

# THE USE OF THE SABINE AND EYRING REVERBERATION TIME EQUATIONS TO CHURCHES

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## ABSTRACT

Reverberation time measurements were taken at several source/receiver locations in 41 Catholic churches in Portugal built in the last fourteen centuries, using the impulse response method. The use of the Sabine and Eyring reverberation time equations was tested to estimate the measured RTs in this sample of churches. The effect of coupled spaces was analyzed and a new algorithm for the application of the Sabine equation in churches was developed producing an average of 16% in the differences between the predicted and measured RTs compared to 71% using the standard Sabine equation. Coupled spaces were found to act as *windows* with a characteristic absorption coefficient depending on their dimensions. The recesses in churches were grouped in three types: main altar area, chapels and lateral aisles, each having a particular acoustical behavior. It was found that those recesses only acted as coupled spaces if their *length/opening\_width* > 0.6 or if the *aisle\_width/opening\_height* > 0.4 in lateral aisles. The remaining differences found between the RTs measured and predicted with this new algorithm were hypothesized to be related to what was called a *reverberant ceiling effect*, which is presumed to be due to a two-dimensional reverberant sound field that builds up near a very tall ceiling.

## INTRODUCTION

The reverberation time equations have been the most widely used prediction tools in acoustical design because they are simple to use and usually give reasonable results. The first and perhaps the most widely used reverberation time equation is the Sabine equation (Sabine 1992). In the following years several revised equations were proposed like the Eyring or the Millington equations (Eyring 1930; Millington 1932). The purpose of this study was to test the use of the Sabine and Eyring equations in churches especially when recesses and coupled spaces are present.

The main investigation is focused on the Roman Catholic churches of Portugal. Portugal is one of the oldest European countries and played a prominent role in some of the most significant events in world history. It presents an almost perfect location to trace the history of Catholic Church buildings in the world. Portuguese churches can be considered a representative example of Catholic churches in the world (Gil 1992; DGEMN 1936/64; Azevedo 1985).

This study reports on acoustical field measurements in a major survey of 41 Roman Catholic churches in Portugal that were built between the 6th century and 1993 (Carvalho 1994a). The churches are a sample of 14 centuries of church building in Portugal. Several particular analysis regarding other acoustical subjects using this sample of churches are already available (Carvalho 1994b,c,d,e,f,g, 1995a,b).

The churches were selected to represent the main architectural styles found throughout Portugal and to represent the evolution of church construction in Portugal. The architectural styles of the churches are presented in Table 1. For more uniformity of the sample and due to the sound power limits of the sound source, only churches with a maximum volume of less than 19000 m<sup>3</sup> were selected for the study.

Acoustical measurements were taken in similar numbers of churches grouped by large periods of history: 12 *Visigothic* or *Romanesque* churches (6th-13th centuries), 16 *Gothic* or *Manueline* churches (13th-16th centuries), 13 *Renaissance*, *Baroque* or *Neoclassic* churches (16th-19th centuries) and 4 *Contemporary* churches (20th century). The main architectural features of these churches are displayed in Table 2.

TABLE 1 - Architectural styles of churches tested.

1 - VISIGOTHIC	(6th-11th centuries)	5 - RENAISSANCE	(16th-17th centuries)
2 - ROMANESQUE	(12th-13th centuries)	6 - BAROQUE	(17th-18th centuries)
3 - GOTHIC	(13th-15th centuries)	7 - NEOCLASSIC	(18th-19th centuries)
4 - MANUELINE	(15th-16th centuries)	8 - CONTEMPORARY	(20th century)

TABLE 2 - Simple statistics for all churches tested.

ARCH. FEATURE	Min.	Max.	Mean	Median
VOLUME (m <sup>3</sup> )	299	18674	5772	3918
AREA (m <sup>2</sup> )	56	1031	450	427
MAX. HEIGHT (m)	7	39	15	13
MAX. LENGTH (m)	12	62	33	31
WIDTH NAVE (m)	4	38	13	11

The method used to calculate the Reverberation Time (RT30) is based on the integrated impulse-response method described by Schroeder in 1965. A limited-bandwidth noise-burst is generated and transmitted into the church by a loudspeaker via an amplifier. The room's response to the noise-burst (called the *impulse response*) is then sampled from the RMS detector output of the sound level meter (Brüel & Kjær 1990).

Rather than a pistol, a loudspeaker emitting noise (short noise pulse bursts) in 3/2 octave frequency bands (to ensure that the received noise-burst is of 1/1 octave bandwidth) was used as sound source. For a specific power amplifier this system allows more energy to be transmitted into the room than with a pistol. This advantage is especially important when background noise is present. The pistol is a very powerful and practical sound source. However, its shots usually lack energy in the lower frequency bands and reproducibility (Brüel & Kjær 1988). Moreover, a pistol shot may be of too short duration to allow the noise to attain a steady level in the room (Brüel & Kjær 1980).

