Coordinated Path-Following Control for Nonlinear Systems with Logic-Based Communication

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***Abstract*— We address the problem of designing decentralized feedback laws to force the outputs of decoupled nonlinear systems (agents) to follow geometric paths while holding a desired formation pattern. To this effect we propose a general framework that takes into account *i)* the topology of the communication links among the agents, *ii)* the fact that communications do not occur in a continuous manner, and *iii)* the cost of exchanging information. We provide conditions under which the resulting overall closed loop system is input-to-state stable and apply the methodology for two cases: agents with nonlinear dynamics in strict feedback form and a class of underactuated vehicles. Furthermore, we address explicitly the case where the communications among the agents occur with non-homogenous, possibly varying delays. A coordinated path-following algorithm is derived for multiple underactuated autonomous underwater vehicles. Simulation results are presented and discussed.**

I. Introduction

Motivated by challenging practical applications, the problem of coordinated path-following (CPF) control has recently come to the forum. See for example [1]–[4] and the references therein for an introduction to the subject and an overview of important theoretical issues that warrant consideration. The essence of the problem can be best explained by focusing on a simple mission scenario with n autonomous vehicles: given n spatial paths, derive control laws to steer the vehicles along the paths at a “common” desired speed profile, while holding a specified formation pattern.

Different approaches to the solution of this and similar problems have been reported in the literature [5]–[12]. The common strategy to solve the problem of CFP is to partially decouple it into two tasks: *i)* path-following (PF), where the objective is to find local closed loop control laws to steer each vehicle to its assigned path at a nominal reference speed, and *ii)* multiple vehicle coordination, where the goal is to adjust the reference speeds of the vehicles about the desired formation speed, so as to reach formation. Presently, many schemes are available for PF control. However, the coordination problem lacks a complete solution addressing explicitly practical constraints that arise from the characteristics of the supporting inter-vehicle communication network. For example, underwater applications require that a fleet of vehicles coordinate themselves by exchanging information over low bandwidth, short range communication channels that are plagued with intermittent failures, multi-path effects, and distance-dependent delays. Currently, no CPF algorithms are known that will yield guaranteed performance in the presence of such stringent constraints occurring simultaneously. Some of these issues, however, have been partially addressed in the literature by exploiting the use of graph theory to model the topology of the underlying communication network and Lyapunov-based tools to deal with intermittent failures or switching topologies [1], [3], [4], [11].

Inspired by the above results, this paper proposes a framework for CPF that applies to a very general class of nonlinear systems and takes into account the topology of the (bidirectional) communication links among such systems, explicitly. Furthermore, the paper addresses also the fact communications among systems do not occur in a continuous manner but take place at discrete instants of time instead. As such, the results in the paper go well beyond those obtained in [1], [4] where it was tacitly assumed that the flow of information is continuous, even though it may exhibit intermittent interruptions. The main contribution of the paper lies in a new proposed control architecture that *aims to reduce the frequency at which information is exchanged among the systems involved by incorporating a logic-based communication*. To this effect, we borrow from and expand some of the key ideas exposed in [13], [14] where decentralized controllers for a distributed system were derived by using for each system its local state information, together with estimates of the states of the systems it communicates with. Here, we introduce the key constraint that a vehicle is only allowed to communicate with a set of immediate neighbors. With the scheme adopted, each vehicle decides when to transmit information to the neighbors by comparing its actual state with its estimate “as perceived” by the neighboring systems, and transmitting data when the “difference” between the two exceeds a certain level. Thus, the systems communicate at discrete instants of time, asynchronously.

With the framework proposed, we provide conditions under which the CPF control architecture obtained by putting together path-following, coordination, and logic-based communication strategies solves the CPF problem with guaranteed stability, convergence, robustness and performance in the presence of disturbances, sensor noise, and strict communication constraints. The mathematical tools adopted borrow from graph theory and the theory of ISS systems.

To illustrate the scope of the methodology proposed, we solve the CPF problem for nonlinear systems in strict feedback form and for a class of underactuated vehicles. One of the key contributions is the fact that, to the best of our knowledge, and contrary to CPF algorithms described in the literature, we address explicitly the case where the communications among systems occur with non-homogenous,

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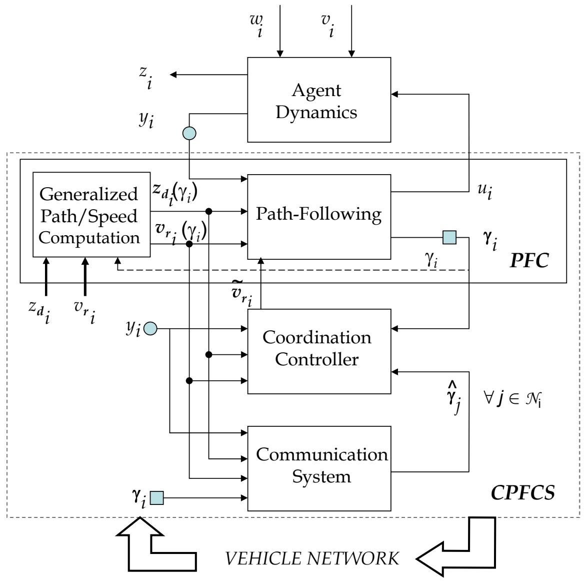

Fig. 1. Coordinated path-following control system architecture.

possibly varying delays.

Due to space limitations most of the proofs are omitted. These can be found in [15]. We refer the reader to [1]–[4] and the references therein for background material on CPF.

Notation: $|\cdot|$ denotes the standard Euclidean norm of a vector in $\mathbb{R}^n$ and $\|u\|_I$ is the (essential) supremum norm of a signal $u : [0, \infty) \to \mathbb{R}^n$ on an interval $I \subset [0, \infty)$. We let $\mathcal{I} := \{1, \ldots, n\}$, $z_{d_i}^{\gamma_i} := \frac{\partial z_{d_i}}{\partial \gamma_i}$ $z_{d_i}^{\gamma_i^2} := \frac{\partial^2 z_{d_i}}{\partial^2 \gamma_i}$, $[a_i]_{i \in \mathcal{I}} := \operatorname{col}(a_1, \ldots, a_n)$, and $a \oplus b := \max\{a, b\}$.

II. Coordinated Path-Following Control System

This section proposes a CPF control architecture for a group of n decoupled agents Σ_i, $i \in \mathcal{I}$ modeled by general systems of the form

$$\Sigma_i : \quad \dot{x}_i = \mathcal{F}_i(x_i, u_i, w_i), \tag{1a}$$

$$y_i = \mathcal{H}_i(x_i, u_i, v_i), \tag{1b}$$

$$z_i = \mathcal{J}_i(x_i, u_i), \tag{1c}$$

where $x_i \in \mathbb{R}^{n_i}$ denotes the state of agent i, $u_i \in \mathbb{R}^{m_i}$ its control input, $y_i \in \mathbb{R}^{p_i}$ its measured noisy output, w_i an input disturbance, and v_i measurement noise. The output $z_i \in \mathbb{R}^{q_i}$ is a signal that we require to reach and follow a desired feasible path $z_{d_i}(\gamma_i) : \mathbb{R} \to \mathbb{R}^{q_i}$ parameterized by $\gamma_i \in \mathbb{R}$. The following notation is required: $\mathbf{z_{d_i}}(\gamma_i) := \operatorname{col}\left(z_{d_i}(\gamma_i), z_{d_i}^{\gamma_i}(\gamma_i), \ldots z_{d_i}^{\gamma_i^k}(\gamma_i)\right)$, $\mathbf{v_{r_i}}(\gamma_i, t) := \operatorname{col}\left(v_{r_i}(\gamma_i, t), v_{r_i}^{\gamma_i}(\gamma_i, t) \ldots v_{r_i}^{\gamma_i^l}(\gamma_i, t)\right)$, and $\boldsymbol{\gamma}_i := \operatorname{col}(\gamma_i, \gamma_i^{(1)} \ldots \gamma_i^{(n)})$. The architecture for the general coordinated path-following control system (CPFCS) proposed in this paper is shown in Fig. 1. It consists of three interconnected subsystems:

