Selecting the Optimal Signals in Phasor Measurement Unit-based Power System Stabilizer Design

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***Abstract*—Phasor Measurement Unit (PMU) provides beneficial information for dynamic power system stability, analysis and control. One main application of such useful information is data-driven control. This paper is devoted to presenting an approach for optimal signal selection in PMU-based power system stabilizer (PSS) design. In this paper, for selecting the optimal input and output signals for PSS, an algorithm is suggested in which the combination of clustering the generators and the buses of the system with ICA, modal analysis and PCA techniques is used. The solution for optimal PSS input-output selection is found to increase the observability and damping of the power system. This method is simulated on a 68 buses system with 16 machines. To compare the results with the previous methods, the system is simulated and the results of two previously-developed algorithms are compared with the proposed approach. The results show the benefit of the suggested method in reducing the required signals, which lowers the number of required PMUs while the system damping is not deteriorated.**

Keywords— lustering, ICA, Modal Analysis, PCA Technique, Phasor Measurement Unit

NOMENCLATURE

A, B, C	State, input, ouput matrices.
C_j	*j*th cluster center.
$C_{m\times n}$	Covariance of normalized feature matrix.
$C(S)$	Gain of PSS with a lead-lag form
c	Number of clusters.
D	Feedforward matrix.
d	Number of data.
d_i	Distance of x_i from the nearest cluster center.
F	Transferred modal of state matrix.
$f_{c_i}(k)$	Modal controllability factors of *i*th mode.
$f_{o_i}(l)$	Modal observability factors of *i*th mode.
G	Transferred modal of input matrix.
H	Transferred modal of output matrix.
k	Number of principle components.
K	PSS gain
l_i, l_o	Input and output signals.
m	Number of patterns.
m_i	Number of inputs
n	Number of dimension of features vector.
P_j	Principle component of raw features vectors.
p	Number of outputs.
T_i	Time constants of the PSS model
u	Input or control vector.
$V_{n\times k}$	Raw features vectors.
v_i	Eigenvectors of the covariance matrix.
$X_{m\times n}$	Data set or features matrix.
$X'_{m\times n}$	Normalized feature matrix.
x	State vector.
x_i	*i*th element of data.
x_j	*j*th element of features vector.
x_{ij}	*ij*th elements of features matrix.
x'_{ij}	Average of *ij*th elements of features matrix.
y	Output vector.
Z	Transferred modal of state vector
θ	Minimum threshold value as the accuracy of approximation
λ_i	Eigenvalues of the covariance matrix.
Φ	Right modal matrices of the system.
Ψ	Left modal matrices of the system.

I. INTRODUCTION

A. Motivation

With the advent of PMU in the last decades, extensive approaches in power system monitoring, state estimation, stability and control has been developed [1-3]. One interesting application of PMU signals is to be utilized in PSS online tuning [2] which has merits over decentralized ones [4] due to WAMS.

Applying PMUs in designing PSS is a solution to increase observability in previous monitoring systems [5-7]. The signals of PMUs which are installed on the buses of network are received by the universal PSSs. Because of the high cost of instrument installation such as communication platform, the number of them must be minimized [8].

The location of the universal measurement signals (the inputs of PSS) must be defined in order to have most observability with less number of PMUs. Similarity, the location of the universal control signals (the outputs of PSS) is defined so that the decreased control signals are selected optimally to ensure the most controllability issue [9]. The less number of measurement signals in the network results in less communication links, while the less number of control loops leads to reduce their interactions and system complexity.

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B. Relevant literature

The recent technology in PSS design is devoted to select appropriate PMU signals for controlling purposes [10-11]. The methods discussed in literature for choosing the PSS inputs can be divided in two categories: geometric criteria of observability and controllability based on modal analysis [12-14] and relative criteria based on residuals and exploiting heuristic optimization methods for finding optimal criteria [15-16]. Though, reviewing the state-of-the-art methods show that the final number of the signals are still high and therefore the implementation of the exiting results is not costly effective. Therefore, there still more efforts is needed to optimally reduce the number of signals as well as maintaining the overall performance. In [17], modal analysis is used for selecting both of the input and output signals. In this method, the number of selected signals are usually high which increase the complexity of calculations.