The receiving section consisted of one 1/2" diameter microphone (which changed position throughout the room) and a sound level meter with a 1/1-octave filter set. A filter centered on the same frequency as the filter in the transmitting section reduces the influence of background noise.

The procedure was commanded by specific control software (*Room Acoustics*) using a notebook computer *in loco*. The loudspeaker was placed at two sound source locations in each church: one in front of the altar to standardize the measurements and to be able to compare results among churches and another in the center of the main floor to simulate the sound of the congregation. The sound source was positioned at 0.8 m above the floor and at a 45° angle with the horizontal plane. That angle was chosen to transmit more energy into the room volume, to try to better excite the reverberant field of the church. This loudspeaker position also gave more omnidirectionality to the sound source by locating the sides of the loudspeaker with less directivity such as in the back, facing the floor. A *diffuser*, a conical piece snap-locked onto the front of the cabinet, was used to render the measured results less dependent on the position and angle of inclination of the cabinet and to lower the directivity coefficient values.

Each measurement was calculated from an ensemble of three and four pulse responses in each position. This number of samples was chosen considering the high quality of the reproducibility of the sound source used, the number of samples used in the recent past of room acoustics as seen in the available literature, and the experience acquired by previous measurements. Five receiver positions were, on average, used, depending on the width of the church. The microphone at each location was placed at 1.30 m above the floor. In total, nearly 8000 values were determined (all combinations of frequency bands and source/receiver locations).

The churches were measured while unoccupied, as the available state of the art does not allow easy and practical acoustical measurements to be made in an occupied room. The high noise level of the sound source and the long duration of the measurements make the presence of a quiet congregation almost impossible. Furthermore, the use of absorptive materials to simulate the presence of people is also impractical due to the huge amount needed. In addition, most of the available bibliographic data were determined for unoccupied conditions. Therefore consistency of data is useful for possible comparison purposes.

However, another perspective is possible: In the past (until a few centuries ago) there were no pews or chairs for people in the churches. For that reason, the total absorption in today's unoccupied churches with a large number of pews may not be greatly different in some frequency bands, particularly the higher bands from the acoustical conditions of the churches in the past with no pews and a smaller congregation. The difference can be seen then in another dimension, *time* - almost as an exercise of *archaeological* acoustics.

Equipment from the Acoustical Laboratory of the University of Porto College of Engineering was used: a sound level meter by *Brüel & Kjær (B&K)*, type 2231; a 1/3-1/1 octave filter set by *Brüel & Kjær*, type 1625; a *Room Acoustics* module by *B&K*, type BZ7109; a sound source *B&K*, type 4224; a 1/2" diameter microphone by *B&K*; a notebook computer by *Compaq*, LTE 386-25 MHz; and *Room Acoustics* an application software by *B&K*, VP7155.

### SABINE AND EYRING EQUATIONS

In this study two classical equations, the Sabine and the Eyring, for the prediction of RT were applied to the 41 churches measured.

SABINE EQUATION	$RT = 0.16 V / A$
EYRING EQUATION	$RT = 0.16 V / [A_{air} - S_T \log_n (1 - \alpha_{avg})]$

where:

V - Volume (m <sup>3</sup> );	RT - Expected Reverberation Time (s);
A - Total Absorption (m <sup>2</sup> );	$\alpha_{avg}$ - Absorption Coefficient (avg. all surfaces);
A <sub>air</sub> - Air Absorption (m <sup>2</sup> );	S <sub>T</sub> - Surfaces Total Area (m <sup>2</sup> ).

The Appendix A presents the results for the application of the Sabine and Eyring equations to this sample of churches. The predicted results (Table A.1) for the RT are slightly better (near 13%) with the Eyring equation than with the Sabine equation but nevertheless, there are huge differences between measured and estimated RTs. The differences are due to the presence, in some churches, of chapels and other deep spaces that act as coupled spaces.

### ANALYSIS BETWEEN *RT REAL* AND *RT EXPECTED*

The measured RT values (*RT real*) and the predicted values using the Sabine or Eyring equations are plotted in the Figure 1 jointly with linear regression models using only freq. = 500 and 1000 Hz in the averaging process. The Pearson correlation coefficients (R) are presented in Table 3.

TABLE 3 - Matrix of Pearson correlation coefficients (R)

	RT(real)
RT(SABINE_VOL.TOTAL)	0.722
RT(SABINE_VOL.NAVE)	0.746
RT(EYRING_VOL.TOTAL)	0.717
RT(EYRING_VOL.NAVE)	0.743

Figure 2 shows the graphical representation of the *RT real* versus the *RT calculated* with the Sabine equation using the Volume Total and the Volume of the Nave only, together with the linear regression models. The fit of the linear regression line is clearly not perfect, therefore a new approach was tested and presented in Figure 3. Those two plots display the RT(Sabine) and RT(Eyring), using the Volume Total, with two linear models: one for the  $RT_{Real} = RT_{Expected}$  and the other for the best linear fit regarding the points that are not close to the previous line. The equations of these trends are:

$RT = 0.501 * RT(SABINE\_VOL.TOT)$	$R^2 = 0.968$	(Figure 3a)
$RT = 0.538 * RT(EYRING\_VOL.TOT)$	$R^2 = 0.976$	(Figure 3b)

This approach seems to give a good approximation for the data. The justification for the use of one or the other lines is based upon whether or not there are deep recesses such as chapels or altars that act as coupled spaces present in the churches. All the churches close to the  $RT_{real} = RT_{expected}$  are those without

deep recesses. Therefore the prediction equation gives a good approximation of the results. The others are churches with chapels that act as coupled spaces *artificially* increasing the absorption of the room.