Path-following controller — a dynamical system whose inputs are a path z_{d_i}, a desired speed profile v_{r_i} that is common to all agents, and the agent's output y_i. Its output is the agent's input u_i, computed so as to make it follow the path at the assigned speed. In preparation for its connection to the coordination controller, this system produces also a generalized path $\mathbf{z_{d_i}}(\gamma_i)$, a generalized speed profile $\mathbf{v_{r_i}}(\gamma_i)$, and a generalized path-variable $\boldsymbol{\gamma}_i$. Further, it accepts corrective speed action from the coordination controller via the signal $\tilde{v}_{r_i}$. Notice that the dynamics of the parameterizing variable $\boldsymbol{\gamma}_i$ are defined internally at this stage and play the role of an extra design knob to tune the performance of the PF control law.

Coordinated controller — a dynamical system whose inputs are the plant output y_i, the desired generalized path $\mathbf{z_{d_i}}$ and speed profile $\mathbf{v_{r_i}}$, the generalized path-variable $\boldsymbol{\gamma}_i$, and estimates $\hat{\boldsymbol{\gamma}}_j$ of the generalized coordination states $\boldsymbol{\gamma}_j$; $j \in \mathcal{N}_i$ where $\mathcal{N}_i$, denotes the set of agents that agent i communicates with. Its output is the correction speed signal $\tilde{v}_{r_i}$ which is used to synchronize agent i with its neighbors.

Logic-based communication system — a logic-based dynamical system that establishes an interface with the network through which the agents communicate. Its inputs are the agents' output y_i, the generalized desired path $\mathbf{z_{d_i}}$ and speed profile $\mathbf{v_{r_i}}$, and the generalized path variable $\boldsymbol{\gamma}_i$. Its output is the estimate $\hat{\boldsymbol{\gamma}}_j$ of the generalized coordination states $\boldsymbol{\gamma}_j$; $j \in \mathcal{N}_i$.

We now describe in more detail how this circle of ideas can be formalized using dynamical system concepts. We also discuss what properties every subsystem should satisfy in order to obtain a CPFCS that solves the CPF problem and is robust to input disturbances and measurement noise.

A. Path-following controller

Definition 1: Consider a set of n agents Σ_i, $i \in \mathcal{I}$ with dynamics (1) and let $\mathcal{Z}_{d_i}$ and $\mathcal{V}_{r_i}$ be the classes of admissible paths and speed profiles, respectively. We say that a given set of controllers given by $\Sigma_{PFi}; i \in \mathcal{I}$

$$\Sigma_{PFi} : \ \dot{x}_{PFi} = \mathcal{F}_{PFi}(x_{PFi}, y_i, \mathbf{z_{d_i}}, \mathbf{v_{r_i}}, \tilde{v}_{r_i}), \tag{2a}$$

$$u_i = \mathcal{H}_{PFi}(x_{PFi}, y_i, \mathbf{z_{d_i}}, \mathbf{v_{r_i}}, \tilde{v}_{r_i}) \tag{2b}$$

solves *robustly the output path-following problem* if for every path $z_{d_i} \in \mathcal{Z}_{d_i}$ and prescribed speed profile $v_{r_i} \in \mathcal{V}_{r_i}$, there exist functions $\sigma_w^e, \sigma_v^e, \sigma_{\tilde{v}_r}^e, \sigma^e \in \mathcal{K}_\infty$, $\beta^e \in \mathcal{KL}$ and a signal error $\mathbf{e}$ such that for each initial condition $\mathbf{x^0} := [\operatorname{col}(x_i(0), x_{PFi}(0))]_{i \in \mathcal{I}}$ and bounded signals $w := [w_i]_{i \in \mathcal{I}}$, $v := [v_i]_{i \in \mathcal{I}}$, and $\tilde{v}_r := [\tilde{v}_{r_i}]_{i \in \mathcal{I}}$, all the states of the closed-loop system (1)–(2), $\forall i \in \mathcal{I}$ with exception of $\gamma_i(t)$ are bounded, the path-following errors

$$e_i(t) := z_i(t) - z_{d_i}(\gamma_i(t)), \quad \forall i \in \mathcal{I}$$

and the speed errors

$$e_{\dot{\gamma}_i}(t) := \dot{\gamma}_i(t) - v_{r_i}(\gamma_i, t), \quad \forall i \in \mathcal{I}$$

satisfy the detectability condition

$$|e_i(t)| \oplus |e_{\dot{\gamma}_i}(t)| \leq \sigma^e(\|\mathbf{e}\|_{[0,t]}), \quad \forall i \in \mathcal{I} \tag{3}$$

and $\mathbf{e}$ is input-to-output stable (IOS) with respect to w, v, and $\tilde{v}_r$, that is,

$$|\mathbf{e}(t)| \leq \beta^e(|\mathbf{x^0}|, t) \oplus \sigma_w^e(\|w\|_{[0,t]}) \oplus \sigma_v^e(\|v\|_{[0,t]}) \oplus \sigma_{\tilde{v}_r}^e(\|\tilde{v}_r\|_{[0,t]}). \tag{4}$$

□

B. Coordination controller

Definition 2: Consider a set of n agents Σ_i, $i \in \mathcal{I}$ with dynamics (1), the corresponding paths $z_{d_i} \in \mathcal{Z}_{d_i}$, and formation speed assignments $v_{r_i} \in \mathcal{V}_r$. Assume that $\boldsymbol{\gamma}_i$ and $\boldsymbol{\gamma}_j$, $\forall j \in \mathcal{N}_i$ are available to agent i and let $\bar{\boldsymbol{\gamma}}_i := \mathrm{col}(\boldsymbol{\gamma}_i, [\boldsymbol{\gamma}_j]_{j\in\{\mathcal{N}_i\}})$. Let $\bar{\tilde{\boldsymbol{\gamma}}}_i$ be a bounded error signal of the form $\bar{\tilde{\boldsymbol{\gamma}}}_i := \mathrm{col}(0, [\tilde{\boldsymbol{\gamma}}_j]_{j\in\mathcal{N}_i})$. We say that a set of coordination controllers Σ_{CCi}, $i \in \mathcal{I}$

$$\Sigma_{CCi}: \ \dot{x}_{CCi} = \mathcal{F}_{CCi}(x_{CCi}, y_i, \mathbf{z_{d_i}}, \mathbf{v_{r_i}}, \bar{\boldsymbol{\gamma}}_i + \bar{\tilde{\boldsymbol{\gamma}}}_i), \quad (5a)$$
$$\tilde{v}_{r_i} = \mathcal{H}_{CCi}(x_{CCi}, y_i, \mathbf{z_{d_i}}, \mathbf{v_{r_i}}, \bar{\boldsymbol{\gamma}}_i + \bar{\tilde{\boldsymbol{\gamma}}}_i) \quad (5b)$$

