the renewable power output deviations. Denote $\mathcal{Q} := \{q^c \mid \text{(1c) and (1d) are met}\}$, $\mathcal{D}_k := \{\Delta d_k \mid \underline{D}_k \le d_k + \Delta d_k \le \overline{D}_k\}$ for all $k \in \mathcal{I}\cup\mathcal{J}$.

Proposition 1. The energy sharing game $\mathcal{G}=\{\mathcal{K},\mathcal{X},\Pi\}$ has at least one GNE if (6) is feasible, and $(\Delta d^*, q^*, q^{c*}, \delta^*)$ is an GNE if and only if Δd^* is the unique optimal solution of

$$\min_{\Delta d_k, \forall k \in \mathcal{I}\cup\mathcal{J}} \ \sum\nolimits_{k\in\mathcal{I}\cup\mathcal{J}} f_k(\Delta d_k) \tag{6a}$$

$$\text{s.t.} \ q_k^c = \begin{cases} d_k + \Delta d_k, \forall k \in \mathcal{I} \\ d_k + \Delta d_k - w_k - \Delta w_k, \forall k \in \mathcal{J} \end{cases} : \eta_k \tag{6b}$$

$$\Delta d_k \in \mathcal{D}_k, \forall k \in \mathcal{I}\cup\mathcal{J} \tag{6c}$$

$$q^c \in \mathcal{Q} \tag{6d}$$

with $\delta_k^* = -\hat{\eta}_k/\tau$ for all $k \in \mathcal{I}\cup\mathcal{J}$, where $\hat{\eta}$ is the value of dual variable at optimum, and $q_k^{c*} = q_k^* + \delta_k^*$ with

$$q_k^* = \begin{cases} d_k + \Delta d_k^* - \delta_k^* \ , \ \forall k \in \mathcal{I} \\ d_k + \Delta d_k^* - w_k - \Delta w_k - \delta_k^* \ , \ \forall k \in \mathcal{J} \end{cases} \tag{7}$$

The proof of Proposition 1 can be found in Appendix B. It offers a more convenient way to calculate and analyze the energy sharing game equilibrium by solving (6). As problem (6) minimizes the total disutility of all users, Δd^* is social optimal. Take the real-time renewable power output deviation Δw as parameters, problem (6) can be regarded as a quadratic multi-parametric optimization problem.

C. Linearization and compact form

For the sake of analysis, the objective function is linearized via a convex combination approach [38]. Take a univariate convex objective function $z = g(\xi)$, $\xi_l \le \xi \le \xi_u$ for example. The function is evaluated at some discrete points $\xi_1, \cdots, \xi_K$, and $z_1 = g(\xi_1), \cdots, z_K = g(\xi_K)$. By introducing variables $\sigma_1, \cdots, \sigma_K \ge 0$ and $\sum_{k=1}^K \sigma_k = 1$, ξ and $g(\xi)$ in the optimization problem can be replaced with linear functions $\sum_{k=1}^K \sigma_k \xi_k$ and $\sum_{k=1}^K \sigma_k z_k$ in σ, respectively. Applying this technique to (6a), problem (6) turns into a multi-parametric linear program (MP-LP) with the compact form as:

$$\begin{aligned} v(\theta) = \min_x \ \ & c^\top x \\ \text{s.t.} \ \ & Ax \le t + B\theta \end{aligned} \tag{8}$$

where $x \in \mathbb{R}^n$ and $\theta \in \mathbb{R}^p$ are vectors of decision variables and parameters, respectively; $A \in \mathbb{R}^{m\times n}$, $t \in \mathbb{R}^m$, $B \in \mathbb{R}^{m\times p}$ and

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TABLE I
ELEMENT OF THE ENERGY SHARING GAME

Players $\mathcal{K}$	Actions $\mathcal{X}$	Cost functions Π
consumer $k=1,\cdots,I$	$\mathcal{X}_k(q_k^c,\delta_k):=\{(\Delta d_k,q_k)\mid$ (2b) and (2c) are met.$\}$	$\Pi_k(q_k^c,\delta_k):=f_k(\Delta d_k)+\tau/2(q_k^c-q_k)^2$
prosumer $k=(I+1),\cdots,(I+J)$	$\mathcal{X}_k(q_k^c,\delta_k):=\{(\Delta d_k,q_k)\mid$ (3b) and (3c) are met.$\}$	$\Pi_k(q_k^c,\delta_k):=f_k(\Delta d_k)+\tau/2(q_k^c-q_k)^2$
operator $k=I+J+1$	$\mathcal{X}_k(\Delta d,q):=\{(q^c,\delta)\mid$ (1b)-(1d) are met.$\}$	$\Pi_{I+J+1}:=\sum_{k\in\mathcal{I}\cup\mathcal{J}}\delta_k^2$

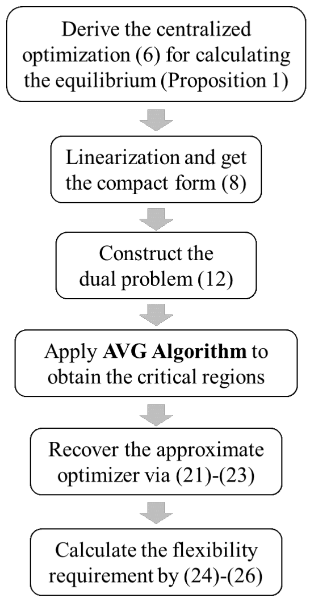

Fig. 2. A linear program (8) for computing the market equilibrium is derived based on Proposition 1 and linearization techniques. Then AVG algorithm is applied to calculate the flexibility requirement.

$c\in\mathbb{R}^n$ are input data. The decision variables x is bounded due to the lower and upper bound constraints. Denote $v(\theta)$ and $x^*(\theta)$ as the optimal value and optimal solution associated with given parameter θ, respectively.

Based on the above MP-LP (8), in the following, we develop an adaptive vertex generation (AVG) algorithm to approximate the optimal cost and the demand adjustment at equilibrium under different renewable output deviations and quantify the flexibility requirements of individual users.

III. Flexibility Characterization

In this section, we first define the flexibility requirement of individual users under energy sharing; then, an adaptive vertex generation algorithm is developed to construct a piecewise linear estimator of the optimal total cost and the demand adjustment at equilibrium subject to a given error tolerance, based on which we calculate the flexibility requirement. The processing framework of the algorithms is shown in Fig. 2.

