A planar path following controller for underactuated marine vehicles

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Abstract—**Inspired by the previous work of Aicardi et al.[1] a path following controller for underactuated planar vehicles is designed by adopting a polar-like kinematic model of the system. The solution proposed does not generally guarantee null asymptotic path following error, but only its boundedness below an adjustable upper threshold. However, knowledge of the path curvature is not necessary, thus resulting in a much easier solution to implement when compared to the alternative ones available in the literature. Indeed controllers achieving perfect path following usually require not only the use of the path curvature, but also of its derivative with respect to the curvilinear abscissa. Furthermore, the proposed solution may be applied to the control of unicycle as well as underactuated marine vehicle models. Simulation results illustrate the performance of the algorithm proposed for path following.**

I. Introduction

The task of designing path following and trajectory tracking controllers for underactuated marine vehicles is very challenging and has received increasing attention in the past few years. Trajectory tracking deals with the case where a vehicle must track a time-parameterized reference. Path following refers to the problem of making a vehicle converge to and follow a given path, without any temporal specifications. Among the most relevant recent results, a backstepping control design technique is employed in [2] for a surface vessel having an aft propeller along the surge and sway axis, respectively. In this work, control system design is done based on a full nonlinear dynamic model of the ship. The resulting tracking control law is static (i.e.time invariant) and guarantees global convergence of the position error to zero in spite of constant unknown environmental force disturbances, but the orientation of the vehicle is left in open loop. In [3] the kinematics of a surface vehicle are considered and a time-varying control law for the surge and yaw inputs is designed such that the pose (i.e. position and orientation) error of the vehicle with respect to a reference trajectory of constant curvature is practically globally exponentially stabilized to zero. Remarkably it is not necessary for the reference linear speed to be nonzero, i.e. the same control law guarantees global practical stabilization if the reference trajectory should degenerate to a constant pose. In [4] a similar model is considered and a static control law is designed that guarantees exponential, semi-global convergence of the surface vessel to a desired reference trajectory. The major limit of this solution is that the reference angular velocity is required to be always nonzero, namely straight lines are not valid reference courses. In [5] a 2D path following controller is designed for the dynamic model a marine vehicle. This static solution takes explicitly into account constant but unknown currents and it guarantees global convergence of the pose error to zero. As explicitly shown in [6], reference paths are not required to have constant curvature. Remarkably the same design methodology may be extended to solve also the 3D path following problem as addressed in [7]. In [8] a static solution to the 2D problem of stabilizing the kinematic model of a marine vehicle on a linear course is proposed. The control law is designed exploiting a potential field -like idea which has been successfully employed to solve also the unicycle pose regulation problem [9]. Convergence to the reference linear course is global and has exponential rate, but environmental disturbances are not explicitly taken into account. In [10] [11] a position (not pose) tracking law is designed for the dynamic model of a surface vessel having a stern propeller and a

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rudder. The solution is static and explicitly takes into account a constant unknown disturbance force. Interestingly the idea behind the design of this solution is quite close to the one adopted in this paper: a *virtual reference point* (VRP) is defined at the bow or ahead of the vessel and the error variable that is globally exponentially stabilized to zero is precisely the distance between the VRP and a reference point moving along the reference path. The heading of the vessel is left in open loop, thus the vessel may rotate around if the VRP is not suitably chosen with respect to the center of mass of the vessel.

It should be noticed that some of the above references refer to trajectory tracking controllers, whereas others refer to path following ones. Nevertheless they all share a common property: the reference curvature and its derivative are generally required to compute the control inputs. This might be a quite demanding issue in real applications. The ultimate reason for this drawback lays in the extreme ambition of the control objective, namely perfect tracking (viz. perfect path following). The solution described in this paper originates from the observation that if the objective is relaxed from perfect path following to convergence inside a "tube" of non null diameter centered on the reference path, the steering law will not require explicit use of either the curvature κ or its spatial derivative $d\kappa/ds$. On the other hand in the great majority of real applications perfect path following is actually not necessary. In marine application, moreover, it would not be practically realizable anyhow because of the limited accuracy of the vehicles position relative to the reference path. The main idea underlying the proposed approach is thus to design a kinematic "tube" path following controller adopting a set of polar-like variables. It is shown that if the reference path has bounded curvature there exist initial conditions for the vehicles position such that convergence is achieved without any singularity in the control signals.

The paper is organized as follows. Section II introduces the marine vehicle model. Section III describes a solution to the problem of path following assuming known currents. In Section IV, convergence and stability of the of proposed solution are analyzed. Section V addresses the problem of current estimation, while Section VI analyzes the combined observer-controller system. Section VII provides results of simulations. Finally, Section VIII contains the main conclusions.