solves *robustly the coordination problem* if there exist functions $\beta^{\xi} \in \mathcal{KL}$, $\sigma^{\xi}, \sigma^{\xi}_{v}, \sigma^{\xi}_{\gamma}, \sigma^{\xi}_{ij}, \sigma^{\xi}_{e} \in \mathcal{K}_\infty$ and a coordination error signal ξ that satisfies the detectability property

$$\max_{i\in\mathcal{I};j\in\mathcal{N}_i} |\gamma_i(t) - \gamma_j(t)| \leq \sigma^{\xi}(\|\xi\|_{[0,t)}) \quad (6)$$

and the evolution of $\boldsymbol{\xi}(t) := \mathrm{col}(\xi, [x_{CCi}]_{i\in\mathcal{I}}, [\tilde{v}_{r_i}]_{i\in\mathcal{I}})$ satisfies, for each initial condition $\mathbf{x^0_{\xi}} := \big[\mathrm{col}(x_i(0), x_{PFi}(0), x_{CCi}(0))\big]_{i\in\mathcal{I}}$,

$$\begin{aligned}|\boldsymbol{\xi}(t)| \leq \beta^{\xi}(|\mathbf{x^0_{\xi}}|, t) \oplus \sigma^{\xi}_{v}(\|v\|_{[0,t]}) \oplus \sigma^{\xi}_{\gamma}(\|\bar{\tilde{\boldsymbol{\gamma}}}\|_{[0,t]}) \\ \oplus \max_{i\in\mathcal{I};j\in\mathcal{N}_i} \sigma^{\xi}_{ij}(\|\mathbf{v_{r_i}} - \mathbf{v_{r_j}}\|_{[0,t)}) \oplus \sigma^{\xi}_{e}(\|\mathbf{e}\|_{[0,t)}),\end{aligned} \quad (7)$$

where $v := [v_i]_{i\in\mathcal{I}}$, $\bar{\tilde{\boldsymbol{\gamma}}} := [\bar{\tilde{\boldsymbol{\gamma}}}_i]_{i\in\mathcal{I}}$ and $\mathbf{e} := [\mathbf{e_i}]_{i\in\mathcal{I}}$. □

C. Logic-based communication system

Inspired by the communication logic proposed in [14], each communication subsystem is composed by a bank of estimators and a communication logic. The estimators run open-loop most of the time but are sometimes reset (not necessarily periodically) to correct their state when measurements are received through the network. The communication logic is responsible for determining for each agent, using an internal estimator, how well the other agents from the communication topology can predict the value of its local coordination state and decide when it should communicate the actual measured value to its neighbors. As in [14], the banks of estimators running in the different agents are synchronized, that is, the state estimate of each agent is the same as that of the corresponding neighbors.

Definition 3: Consider a set of n agents Σ_i, $i \in \mathcal{I}$ with dynamics (1), the corresponding paths $z_{d_i} \in \mathcal{Z}_{d_i}$, and formation speed assignments $v_{r_i} \in \mathcal{V}_r$. Assume that $\boldsymbol{\gamma}_i$, y_i, x_{PFi}, x_{CCi} are continuously available to agent i and $\boldsymbol{\gamma}_j$; $\forall j \in \mathcal{N}_i$ is available asynchronously through the network system. Let $t^{[i]}_k; k \geq 0$ indicate the instants of data transmission. We say that a given set of logic-based impulse dynamical systems $\Sigma_{SEi}; i \in \mathcal{I}$ defined as:

For $t^{[i]}_k \leq t < t^{[i]}_{k+1}$

$$\dot{\hat{x}}_{SEi} = \mathcal{F}_{SEi}(\hat{x}_{SEi}, \hat{x}_{CCi}, \hat{x}_{PFi}, \hat{y}_i, \mathbf{z_{d_i}}, \mathbf{v_{r_i}}, \hat{\boldsymbol{\gamma}}_i) \quad (8a)$$
$$\dot{\hat{\gamma}}_{SEi} = \mathcal{H}_{SEi}(\hat{x}_{SEi}, \hat{x}_{CCi}, \hat{x}_{PFi}, \hat{y}_i, \mathbf{z_{d_i}}, \mathbf{v_{r_i}}, \hat{\boldsymbol{\gamma}}_i) \quad (8b)$$

At $t = t^{[i]}_{k+1}$

$$\hat{x}_{SEi}(t^{[i]}_{k+1}) = x_{SEi}(t^{[i]}_{k+1}) \quad (9)$$

solves *robustly the communication problem* if for every $i \in \mathcal{I}$

$$|\tilde{\boldsymbol{\gamma}}_j(t)| \leq \epsilon_i, \quad \forall j \in \mathcal{N}_i, \forall t \geq 0, \quad (10)$$

where $\tilde{\boldsymbol{\gamma}}_j := \hat{\boldsymbol{\gamma}}_j - \boldsymbol{\gamma}_j$. □

At this point, for the sake of generality, we purposely avoid discussing the mechanism for generation of communication times $t^{[i]}_k$. This will be done later in the paper.

We now state the main result of this section.

Theorem 1: Consider the overall closed-loop system Σ_{CL} composed by n agents of the form (1) and the CPFCS defined by (2), (5) and (8)–(9). Suppose that each PF controller Σ_{PFi} and coordinated controller Σ_{CCi} solve robustly the output path-following and coordination problem, respectively, that is, inequalities (3)–(4), (6)–(7) hold. Suppose further that the logic-based communication system satisfies (10) $\forall i \in \mathcal{I}$ and that there exists $r_0 \geq 0$ such that

$$\sigma^{e}_{\tilde{v}_r} \circ \sigma^{\xi}_{e}(r) < r, \quad \forall r > r_0. \quad (11)$$

Then, the overall closed-loop system solves robustly the CPF problem. Stated mathematically, for every agent $i \in \mathcal{I}$, path $z_{d_i} \in \mathcal{Z}_{d_i}$ and prescribed speed profile $v_r = v_{r_1} = \cdots = v_{r_n} \in \cap_{i\in\mathcal{I}}\mathcal{V}_{r_i}$, there exist functions $\sigma^{\bar{e}}, \sigma^{\bar{e}}_{w}, \sigma^{\bar{e}}_{v} \in \mathcal{K}_\infty$, $\beta^{\bar{e}} \in \mathcal{KL}$, a positive scalar ε, and a signal error $\bar{\mathbf{e}}$ such that for each initial condition $\mathbf{x^0_e} := \big[\mathrm{col}(x_i(0), x_{PFi}(0), x_{CCi}(0))\big]_{i\in\mathcal{I}}$ and bounded signals $w := [w_i]_{i\in\mathcal{I}}$ and $v := [v_i]_{i\in\mathcal{I}}$, all the states of the closed-loop system Σ_{CL} with exception of $\gamma(t) := [\gamma_i]_{i\in\mathcal{I}}$ are bounded, the path-following errors, speed errors, the coordination errors satisfy the detectability condition

$$\max_{i\in\mathcal{I}} \big(|e_i(t)| \oplus |e_{\dot{\gamma}_i}(t)| \oplus \max_{j\in\mathcal{N}_i}(\gamma_i - \gamma_j)\big) \leq \sigma^{\bar{e}}(\|\bar{\mathbf{e}}\|_{[0,t]}), \quad (12)$$

and $\bar{\mathbf{e}}$ is input-to-output practically stable (IOpS) with respect to w and v, that is,

$$|\bar{\mathbf{e}}(t)| \leq \beta^{\bar{e}}(|\mathbf{x^0_e}|, t) \oplus \sigma^{\bar{e}}_{w}(\|w\|_{[0,t]}) \oplus \sigma^{\bar{e}}_{v}(\|v\|_{[0,t]}) \oplus \varepsilon \quad (13)$$