Although much research has been done in this regard, the problem is still remained unsolved and more professional methods for input-output selection in large-scale power systems are needed.

C. Contribution

To determine the optimal input-output signals of PSS from PMU signals, this paper presents a novel algorithm to find the optimum signals and minimize the number of required signals. First the buses of system are clustered optimally with the Imperial Competitive Algorithm (ICA) as an optimization problem. Next, for each bus cluster, the generators of them are clustered. As the number of the received data is very high, then it is necessary to use Principal Component Analysis (PCA) technique in each clustering process to decrease the time of calculation. Finally, the limited number of generators are selected as a representation of the whole system which are candidate for input and output signals. At last, by applying modal analysis, the optimal number of signals are obtained. Compared to the state-of-the-art approaches, the method results in the least number of input-output signals while the overall performance is still satisfactory.

D. Paper structure

The structure of this paper is as follows. The proposed approach is described in section II which consists of five sections. PCA technique, the formulation of optimal clustering and Modal analysis are described in section A to C. The suggested algorithm for selecting the optimal input and output signals of PSS based on PMU data is presented in D. The suggested approach is applied on 68 bus test system in section III and followed by conclusion in section IV.

II. PSS Signal Selection Preliminaries

In this paper, an algorithm based on candidate input and output signals is suggested. The information of the buses and generators of a power system is extracted to find the best input and output signals of power system stabilizer (PSS) candidate for input and output signals. At last, by applying modal analysis, the optimal number of signals are obtained. The overall design procedure of the proposed approach is illustrated in Fig. 1.

In the proposed approach, first, the buses of system are clustered optimally with the Imperial Competitive Algorithm (ICA). Next, in each bus cluster, generators are clustered optimally.

Please note that according to the huge amount of data, clustering is done in two steps such that the optimum number of generators for PSS allocation is determined. To reduce the number of received data, principle component analysis (PCA) technique is proposed in both clustering steps. Finally, a limited number of generators are selected for PSS allocation. In the end, Modal analysis is applied on the system to determine the input/output pairs of the signal. Successive usage of clustering technique on the reduced data results in a smaller number of required controllers compared to the previous technique.

The calculated (input) output signals are devoted to those generators that are selected in the steps of algorithm in Fig. 1 and they are applied using data from PMUs. The procedure is shown in Fig. 2. To clarify the methodology, the theories are elaborated in the following subsections.

A. Principle Component Analysis

Principle component analysis (PCA) is a known method for reducing the dimension of data or feature without loss of information. The aim of this method is to convert main features to a smaller number of features with a linear combination of main features. Usually the subset of k principle component contains the information similar to the main data set. The algorithm is described as follows [18]:

Assume a set of n-dimension features vector $x_j (j = 1,2,\dots,n)$ in which each feature has m patterns. Therefore, the data set is a $m \times n$ matrix ($X_{m\times n}$). Then, the steps of PCA technique are as follows:

1- The average of each dimension is calculated.

$$x'_{ij} = x_{ij} - \frac{1}{m}\sum_{i=1}^{m} x_{ij} \quad (1)$$

2- Each feature is subtracted from its mean and the normalized feature matrix $X'_{m\times n}$ is created.

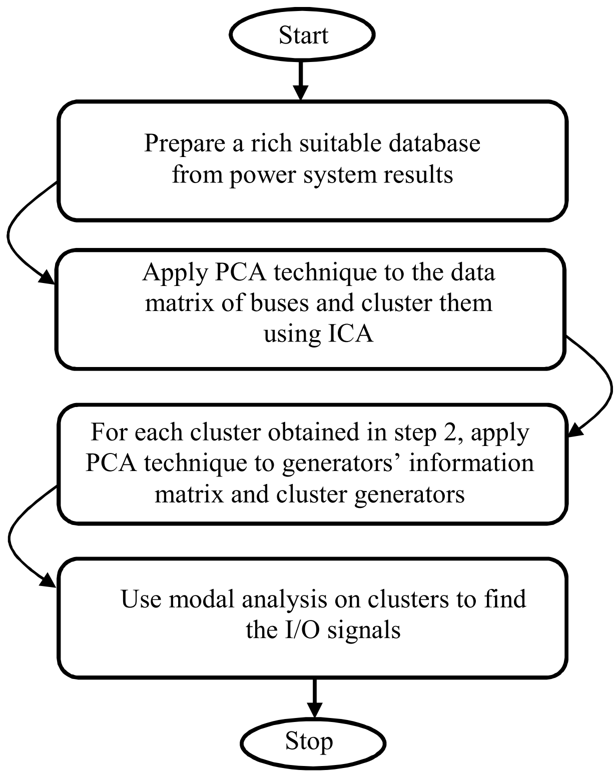

Fig. 1. The overall schematic of the proposed approach.