A. Definition of flexibility requirement

Let $\bar{\Theta}$ be the maximum set of θ such that problem (8) has at least one feasible solution x, known as the dispatchable region [22]. We assume $\theta\in\Theta$, a bounded polyhedral subset of $\bar{\Theta}$. The flexibility requirement of each user is defined below.

Definition 2. (Flexibility Requirement) Suppose $\Delta d_k^*(\theta),\forall k\in\mathcal{I}\cup\mathcal{J}$ are users' demand adjustment at equilibrium under renewable deviation θ. The flexibility requirement of user $k\in\mathcal{I}\cup\mathcal{J}$ is defined as a region $[\underline{r}_k^d,\bar{r}_k^d]$ with

$$\underline{r}_k^d=\min\{\Delta d_k^*(\theta),\forall\theta\in\Theta\} \tag{9}$$

and

$$\bar{r}_k^d=\max\{\Delta d_k^*(\theta),\forall\theta\in\Theta\} \tag{10}$$

The flexibility in this paper is a kind of *reserve capacity* [34]. It defines the backup adjustable capacity the demand can provide in the occurrence of renewable generation output deviations due to uncertainty. Since $\Delta d^*(\theta)$ involves solving problem (6), getting $[\underline{r}_k^d,\bar{r}_k^d]$ directly by definition can be difficult. In this paper, an MP-LP based approach is proposed to obtain the flexibility requirement. To be specific, we divide Θ into non-overlapping critical regions $\mathrm{CR}_1,\cdots,\mathrm{CR}_N$ such that in each region CR_i, the optimal value $v(\theta)$ and optimal solution $x^*(\theta)$ are linear functions in the parameter vector θ. Then (9)-(10) is equivalent to finding the maximum/minimum point of a piecewise linear function, which is easier to solve.

B. Adaptive vertex generation algorithm

The majority of current MP-LP related works identifies the critical regions by leveraging the graph of optimal bases based on which the expression of $x^\star(\theta)$ and $v(\theta)$ are obtained [28]. However, degeneracy that indicates the existence of multiple primal or dual optimal solutions may cause difficulties in building the critical regions [39]. In this paper, we develop an adaptive vertex generation algorithm to approximate $v(\theta)$ via dual variables without the information of critical regions first, and then retrieve the critical regions and $x^*(\theta)$ accordingly. In this way, the impact of degeneracy can be avoided. The details of the algorithm are as follows:

1) Lower bound of $v(\theta)$ from dual problem

Suppose the optimal value function $v(\theta)$ can be represented by a piecewise linear function

$$v(\theta)=\begin{cases}m_1+n_1^\top\theta, & \theta\in\mathrm{CR}_1\\ m_2+n_2^\top\theta, & \theta\in\mathrm{CR}_2\\ \vdots & \\ m_N+n_N^\top\theta, & \theta\in\mathrm{CR}_N\end{cases} \tag{11}$$

where $m_1,\cdots,m_N$ are constant scalars; $n_1,\cdots,n_N$ are constant vectors. For any fixed θ, the dual problem of (8) is

$$\begin{aligned} v(\theta)=\max_{\gamma}\quad & \gamma^\top(t+B\theta)\\ \text{s.t.}\quad & A^\top\gamma=c,\ \gamma\leq 0\end{aligned} \tag{12}$$

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where γ is the dual variable, and its feasible region is denoted by $\Gamma = \left\{\gamma \middle| A^\top \gamma = c, \gamma \le 0\right\}$. According to strong duality, the optimal value of (12) is bounded and equals to $v(\theta)$ for any given θ. Since $v(\theta)$ is finite, the dual optimal solution γ must belong to the set of extreme points of Γ [vert(Γ) for short], although Γ may be unbounded and contain extreme rays. In this regard, by enumerating the vertices of Γ, $v(\theta)$ can be equivalently expressed as

$$\begin{gathered} v(\theta) = \max_i \left\{\gamma_i^\top t + \gamma_i^\top B\theta\right\} \\ \{\gamma_1, \gamma_2, \cdots\} \in \text{vert}(\Gamma) \end{gathered} \tag{13}$$

Comparing (13) and (11), coefficients m_i and n_i and dual variable γ_i have the following relation

$$m_i = \gamma_i^\top t,\ n_i = \gamma_i^\top B \tag{14}$$

Nevertheless, vertex enumeration is an exhaustive task, and only a small fraction of vertices correspond to valid pieces in (11) and (13), but we do not know which one in vert(Γ) appears in (13) in advance. Let $\underline{\Gamma}$ be a subset of vert(Γ), and

$$\underline{v}(\theta) = \max_i \left\{\gamma_i^\top t + \gamma_i^\top B\theta\right\}, \{\gamma_1, \gamma_2, \cdots\} \in \underline{\Gamma} \tag{15}$$

Hence, $\underline{v}(\theta)$ is an underestimator of $v(\theta)$. The approximation quality depends on the choice of $\underline{\Gamma}$. In the following, an adaptive vertex generation algorithm is developed to construct $\underline{\Gamma}$ so that the gap between $\underline{v}(\theta)$ and $v(\theta)$ can be small.

2) Recover the critical regions

Suppose (15) is a non-redundant representation. Intuitively, if the value of any piece $\gamma_i^\top t + \gamma_i^\top B\theta$ can reach maximum for some $\theta \in \Theta$, (15) is non-redundant. The exact definition and redundancy elimination methods are in Appendix C. Then the critical region associated with each piece can be retrieved from (15). Recall the original piecewise linear form (11) and the point-wise maximum form (15), the value of $m_i + n_i^\top\theta$ achieves maximum in CR_i, i.e.:

$$\text{CR}_i = \left\{\theta \middle| m_i + n_i^\top \theta \ge m_{[-i]} + n_{[-i]}^\top \theta\right\} \tag{16}$$

where $[-i] = \{1, \cdots, i-1, i+1, \cdots, n\}$ stands for the set of complementary indexes of i, so CR_i is described by $n-1$ inequalities in (16). It can be represented in a matrix form (17), where $H \in \mathbb{R}^{(n-1)\times p}$, $h \in \mathbb{R}^p$.

$$\text{CR}_i = \{\theta | H\theta \le h\} \tag{17}$$

For notation conciseness, index i of critical region for matrix H and vector h are omitted; $H_j\theta \le h_j$, $j = 1, \cdots, (n-1)$ represent the individual constraints in CR_i.