Proof: From (4) and (7) we conclude that the path-following and coordinated controllers can be viewed as two interconnected IOS systems with outputs $\mathbf{e}$ and $\tilde{v}_r := [v_{r_i}]_{i\in\mathcal{I}}$, respectively. An application of the small-gain theorem in [16] implies that if (11) holds then the interconnection is practically IOS. We can then conclude (13) since $\mathbf{v_{r_i}} - \mathbf{v_{r_j}} = 0$ and $\bar{\tilde{\boldsymbol{\gamma}}}$ is bounded. Inequality (12) follows using the detectabilities conditions (3) and (6). ■

III. Coordinated path-following for nonlinear systems in strict feedback form

In [17], Skjetne et al. considered the path-following problem for a nonlinear plant Σ_i in strict feedback form of vector relative degree n_i of the form

$$\begin{aligned}\dot{x}_{1_i} &= G_{1_i}(x_{1_i})x_{2_i} + f_{1_i}(x_{1_i}) + W_{1_i}(x_{1_i})\delta_{1_i}(t), \\ \dot{x}_{2_i} &= G_{1_i}(\bar{x}_{2_i})x_{3_i} + f_{2_i}(\bar{x}_{2_i}) + W_{2_i}(\bar{x}_{2_i})\delta_{2_i}(t), \\ &\vdots \\ \dot{x}_{n_i} &= G_{n_i}(\bar{x}_{n_i})u_i + f_{n_i}(\bar{x}_{n_i}) + W_{n_i}(\bar{x}_{n_i})\delta_{n_i}(t), \\ y_i &= h_i(x_{1_i}),\end{aligned} \quad (14)$$

where $x_{j_i} \in \mathbb{R}^{m_i}$, $j = 1, \ldots, n_i$, are the states, $y_i \in \mathbb{R}^{m_i}$ is the output, $u_i \in \mathbb{R}^{m_i}$ is the control, and δ_{j_i} are unknown bounded disturbances. The matrices $G_{j_i}(\bar{x}_{j_i})$ and $h_i^{x_{1_i}}(x_{1_i}) := (\partial h_i/\partial x_{1_i})(x_{1_i})$ are invertible for all $\bar{x}_{j_i} := \mathrm{col}(x_{1_i}, \ldots, x_{j_i})$, $h_i(x_{1_i})$ is a diffeomorphism, and G_{j_i}, f_{j_i}, and W_{j_i} are smooth. The paths $z_{d_i}(\gamma_i) \in \mathcal{Z}_{d_i}$ and speed assignments $v_{r_i}(\gamma_i, t) \in \mathcal{V}_{r_i}$ considered are such that $\mathcal{Z}_{d_i}$ is the space of continuous uniformly bounded functions with its n_i derivatives uniformly bounded on $\mathbb{R}^{m_i}$, and $\mathcal{V}_{r_i}$ the space of uniformly bounded functions in γ_i and t with their $n_i - 1$ partial derivatives uniformly bounded in γ_i and t.

In this section we illustrate the CPF controller architecture for a group of n agents with dynamics (14). We will use the results in [17] for the PF controller.

A. Path-following controller

Following the design in [17], the n_i^{th} step of backstepping yields a closed-loop system of the form

$$\begin{aligned}\dot{\chi}_i &= A_i(\bar{x}_{n_i}, \gamma_i, t)\chi_i + b_i(\bar{x}_{n-1_i}, \gamma_i, t)\omega_i \\ &\quad + W_i(\bar{x}_{n_i}, \gamma_i, t)\Delta_{n_i}(t), \\ \dot{\gamma}_i &= v_{r_i}(\gamma_i, t) - \omega_i,\end{aligned}$$

where $\chi_i \in \mathbb{R}^{n_i m_i}$ is a set of new variables that include the path-following error, $\Delta_{n_i} := \mathrm{col}(\delta_{1_i}, \ldots, \delta_{n_i})$, ω_i is an input signal to be designed, and the functions $W_i \in \mathbb{R}^{n_i m_i \times n_i m_i}$ and $b_i \in \mathbb{R}^{n_i m_i}$ are uniformly bounded in their arguments. From the backstepping design described in [17] it can be concluded that for every given constant $\rho_i > 0$ we can choose sufficiently large gains such that the matrix $A_i \in \mathbb{R}^{n_i m_i \times n_i m_i}$ satisfies

$$P_i\big(A_i + \frac{\rho_i}{2}\big) + \big(A_i + \frac{\rho_i}{2}\big)' P_i \le -Q_i \tag{15}$$

for some matrices $P_i = P_i' > 0$ and $Q_i = Q_i' > 0$.

We now modify the design in [17] to satisfy (3)–(4). Set

$$\omega_i = \tilde{v}_{r_i} + \bar{\omega}_i, \tag{16}$$

where $\bar{\omega}_i$ is a feedback term to be designed such that $\bar{\omega}_i \to 0$ as $t \to \infty$. Examples of $\bar{\omega}_i$ are the tracking update law, gradient update law, or filtered-gradient update law as described in [17], that is,

i) $\bar{\omega}_i = 0,$

ii) $\bar{\omega}_i = -\mu_i \tau_{n_i}(\bar{x}_{n_i}, \gamma_i, t), \quad \mu_i \ge 0$

iii) $\dot{\bar{\omega}}_i = -\lambda_i(\rho_i \bar{\omega}_i + \mu_i \tau_{n_i}(\bar{x}_{n_i}, \gamma_i, t)), \quad \lambda_i, \mu_i \ge 0$

where $\tau_{n_i}(\bar{x}_{n_i}, \gamma_i, t)$ is the n_ith tuning function defined in [17, equation (35)]. From the results in [17], property (15), and the fact that $e_{\dot{\gamma}} = -\tilde{v}_{r_i} - \bar{\omega}_i$ we conclude the following Lemma.

Lemma 1: The state-feedback controllers proposed in [17], together with (16) solve robustly the path-following problem and inequalities (3)–(4) hold with $\mathbf{e} := [\chi_i]_{i\in\mathcal{I}}$ for the tracking and gradient update law or $\mathbf{e} := [\mathrm{col}(\chi_i, \bar{\omega}_i)]_{i\in\mathcal{I}}$ for the filtered-gradient update law, $v \equiv 0$, and $w := [\Delta_{n_i}]_{i\in\mathcal{I}}$. Moreover, for fixed r, the term $\sigma^e_{\tilde{v}_r}(r)$ satisfies $\lim_{|\rho|\to\infty} \sigma^e_{\tilde{v}_r}(r) = 0$, where $\rho := [\rho_i]_{i\in\mathcal{I}}$. □

Remark 1: In (16), we impose relative degree one from γ_i to $\tilde{v}_{r_i}$. Other designs are possible. See [15] for the case of relative degree two.

B. Coordinated controller

This section details the development of the coordination controller subsystem. To this effect, we first recall some key concepts from algebraic graph theory.