3- The covariance of normalized feature matrix is calculated as below:

$$C_{m\times n} = \frac{1}{n-1} X' X'^T \quad (2)$$

4- The eigenvalues and eigenvectors of the covariance matrix are calculated.

$$C v_i = \lambda_i v_i \,, i = 1,2,\dots,n \quad (3)$$

5- The principle components are the eigenvectors of the largest eigenvalues. The first k eigenvectors ($k \le n$) related to the k largest eigenvalues are selected to represent the raw feature vectors with low dimension.

$$V_{n\times k} = [v_1, v_2, \dots, v_k] \quad (4)$$

The amount of k is defined by a minimum threshold value, θ as the accuracy of approximation of the k largest eigenvectors:

$$\frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{n} \lambda_i} \ge \theta \quad (5)$$

6- According to $V_{n\times k}$, the low dimensional feature vectors are computed as principle component of raw ones:

$$P_j = V^T x_j^T \;, j = 1,2,\dots,k \quad (6)$$

B. Formulation of the Clustering as an optimization Problem

Clustering means identifying the homogeneous groups of data which are called clusters. Data of each cluster must be alike and be different from the other clusters. The concept of distance is used as a basis of data similarity such as Euclidean distance which is a most usable similarity criterion. The similar data has the less distance from each other. As the distance is inversely proportional to the similarity, for clustering it is needed to minimize the distance of the data.

Assume that d data , x_i ($i = 1,2,\dots,d$) will be sectionalized to c cluster. Consider C_j ($j = 1,2,\dots,c$) as the cluster center and d_i as distance of x_i from the nearest cluster center. The goal is minimizing the total of these distances. The clustering problem as an optimization problem is mathematically defined as follows:

$$\min \sum_{i=1}^{d} \min_j \left\| x_i - C_j \right\|_2 \quad (7)$$

$$i = 1,2,\dots,d \quad j = 1,2,\dots,c$$

This minimization problem is solved by Imperialist Competitive Algorithm (ICA) which is briefly defined in the following. ICA is inspired by a social phenomenon. In other words, a historical phenomenon of imperialist is analysed and presented as a powerful optimization algorithm by modelling mathematically [19].

ICA starts with an initial population called country. Some of the countries are selected as imperialist. The others named colony are divided among the imperialists. The number of colonies of each imperialist depends on how much powerful it is. Then, the colonies start to move toward the related imperialist and the imperialistic competition begins. Along the competition, if a colony gain more power than its imperialist, the position of them is exchanged and the colony becomes an imperialist. If an imperialist releases all its colonies, it becomes a powerless country and will be eliminated.

The competition is continued until all the colonies are assigned to the only one imperialist which is the most powerful. In the process of moving countries, a colonial country competitive is shown in Fig. 3.

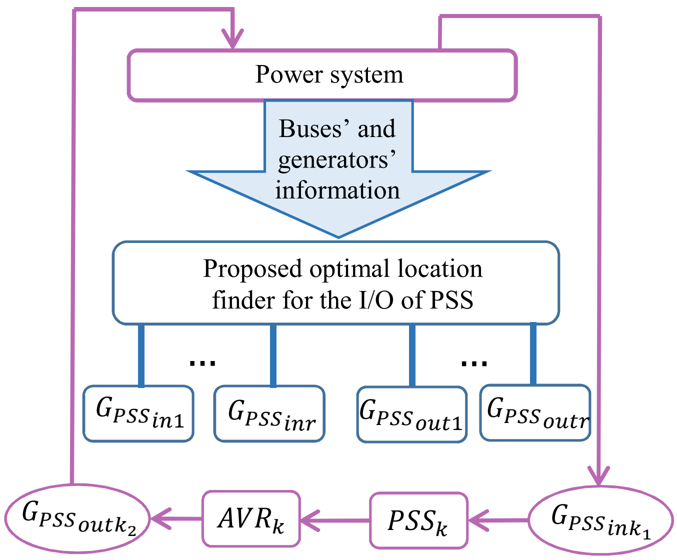

Fig. 2. The overall power system with the proposed approach.