3) Overall algorithm

Suppose we already have an approximation (15) for the optimal value function, as well as critical regions retrieved from (16), then we have to investigate the quality of this approximation in each CR_i. The task comes down to evaluating the maximal approximation error through

$$\begin{aligned} \varepsilon_i = \max\ & v(\theta) - \underline{v}(\theta) \\ \text{s.t.}\ & \theta \in \text{CR}_i \end{aligned} \tag{18}$$

where $v(\theta)$ is the true optimal value function of problem (8), which is unknown. ε_i must be nonnegative because $\underline{v}(\theta)$ is an underestimator. First, due to strong duality, we have

$$\varepsilon_i = \max_{\theta \in \text{CR}_i} \left\{\max_{\gamma\in\Gamma} \gamma^\top (t + B\theta) - (\gamma_i^\top t + \gamma_i^\top B\theta)\right\} \tag{19}$$

Problem (19) is nonconvex due to the product term $\gamma^\top B\theta$ in the objective function. Since the parameter space has a low dimension, we can solve (19) by numerating the vertex of $\text{CR}_i = \{\theta | H\theta \le h\}$, which is

$$\begin{aligned} \varepsilon_i = \max_i \max_\gamma\ & (\gamma - \gamma_i)^\top (t + B\theta_i) \\ \text{s.t.}\ & \{\theta_1, \theta_2, ...\} \in \text{vert}(\text{CR}_i) \\ & A^\top \gamma = c,\ \gamma \le 0 \end{aligned} \tag{20}$$

Another approach to solve (19) is the big-M method [40], but our approach is more robust since it avoids the difficulty of choosing a proper M. The proposed adaptive vertex generation algorithm is summarized in Algorithm 1. The output is an ε-optimal underestimator of the true optimal value function.

Algorithm 1 : Adaptive Vertex Generation Algorithm

1: Initiation: Error tolerance $\varepsilon > 0$; parameter set Θ; initial sampled parameters $\theta_1, \cdots, \theta_n$;
2: Underestimation
 For $i = 1 : n$
  Solve LP (12) with θ_i and the solution is γ_i^*.
  Update $\underline{\Gamma} \leftarrow \underline{\Gamma} \cup \gamma_i^*$ if $\gamma_i^* \notin \underline{\Gamma}$.
 end
 Construct $\underline{v}(\theta)$ by (15); retrieve CR_1, $\cdots$, CR_n by (16).
3: Check approximation error
 For each critical region i
  Solve LP (20) with CR_i. Record ε_i^*, γ^*.
  If $\varepsilon_i^* > \varepsilon$, update $\underline{\Gamma} \leftarrow \underline{\Gamma} \cup \gamma^*$.
 end
 If $\varepsilon_i^* \le \varepsilon$, $\forall i$, terminate.
4: Update $\underline{v}(\theta)$ by (15) with the current $\underline{\Gamma}$, and remove redundant pieces (see Appendix C).
5: Retrieve critical regions from the current $\underline{v}(\theta)$; remove redundant constraints (Appendix C), and go step 3.

Remark 3: Because the set vert(Γ) has finite elements, Algorithm 1 must terminate in a finite number of steps. Nonetheless, as only a small fraction of elements in vert(Γ) corresponds to the pieces in the optimal value function, Algorithm 1 is efficient in practice. In contrast to the existing approaches where critical regions are obtained from the analysis of optimal bases invariancy, in Algorithm 1, critical regions are retrieved from the optimal value function. Therefore, model degeneracy does not affected the proposed algorithm.

C. Calculation of flexibility requirement

To obtain the flexibility requirement of individual users, we need to recover an approximate optimizer $x^*(\theta)$ as a function of θ. In CR_i, the optimal value is $v(\theta) = \gamma_i^\top t + \gamma_i^\top B\theta$, the complementarity and slackness condition of (8) implies

$$\gamma_{ij}^* < 0 \rightarrow (Ax^* - B\theta - t)_j = 0 \tag{21}$$

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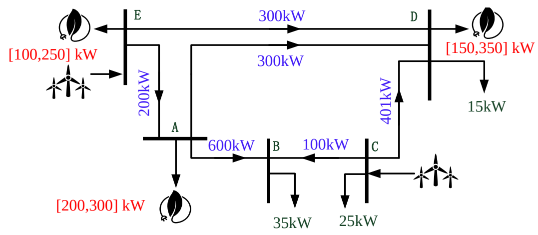

Fig. 3. Topology and parameters of the 5-bus system. The adjustable ranges of elastic demands are in red, the line flow limits are in blue, and the fixed demands are in dark green.

Let $A'_i/B'_i/t'_i$ denote the rows of $A/B/t$ corresponding to the indexes of nonzero elements in γ_i^*, we obtain a set of linear equations:

$$A'_i x^* = t'_i + B'_i \theta, \;\; \theta \in \mathrm{CR}_i \tag{22}$$

If A'_i is invertible, the optimizer is obtained by

$$x^*(\theta) = (A'_i)^{-1}(t'_i + B'_i\theta), \;\; \theta \in \mathrm{CR}_i \tag{23}$$

Otherwise, we can calculate the flexibility requirement $[\underline{r}^d_{ik}, \overline{r}^d_{ik}]$ by the following optimization problems. Without loss of generality, suppose the first $I+J$ terms in x correspond to the decision variables $\Delta d_k, \forall k = 1,...,(I+J)$.

$$\underline{r}^d_{ik} = \min_{\theta \in \mathrm{CR}_i} \; x_k \tag{24a}$$
$$\text{s.t. } A'_i x = t'_i + B'_i \theta \tag{24b}$$
$$\underline{D}_k \le d_k + x_k \le \overline{D}_k, \forall k = 1,...,(I+J) \tag{24c}$$

and

$$\overline{r}^d_{ik} = \max_{\theta \in \mathrm{CR}_i} \; x_k \tag{25a}$$
$$\text{s.t. } A'_i x = t'_i + B'_i \theta \tag{25b}$$
$$\underline{D}_k \le d_k + x_k \le \overline{D}_k, \forall k = 1,...,(I+J) \tag{25c}$$

Therefore, the flexibility requirement of user $k \in \mathcal{I} \cup \mathcal{J}$ over the entire parameter set θ is $[\underline{r}^d_k, \overline{r}^d_k]$, where

$$\underline{r}^d_k = \min_i \{r^d_{ik}\}, \;\; \overline{r}^d_k = \max_i \{r^d_{ik}\} \tag{26}$$

In general, the obtained critical regions $\mathrm{CR}_i, \forall i$ can provide geometry information of how uncertainty be handled under energy sharing, while the flexibility requirement $[\underline{r}^d_k, \overline{r}^d_k], \forall k$ offers a quantitative metric.