Let $\mathcal{N}_i$ be the index set of the vehicles that vehicle i communicates with (the so called neighboring set of vehicle i). We assume that the communication links are bidirectional, that is, $i \in \mathcal{N}_j \Leftrightarrow j \in \mathcal{N}_i$. Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ be the undirected graph induced by the inter-vehicle communication network, with $\mathcal{V}$ denoting the set of n nodes (each corresponding to an agent) and $\mathcal{E}$ the set of edges (each standing for a data link). We say that $\mathcal{G}$ is connected when there exists a path connecting every two nodes in the graph. The adjacency matrix of a graph, denoted A, is a square matrix with rows and columns indexed by the nodes such that the i, j-entry of A is 1 if $j \in \mathcal{N}_i$ and zero otherwise. The degree matrix D of a graph $\mathcal{G}$ is a diagonal matrix where the i, i-entry equals $|\mathcal{N}_i|$, the cardinality of $\mathcal{N}_i$. The Laplacian of a graph is defined as $L := D - A$. Thus, L is symmetric and its every row sums equal zero, that is, $L\mathbf{1} = \mathbf{0}$, where $\mathbf{1} := [1]_{n\times 1}$ and $\mathbf{0} := [0]_{n\times 1}$. If $\mathcal{G}$ is connected, L has a simple eigenvalue at zero with an associated eigenvector $\mathbf{1}$ and the remaining eigenvalues are all positive.

Consider now the coordination control problem with a communication topology defined by a graph $\mathcal{G}$. Using a Lyapunov-based design, we propose a decentralized feedback law for $\tilde{v}_{r_i}$ as a function of the information obtained from the neighboring agents. Following [4], we introduce the error vector

$$\xi := L_K \gamma, \quad L_K := I - \frac{1}{\mathbf{1}' K^{-1} \mathbf{1}} \mathbf{1}\mathbf{1}' K^{-1},$$

where $\gamma := [\gamma_i]_{i\in\mathcal{I}}$, $\mathbf{1} := [1]_{i\in\mathcal{I}}$, and $K > 0$ is a diagonal matrix. See in [4], [15] some key properties of the error vector ξ. With the path-following proposed above, the dynamics of the coordination subsystem can be written in vector form as

$$\dot{\gamma} = v_r + \tilde{v}_r + \bar{\omega}, \tag{17}$$

where $v_r := [v_{r_i}]_{i\in\mathcal{I}}$, $\bar{\omega} := [\omega_i]_{i\in\mathcal{I}}$, and $\tilde{v}_r := [\tilde{v}_{r_i}]_{i\in\mathcal{I}}$. Consider the control Lyapunov function $V := \frac{1}{2}\xi' K^{-1}\xi$. Computing its time-derivative yields

$$\dot{V} = \xi' K^{-1} L_K (v_r + \tilde{v}_r + \bar{\omega}).$$

To make ξ ISS with respect to inputs $L_K v_r$ and $\bar{\omega}$, and assuming that measurements of the coordination states $\gamma_j; \forall j \in \mathcal{N}_i$ are available continuously, a natural choice would be $\tilde{v}_r = -KL\xi = -KLL_K\gamma = -KL\gamma$. In this case, the dominator of $\dot{V}$, $-\xi' K^{-1} L_K K L \xi = -\xi' L \xi$, is negative definite provided that the Graph that models the constraints imposed by the communication topology among the agents is connected. To reduce the communication rate using a logic-based dynamical system, we will lift the assumption that each agent receives information from its neighborhoods continuously. We assume instead that it relies on estimated values. In this case, the coordination feedback law becomes

$$\tilde{v}_r = -K(D\gamma - A\hat{\gamma}), \tag{18}$$

or equivalently, $\tilde{v}_{r_i} = -k_i \sum_{j\in\mathcal{N}_i} \gamma_i - \hat{\gamma}_j$, (the so-called neighboring rule) where k_i denotes the ith diagonal element of K_i. In (18), D and A denote the degree and adjacency matrices of the graph $\mathcal{G}$, respectively. The time-derivative of V_1 is

$$\dot{V}_1 = -\xi' L\xi + \xi' K^{-1} L_K(v_r + \bar{\omega}) + \xi' K^{-1} L_K K A\tilde{\gamma},$$

where $\tilde{\gamma} := \hat{\gamma} - \gamma$. Provided that $\mathcal{G}$ is connected, it is now straightforward to conclude that ξ is ISS with respect to the inputs $L_K v_r$ (which is zero if $v_{r_1} = \cdots = v_{r_n}$), $\bar{\omega}$, and $\tilde{\gamma}$. This leads to the following result.

Lemma 2: The coordination law (18) solves robustly the coordination problem. Inequalities (6)–(7) hold with $v \equiv 0$, $\tilde{\bar{\boldsymbol{\gamma}}} = \tilde{\gamma}$, $\mathbf{v_{r_i}} = v_{r_i}$, and $\sigma_e^{\xi}(r) \equiv 0$ for the tracking update law. For the gradient and filtered-gradient update laws the term $\sigma_e^{\xi}(r)$ satisfies, for fixed r, $\lim_{\underline{k}\to\infty} \sigma_e^{\xi}(r) = 0$, where $\underline{k} := \min_{i\in\mathcal{I}} k_i$. □

C. Logic-based communication system

1) No delayed information: Let $t_k^{[i]}$, $k \geq 0$ denote the instants of time at which agent i transmits or receives data from its neighborhoods. Following the procedure described in Section II and taking account the dynamic equations of the coordination subsystem, we propose for each agent i the following logic-based communication system:

- For $t_k^{[i]} \leq t < t_{k+1}^{[i]}$

Internal estimator: Synchronized estimators:

$$\dot{\hat{\gamma}}_i = \hat{v}_{r_i} + \hat{\tilde{v}}_{r_i} + \hat{\bar{\omega}}_i \qquad \dot{\hat{\boldsymbol{\gamma}}}_i = \hat{\mathbf{v}}_{\mathbf{r_i}} + \hat{\tilde{\mathbf{v}}}_{r_i} + \hat{\bar{\boldsymbol{\omega}}}_i \tag{19a}$$

$$\dot{\hat{v}}_{r_i} = 0 \qquad \dot{\hat{\mathbf{v}}}_{\mathbf{r_i}} = 0 \tag{19b}$$

$$\dot{\hat{\bar{\omega}}}_i = 0 \qquad \dot{\hat{\bar{\boldsymbol{\omega}}}}_i = 0 \tag{19c}$$

- For $t = t_{k+1}^{[i]}$

$$\hat{\gamma}_i = \gamma_i,\ \hat{v}_{r_i} = v_{r_i},\ \hat{\bar{\omega}}_i = \bar{\omega}_i,$$
$$\hat{\boldsymbol{\gamma}}_i = \boldsymbol{\gamma}_i,\ \hat{\mathbf{v}}_{\mathbf{r_i}} = \mathbf{v}_{\mathbf{r_i}},\ \hat{\bar{\boldsymbol{\omega}}}_i = \bar{\boldsymbol{\omega}}_i$$

where $\hat{\tilde{v}}_{r_i} = -k_i \sum_{j\in\mathcal{N}_i} \hat{\gamma}_i - \hat{\gamma}_j$, $\hat{\boldsymbol{\gamma}}_i := [\hat{\gamma}_j]_{j\in\mathcal{N}_i}$, $\hat{\mathbf{v}}_{\mathbf{r_i}} := [\hat{v}_{r_j}]_{j\in\mathcal{N}_i}$, $\hat{\tilde{\mathbf{v}}}_{r_i} := [\hat{\tilde{v}}_{r_j}]_{j\in\mathcal{N}_i}$, and $\hat{\bar{\boldsymbol{\omega}}}_i := [\hat{\bar{\omega}}_j]_{j\in\mathcal{N}_i}$. To simplify the estimators, we have chosen (19b)–(19c) instead of using a copy of the corresponding dynamics models. To solve robustly the communication problem (see Definition 3) we introduce the communication index $S_i(\tilde{\gamma}_i) := c_i\tilde{\gamma}_i^2$, $c_i > 0$, $\tilde{\gamma}_i := \hat{\gamma}_i - \gamma_i$ and use the following logic: agent i transmits to its neighborhoods a message composed by $\{\gamma_i,\ v_{r_i},\ \bar{\omega}_i\}$ at time $t_k^{[i]}$ when $\lim_{t\uparrow t_k^{[i]}} S_i\big(\tilde{\gamma}_i(t)\big) \geq 1$. Note that the post-reset value of $\tilde{\gamma}_i$ is $\tilde{\gamma}_i(t_k^{[i]}) = 0$. Consequently, $\tilde{\gamma}_i \in \{\tilde{\gamma}_i \in \mathbb{R} : S(\tilde{\gamma}_i) \leq 1\}$ and, hence, (10) holds.