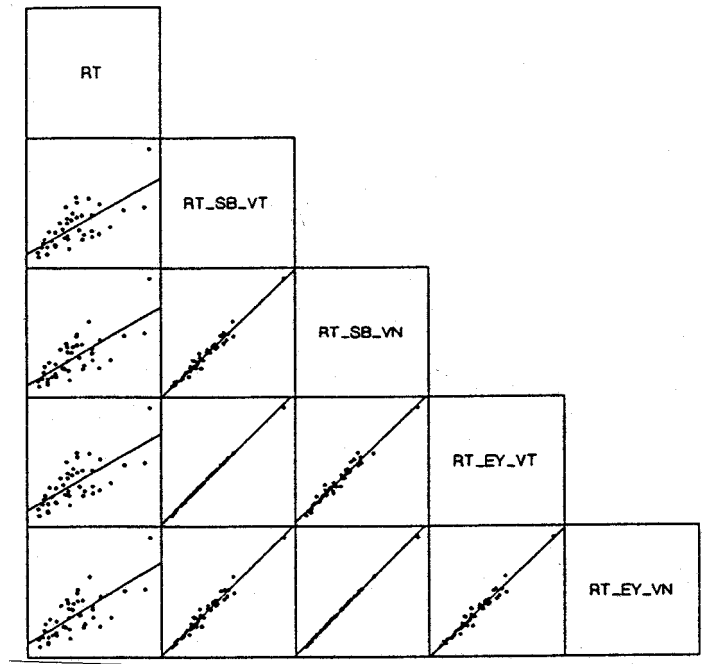


Figure 1 - Casement plot among measured and predicted RTs with linear regression models using freq. = 500 & 1000 Hz. SB-Sabine, EY-Eyring, VT-using volume total, VN-using volume of the nave.

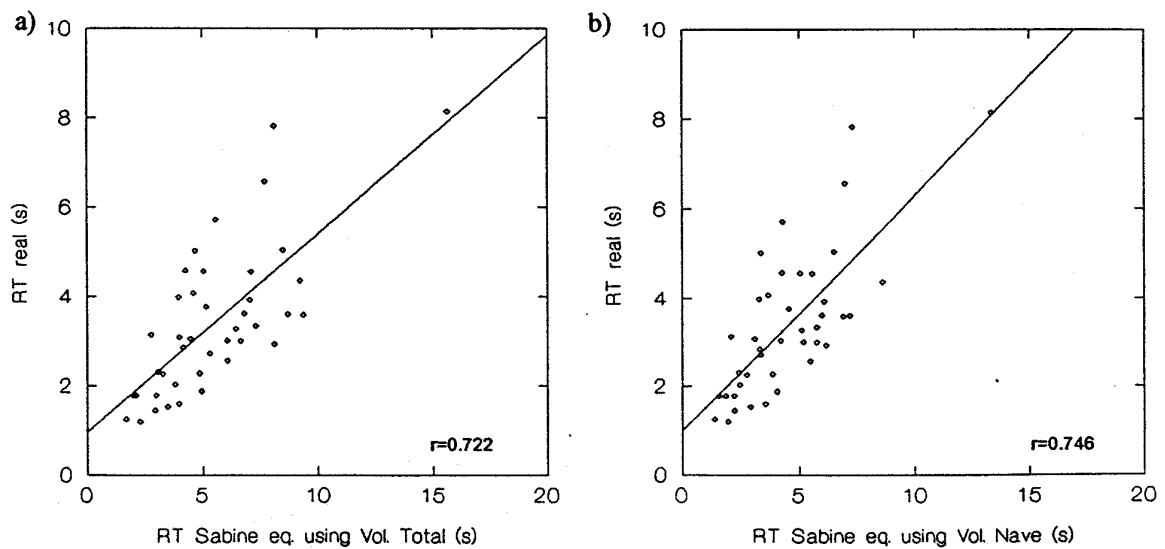


Figure 2 - Plots of measured (y axis) and predicted (x axis) RTs with linear regression models and Pearson correlation coefficients using the Sabine equation calculated with different volumes (41 points = 41 churches a) Using the total volume; b) Using the volume of the nave.