IV. Case Studies

We first use a 5-bus system for illustration; then 69-bus and 123-bus microgrid systems are tested to show the scalability of the proposed model and algorithm. Data is available in [41].

A. 5-bus system

The topology of the 5-bus system is given in Fig. 3 with other parameters in TABLE II-III. There are three users with elastic demands and two renewable generators at nodes C and E, respectively, so $\theta = [\Delta w_1, \Delta w_2]^\top$. The maximum set

[1]Wind farms locate at buses C and E with $w_1 = 220$ kW, $w_2 = 450$ kW.
[2]170 kW elastic demand and 15 kW inelastic demand.

TABLE II
System Parameter of the 5-bus System

From	To	X	F_l
A	B	0.0281	600
A	D	0.0304	300
A	E	0.0064	200
B	C	0.0108	100
C	D	0.0297	401
D	E	0.0297	300

TABLE III
Demand Data of the 5-bus System[1]

Node	d_k (kW)	Bound	α_k (\$/kW^2)	β_k (\$/kW)	ζ_k (\$)
A	230	[200, 300]	0.003	1.80	255.30
B	35	-	-	-	-
C	25	-	-	-	-
D	185 [2]	[150, 350]	0.006	2.76	295.80
E	200	[100, 250]	0.005	2.56	312.00

of parameters $\bar{\Theta}$ is the dispatchable region [22], the grey region in Fig. 4. The θ can vary within the rectangle subregion Θ. Applying Algorithm 1, we can obtain six critical regions as in Fig. 5. Note that the quadratic objective function (6) is linearized by 5 segments. The approximate optimal cost function $\underline{v}(\theta)$ in each critical region is as follows.

$$\underline{v}(\theta) = \begin{cases} 814.92 + 4.00\Delta w_1 + 2.81\Delta w_2, & \theta \in \mathrm{CR}_1 \\ 835.39 + 2.13\Delta w_1 + 2.81\Delta w_2, & \theta \in \mathrm{CR}_2 \\ 810.04 + 4.50\Delta w_1 + 2.31\Delta w_2, & \theta \in \mathrm{CR}_3 \\ 837.26 + 2.01\Delta w_1 + 2.31\Delta w_2, & \theta \in \mathrm{CR}_4 \\ 819.08 + 1.92\Delta w_1 + 1.92\Delta w_2, & \theta \in \mathrm{CR}_5 \\ 780.51 + 5.07\Delta w_1 + 1.93\Delta w_2, & \theta \in \mathrm{CR}_6 \end{cases} \tag{27}$$

Following the procedure in Section III-C, we can get the demand adjustment at equilibrium in each critical region as well. Take CR_4 as an example, we have $\forall [\Delta w_1, \Delta w_2]^\top \in \mathrm{CR}_4$:

$$\begin{aligned} \Delta d_1^* &= 18.75 + 0.76\Delta w_1 \\ \Delta d_2^* &= -20.00 \\ \Delta d_3^* &= -3.75 + 0.24\Delta w_1 + \Delta w_2 \end{aligned} \tag{28}$$

If we choose a scenario inside CR_4, say $[\Delta w_1, \Delta w_2] = [-10, -20]^\top$kW, then according to (27), the approximate optimal cost is \$770.96 ($= 837.26 - 2.01 \times 10 - 2.31 \times 20$); according to (28), the approximate demand adjustments at equilibrium are $\Delta d_1^* = 11.10$ kW, $\Delta d_2^* = -20$ kW, $\Delta d_3^* = -26.10$ kW. Both the optimal cost and demand adjustments are the same as the one we get by solving problem (8) directly with given parameters $\theta = [\Delta w_1, \Delta w_2] = [-10, -20]^\top$kW. This validates the effectiveness of our algorithm. The optimal cost of the original quadratic problem (6) is \$767.24, so the relative error is only 0.48%, showing the linearization is accurate enough. Moreover, the obtained $\Delta d_k^*, \forall k = 1,2,3$ are the same as well. We calculate the flexibility requirement of each user, shown in TABLE IV. To accommodate the volatile renewable generation, all users need to have the ability to both increase

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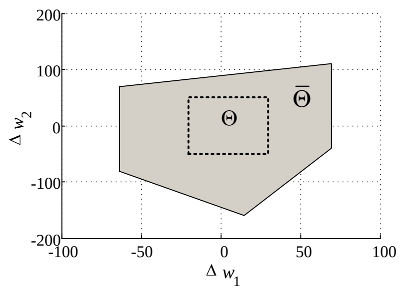

Fig. 4. Dispatchable region $\bar{\Theta}$ (maximum set of θ such that problem (8) has at least one feasible solution) and parameter range Θ (we assume $\theta \in \Theta$) of the 5-bus system.

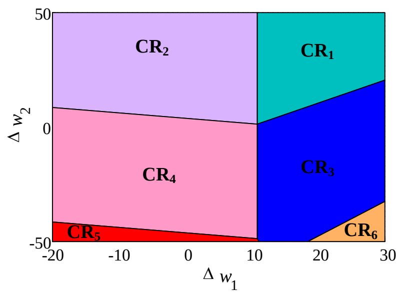

Fig. 5. Critical Regions of the 5-bus system. The parameter range is divided into six critical regions and within each region, the optimal cost is a linear function in θ given in (27).

and decrease their demand. User 3 requires the most flexibility since the value of $\overline{r}_3^d - \underline{r}_3^d$ is the largest.

TABLE IV
FLEXIBILITY REQUIREMENT OF 5-BUS SYSTEM

Elastic demand	1	2	3
d_k (kW)	230	170	200
$\underline{r}_k^d$ (kW)	-8.16	-20.00	-70.24
$\overline{r}_k^d$ (kW)	27.11	36.75	48.83
$\overline{r}_k^d - \underline{r}_k^d$ (kW)	35.27	56.75	119.07

Given $[\Delta w_1, \Delta w_2] = [-10, -20]^\top$kW, the equilibrium of the energy sharing game (4)-(5) can be reached via a best-response algorithm. The change of Δd and δ during iterations are shown in Fig. 6 and Fig. 7, respectively. We can find that the demand adjustments converge to the above approximate values. The value of dual variables of problem (6) at optimum can be retrieved by γ_i in CR_i, i.e. $\eta_1^* = -1.92$, $\eta_2^* = -2.08$, $\eta_3^* = -2.31$, which are very close to the value of $-\delta/\tau$ plotted in Fig. 7. The above analysis validates Proposition 1.