2) Delayed information: We now consider the case where the communication channels have time-varying and nonhomogeneous delays. The delays are not known a priori but we assume that all the agents have synchronized clocks. Thus, each agent can compute the time-delay when the time-tagged data arrives. Consider the following situation: agent i sends data to j at time $t_k^{[i]}$, and agent j receives it at time $t_k^{[i]} + \tau_k^{ij}$. Then, immediately afterwards, at time $t_k^{[j]} = t_k^{[i]} + \tau_k^{ij}$, agent j sends to i the computed time-delay τ_k^{ij}. Agent i receives it at time $t = t_k^{[j]} + \tau_k^{ji} = t_k^{[i]} + \tau_k^{ij} + \tau_k^{ji}$. We assume that

$$|\tau_k^{ji} - \tau_k^{ij}| \leq \bar{\tau}, \quad \forall i, \forall j \in \mathcal{N}_i, \forall k \geq 0$$

where $\bar{\tau} > 0$ is a small value known a priori. Loosely speaking, this assumption means that the time-delays in consecutive communications in a bidirectional link are roughly the same.

We now describe the communication system, which is slightly different from the one in the case of non-delayed information. Due to the presence of nonhomogeneous and time-varying delays, each agent i must have $|\mathcal{N}_i|$ internal and synchronized estimators, that is, one set of estimators per each communication link. For example, if agent i can only communicate with agents j and ℓ, i.e. $\mathcal{N}_i = \{j, \ell\}$, then the internal estimator of agent i consists of the following systems:

$$\begin{aligned} &\dot{\hat{\gamma}}_i^{[j]} = \hat{v}_{r_i}^{[j]} + \hat{\tilde{v}}_{r_i}^{[j]} + \hat{\bar{\omega}}_i^{[j]}, && \dot{\hat{\gamma}}_i^{[\ell]} = \hat{v}_{r_i}^{[\ell]} + \hat{\tilde{v}}_{r_i}^{[\ell]} + \hat{\bar{\omega}}_i^{[\ell]}, \\ &\dot{\hat{v}}_{r_i}^{[j]} = 0, && \dot{\hat{v}}_{r_i}^{[\ell]} = 0, \\ &\dot{\hat{\bar{\omega}}}_i^{[j]} = 0, && \dot{\hat{\bar{\omega}}}_i^{[\ell]} = 0. \end{aligned}$$

The communication logic must also to be changed: Suppose that at time $t_k^{[i]}$ agent i transmits to agent ℓ a message, which contains the following data: $\{t_k^{[i]}, \gamma_i(t_k^{[i]}), v_{r_i}(t_k^{[i]}), \bar{\omega}_i(t_k^{[i]})\}$. Then, the internal estimator $\hat{\gamma}_i^{[\ell]}$ cannot be immediately updated. This is because we must guarantee that the value of the state estimate $\hat{\gamma}_i^{[\ell]}$ will always remain equal to the corresponding state estimate running in agent ℓ. To this effect, both estimates can only be updated at time $t = t_k^{[i]} + \bar{\tau}_k^{i\ell}$, where $\bar{\tau}_k^{i\ell} := 2\tau_k^{i\ell} + \bar{\tau}$, because although agent ℓ receive the data at time $t = t_k^{[i]} + \tau_k^{i\ell}$, agent i only knows the delay $\tau_k^{i\ell}$ at time $t = t_k^{[i]} + \tau_k^{i\ell} + \tau_k^{\ell i}$. Upon receiving $\tau_k^{i\ell}$, the coordination state estimate $\hat{\gamma}_i^{[\ell]}$ running in agent i and the corresponding dual estimate running in agent ℓ should be updated at time $t = t_k^{[i]} + \bar{\tau}_k^{i\ell}$ to

$$\begin{aligned} \hat{\gamma}_i^{[\ell]}(t_k^{[i]} + \bar{\tau}_k^{i\ell}) &= e^{-2|\mathcal{N}_i|k_i(\tau_k^{i\ell}+\bar{\tau})}\gamma_i(t_k^{[i]}) \\ &+ \int_{t_k^{[i]}}^{t_k^{[i]}+\bar{\tau}_k^{i\ell}} e^{-2|\mathcal{N}_i|k_i(t_k^{[i]}+\bar{\tau}_k^{i\ell}-\sigma)}\big(v_{r_i}(t_k^{[i]}) + \bar{\omega}_i(t_k^{[i]})\big)\, d\sigma \end{aligned} \tag{20}$$

With the above procedure, we guarantee that the estimators are always synchronized. Equation (20) follows from (17) and (18). Notice that in general $\tilde{\gamma}_i^{[\ell]}(t_k^{[i]} + \bar{\tau}_k^{i\ell})$ will not be zero because v_{r_i} and $\bar{\omega}_i$ may not be constant in the interval $[t_k^{[i]}, t_k^{[i]} + \bar{\tau}_k^{i\ell})$. The estimation error $\tilde{\gamma}_i$ viewed by agent ℓ will be

$$\lim_{t\uparrow t_k^{[i]}+\bar{\tau}_k^{i\ell}} \tilde{\gamma}_i(t) = S_i^{-1}(1) + \int_{t_k^{[i]}}^{t_k^{[i]}+2\tau_k^{i\ell}} \dot{\tilde{\gamma}}_i(\sigma)\, d\sigma, \tag{21}$$

which is bounded assuming that the time-delay is bounded, hence, (10) holds. Equation (21) only holds if the weight of the communication index $S_i(\tilde{\gamma}_i^{[\ell]})$ is selected to be sufficiently small so as to guarantee that the post-reset value of $\tilde{\gamma}_i$ satisfies $S_i(\tilde{\gamma}_i) < 1$.

D. Stability analysis

Using the results in Lemmas 1–2, the properties of the communication system described above, and applying Theorem 1 we conclude the following result.

Theorem 2: Consider the overall closed-loop system Σ_{CL} composed by n agents of the form (14) and the proposed CPF controller. For sufficiently large path-following control gains or coordination control gains, the overall closed-loop system solves robustly the CPF problem. In the presence of delayed information, this result holds true for sufficiently small time-delays or weights of the communication indexes. □

Remark 2: For the particular case of the path-following controller with $\bar{\omega} = 0$ (tracking update law), it turns out that the CPF controller takes a cascaded form of two ISS systems, where the output of the coordination subsystem is the input of the PF controller. In this case, there is no need to use the small-gain theorem and, hence, no constraints on the gains are required.