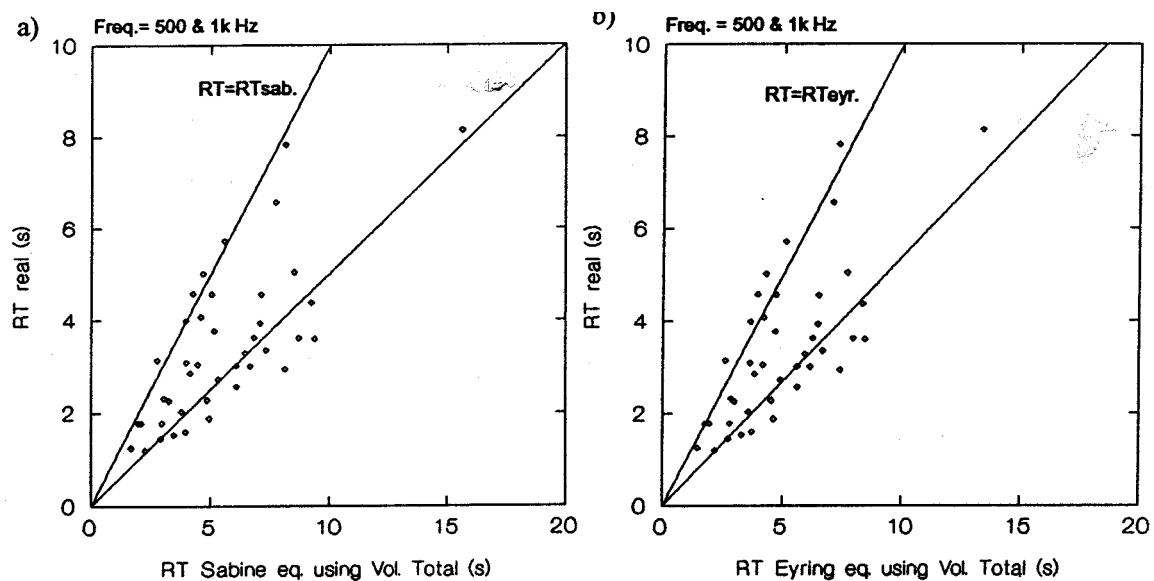


Figure 3 - Plots of measured ( $y$  axis) and predicted ( $x$  axis) RTs using the Sabine and the Eyring equations with two linear trends, one for the  $RT_{\text{REAL}} = RT_{\text{EXPECTED}}$  and the other for the best linear fit regarding the points that are not close to the previous line. a) Using the Sabine equation; b) Using the Eyring equation.

### COUPLED SPACES

The subdivision of the volume into a number of smaller volumes coupled together, results in very low RT without the addition of absorptive materials. Deep lateral chapels and even in certain cases, the main altar area (apse), can act as coupled spaces. This will entirely transform the analysis and application of the prediction equations.

The border between those coupled spaces and the main room acts as an absorptive surface with an indeterminate absorption coefficient  $\alpha$ . Some authors have tried to determine values for the  $\alpha$  of the recesses and coupled spaces in churches. Tzekakis using measurements in eight Greek orthodox churches in Thessaloniki, found that the openings must have an  $\alpha$  above 0.5. Shankland presents values between 0.38 and 0.67 using the results of measurements in four basilicas in Rome.

Cremer states that if the equivalent absorption area of room 2 - the smaller room is much smaller than the area of the opening between rooms ( $S_{12}$ ), the two rooms can be treated as one. This approach was taken in the produced Table A.1. In other words, these rooms were not considered as coupled spaces because the interior absorption in the chapels or main altar is usually much smaller than the opening area because the walls, ceilings and part of the floors are made of stone. This approach did not produce satisfactory results.

Cremer also states, as a *rule of thumb*, that if the boundary area covered with absorptive materials in the coupled room ( $S_a$ ) exceeds that of the coupling area to the main room ( $S_c$ ), it should be treated as an open window ( $\alpha = 1$ ); if not, the coupled room (room 2) should be treated as part of the main room. Using that rule and considering that all chapels and the main altar area (apse) have at least a  $S_a = S_c$  due to the wood-carved altars that fill one of the walls entirely and freely supposing that the wood-carving is an absorptive material a new spreadsheet was calculated using an  $\alpha = 0.9$  in all openings to chapels or to the main altar area (Table A.2). This approach did not produce satisfactory results. The answer seems to indicate the use of different  $\alpha$ 's for the main altar area (apse) and for the lateral chapels.

In many of the churches, the chapels cannot be considered as coupled rooms due to their size or shape. As Kuttruff states, the necessity of considering coupling effects when calculating the RT arises if the area of the coupling aperture is substantially smaller than the total wall area of a partial room. Another explanation can be in the lack of diffusion that happens in some of the churches, especially those having very simple geometric shapes and extremely non-uniform distribution of absorption on their walls.

Neither the Sabine nor the Eyring equations provided a very good prediction of the measured RT. The use of the Total Volume or only the Nave Volume of each church in the RT calculation in one of those equations gave a Pearson correlation coefficient of approximately 0.73.