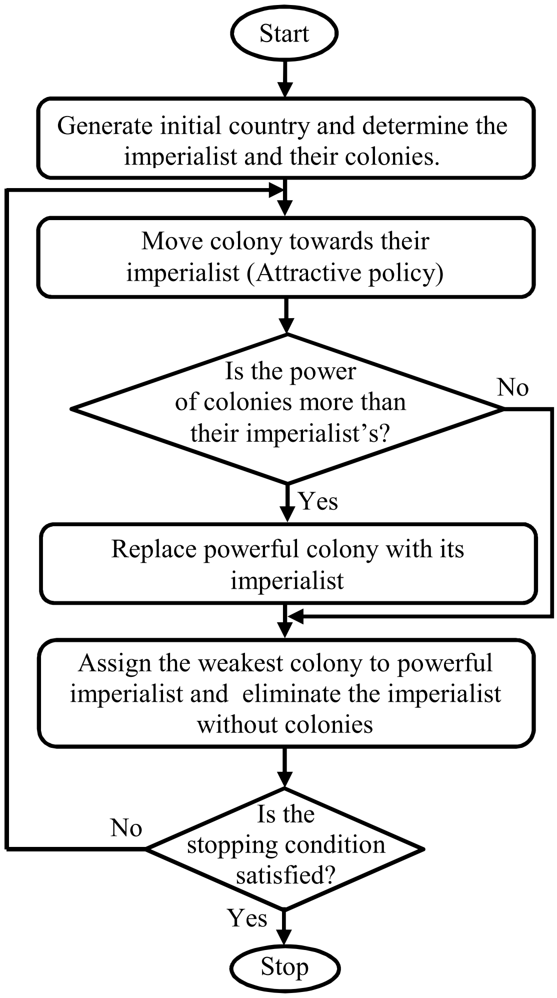

Fig. 3. Flowchart of ICA [19].

C. Modal Analysis for Controlability and Observability

The transferred modal of a linear system is defined as follows. It is based on modal transform.

$$\dot{x} = Ax + Bu \;\rightarrow\; \dot{z} = Fz + Gu \quad (8)$$

$$y = Cx + Du \;\rightarrow\; y = Hz + D \quad (9)$$

$$F = \Phi^{-1} A \Phi \quad (10)$$

$$G = \Phi^{-1} B = \Psi B \quad (11)$$

$$H = C\Phi \quad (12)$$

where Φ and Ψ are right and left modal matrices of the system, respectively. If the row ith of G is equal to zero, it means that the inputs do not affect the ith mode. Then, the elements of G are defined as modal controllability factors. Similarly, if the ith column of H is equal to zero, it means that the ith mode of outputs is not observable and its elements are modal observability factors.

$$f_{c_i}(l_i) = \psi_i b_k \ , l_i = 1,2, \dots, m_i \quad (13)$$

$$f_{o_i}(l_o) = c_l \phi_i \ , l_o = 1,2, \dots, p_o \quad (14)$$

where l_i, l_o and i are inputs, outputs and their modes of system and m_i and p_o are the number of inputs and outputs.

According to the modal analysis, the best selection for inputs and outputs are the signals which have the maximum observability and controllability modal factors (According to the elements in G and H matrices). The signal with the highest observability factor is the best candidate for being selected as input, and similarly, the signal with the highest controllability factor is the most appropriate one as an output. Briefly, the PSS input and output are selected based on the signals with highest observability and controllability, respectively.

D. Suggested Algorithm for Selecting Optimal Input and Output Signals of PSS Based on Data of PMU

The flowchart of the proposed algorithm which is shown in Fig. 4. First, a rich suitable database is prepared. For this purpose, the different faults are applied to power system and all data which are received directly from PMUs such as the angle of bus voltage and the generators' rotor angles are saved. Then, the data matrix of buses is constructed. To reduce the number of received data, PCA technique (as described in section A) is applied to the data matrix of buses. According to B, the optimal clustering is done with ICA on the reduced data matrix.