IV. Coordinated path-following for a class of underactuated vehicles

Consider an underactuated vehicle modeled as a rigid body subject to external forces and torques. Let $\{\mathcal{I}\}$ be an inertial coordinate frame and $\{\mathcal{B}\}$ a body-fixed coordinate frame whose origin is located at the center of mass of the vehicle. The configuration (R, p) of the vehicle is an element of the Special Euclidean group $SE(3) := SO(3) \times \mathbb{R}^3$, where $R \in SO(3)$ is a rotation matrix that describes the orientation of the vehicle by mapping body coordinates into inertial coordinates, and $p \in \mathbb{R}^3$ is the position of the origin of $\{\mathcal{B}\}$ in $\{\mathcal{I}\}$. Denoting by $v \in \mathbb{R}^3$ and $\omega \in \mathbb{R}^3$ the linear and angular velocities of the vehicle relative to $\{\mathcal{I}\}$ expressed in $\{\mathcal{B}\}$, respectively, the following kinematic relations apply:

$$\dot{p} = Rv, \quad \dot{R} = RS(\omega), \tag{22}$$

where $S(\cdot)$ is a function from $\mathbb{R}^3$ to the space of skew-symmetric matrices $\mathcal{S} := \{M \in \mathbb{R}^{3\times 3} : M = -M'\}$. We consider here underactuated vehicles with dynamic equations of motion of the following form:

$$\mathbf{M}\dot{v} = -S(\omega)\mathbf{M}v + f_v(v, p, R) + g_v u_v \tag{23a}$$

$$\mathbf{J}\dot{\omega} = -S(v)\mathbf{M}v - S(\omega)\mathbf{J}\omega + f_\omega(v, \omega, p, R) + G_\omega u_\omega \tag{23b}$$

where $\mathbf{M} \in \mathbb{R}^{3\times 3}$ and $\mathbf{J} \in \mathbb{R}^{3\times 3}$ denote constant symmetric positive definite mass and inertia matrices; $u_v \in \mathbb{R}$ and $u_\omega \in \mathbb{R}^3$ denote the control inputs, which act upon the system through a constant nonzero vector $g_v \in \mathbb{R}^3$ and a constant nonsingular matrix[1] $G_\omega \in \mathbb{R}^{3\times 3}$, respectively; the terms $-S(\omega)\mathbf{M}v$ in (23a) and $-S(v)\mathbf{M}v - S(\omega)\mathbf{J}\omega$ in (23b) are the rigid-body Coriolis terms, and the $\mathcal{C}^1$ functions $f_v(\cdot)$, $f_\omega(\cdot)$ represent all the remaining forces and torques acting on the body. For the special case of an underwater vehicle, $\mathbf{M}$ and $\mathbf{J}$ also include the so-called hydrodynamic added-mass M_A and added-inertia J_A matrices, respectively, i.e., $\mathbf{M} = M_{RB} + M_A$, $\mathbf{J} = J_{RB} + J_A$, where M_{RB} and J_{RB} are the rigid-body mass and inertia matrices, respectively.

In [18] we proposed a solution to the path-following problem for underactuated autonomous vehicles described by (22)–(23) in the presence of possibly large modeling parametric uncertainty.

If we select the same update law for $\ddot{\gamma}_i$ as in [18] but adding the additional term $\tilde{v}_{r_i}$, the coordination subsystem can be written in vector form as

$$\ddot{\gamma} = f_\gamma(\chi, \gamma, \dot{\gamma}) + \tilde{v}_r, \tag{24}$$

where $f_\gamma(\cdot)$ represents right-hand-side of the path-following update law in [18]. The following result holds.

Lemma 3: The state-feedback controller proposed in [18] together with (24) solve robustly the path-following problem with condition (4) modified to an IOpS inequality. Moreover, if all the control gains are scaled by a factor $\rho > 0$, then, for fixed r, the gain function $\sigma^e_{\tilde{v}_r}(r)$ satisfies $\lim_{\rho\to\infty} \sigma^e_{\tilde{v}_r}(r) = 0$. □

From (24), following steps similar to the ones described in [15] for the coordination, it is straightforward to obtain a decentralized feedback law of the form

$$\tilde{v}_r = -K(D\dot{\gamma} - A\dot{\hat{\gamma}}) - K_2\big(\dot{\gamma} - v_r\mathbf{1} + K(D\gamma - A\hat{\gamma})\big), \tag{25}$$

where $K > 0$ and $K_2 > 0$ are diagonal matrices, and to conclude that it solves robustly the coordination problem. Further, for fixed r, $\lim_{\underline{k_2}\to\infty} \sigma^\xi_e(r) = 0$.

We can now conclude the following.

Theorem 3: Consider the overall closed-loop system Σ_{CL} composed by n underactuated vehicles of the form (22)–(23) and the CPF controller with the PF control law in [18] together with (24), the coordinated controller (25), and a robustly logic-based communication system as defined in Definition 3. For sufficiently large path-following control gains or coordination control gains, the overall closed-loop system solves robustly the CPF problem. □

A. CFP of underwater vehicles in 3-D space

Consider an ellipsoidal shaped underactuated autonomous underwater vehicle (AUV) not necessarily neutrally buoyant. Let $\{\mathcal{B}\}$ be a body-fixed coordinate frame whose origin is located at the center of mass of the vehicle and suppose that we have available a pure body-fixed control force τ_u in the x_B direction, and two independent control torques τ_q and τ_r about the y_B and z_B axes of the vehicle, respectively. The kinematics and dynamics equations of motion of the vehicle can be written as (22)–(23) (see [18] for details).

Computer simulations were done to illustrate the performance of the CPF controller proposed, when applied to a group of three AUVs ($n = 3$). The numerical values used for the physical parameters match those of the *Sirene* AUV (see details in [15]).

The AUVs are required to follow paths of the form

$$\mathbf{p}_{d_i}(\gamma_i) = \left[c_i \cos(\tfrac{2\pi}{T}\gamma_i + \phi_d), c_i \sin(\tfrac{2\pi}{T}\gamma_i + \phi_d), d\gamma_i\right],$$

with $c_1 = 20\,\mathrm{m}$, $c_2 = 15\,\mathrm{m}$, $c_3 = 25\,\mathrm{m}$, $d = 0.05\,\mathrm{m}$, $T = 400$, and $\phi_d = -\frac{3\pi}{4}$. The initial conditions are $\mathbf{p}_1 = (10\,\mathrm{m}, -15\,\mathrm{m}, -5\,\mathrm{m})$, $\mathbf{p}_2 = (5\,\mathrm{m}, 0\,\mathrm{m}, 0\,\mathrm{m})$, $\mathbf{p}_3 = (20\,\mathrm{m}, -25\,\mathrm{m}, 5\,\mathrm{m})$, $R_1 = R_2 = R_3 = I$, and $v_1 = v_2 = v_3 = \omega_1 = \omega_2 = \omega_3 = \mathbf{0}$. The reference speed was set to $v_r = 1[\mathrm{sec}^{-1}]$. The vehicles are also required to keep a formation pattern that consists of having them aligned along a common horizontal line. Furthermore, AUV 1 is allowed to communicate with AUVs 2 and 3, but the latter two do not communicate between themselves directly. To further illustrate the behavior of the proposed CPF control

[1] See [18, Remark 4] for the special case of $G_\omega \in \mathbb{R}^{3\times 2}$.