II. The Model

In the absence of currents, the vehicle kinematics are described by:

$$\dot{x} = u\cos\psi - v\sin\psi \tag{1}$$
$$\dot{y} = u\sin\psi + v\cos\psi \tag{2}$$
$$\dot{\psi} = r. \tag{3}$$

where, following standard notation, u (surge speed) and v (sway speed) are the body fixed frame components of the vehicle's velocity relative to the water, x and y are the cartesian coordinates of its center of mass, ψ defines its orientation, and r is the vehicle's angular speed. In the presence of a constant current, the variables u and v in the equations above should be replaced by $u \longrightarrow u + V_c\cos(\sigma-\psi)$ and $v \longrightarrow v + V_c\sin(\sigma-\psi)$ respectively, where V_c is the current's intensity and σ defines its absolute direction. With reference to figure (1), consider a target frame $< T >$ moving at velocity $\dot{s}$ along a desired reference path having curvature $\kappa = \omega_r/\dot{s} : |\kappa| < 1/R$ for some positive R, where $\omega_r = \dot{\phi}$ is the angular velocity of the target frame. The evolution of the position and orientation of the underactuated vehicle with respect to the target frame may be described by the polar like variables $e = \sqrt{x^2+y^2}, \alpha, \phi$ and θ that are defined in accordance to figure (1). Notice that by construction $\alpha + \psi = \phi + \theta$. The state variables are e, α, θ and v and their dynamics are given by:

$$\begin{aligned}\dot{e} &= -(u + V_c\cos(\sigma-\psi))\cos\alpha + \dot{s}\cos\theta \\ &\quad -(v + V_c\sin(\sigma-\psi))\sin\alpha \end{aligned}\tag{4}$$
$$\begin{aligned}\dot{\alpha} &= -r + \frac{(u + V_c\cos(\sigma-\psi))\sin\alpha}{e} \\ &\quad -\frac{(v + V_c\sin(\sigma-\psi))\cos\alpha}{e} - \frac{\dot{s}\sin\theta}{e}\end{aligned}\tag{5}$$
$$\begin{aligned}\dot{\theta} &= -\omega_r + \frac{(u + V_c\cos(\sigma-\psi))\sin\alpha}{e} \\ &\quad -\frac{(v + V_c\sin(\sigma-\psi))\cos\alpha}{e} - \frac{\dot{s}\sin\theta}{e}\end{aligned}\tag{6}$$
$$\dot{v} = -a\,u\,r - k_v v - k_{v|v|}v|v| \tag{7}$$

where $a = m_{11}/m_{22}$, $k_v = |Y_v|/m_{22}$, $k_{v|v|} = |Y_{v|v|}|/m_{22}$. The symbols m_{11} and m_{22} capture the effect of mass and added mass terms, whereas Y_v and $Y_{v|v|}$ are hydrodynamic derivatives [12]. Notice that not only the equations (5) and (6) for $\dot{\alpha}$ and $\dot{\theta}$ are singular in $e = 0$, but α and θ are themselves not defined in $e = 0$.

III. Path following controller problem

With the above notation, the problem considered in this paper can be formulated as follows. Given the underactuated marine vehicle with kinematics equations (1-3) and sway dynamics (7), together with a reference path to be followed, derive a feedback control law for u and r so that: *i*) the angle α converge to zero, and *ii*) the vehicle center of mass converge to and remain inside a tube centered around that path with a nonzero desired speed. In order to design a suitable controller to accomplish this task, assume that $e(t) \neq 0 : \forall t$. The closed loop validity of this hypothesis will be discussed in the following. As far as the angular velocity is concerned, r may be designed to yield

$$\dot{\alpha} = -\gamma_\alpha\,\alpha \quad : \quad \gamma_\alpha > 0 \tag{8}$$

in closed loop, namely:

$$\begin{aligned} r &= +\frac{(u + V_c\cos(\sigma-\psi))\sin\alpha}{e} - \frac{\dot{s}\sin\theta}{e} \\ &\quad -\frac{(v + V_c\sin(\sigma-\psi))\cos\alpha}{e} + \gamma_\alpha\,\alpha\end{aligned}\tag{9}$$