A different approach was then tested using two linear trends: one for the  $RT_{real} = RT_{expected}$  and the other for the best fit regarding the points that were not close to the previous line. All the churches close to the  $RT_{real} = RT_{expected}$  were those without deep recesses. The others were churches with coupled spaces that *artificially* increased the absorption of the room. Therefore the importance of the coupled spaces justified the search for a new approach in using the Sabine equation in these situations.

### NEW ALGORITHM

**Method** Lateral chapels, the main altar (apse) and lateral aisles, can in certain cases act as coupled spaces. This will entirely transform the analysis and application of the Sabine equation. A new algorithm for use in the Sabine equation considering the existence of coupled spaces was developed. An absorption coefficient for the opening of each coupled space ( $\alpha_{CS}$ ) was calculated depending on the geometric characteristics of the specific coupled space. With that  $\alpha_{CS}$  a new Total Absorption for the church was calculated and the Sabine equation was used with the appropriate Final Volume. *Volume Total* was used if no coupled spaces and *Volume Nave* was used if chapels and main altar are coupled spaces, etc..

$$RT_{SABINE} = 0.16 V. Final / A$$

where:  $V$  - Volume ( $m^3$ ),  $\alpha_{CS}$  - Absorption coefficient (coupled space),  
 $A$  - Total absorption ( $m^2$ ) =  $\sum A_i + \sum \alpha_{CSj} S_j$ ,  $S$  - Coupled space opening surface area ( $m^2$ ).

As Kuttruff states, the necessity of considering coupling effects when calculating the RT arises if the area of the coupling aperture is substantially smaller than the total wall area of the partial (or coupled) room. Using this idea, a geometrical parameter was found to *weight* the *degree of coupling* of a specific partial room to the main room volume. Using Figure 4 (where  $l$ ,  $w$  and  $h$  are the length, width and height) by Kuttruff's rule, it is a coupled space if

$$S_{12} < k \cdot S_2 \quad \text{where } k = \text{constant} > 1 \text{ and } S_2 = \sum S_{2i} \text{ (3 walls in room 2 - the coupled room)}$$

then

$$w \cdot h < k (2 \cdot l + w_2) h$$

or

$$k > w / (2 \cdot l + w_2) \approx 1 / [(2 \cdot l/w) + 1], \text{ because } (w_2 / w) \approx 1$$

or finally

$$l/w > (k - 1) / 2, \quad k > 1$$

If

$$k = 2, \quad l/w > 0.5 \quad \dots \quad k = 3, \quad l/w > 1.0 \quad \dots$$

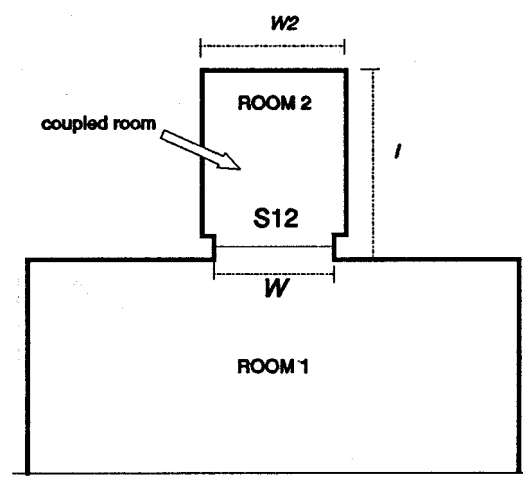


Figure 4 - Plan sketch of a general church with a coupled space (not to scale).  $l$ -length,  $S_{12}$ -opening surface area,  $w$ -opening width,  $w_2$ -coupled space width, Room 1-main room, Room 2-coupled room.

Therefore,  $l/w$  appears as a good parameter to characterize a coupled space. Then,  $\alpha_{CS} = f(l/w)$ . This function  $f$  must be restricted to the limits of  $\alpha_{CS}$ . That is, it must be between 0 and 1. The TANH (hyperbolic tangent) was chosen with an  $x$  axis shift to eliminate the presence of negative  $\alpha_{CS}$ 's. Therefore, the final transfer function is:

$$\alpha_{CS} = \tanh [ a (l/w - b) ]$$

TABLE 4 - Coefficients ( $a$  and  $b$ ) to use in new algorithm to account for the coupled spaces effect in the use of the RT Sabine equation

Type of Coupled Space	$a$	$b$
CH - CHAPELS	0.007	0
MA - MAIN ALTAR (APSE)	0.985	0.6
LA - LATERAL AISLES	0.0118	-14

Table 4 presents the best parameters  $a$  and  $b$  that were found by experimentation, using the 41 church sample. Other general rules in the use of this algorithm are presented below.