After bus clustering, the data matrix of generators is constructed for each cluster of buses. And similarly, PCA technique is applied to each data matrix of generators and the generators of each bus cluster are clustered optimally. Finally, by using modal analysis on every cluster of generators, the optimal input and output signals are found. The calculated (input) output signals are devoted to those generators that are used to gather database on PMUs and used as the input/output of PSSs. The number of inputs and outputs is equal to the number of final generators' clusters because in each cluster, one generator is selected as a location for installing PSS (input) and one generator is selected as a location for applying control signal of PSS (output) with modal analysis. It is assumed that each PSS has a lead-lag form as follows:

$$C(S) = 1.4\, K \frac{(T_1 s+1)(T_3 s+1)}{(T_2 s+1)(T_4 s+1)} \quad (15)$$

As can be seen in Fig. 5, the power system is stabilized by designing PSS sending or receiving data from the different parts of the power system. This strategy can save cost while the performance is not deteriorated.

III. Simulation Results

The suggested algorithm is simulated on the 68 buses system with 16 machines [20], shown in Fig. 6. The first step is gathering the rich and suitable data from buses and generators through installed PMUs [21-22]. For this purpose, the test system is implemented in MatNetEig [23-24] and 391 different faults such as three phase fault, single/double/triple phase to ground fault, short circuit, loss of load and line outage are applied to the most of the buses and lines. Then, voltage angle of each bus in all 391 cases construct a column of the data matrix of buses. Therefore, a 6120×68 matrix lead to use PCA technique necessarily. The final reduced matrix is a 34×68 matrix which help to decrease the complexity of calculation. The results of bus clustering are shown in Table I.

For generator clustering, the data matrix is generated by saving the rotor angle of 391 cases. It is a 7480×16 matrix which is reduced to a 15×16 matrix with PCA. The results of the generators clustering in each bus cluster are shown in Table II. Finally, the generators are divided to 9 clusters. Therefore, there are 9 pairs of inputs and outputs.

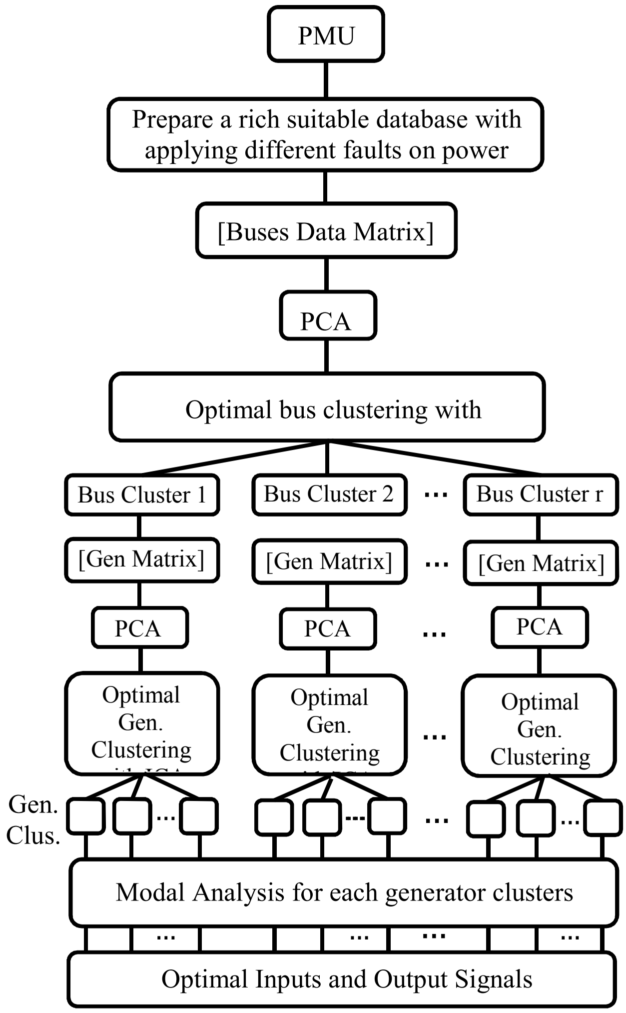

Fig. 4. Flowchart of the proposed algorithm.