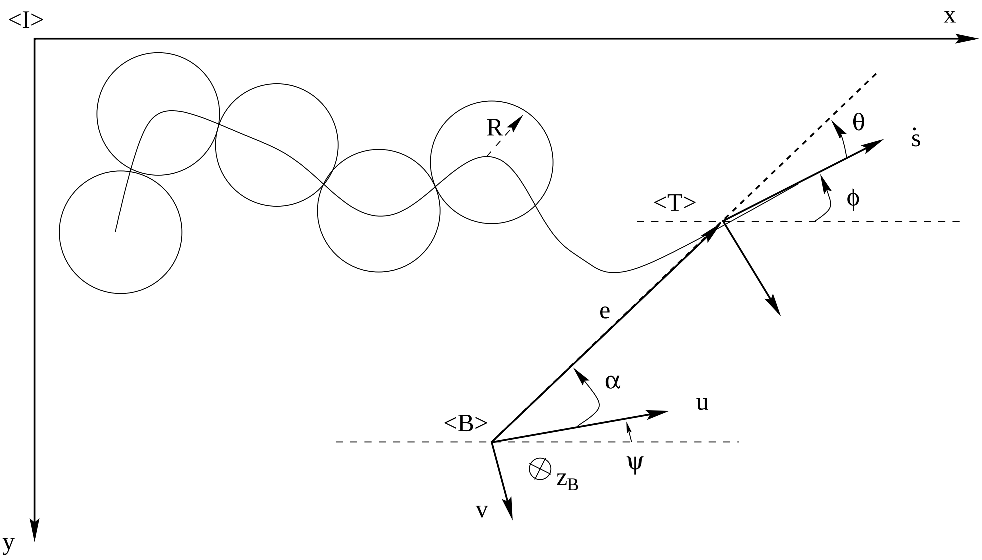

Fig. 1. The model

where the current intensity and direction are assumed (for the moment) known. As far as u is concerned, let $\bar{e}$ be a constant such that $0 < \bar{e} < R$. A direct inspection of equation (4) suggests to fix:

$$u = -V_c \cos(\sigma - \psi) + f(t) \cos\theta \tag{10}$$

where $0 < f(t) < f_{\max}$ (the applied thrust is assumed to be positive and bounded) is a function to be specified. In particular intuitive physical considerations suggest that $f(t) > V_c$ in order to make the path following physically possible: the convergence analysis outlined in the next sections confirms that indeed $f(t) > V_c$ is a necessary condition. Replacing the above equation (10) in (4)

$$\dot{e} = (-f(t)\cos\alpha + \dot{s})\cos\theta - (v + V_c \sin(\sigma - \psi))\sin\alpha. \tag{11}$$

In order to guarantee asymptotic convergence of e to $\bar{e}$, $\dot{s}$ and f may be chosen such to satisfy:

$$-f(t)\cos\alpha + \dot{s} = -\gamma_e\,(e - \bar{e})\cos\theta : \gamma_e > 0 \tag{12}$$

The target velocity $\dot{s}$ may be thus fixed to:

$$\dot{s} = +f(t)\cos\alpha - \gamma_e\,(e - \bar{e})\cos\theta \tag{13}$$

yielding in closed loop

$$\dot{e} = -\gamma_e\,(e - \bar{e})\,\cos^2\theta - (v + V_c \sin(\sigma - \psi))\sin\alpha. \tag{14}$$

Notice that $\dot{s}$ may be negative, namely the target may move "backwards" towards the vehicle. Qualitatively this will occur if either the vehicle and the target are very distant from each other and the vehicle has a low velocity ($\gamma_e\,|(e - \bar{e})| >> f(t)$) or if the vehicle is pointing in the "opposite" direction of the target ($|\alpha| > \pi/2$) at high speed compared to $\gamma_e\,|(e - \bar{e})|$. Summarizing the control signals are given by

$$\begin{aligned} u &= -V_c \cos(\sigma - \psi) + f(t)\cos\theta && (15)\\ r &= \gamma_\alpha\,\alpha + \frac{1}{e}(f(t)\cos\theta\sin\alpha - (v + V_c\sin(\sigma - \psi))\cos\alpha \\ &\quad -(f(t)\cos\alpha - \gamma_e\,(e - \bar{e})\cos\theta)\sin\theta) && (16)\end{aligned}$$