CHAPELS are only considered as coupled spaces if  $l/w > 0.6$ .  $l/w$  is the average of all  $(l/w)_{chapel i}$  weighted by their opening surfaces  $S_i$ . This is the area of the vertical plan that is the border between the chapel and the main volume of the church. The total interior absorption should be included. In the simplified version of this method, this absorption is sufficient in the account of the total absorption for this type of coupled space. If the chapels are *inside* the lateral aisles area, they should be omitted if that volume is also omitted as referred below if  $l/w_{(lateral\ aisles)} > 0.70$ .

MAIN ALTAR (APSE) is only considered as coupled spaces if  $l/w > 0.6$ . The total interior absorption should be accounted for (normally this is a very small quantity).

LATERAL AISLES are only considered as coupled spaces if  $l/w > 0.4$ . In this type of coupled space the parameters  $l$  and  $w$  are defined as seen in Figure 5 where  $l$  = width of lateral aisle and  $w$  = height of each opening. The volume of the Lateral Aisles is only excluded of the Total Volume of the church if  $l/w > 0.70$ :

Volume Final = Volume Nave - Volume Lateral Aisles if  $l/w > 0.70$  or

Volume Final = Volume Nave if  $l/w \leq 0.70$ . The total interior absorption should be included.

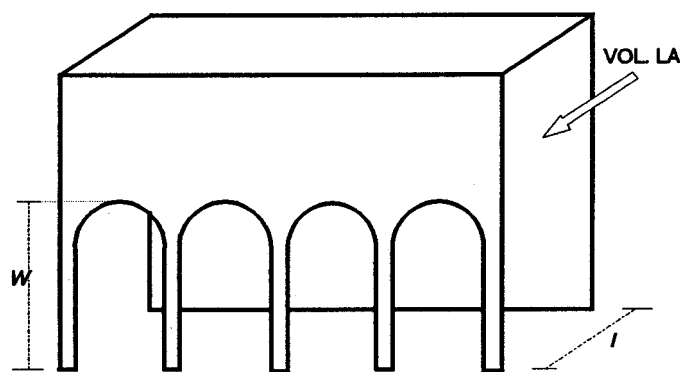


Figure 5 - 3-D sketch of lateral aisles in a general church (not to scale).  $l$ -width of lateral aisle,  $w$ -width of opening to lateral aisle,  $vol_{LA}$ -volume of lateral aisle

**Results** The results of this algorithm applied to the 41 churches are presented in Appendix A.3 and summarized in Figure 6. An average of 16% between measured and predicted RT was found for the total 41 churches. This is a huge improvement from the 71% average absolute difference found without the use of this algorithm (see Table A.2).

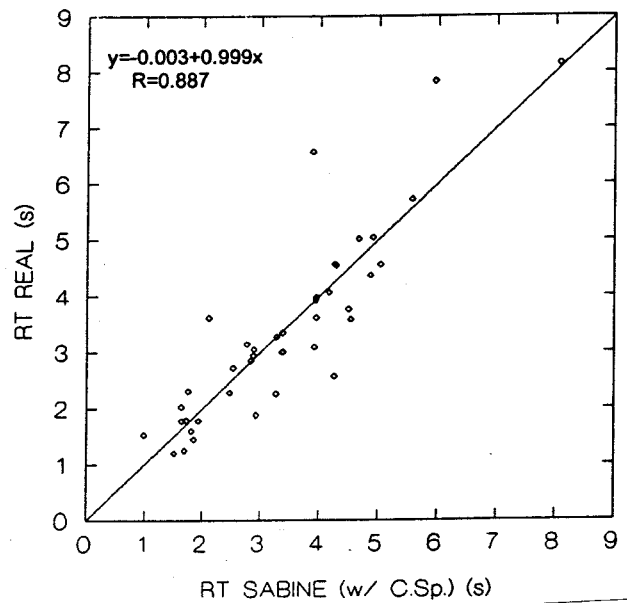


Figure 6 - Plot of measured (y axis) vs. predicted (x axis) RTs with linear prediction line and Pearson correlation coefficient.

Using *seconds*, the average of the absolute differences is 0.49 s in the RT expected, which can be considered a very good result due to the large values for the RTs involved. Figure 6 presents the plot of the  $RT_{REAL}$  vs  $RT_{SABINE}(w/ CS)$  and the prediction line. This prediction linear equation ( $RT_{REAL} = - 0.003 + 0.999 RT_{SABINE}$ ) with  $R = 0.887$  is very close to the ideal  $RT_{REAL} = RT_{SABINE}$ . The differences found between  $RT_{REAL}$  and  $RT_{SABINE}$  are slightly correlated with the height of the churches. The Pearson correlation coefficient between the  $\Delta RT$  and the fifteen architectural parameters used are in Table 5.

TABLE 5 - Pearson correlation coefficients (R) between  $\Delta RT$  and the fifteen architectural parameters.