B. 69-bus system

A modified 69-bus microgrid [42] is tested with topology in Fig. 8. There are three renewable generators connected to nodes 9, 30, and 60, respectively. Suppose their real-time output may deviate within $[-30, 30]$ kW. Applying Algorithm 1, the change of the maximum approximation error (defined in (18)) during iterations is shown in Fig. 9. The error decreases fast and the algorithm can output 10 critical regions (as in

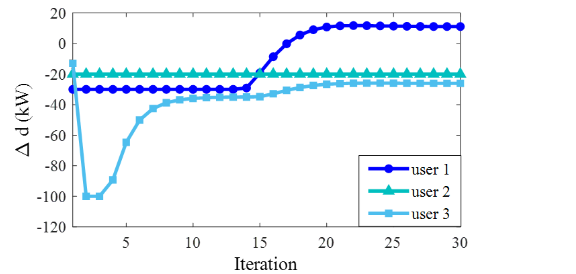

Fig. 6. Change of Δd during iterations and the energy sharing market reaches an equilibrium after about 20 iterations.

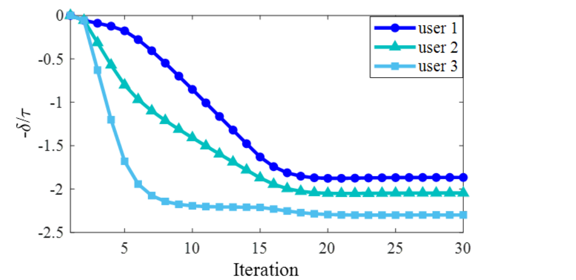

Fig. 7. Change of δ during iterations and $\frac{\delta^*}{\tau} = -\eta^*$ validates Proposition 1.

Fig. 10) after three iterations. The approximate optimal cost function $\underline{v}(\theta)$ in each critical region is

$$\underline{v}(\theta) = \begin{cases} 353.05 + [2.90, 2.90, 2.90]\theta, & \theta \in \mathrm{CR}_1 \\ 489.83 + [4.58, 4.58, 4.58]\theta, & \theta \in \mathrm{CR}_2 \\ 527.73 + [5.21, 5.21, 5.21]\theta, & \theta \in \mathrm{CR}_3 \\ 538.89 + [5.52, 5.52, 5.52]\theta, & \theta \in \mathrm{CR}_4 \\ 549.33 + [6.00, 6.00, 6.00]\theta, & \theta \in \mathrm{CR}_5 \\ 560.98 + [8.09, 8.09, 8.09]\theta, & \theta \in \mathrm{CR}_6 \\ 548.24 + [9.52, 9.52, 9.52]\theta, & \theta \in \mathrm{CR}_7 \\ 485.84 + [11.47, 11.47, 11.47]\theta, & \theta \in \mathrm{CR}_8 \\ 361.11 + [13.32, 13.32, 13.32]\theta, & \theta \in \mathrm{CR}_9 \\ -9.46 + [17.81, 17.81, 17.81]\theta, & \theta \in \mathrm{CR}_{10} \end{cases} \quad (29)$$

To examine the accuracy of the approximation, let's take CR_4 as an example. Following the procedure in Section III-C, the demand adjustments at equilibrium are:

$$\Delta d_1^* = -17,\ \Delta d_2^* = -9,\ \Delta d_3^* = -4,\ \Delta d_4^* = 8$$
$$\Delta d_5^* = 5,\ \Delta d_6^* = 6 + \Delta w_1 + \Delta w_2 + \Delta w_3 \quad (30)$$

For scenario $\theta = [-30, -30, 30]^T$ kW $\in \mathrm{CR}_4$, according to (29) and (30), the optimal cost is \$373.29 and $\Delta d_1^* = -17$ kW, $\Delta d_2^* = -9$ kW, $\Delta d_3^* = -4$ kW, $\Delta d_4^* = 8$ kW, $\Delta d_5^* = 5$ kW, $\Delta d_6^* = -24$ kW, which are the same as the one obtained by solving problem (6). Similarly, we can calculate the flexibility requirement for each elastic demand as in TABLE V. We notice that while elastic demands 2, 3, 4 still have some redundancy, the adjustable ability of elastic demands 1, 5, 6 has been fully exploited as $d_k + \underline{r}_k^d = \underline{D}_k$ and $d_k + \overline{r}_k^d = \overline{D}_k$, for $k = 1, 5, 6$. Therefore, in long-term planning, we may put more emphasis on increasing the flexibility of demands 1, 5, 6.

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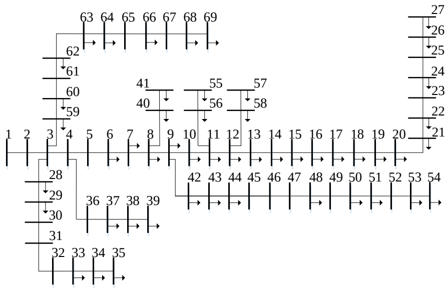

Fig. 8. Topology of the 69-bus system. Three wind farms are connected to nodes 9, 30, and 60; six elastic demands are located at nodes 12, 23, 32, 42, 53, and 62.

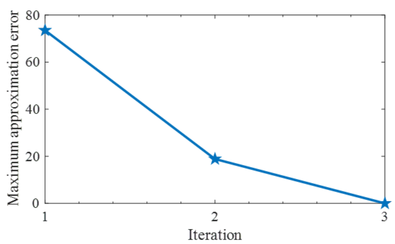

Fig. 9. The maximum approximation error of the AVG algorithm during iterations. The algorithm can output the critical regions after three iterations.

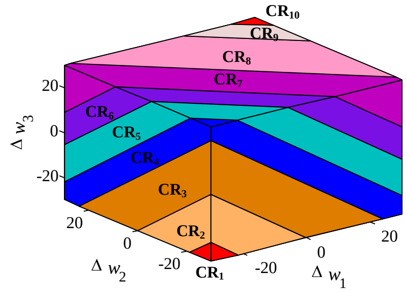

Fig. 10. Critical region of 69-bus system. The parameter range is divided into ten critical regions and within each region, the optimal cost is a linear function in θ given in (29).