A. Boundedness of relevant signals

The function $f(t)$ and the current velocity V_c are bounded by hypothesis. Consequently so is u. As long as $e > \varepsilon > 0$ for some ε then v and r will be bounded. This follows by noticing that as long as $e > \varepsilon > 0$, replacing (15) and (16) in (7) the following holds: $\lim_{|v|\to\infty} v\dot{v} = -\infty$. This implies that both v and the control signal r given by (16) are bounded if $e > \varepsilon > 0$. The point is now to show that there indeed exist initial conditions for $e_{|t=0}$ such that $e > \varepsilon > 0 \;\; \forall\, t \geq 0$. Call $\tilde{e} = e - \bar{e}$ and $|v_m|$ the maximum value of $|v + V_c\sin(\sigma - \psi)|$ when $e(t) \neq 0$. Assume $\tilde{e}_{|t=0} > 0$. The closed loop dynamics of $\tilde{e}$ can be derived from equation (14) and satisfies the following:

$$\dot{\tilde{e}} = -\gamma_e\,\tilde{e}\cos^2\theta - (v + V_c\sin(\sigma - \psi))\sin\alpha \geq -\gamma_e\,\tilde{e} - |v_m||\alpha|. \tag{17}$$

Considering the closed loop dynamics of α given by equation (8) (which can be assumed valid as long as $e > \varepsilon > 0$), the solution of the differential equation

$$\dot{z} = -\gamma_e\,z - |v_m||\alpha| \tag{18}$$

for $t \geq 0$ is given by

$$\begin{aligned} z(t) &= \exp(-\gamma_e\,t)\,z_{|t=0} \\ &- \frac{|v_m||\alpha_{|t=0}|}{\gamma_e - \gamma_\alpha}\Big[\exp(-\gamma_\alpha\,t) - \exp(-\gamma_e\,t)\Big]. \end{aligned} \tag{19}$$

Notice that if $\gamma_\alpha > \gamma_e$ and $z_{|t=0} > |v_m||\alpha_{|t=0}|/|(\gamma_e - \gamma_\alpha)|$ then $z(t) > 0 \;\; \forall\, t \geq 0$ and $z(t) \to 0$ exponentially, namely $z(t)$ will converge to zero monotonically with exponential rate. Given this result, by applying the comparison Lemma [13, Chp. 2, p. 84] to inequality (17) it follows that

$$\gamma_\alpha \;>\; \gamma_e \;>\; 0 \tag{20}$$

$$e_{|t=0} \;>\; \max\left\{\bar{e},\, \frac{|v_m||\alpha_{|t=0}|}{|\gamma_e - \gamma_\alpha|}\right\} \tag{21}$$

guarantee

$$e(t) > \bar{e} \;\; \forall \;\; t \geq 0. \tag{22}$$

The physical interpretation of these results is straightforward: intuitively $\gamma_\alpha > \gamma_e$ guarantees that α converges to zero faster than $\tilde{e}$; once that this has occurred it is apparent from figure (1) that the sway component v of the vehicles velocity will not be able to "push" the vehicle over the point $e = 0$ (actually not even over $e = \bar{e}$) and the surge component of the velocity is regulated such that e is not larger than $\bar{e}$ (equation (14)). The condition (21) guarantees that during the convergence of α to zero, e does not reach $\bar{e}$.

IV. Convergence and stability analysis

Having proven that $e(t) > \bar{e} \;\; \forall \;\; t \geq 0$ as long as conditions (20) and (21) hold is sufficient to guarantee that the steering control (16) and v are bounded and thus that α indeed obeys equation (8) at all times. It must still be proven that $e \rightarrow \bar{e}$. A preliminary inspection of equation (14) suggests that as long as $|\theta| = n\,\pi/2$ is not an equilibrium for any positive integer n then e should indeed converge to $\bar{e}$. In order to proof this formally the following assumptions is first made:

Assumption: (23)

$\exists \;\; t^* > 0$ and $\theta^* \in [0, \pi/2) : \theta(t) \in (-\theta^*, \theta^*) \;\; \forall \, t \geq t^*.$

Although a proof of this assumption for an arbitrary path of bounded curvature is not provided, notice that it is most reasonable to consider it valid on the basis of physical considerations: with reference to figure (1) consider the situation in which $\gamma_e\,|(e - \bar{e})|\cos\theta$ is negligible with respect to $f(t)$ and α has converged to zero (which holds globally exponentially). In this kind of situation the target frame will be moving with a speed $\dot{s} \approx f(t)$ and $|\theta|$ could be kept larger than $\pi/2$ only if the total inertial speed of the vehicle was approximately equal (in the vector sense) to the one of the target. But such circumstance is prevented by the facts that v is not actuated and highly damped and by the initial hypothesis that $f(t)$ is larger than V_c. To make this point more precise, consider the closed loop equation of θ in the limit $\alpha = 0$, namely:

$$\dot{\theta} = -\frac{v + V_c \sin(\sigma - \psi)}{e} - \left(\kappa + \frac{\sin\theta}{e}\right)(f(t) - \gamma_e(e - \bar{e})\cos\theta). \tag{24}$$

According to the above equation, if the reference path should be a straight line, the only "driving" input for θ would be the first term, related to the velocity along the sway axis, while for all $|\theta| \in [\pi/2, \pi]$ the second one is a nonlinear damping term as can be seen by the facts that $e > \bar{e} \;\; \forall \; t \geq 0$, $\cos\theta \leq 0$ and the sign of $\sin\theta$ is the same of θ if $|\theta| \in [\pi/2, \pi]$.