ARCHIT. PARAMETERS	R	ARCHIT. PARAMETERS	R
SEATS	-0.115	HEIGHT NAVE	-0.154
VOLUME TOTAL	-0.098	WIDTH NAVE	-0.181
VOLUME NAVE	-0.120	WIDTH AVERAGE	-0.134
AREA TOTAL	-0.102	V. TOTAL/AREA TOTAL	-0.086
AREA NAVE	-0.095	ALPHA AVERAGE	0.117
LENGTH MAXIMUM	-0.130	R_LOCAL	-0.014
LENGTH NAVE	-0.085	ABSORPTION TOTAL	-0.021
HEIGHT MAXIMUM	<b>-0.209</b>		

Figure 7 shows the plot of the *RT Differences* (in second) versus the *Maximum Height*. The *Maximum Height* appeared as a justification for part of the differences found between  $RT_{REAL}$  and  $RT_{SABINE}$  in a general linear model to predict the  $RT_{REAL}$  with the use of the  $RT_{SABINE}$  together with the fifteen architectural parameters. With an  $\alpha$ -to-enter/remove = 0.15 the result was:

$$RT_{REAL} = - 0.162 + 0.835 RT_{SABINE} + 0.048 HEIGHT\_MAX \quad (R^2 = 0.81)$$

This supports the explanation that the RT differences are due to the lack of diffusion that occurs in some of the churches, especially those having simple geometric shapes and extremely non-uniform distribution of absorption on their walls. This occurs in rectangular churches with smooth, reflecting walls and a tall ceiling. The absorption is mainly concentrated on the ceiling if it is wood or/and on the floor if it is wood or if wooden pews are used. In this case a two-dimensional reverberant sound field can be built.

Generally, the higher the ceiling, the longer the RT. The higher ceiling can almost act as a *reverberant chamber* included in the main room. This will only happen if the ceiling is non absorptive, that is, if it is not made of wood (in this sample of churches). To check this hypothesis the 41 churches



were grouped according to their ceiling type (*wood* and *non wood*). The Pearson correlation coefficients were then calculated between these two groups and the  $\Delta RT$ . The results are found in Table 6.

Figure 8 shows the RT differences grouped by the two groups of ceiling type with the standard error interval. An ANOVA test was calculated to determine if these two groups of ceiling types were statistically different. It was found that, at a level of probability (p-value) higher than 0.12, the two groups were statistically different. Therefore it can be concluded that there are enough data to support the idea that a *reverberant ceiling effect* may play a role in the differences found between the RT real and the RT calculated by the Sabine equation. Therefore a new and future improvement in the proposed algorithm will be to consider that *reverberant ceiling effect* included in the total absorption parameter in the Sabine Equation or in the prediction value for the RT (as a  $\Delta RT$ ).

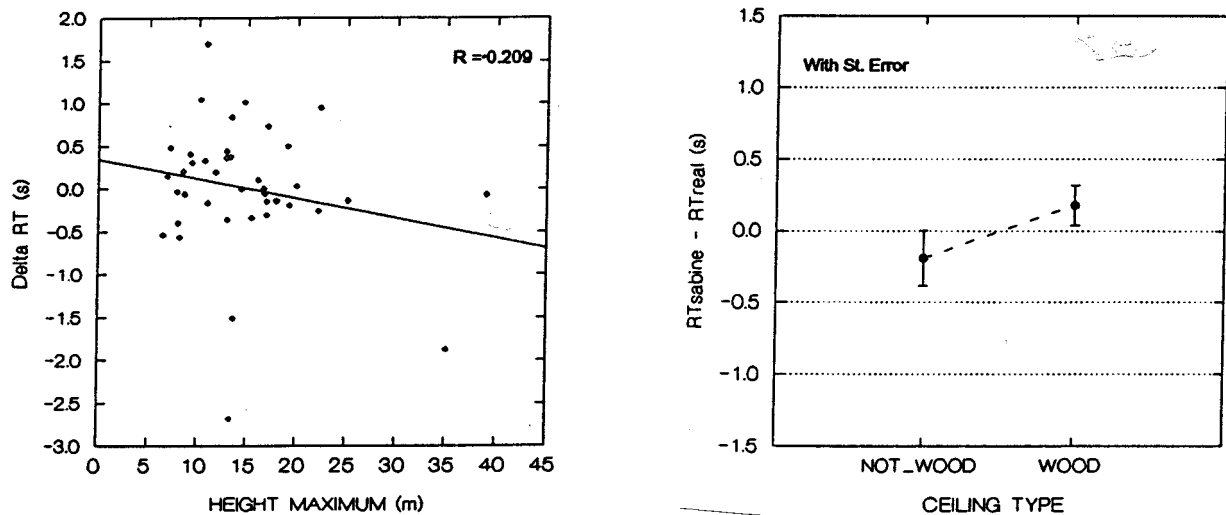


Figure 7 (left) - Plot of the RT differences ( $\Delta RT = RT_{REAL} - RT_{SABINE}$ ) vs. Maximum Height of each church with linear regression line and correlation coefficient (41 points = 41 churches).