TABLE V
FLEXIBILITY REQUIREMENT OF 69-BUS SYSTEM

Elastic demand	d_k (kW)	$\underline{r}_k^d$ (kW)	$\overline{r}_k^d$ (kW)	Range
1	40	-30	10	[10, 50]
2	20	-10.09	15	[0, 35]
3	30	-25	24	[5, 70]
4	10	-2	10	[0, 20]
5	10	-5	10	[5, 20]
6	40	-30	10	[10, 50]

To test the practicability of the proposed energy sharing mechanism, we compare the computational time under different numbers of users in TABLE VI. Our proposed mechanism focuses on the real-time market, so the computational time analysis is over an hour. We begin with the case with 69 users, where each node has one representative user. Then, we divide the demand at each node randomly to 5 (or 10) users and get the case with 345 (or 690) users. As we can see, the computational time does not change much when the number of users increases as the decision-making for each user can be done in parallel. Moreover, it only takes about 30 seconds to converge, so our proposed mechanism is still applicable in a market with smaller time resolution such as 15 minutes. The sequence of demand adjustment generated by the energy sharing mechanism was given in Fig. 11. The demand adjustments converge to the value at equilibrium (i.e. $\Delta d_1^* = -17$kW, $\Delta d_2^* = -9$kW, $\Delta d_3^* = -4$kW, $\Delta d_4^* = 8$kW, $\Delta d_5^* = 5$kW, $\Delta d_6^* = -24$kW) after a few iterations. This shows the effectiveness of the proposed method.

To show the scalability of Algorithm 1, we test its performance under different numbers of renewable generators and users in TABLE VII. The impact of the number of users on the computational time is marginal. The time needed increases with the number of RGs, but in this paper, we only consider the distributed renewable generators with a relatively large scale (say about 500kW) while the uncertainties from small-scale units are neglected. Usually, in a residential community, there are few such relatively large-scale units so our proposed method can still be applied.

C. 123-bus system

A larger distribution network, the modified IEEE-123 bus system, is tested to show the capability of the proposed model. The topology of the system is shown in Fig. 12. Suppose the real-time outputs of renewable generators may vary within [-40,40]kW. We run Algorithm 1 and get 12 critical regions in 128.97s, as shown in Fig. 13. The approximate optimal cost function $\underline{v}(\theta)$ in each critical region is

$$\underline{v}(\theta) = \begin{cases} 803.74 + [4.71, 4.71, 4.71]\theta, & \theta \in \text{CR}_1 \\ 853.97 + [5.21, 5.21, 5.21]\theta, & \theta \in \text{CR}_2 \\ 885.45 + [5.66, 5.66, 5.66]\theta, & \theta \in \text{CR}_3 \\ 909.57 + [6.24, 6.24, 6.24]\theta, & \theta \in \text{CR}_4 \\ 953.97 + [7.12, 7.12, 7.12]\theta, & \theta \in \text{CR}_5 \\ 952.46 + [8.09, 8.09, 8.09]\theta, & \theta \in \text{CR}_6 \\ 953.3 + [8.30, 8.30, 8.30]\theta, & \theta \in \text{CR}_7 \\ 943.22 + [9.42, 9.42, 9.42]\theta, & \theta \in \text{CR}_8 \\ 931.12 + [9.82, 9.82, 9.82]\theta, & \theta \in \text{CR}_9 \\ 852.86 + [11.51, 11.51, 11.51]\theta, & \theta \in \text{CR}_{10} \\ 778.3 + [12.32, 12.32, 12.32]\theta, & \theta \in \text{CR}_{11} \\ 670.27 + [13.32, 13.32, 13.32]\theta, & \theta \in \text{CR}_{12} \end{cases} \tag{31}$$

We take the scenario $\theta = [-40, 40, 40]^T$kW in CR_9 as an example. According to (31), the approximate optimal objective value is $931.12 + [9.82, 9.82, 9.82]\theta = \$1323.92$, which is the same as the real optimal objective value obtained by problem (6). Moreover, we compare the computational time under different system settings in TABLE VIII, and find that the time for a larger system increases but is still reasonable compared to the frequency of market clearing, which is typically every 15 minutes.

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TABLE VI
COMPUTATIONAL TIME OF THE ENERGY SHARING MECHANISM

Number of users	69	345	690
Time (s)	29.74	30.06	30.21

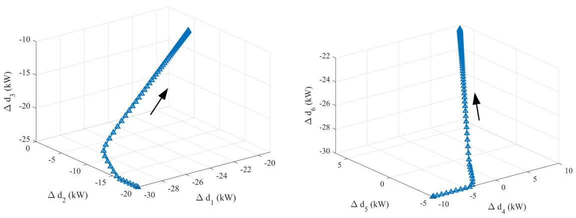

Fig. 11. Change of demand adjustment during iterations.

V. CONCLUSION

The prosperity of distributed renewable generators calls for effective management of end-users to hedge against uncertainty. In this paper, an innovative energy sharing mechanism that can facilitate the energy exchange among end-users with local information is proposed. The interaction among users and the operator is formulated as a generalized Nash game, whose equilibrium is proved to exist and can be obtained by a multi-parametric program parameterized in the renewable generation deviations. To quantify the flexibility requirement of individual users, an alternative vertex generation algorithm is developed to output a piecewise linear estimator of both the optimal cost function and demand adjustment strategies at equilibrium via dual problem. It can avoid the impact of degeneracy since it does not require visiting the graph of degenerate base. Case studies validate the accuracy and scalability of our method.

Future research may further improve the proposed energy sharing mechanism by considering more realistic situations: 1) *Bounded rationality of users.* In this paper, the users are assumed to be fully rational while in practice they may have bounded rationality. This can be analyzed with the prospect theory [43]. 2) *AC power flow model.* Lossless DC power flow model is adopted in this paper for simplification, which shall be extended to incorporate AC power flow model with the help of convex relaxation [44] or linearization techniques [45]. 3) *Diversity of devices.* Different types of flexible devices such as energy storage shall be integrated by changing the proposed model to a multiple time step version. More future research directions can be found in [8].

APPENDIX

A. Penalty Mechanism for the Energy Sharing Market

The hour-ahead sharing quantity for user $k \in \mathcal{I} \cup \mathcal{J}$ is q_k^{c*}, which satisfies $\sum_{k\in\mathcal{I}\cup\mathcal{J}} q_k^{c*} = 0$. If $q_k^{c*} > 0$, the user imports energy and if $q_k^{c*} < 0$, the user exports energy. Suppose the real-time buying (selling) price from (to) the main grid is λ^b (λ^s), the hour-ahead sharing price λ^c can be set as $\lambda^s \le \lambda^c \le \lambda^b$. Since $\sum_{k\in\mathcal{I}\cup\mathcal{J}} q_k^{c*} = 0$, we have $\sum_{k\in\mathcal{I}\cup\mathcal{J}} \lambda^c q_k^{c*} = 0$ which means the energy sharing market is self-budget balanced. Denote the *intra-hour* deviation of renewable generator and load as $\Delta\hat{w}_k$ and $\Delta\hat{d}_k$, respectively. Then the actual renewable generator output is $w_k + \Delta w_k + \Delta\hat{w}_k$ and the actual demand is $d_k + \Delta d_k + \Delta\hat{d}_k$, where w_k and d_k are *day-ahead* predictions, Δw_k and Δd_k are *hour-ahead* deviations. Denote