Given the above assumption, equation (17) describes an exponentially stable system subject to the exponentially decaying perturbation $-(v + V_c \sin(\sigma - \psi))\sin\alpha$, thus $\tilde{e}$ is guaranteed to converge exponentially to zero [13]. At last it should be noticed that if $\bar{e} < 1/\kappa_{max} = R$, being $\kappa_{max} = \max|\kappa|$, from equation (24) it follows that indeed if

$$f(t) > \frac{R\,|v_m|}{R - \bar{e}} \tag{25}$$

then $e = \bar{e} < R \,\cup\, \alpha = 0 \;\; \Rightarrow$

$$\left.\frac{d}{dt}\,\frac{1}{2}\theta^2\right|_{\theta=\pm\pi/2} = (\theta\,\dot{\theta})_{|\theta=\pm\pi/2} < 0 \tag{26}$$

confirming that asymptotically the vehicle will remain strictly inside a tube of radius R centered on the reference path and $|\theta| \in [0, \pi/2)$, namely the vehicle will remain "behind" the origin of the target frame $<T>$. Equation (25) gives a quantitative estimate of how much $f(t)$ should exceed $|v_m| \geq V_c$ in order to achieve the control objective. Summarizing, assuming (23) to hold, the conditions that must be satisfied in order to achieve exponential convergence and stability of $(\tilde{e}, \alpha)^T$ to zero are given by equations (20), (21) and (25).

V. Current estimation

If the currents intensity V_c and direction σ should not be known, their values must be estimated and the terms V_c and σ in equations (15-16) replaced by the estimated ones $\hat{V}_c$ and $\hat{\sigma}$. Following [5], the constant components of the currents velocity $c_x, c_y \;:\; V_c = \sqrt{c_x^2 + c_y^2}$ may be estimated by the observer:

$$\begin{aligned}
\dot{\hat{x}} &= V_{tot}\cos\mu + \hat{c}_x + k_1\,\tilde{x} \;\;:\;\; \tilde{x} = x - \hat{x} && (27)\\
\dot{\hat{y}} &= V_{tot}\sin\mu + \hat{c}_y + k_2\,\tilde{y} \;\;:\;\; \tilde{y} = y - \hat{y} && (28)\\
\dot{\hat{c}}_x &= k_3\,\tilde{x} && (29)\\
\dot{\hat{c}}_y &= k_4\,\tilde{y} && (30)
\end{aligned}$$

being $V_{tot} = \sqrt{u^2 + v^2}$ the norm of the total vehicle velocity with respect to the water and $\mu = \psi + \text{atan2}(v, u)$ its direction with respect to the x axis. As shown in [5], the resulting estimation error dynamics is linear, namely:

$$\begin{aligned}
\dot{\tilde{x}} &= -k_1\,\tilde{x} + \tilde{c}_x \;\;:\;\; \tilde{c}_x = c_x - \hat{c}_x && (31)\\
\dot{\tilde{y}} &= -k_2\,\tilde{y} + \tilde{c}_y \;\;:\;\; \tilde{c}_y = c_y - \hat{c}_y && (32)\\
\dot{\tilde{c}}_x &= -k_3\,\tilde{x} && (33)\\
\dot{\tilde{c}}_y &= -k_4\,\tilde{y} && (34)
\end{aligned}$$

and it is exponentially stable for any $k_i > 0, \forall i = 1, \ldots 4$. The currents intensity and direction may be estimated as $\hat{V}_c = \sqrt{\hat{c}_x^2 + \hat{c}_y^2}$ and $\hat{\sigma} = \text{atan2}(\hat{c}_y, \hat{c}_x)$ being atan2 the four quadrant inverse tangent function. Notice that given the above estimation error dynamics (31-34) the state vector $(\tilde{x}, \tilde{y}, \tilde{c}_x, \tilde{c}_y)^T$ is observable from $(\tilde{x}, \tilde{y})^T$. In particular this means that the initial values $(\tilde{c}_x, \tilde{c}_y)|_{t=0}$ can be computed knowing $(\tilde{x}, \tilde{y})^T$ over a finite time interval $t \in [0, t^\dagger]$: assuming to initialize the estimation filter to $(\hat{c}_x, \hat{c}_y)|_{t=0} = (0, 0)$ the initial value $(\tilde{c}_x, \tilde{c}_y)|_{t=0}$ is precisely equal to the unknown current (c_x, c_y). Thus it is possible to compute the current velocity components (c_x, c_y) measuring the vehicles pose (x, y, ψ) and velocity with respect to the water u, v over a finite time interval $t \in [0, t^\dagger]$. Considering the physics of the problem under investigation, this is