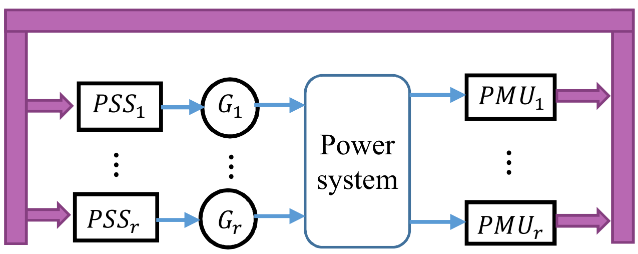

Fig. 5. The conventional structure of universal PSS with data of PMUs.

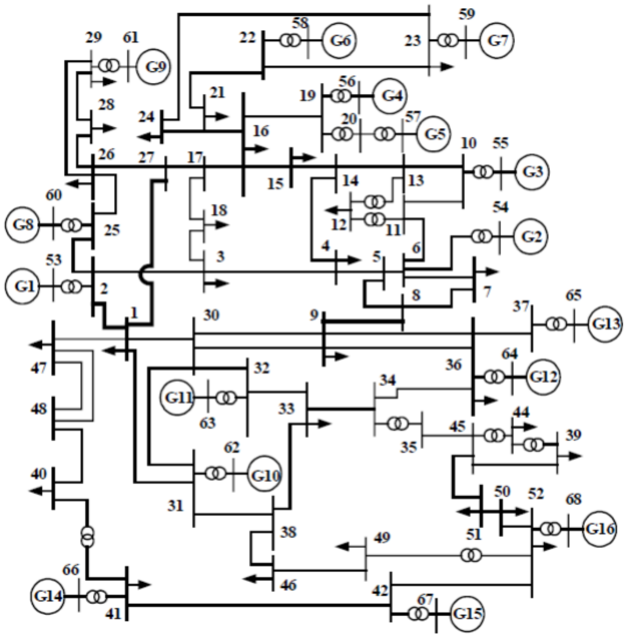

Fig. 6. The single line of the 68 buses test system.

TABLE I. THE RESULTS OF BUS CLUSTERING

Cluster	Buses	Generators' numbers
1	41-42-52-66-67-68	14-15-16
2	19-22-23-29-40-49-50-54-55-56-57-58-59-60-61-62-63	2-3-4-5-6-7-8-9-10-11
3	36-37-39-43-44-65	13
4	1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19-20-21-22-23-24-25-26-27-28-29-30-31-32-33-34-35-38-45-46-47-48-51-53-64	1-12

TABLE II. THE RESULTS OF CLUSTERING GENERATORS

Bus Cluster	Generators of each bus cluster	Clustered generators
1	14-15-16	(16), (15-14)
2	2-3-4-5-6-7-8-9-10-11	(2-5-10), (3-4-6-7-8), (9), (11)
3	13	(13)
4	1-12	(1) , (12)

TABLE III. SELECTED INPUT AND OUTPUT SIGNALS WITH SUGGESTED ALGORITHM

Control signal location (output of PSS)	measurement signal location (input of PSS)
13-1-14-15-16-11-9-2-4	13-12-14-15-16-11-9-5-7

To find the optimal input and output signals, the modal observability and controllability criteria are applied on each cluster. The results of Table III show that there are 3 universal control PSSs on generators 1, 2 and 4 that their output signals are applied to generators 12, 5 and 7 and the other 6 controllers are locally on generators 13, 14, 15, 16, 11 and 9.

To show the effectiveness of the proposed approach, it is compared with the sequential orthogonal (SO) [12] approach and conventional modal approach [14].

Table IV shows the selected input and output signals of the above three approaches. Table IV reveals that there are 15 input and output pairs selected by modal analysis, while the proposed approach results in 9 pairs.

TABLE IV. COMPARISON OF SELECTED INPUT AND OUTPUT SIGNALS IN DIFFERENT ALGORITHMS

Approach	Control signal location (output of PSS)	measurement signal location (input of PSS)
Proposed approach	13-12-14-15-16-11-9-5-7	13-1-14-15-16-11-9-2-4
SO Approach [12]	16-10-8-3-9-2-12-5-7-10-11-4-13-14	16-4-12-11-2-6-8-5-10-7-3-1-14-13
Modal approach [14]	5-16-3-12-8-10-5-2-9-7-8-14-7-5-15	5-2-3-12-11-10-13-11-16-7-9-16-13-13-14

In Table V, the total damping with and without control with different approaches are given. Obviously, the suggested approach concludes better damping and consequently damps the signals oscillations better than the other approaches. The modes of system are shown in Fig. 7 and 8. The voltage step response of all the generators (voltage reference of generator 1) to a disturbance are displayed in Fig. 9 and 10.