TABLE VII
COMPUTATIONAL TIME OF THE AVG ALGORITHM

Cases	3 RGs,69 users	3 RGs,345 users	3 RGs,690 users
Time (s)	103.28	122.34	92.37
Cases	6 RGs,69 users	6 RGs,345 users	6 RGs,690 users
Time (s)	1507.94	1314.58	1644.23

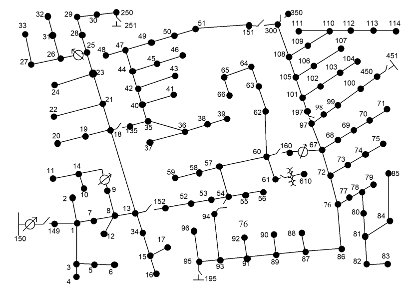

Fig. 12. Topology of the IEEE-123 bus system.

$$\chi_k = \begin{cases} (d_k + \Delta d_k + \Delta\hat{d}_k) - q_k^{c*} & \forall k \in \mathcal{I} \\ (d_k + \Delta d_k + \Delta\hat{d}_k) - (w_k + \Delta w_k + \Delta\hat{w}_k) - q_k^{c*} & \forall k \in \mathcal{J} \end{cases} \tag{A.1}$$

If $\sum_{k\in\mathcal{I}\cup\mathcal{J}} \chi_k > 0$, meaning that we need to buy electricity from the main grid in real-time, the operator's extra cost will be $\lambda^b \sum_{k\in\mathcal{I}\cup\mathcal{J}} \chi_k$ which should be covered by the net payments from users. Then the billing mechanism with penalty for each user can be designed as:

• When $q_k^{c*} > 0$ and $\chi_k > 0$, user k is a buyer and it is buying more in real-time. Therefore, the operator needs to buy from the main grid to supply the additional demand. The user k will pay $\lambda^c(q_k^{c*} + \chi_k) + (\lambda^b - \lambda^c)\chi_k$, where $(\lambda^b - \lambda^c)\chi_k$ is the penalty.

• When $q_k^{c*} > 0$ and $\chi_k < 0$, user k is a buyer and it is buying less in real-time. This won't contribute to extra cost, so no penalty will be imposed and user k will pay $\lambda^c(q_k^{c*} + \chi_k)$.

• When $q_k^{c*} < 0$ and $\chi_k > 0$, user k is a seller and it is selling less in real-time. Therefore, the operator needs to buy from the main grid instead to supply the buyers. The user k will get $-\lambda^c(q_k^{c*} + \chi_k) - (\lambda^b - \lambda^c)\chi_k$, where $(\lambda^b - \lambda^c)\chi_k$ is the penalty.

• When $q_k^{c*} < 0$ and $\chi_k < 0$, user k is a seller and it is selling more in real-time. This won't contribute to extra cost, so no penalty will be imposed and user k will get $-\lambda^c(q_k^{c*} + \chi_k)$.

Overall, the net payment from users is

$$\begin{aligned} & \sum_{k\in\mathcal{I}\cup\mathcal{J}} \lambda^c(q_k^{c*} + \chi_k) + \sum_{k\in\{k|\chi_k>0\}} (\lambda^b - \lambda^c)\chi_k \\ \ge\ & \sum_{k\in\mathcal{I}\cup\mathcal{J}} \lambda^c(q_k^{c*} + \chi_k) + \sum_{k\in\mathcal{I}\cup\mathcal{J}} (\lambda^b - \lambda^c)\chi_k \\ =\ & \lambda^b \sum_{k\in\mathcal{I}\cup\mathcal{J}} \chi_k \end{aligned} \tag{A.2}$$

Therefore, the net payment from users can cover the extra cost for the operator, so that the market can still run in a subsidy-free way. The case when $\sum_{k\in\mathcal{I}\cup\mathcal{J}} \chi_k < 0$ can be discussed in a similar way.

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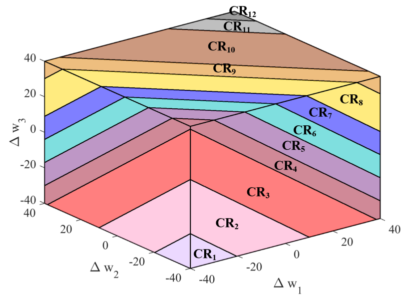

Fig. 13. Critical regions of the 123-bus system. The parameter range is divided into twelve critical region, and within each region, the optimal cost is a linear function in θ given in (31).

TABLE VIII
COMPUTATIONAL TIME FOR DIFFERENT SYSTEMS

Setting	5-bus with F_l	5-bus with $2F_l$	5-bus with $4F_l$
Time (s)	22.09	13.89	10.31
Setting	69-bus with F_l	69-bus with $2F_l$	69-bus with $4F_l$
Time (s)	103.28	60.32	64.19
Setting	123-bus with F_l	123-bus with $2F_l$	123-bus with $4F_l$
Time (s)	128.97	76.31	112.97

B. Proof of Proposition 1

Problem (1) can be equivalently written as

$$\begin{aligned}\min_{q_k^c,\forall k\in\mathcal{I}\cup\mathcal{J}} \; & \sum\nolimits_{k\in\mathcal{I}\cup\mathcal{J}} (q_k^c-q_k)^2 \\ \text{s.t. } & q^c\in\mathcal{Q}\end{aligned} \tag{B.1}$$

Suppose $(\Delta d^*, q^*, q^{c*}, \delta^*)$ is an GNE of the sharing game, then q^{c*} is the optimal solution of (B.1). Therefore, we have

$$\sum_{k\in\mathcal{I}\cup\mathcal{J}} (q_k^c-q_k^{c*})(q_k^{c*}-q_k^*)\geq 0, \forall q^c\in\mathcal{Q} \tag{B.2}$$

