Notes on Trajectory-tracking and Path-following Controllers for Constrained Underactuated Vehicles using Model Predictive Control*

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***Abstract*— This document extends [1] with further details on the computation of the terminal cost and terminal set. For the computation of the terminal set, an easy-to-implement method is proposed that exploits the quadratic structure of the Lyapunov function associated to auxiliary law, avoiding the use of sum-of-square optimization. Moreover, the code needed for numerical results is made available on-line.**

I. Computation of terminal cost and terminal set

In the following the same notation and definitions of [1] are adopted.

The computation of the terminal cost proposed in Theorem 1 of [1] is a direct application of the Proposition 27 in [2].

The computation of the terminal set proposed in [1] consists in solving the following optimization problem:

$$\max_{\alpha} \alpha \text{ s.t. } \mathcal{V}(e) \subseteq \mathcal{U},\ \forall e \subseteq \mathcal{L}(V,\alpha) \subseteq \mathcal{E}. \tag{1}$$

where $\mathcal{L}(V,\alpha)$ denotes the α-level set of the function $V(\cdot)$, i.e., $\{e : (1/2)e'e \leq \alpha\}$, and $\mathcal{E}$ is a constraints set for the error vector $e(t)$.

Note that, form

$$\mathcal{V}(e) := \operatorname{conv}\left\{ \begin{pmatrix} b_1\bar{k}_1 \\ \vdots \\ b_{n_u}\bar{k}_{n_u} \end{pmatrix} - \bar{\Delta}Ke,\ b_1,\dots,b_{n_u} \in \{\pm 1\} \right\}$$

we have

$$\begin{aligned} \mathcal{V}(e) \subseteq \mathcal{U} &\iff -\bar{\Delta}Ke \oplus \mathcal{B} \subseteq \mathcal{U} \\ &\iff -\bar{\Delta}Ke \subseteq \mathcal{U} \ominus \mathcal{B} \end{aligned}$$

where $\mathcal{B}$ is defined as follows:

$$\mathcal{B} := \operatorname{conv}\left\{ \begin{pmatrix} b_1\bar{k}_1 \\ \vdots \\ b_{n_u}\bar{k}_{n_u} \end{pmatrix},\ b_1,\dots,b_{n_u} \in \{\pm 1\} \right\}.$$

For the case of polytopic sets $\mathcal{E}$ and $\mathcal{U}$, the optimization problems (1) is reduced to the geometric problem of finding the larges level set of a quadratic function inside a polytope. In fact, let $\mathcal{E} = \{F_e e \leq f_e\}$ and $\mathcal{U} \ominus \mathcal{B} = \{F_u u \leq f_u\}$, then (1) becomes:

$$\max_{\alpha} \alpha \text{ s.t. } \mathcal{L}(V,\alpha) \subseteq \left\{ e : \begin{pmatrix} -F_u\bar{\Delta}K \\ F_e \end{pmatrix} e \leq \begin{pmatrix} f_u \\ f_e \end{pmatrix} \right\}$$

that can be explicitly solved (see code for more details).

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References

[1] A. Alessandretti, A. P. Aguiar, and C. N. Jones, "Trajectory-tracking and path-following controllers for constrained underactuated vehicles using Model Predictive Control," in *Proc. of the 2013 European Control Conference*, pp. 1371–1376, 2013.

[2] A. Alessandretti, P. A. Aguiar, and C. Jones, "On Convergence and Performance Certification of a Continuous-Time Economic Model Predictive Control Scheme with Time-Varying Performance Index," *Automatica*, vol. 68, pp. 305 – 313, 2016.