TABLE V. SUMMATION OF DAMPING MODES BEFORE AND AFTER EXPLOITING UNIVERSAL PSS

Control	Open-Loop	Local Control	Modal Technique	Suggested approach
damping	0.2777	6.4321	7.1415	8.9727

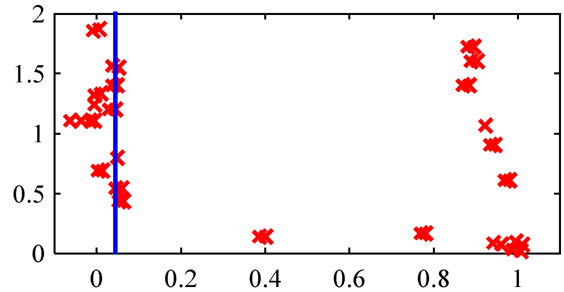

Fig. 7. System modes (Open-loop system).

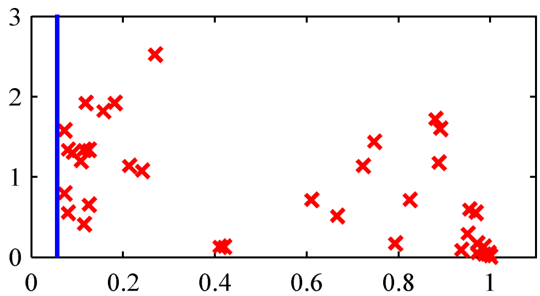

Fig. 8. System modes in case of suggested control.

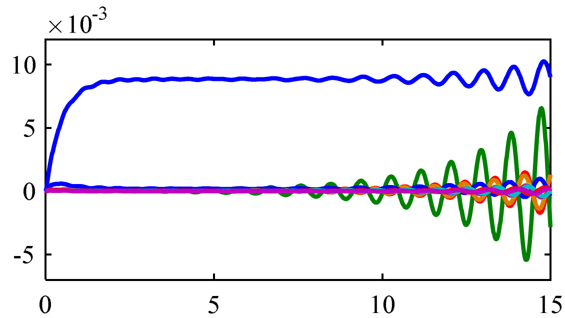

Fig. 9. The response of Open-loop system to disturbance.

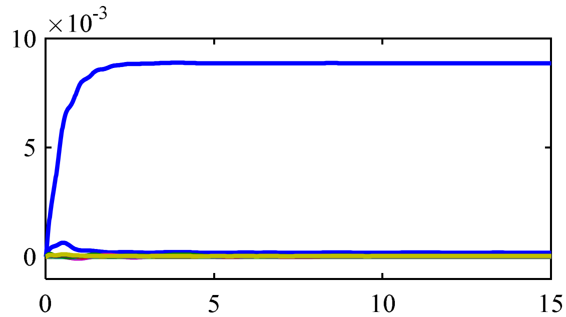

Fig. 10. The response of system to disturbance in case of suggested control.

The results show the effectiveness of the proposed approach in selecting optimal selection for PSS signal which effectively controls the system behaviour in response to disturbances.

IV. Conclusion

In this paper, a method for optimal input and output PSS signals selection based on the data of PMU is proposed. The method utilizes the modal analysis technique, but by clustering buses and generators in the system, a better strategy for signals selection is reached. The combination of generator clustering and bus clustering using modal analysis and PCA techniques results in the desirable damping with the least number of input and output signals. The presented idea exploits the communication facilities through wide area measurement system and increases the quality of PSS. The developed algorithm has many computations which are completely offline and after selecting the appropriate input-output signals, they can be implemented in practice with a low-computational burden. Therefore, it is an appropriate algorithm for being implemented in power systems. The method is applied on a 16 machines test system. Compared to the previous approaches in this area, the suggested algorithm reduces cost and complexity of the monitoring system while the control performance is not deteriorated.

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