Also, $(\Delta d^*, q^*)$ is the optimal solution of problem (2) or (3), which is equivalent to

$$\begin{aligned}\min_{\Delta d_k, b_k} & \begin{cases} f_k(\Delta d_k)+\tau/2(q_k^{c*}+\delta_k^*-d_k-\Delta d_k)^2, \forall k\in\mathcal{I}\\ f_k(\Delta d_k)+\tau/2(q_k^{c*}+\delta_k^*-d_k-\Delta d_k+w_k+\Delta w_k)^2, \forall k\in\mathcal{J}\end{cases}\\ \text{s.t. } & \Delta d_k\in\mathcal{D}_k\end{aligned} \tag{B.3}$$

where $\mathcal{D}_k := \{\Delta d_k \mid \underline{D}_k\leq d_k+\Delta d_k\leq \overline{D}_k\}$. Therefore,

$$f_k(\Delta d_k)-f_k(\Delta d_k^*)-\tau(\Delta d_k-\Delta d_k^*)\delta_k^*\geq 0, \forall k\in\mathcal{I}\cup\mathcal{J} \tag{B.4}$$

Similarly, let $(\Delta\hat{d}, \hat{q}^c)$ be the optimal solution of problem (6) and $\hat{\eta}$ the value of the dual variable. Then, we have $\forall(\Delta d, q^c, \eta)\in\prod_{k\in\mathcal{I}\cup\mathcal{J}}\mathcal{D}_k\times\mathcal{Q}\times\mathbb{R}^{(I+J)}$:

$$\begin{aligned}\left[f_k(\Delta d_k)-f_k(\Delta\hat{d}_k)+(\Delta d_k-\Delta\hat{d}_k)\hat{\eta}_k\right] &\geq 0, \forall k\in\mathcal{I}\cup\mathcal{J}\\ -\sum_{k\in\mathcal{I}\cup\mathcal{J}}(q_k^c-\hat{q}_k^c)(\hat{\eta}_k) &\geq 0\\ \sum_{k\in\mathcal{I}}(\eta_k-\hat{\eta}_k)(d_k+\Delta\hat{d}_k-\hat{q}_k^c) &\\ +\sum_{k\in\mathcal{J}}(\eta_k-\hat{\eta}_k)(d_k+\Delta\hat{d}_k-w_k-\Delta w_k-\hat{q}_k^c) &\leq 0\end{aligned} \tag{B.5}$$

Based on the above optimality conditions, we can analyze the existence and uniqueness of the GNE of the game $\mathcal{G}=\{\mathcal{K},\mathcal{X},\Pi\}$ as follows.

Existence. When problem (6) is feasible, since the objective function (6a) is strictly convex and the constraints constitute a close convex set, problem (6) has a unique optimal solution $(\Delta\hat{d}, \hat{q}^c, \hat{\eta})$. Let $\Delta d^*=\Delta\hat{d}$, $\delta^*=-\hat{\eta}/\tau$, $q^{c*}=\hat{q}^c$, $q^*=\hat{q}^c-\delta^*$. It is easy to check that condition (B.2)-(B.4) are satisfied so that $(\Delta d^*, q^*, q^{c*}, \delta^*)$ is an GNE of the game $\mathcal{G}$.

Uniqueness. Suppose $(\Delta d^*, q^*, q^{c*}, \delta^*)$ is an GNE of the game $\mathcal{G}$, we have $q_k^{c*}=d_k+\Delta d_k^*, \forall k\in\mathcal{I}$, $q_k^{c*}=d_k+\Delta d_k^*-w_k-\Delta w_k, \forall k\in\mathcal{J}$ due to constraint (2b), (3b) and $q^{c*}=\delta^*+q^*$. Let $\Delta\hat{d}=\Delta d^*$, $\hat{q}^c=q^{c*}$, $\hat{\eta}=-\tau\delta^*$, then condition (B.5) is met. Therefore, Δd^* is the optimal solution of (6) and is unique.

C. Redundancy Elimination

Before introducing methods to eliminate redundancy, we first give the following definitions:

Definition B1. (Redundant Constraint) A constraint is said to be *redundant* if removing the constraint does not change the critical region.

Definition B2. (Minimal Representation) Polyhedron (17) is said to be a minimal representation of CR_i if all constraints are non-redundant.

To remove redundant constraints to construct a minimal representation of CR_i, a method based on Nonhomogeneous Farkas Lemma is proposed.

Lemma B1. (Nonhomogeneous Farkas Lemma [46]) In the following two sets

$$\begin{aligned}\mathcal{P}_1 &= \{u|Au\leq t,\ a^\top u>t'\}\\ \mathcal{P}_2 &= \{v|A^\top v=a,\ t'\geq v^\top t,\ v\geq 0\}\end{aligned}$$

Matrix multiplication is compatible in dimension. Then $\mathcal{P}_1$ is empty if and only if $\mathcal{P}_2$ is non-empty.

Consider constraint $H_j\theta\leq h_j$, if it is redundant, the polyhedron $\{\theta|H_{[-j]}\leq h_{[-j]}\}$ defined by the remaining constraints must be a subset of $\{\theta|H_j\theta\leq h_j\}$. In other words, $\{\theta|H_{[-j]}\leq h_{[-j]}\}\cap\{\theta|H_j\theta>h_j\}=\emptyset$. So we have

Theorem B1. Constraint $H_j\theta\leq h_j$ in Polyhedron (17) is redundant if the following LP has a feasible solution

$$v^\top H_{[-j]}=H_j,\ h_j'\geq v^\top h_{[-j]},\ v\geq 0 \tag{C.1}$$

If all constraints in polyhedron (17) are screened, and redundant ones are removed, then the remaining constraints constitute a minimal representation of CR_i. Similarly, we can define and eliminate redundancy in $\underline{v}(\theta)$ as follows.

Definition B3. (Redundant Piece) In the piecewise linear function (15), a piece $\gamma_i^\top b+\gamma_i^\top B\theta$ is said to be redundant if it never reaches maximum for all $\theta\in\Theta$.

Claim B1. The j-th piece in $\underline{v}(\theta)$ is redundant if the j-th constraint in the polyhedron $\text{epi}[\underline{v}(\theta)]$ is redundant. The epigraph of the piecewise linear function (15) is the following polyhedron

$$\text{epi}[\underline{v}(\theta)]=\left\{(\theta,\kappa)\in\mathbb{R}^{p+1}\left|\begin{array}{l}\kappa\geq\gamma_1^\top t+\gamma_1^\top B\theta\\ \vdots\\ \kappa\geq\gamma_n^\top t+\gamma_n^\top B\theta\end{array}\right.\right\} \tag{C.2}$$

Similarly, according to Theorem B1, the redundancy in $\underline{\nu}(\theta)$ can be removed.

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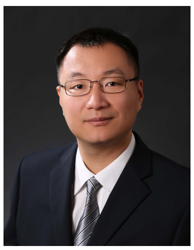

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