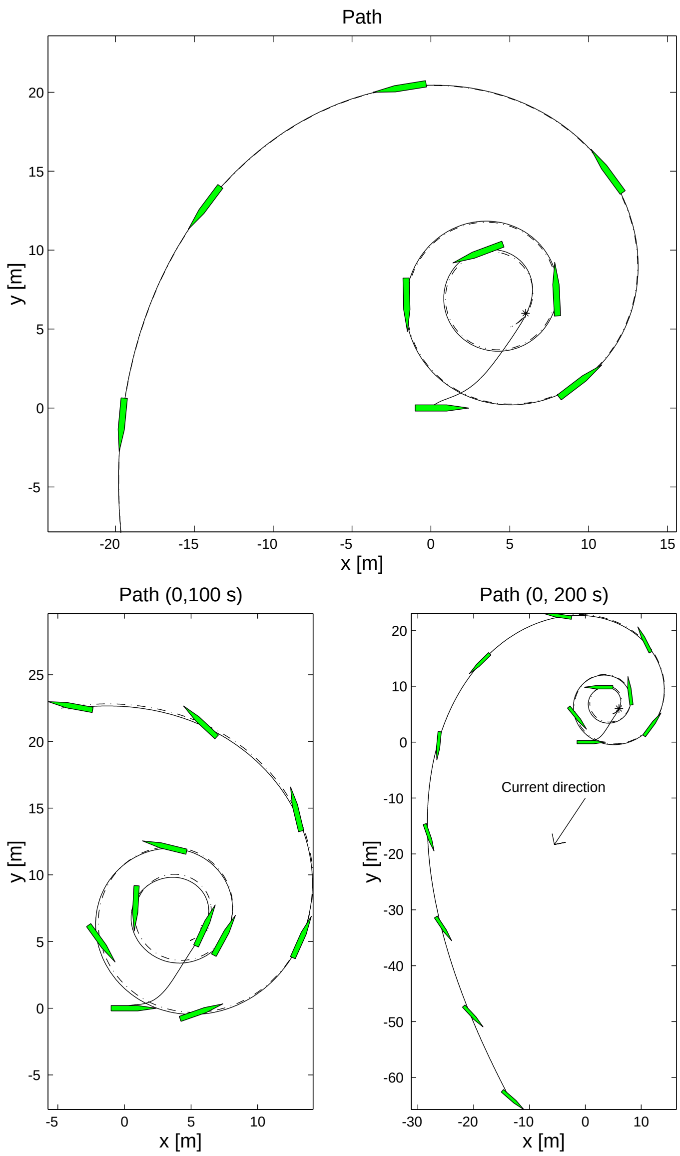

Fig. 2. Paths: no current case (top) and constant current case (other two).