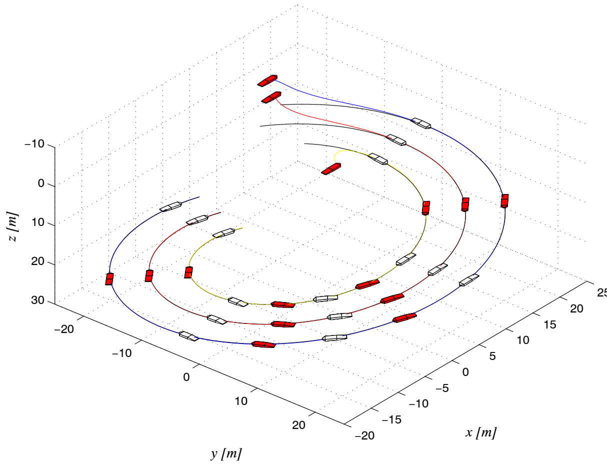

Fig. 2. Coordinated path-following of 3 AUVs, with logic-based communication.

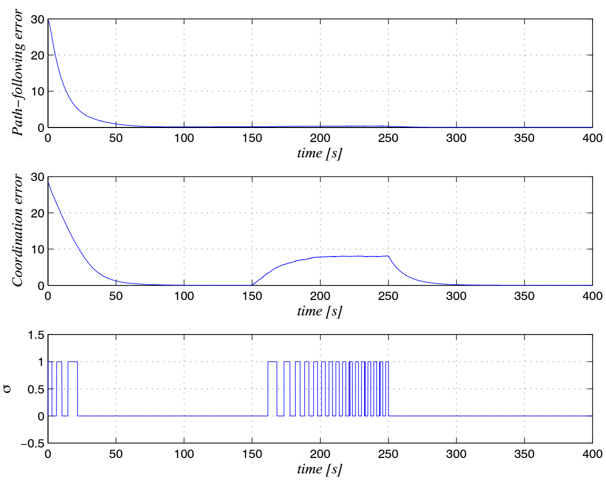

Fig. 3. Path-following error, coordination error $|\gamma_1 - \gamma_2| + |\gamma_1 - \gamma_3|$, and communication signal σ.

architecture, we also force the following scenario: from $t = 150\,s$ to $t = 250\,s$, AUV 1 can only follow the path with $\dot{\gamma}_1 = 0.1$. Figure 2 shows the trajectories of the AUVs and Fig. 3 the evolution of the overall path-following error $\sum_{i=1}^{3} \|p_i - p_{di}\|$, coordination error $|\gamma_1 - \gamma_2| + |\gamma_1 - \gamma_3|$, and the communication signal σ. The signal $\sigma \in \{0, 1\}$ indicates, by switching its value, when there is communication. Before $t = 150\,s$, the vehicles adjust their speeds to meet the formation requirements and the coordination errors converge to zero. Note the reduced number of communications exchanged during that period. In fact, the vehicles only need to communicate a few times during the transient phase. When AUV 1 is forced to slow down from $t \in [150, 250]$ (without transmitting explicitly to its neighborhoods its new reference velocity), the communication rate increases in order to keep the coordination error bounded.

V. Conclusions

We proposed a general decentralized control architecture to address the problem of forcing the outputs of decoupled nonlinear systems (agents) to follow geometric paths while holding a desired formation pattern (coordinated path-following). The architecture takes explicitly into account the topology of the communication links among the agents, the fact that communications do not occur in a continuous manner, and the cost of exchanging information. Conditions under which the overall closed loop system is input-to-state stable were derived. The methodology proposed was illustrated for two cases: agents with nonlinear dynamics in strict feedback form, and a class of underactuated vehicles. The problems that arise when communications among the agents occur with non-homogenous, possibly varying delays was also addressed. In this case, it was required that all the agents have synchronized clocks. Future research will aim at relaxing this requirement.

References

[1] R. Ghabcheloo, A. P. Aguiar, A. Pascoal, C. Silvestre, I. Kaminer, and J. Hespanha, "Coordinated path-following control of multiple underactuated autonomous vehicles in the presence of communication failures," in *Proc. IEEE Conf. Decision and Control (CDC)*, San Diego, CA, USA, Dec. 2006.

[2] R. Ghabcheloo, A. Pascoal, C. Silvestre, and I. Kaminer, "Nonlinear coordinated path following control of multiple wheeled robots with bidirectional communication constraints," *Int. J. Adapt. Control Signal Process*, vol. 21, pp. 133–157, 2007.

[3] I.-A. F. Ihle, M. Arcak, and T. I. Fossen, "Passivation designs for synchronization of path following systems," in *Proc. 45th IEEE Conference on Decision and Control*, San Diego, USA, Dec 2006.

[4] R. Ghabcheloo, A. Aguiar, A. Pascoal, C. Silvestre, I. Kaminer, and J. Hespanha, "Coordinated path-following in the presence of communication losses and time delays," Submitted for publication, 2006.

[5] F. Giuletti, L. Pollini, and M. Innocenti, "Autonomous formation flight," *IEEE Control Systems Magazine*, vol. 20, no. 6, pp. 34–44, Dec. 2000.

[6] D. J. Stilwell and B. E. Bishop, "Platoons of underwater vehicles," *IEEE Control Systems Magazine*, vol. 20, no. 6, pp. 45–52, Dec. 2000.

[7] P. Ögren, M. Egerstedt, and X. Hu, "A control lyapunov function approach to multiagent coordination," in *Proc. of the 40th Conf. on Decision and Contr.*, Orlando, FL, USA, 2001.

[8] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Trans. on Automat. Contr.*, vol. 48, no. 6, pp. 988–1001, June 2003.

[9] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Trans. on Automat. Contr.*, vol. 49, no. 9, pp. 1465–1476, Sept. 2004.

[10] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. on Automat. Contr.*, vol. 49, no. 9, pp. 1520–1533, Sept. 2004.

[11] L. Moreau, "Stability of multiagent systems with time-dependent communication links," *IEEE Transactions on Automatic Control*, vol. 50, no. 2, pp. 169–182, Feb. 2005.

[12] W. Ren, R. W. Beard, and E. M. Atkins, "A survey of consensus problems in multi-agent coordination," in *Proc. of the 2005 Amer. Contr. Conf.*, Portland, OR, USA, June 2005.

[13] J. K. Yook, D. M. Tilbury, and N. R. Soparkar, "Trading computation for bandwidth: Reducing communication in distributed control systems using state estimators," *IEEE Trans. Contr. Syst. Technol*, vol. 10, no. 4, pp. 503–518, 2002.

[14] Y. Xu and J. P. Hespanha, "Communication logic design and analysis for networked control systems," in *Current trends in nonlinear systems and control*, L. Menini, L. Zaccarian, and C. T. Abdallah, Eds. Boston: Birkhäuser, 2006.

[15] A. P. Aguiar and A. Pascoal, "Coordinated path-following control for nonlinear systems with logic-based communication," Institute for Systems and Robotics, Lisbon, Portugal, Tech. Rep., Sept. 2007.

[16] Z. P. Jiang, A. Teel, and L. Praly, "Small-gain theorem for ISS systems and applications," *Math. Control, Signals and Systems*, vol. 7, pp. 95–120, 1994.

[17] R. Skjetne, T. I. Fossen, and P. Kokotović, "Robust output maneuvering for a class of nonlinear systems," *Automatica*, vol. 40, no. 3, pp. 373–383, 2004.

[18] A. P. Aguiar and J. P. Hespanha, "Trajectory-tracking and path-following of underactuated autonomous vehicles with parametric modeling uncertainty," *IEEE Trans. on Automat. Contr.*, vol. 52, no. 8, pp. 1362–1379, Aug. 2007.