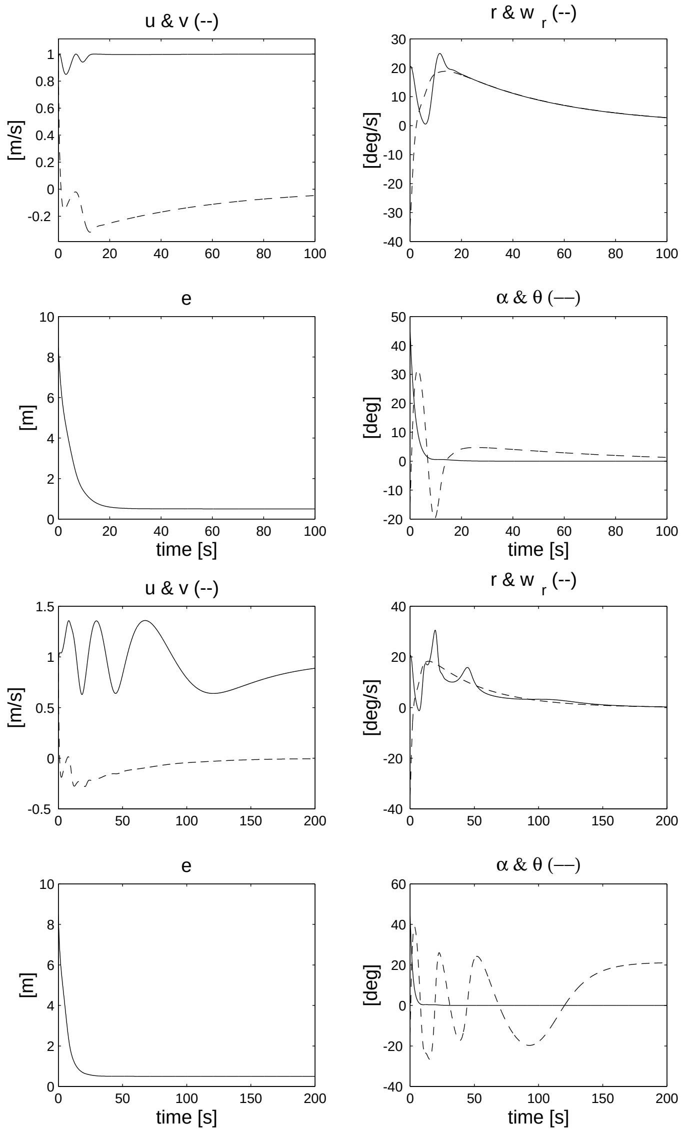

Fig. 3. Results: no current case (first two rows from the top) and constant current case (other two rows)

not surprising: indeed a straightforward application of the galileian velocity composition rule implies that the total (inertial) velocity of the vehicle is given by the superposition of the unknown current velocity plus the velocity with respect to the water (measured) and this inertial velocity may be computed over a finite time interval as the ratio of the traveled distance (measured) over $t^{\dagger}$. Hence one can compute the unknown current by measuring (x, y, ψ) and u, v over a finite time. Of course such open loop finite time estimation will be affected by measurement noise and is valid in the ideal hypothesis that the current is actually strictly constant. Nevertheless the above observation is important as it points out that by performing any manoeuvre of finite duration prior to starting the path following task, one can estimate the constant current velocity components (c_x, c_y) and then use this initial open loop finite time estimated value to initialize the filter given by equations (27-30).

VI. Analysis of the controller-observer system

In the light of the above observations and by exploiting perturbation theory results of exponentially stable systems it can be shown that the combined observer-controller system does guarantee local convergence of $(\tilde{e}, \alpha)^T$ to $(0,0)^T$. Replacing the estimated values $\hat{V}_c$ and $\hat{\sigma}$ to V_c and σ in the control laws given by equations (15-16), the closed loop dynamics of $\tilde{e}$ and α are:

$$\begin{aligned}
\dot{\tilde{e}} &= -\gamma_e\, \tilde{e} \cos^2\theta \\
&- \ (v + V_c \sin(\sigma - \psi)) \sin\alpha + \delta_1(t) \qquad (35)\\
\dot{\alpha} &= -\gamma_\alpha\, \alpha + \delta_1(t)\, \frac{\sin\alpha}{e} - \delta_2(t)\, \frac{\cos\alpha}{e} \qquad (36)\\
\delta_1(t) &= V_c \cos(\sigma - \psi) - \hat{V}_c \cos(\hat{\sigma} - \psi) \qquad (37)\\
\delta_2(t) &= V_c \sin(\sigma - \psi) - \hat{V}_c \sin(\hat{\sigma} - \psi) \qquad (38)
\end{aligned}$$

where given the observer properties described in the previous paragraph, $\lim_{t\to\infty}\delta_1(t) = \lim_{t\to\infty}\delta_2(t) = 0$. Equations (35-36) can be viewed as perturbed versions of equations (8) and (17). In particular the dynamics of the vector $p = (\tilde{e}, \alpha)^T$ can be written as $\dot{p} = f(p) + g(p,t)$ being $f(p)$ the unperturbed system described by equations (8) and (17) and $g(p,t)$ a perturbation described by the above equations (35-36). The unperturbed system $\dot{p} = f(p)$ has been shown to be exponentially stable in section IV and the perturbation $g(p,t)$ can be shown to satisfy the bound $\|g(p,t)\| \leq \gamma(t)\|p\| + \delta(t)\ \forall\ t \geq 0,\ \forall\ \|\tilde{e}\| < \varepsilon\ :\ \varepsilon > 0$ for some nonnegative, continuous, bounded and asymptotically null functions $\gamma(t)$ and $\delta(t)$ that can be chosen to be linear combinations of $|\delta_1(t)|$ and $|\delta_2(t)|$ given in equations (37-38). As a consequence standard perturbation theory results of exponentially stable systems may be applied, in particular under the above conditions Lemma 5.7 [13, Chp. 5, p. 235] guarantees that if $\|p|_{t=0}\|$ and $\sup_{t\geq 0}\delta(t)$ are sufficiently small, then $\|p(t)\|$ will remain bounded at all times by a quantity depending on $\|p|_{t=0}\|$ and $\sup_{t\geq 0}\delta(t)$. Considering that *i*): $\delta(t)$ is a linear combination of $|\delta_1(t)|$ and $|\delta_2(t)|$, *ii*): the exponential convergence properties of the current estimator filter and *iii*): that according to the observations made in the previous paragraph, $\hat{V}_c$ and $\hat{\sigma}$ may be initialized arbitrarily close to the real values, this is enough to guarantee that e will not reach the singular value $e = 0$ as long as $\|p|_{t=0}\| = \|\,(\tilde{e}, \alpha)^T_{t=0}\,\|$ and $(\tilde{c}_x, \tilde{c}_y)|_{t=0}$ are sufficiently small.

At last notice that if $e(t) \neq 0\ \forall\ t \geq 0$ as guaranteed by the above analysis, the solution of the system $\dot{p} = f(p) + g(p,t)$ will be bounded at all times. Moreover thanks to the global exponential stability and convergence properties of the current estimator filter, the perturbation $g(p,t)$ is asymptotically null, thus the largest invariant set of the vector $(\tilde{e}, \alpha, \tilde{x}, \tilde{y}, \tilde{c}_x, \tilde{c}_y)^T$ is the origin. Considering now the following Lemma [13, Chp. 3, p. 114]:

Lemma: If a solution $x(t)$ of $\dot{x} = f(x)$ is bounded and belongs to D for $t \geq 0$, then its positive limit set L^+ is a nonempty, compact, invariant set. Moreover $x(t) \rightarrow L^+$ as $t \rightarrow \infty$.

It follows that $\lim_{t\to\infty}(\tilde{e}, \alpha, \tilde{x}, \tilde{y}, \tilde{c}_x, \tilde{c}_y)^T = (0,0,0,0,0,0)^T$.

VII. Simulation examples

Results relative to two simulations are reported. The first refers to the case of no currents, the second to a constant current of velocity $c_x = -0.2\,[m/s], c_y = -0.3\,[m/s]$. The numerical values of the relevant parameters common to the two cases are: $a = 1; k_v = 1\,[s^{-1}]; k_{v|v|} = 1\,[m^{-1}]$; (sway dynamics) $\bar{e} = 1/2\,[m]; \gamma_\alpha = 1/2\,[s^{-1}]; \gamma_e = 1/4\,[s^{-1}]; f(t) = 1\,[m/s]$; (control law parameters) . The initial pose of the vehicle was $(x_0, y_0, \psi_0) = (0,0,0)$, the target path was generated by a "virtual" unicycle starting in $(x_{r0}, y_{r0}, \psi_{r0}) = (6\,[m], 6\,[m], 60\,[deg])$ and having a curvature given by $k_r(t) = (1/2)\exp(-t/\tau)\ \ :\ \ \tau = 42.9\,[s]$ such that the "virtual" reference unicycle had angular velocity given by $w_r(t) = k_r(t)\,\dot{s}(t)$. The sway velocity was initialized to $v_0 = 1\,[m/s]$ in both cases. In the simulation with constant current, the observer gains where chosen as $k_1 = k_2 = 2.2\,[s^{-1}], k_3 = k_4 = 1\,[s^{-1}]$ and the observer states were initialized to to $\hat{x}_0 = x_0, \hat{y}_0 = x_0, \hat{c}_{x0} = \hat{c}_{y0} = 0$.

VIII. Conclusions

The most remarkable property of the proposed steering control law is that according to equation (16) r is computed without needing to know or to estimate either κ or $d\kappa/ds$ which is indeed required by other tracking laws reported in the literature. Moreover in the limit $\alpha = \dot{\alpha} = 0$ (which occurs exponentially in time) it can be seen from the state equations (5) and (6) that r will converge to $\omega_r + \dot{\theta}$ in spite of the fact that no explicit use of ω_r is made in the designed law (16) for r. A second noteworthy property of the proposed solution is that according to equation (15), the commanded linear velocity will naturally tend to decrease for increasing values of θ thanks to the $\cos\theta$ term multiplying $f(t)$. Thus if $f(t)$ should be kept constant, this would imply to slow down in curves when the misalignment variable θ is more likely to grow.

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