Figure 8 (right) - Analysis of the effect of ceiling type (*wood* and *non wood*) differences ( $RT_{REAL} - RT_{SABINE}$ ). Mean values for all the churches in each ceiling type are shown with one standard error confidence interval.

TABLE 6 - Pearson correlation coefficients (R) between  $\Delta RT$  and *Height maximum*

TYPE OF CEILING	NUMBER OF CHURCHES	R	
		$\Delta RT$ in second	$\Delta RT$ in percentage
WOOD	22	0.030	-0.004
NON WOOD	19	-0.216	-0.154

**Frequency Average Options.** Seven options of frequency band averaging to obtain a representative single number for each church parameter, were tested to compare the predicted RT by the use of the Sabine equation including the coupled spaces algorithm with the real RT measured. The Pearson correlation coefficients are displayed in Table 7. The chosen method of using only the 500 and 1000 Hz octave bands in the frequency averaging process appears as the best (or almost the best ...). However the differences among the options ( $\Delta R < 0.02$ ) are not significant.

TABLE 7 - Pearson correlation coeff. for RT(Sabine) vs. seven options of frequency averaging methods.

	RT (SABINE)
RT (REAL) Freq.= 125-1k Hz	0.870
RT (REAL) Freq.= 2 & 4 kHz	0.875
RT (REAL) All Frequencies	0.879
RT (REAL) Freq.= 250-2k Hz	0.884
RT (REAL) Freq.= 500-2k Hz	0.887
RT (REAL) Freq.= 500 & 1k Hz	<b>0.887</b>
RT (REAL) Freq.= 500-4k Hz	0.888

**Simplified Method** A simplified method of the new algorithm presented above is now described. The  $\alpha_{CS}$  of the CHAPELS should be equal to 0. The interior absorption of each chapel is normally sufficient to consider the effect of chapels in the overall absorption of the church. Therefore an  $\alpha_{CS (CHAPELS)} = 0$  can be used as a simplification. The  $\alpha_{CS}$  of the LATERAL AISLES should be equal to 0.17. The Lateral Aisles (LA) have very similar proportions relatively to the church main volume. Therefore an  $\alpha_{CS (LATERAL AISLES)} = 0.17$  can be used as a simplification if  $l/w > 0.4$ . Then the apse or Main Altar area (MA) will be the only coupled space to be considered if  $l/w > 0.60$  in this simplified version of the algorithm presented.

$$A_{CS} = \alpha_{CS (CH)} \cdot S_{CH} + \alpha_{CS (LA)} \cdot S_{LA} + \alpha_{CS (MA)} \cdot S_{MA}$$

$$A_{CS} = 0 \cdot S_{CH} + 0.17 \cdot S_{LA} + \tanh [ 0.985 ( l/w - 0.6 ) ]. S_{MA}$$

## SUMMARY

The use of the Sabine and Eyring reverberation time equations was tested to estimate the measured reverberation times in this sample of churches. The Eyring equation gives slightly better results than the Sabine equation in predicting the RT when the effect of coupled spaces is not considered. Two trends were clearly distinguishable in the RT values indicating a need for the analysis of the coupled spaces in the prediction of RT in churches that could better explain that difference between measured and predicted RTs. The effect of coupled spaces was analyzed and a new algorithm for the application of the Sabine equation in churches was developed producing an average of 16% in the differences between the reals and predicted RTs compared to a 71% difference using the standard Sabine equation. Coupled spaces (CS) were found to act as *windows* with a characteristic  $\alpha$  depending on their dimensions. The recesses in churches were grouped in three types: main altar area (apse), chapels and lateral aisles. Each type of coupled space has a particular acoustical behavior with different  $a$  and  $b$  parameters in the calculated equation. There are two major reasons that three types of coupled spaces are needed. The first reason is the relative position of the sound source to the coupled space, that is, concerning the direction from which the sound enters the coupled space. Second is the volume of the coupled space relative to the volume of the main room. It was found that those recesses only acted as coupled spaces if their *length / opening\_width* > 0.6 or if the *aisle\_width / opening\_height* > 0.4 in lateral aisles. The remaining differences found between the measured RTs and the predicted RTs with this new algorithm were hypothesized to be related to what was called a *reverberant ceiling effect* which is presumed to be due to a two-dimensional reverberant sound field that builds up near a very tall ceiling in churches.

## ACKNOWLEDGMENTS

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## APPENDIX A

This Appendix presents the results of the application of the Sabine and Eyring equations to this ample of churches. Table A.1 presents the results concerning the direct application of these two equations to the 41 churches measured. The effect of coupled spaces such as chapels, apses, etc. were not considered in the calculation. For each church, two RT values are given using *Volume Total* (VT) and *Volume Nave* (VN) in the prediction equation. Nave stands for the area of the church excluding the lateral chapels and the main altar/apse. For both the Sabine and Eyring equations, there are two columns in the Table where the differences between the RT measured *in loco* and the expected RT calculated by the Sabine or Eyring equations are computed. The average of those 41 differences (*AVGabs*) calculated using the absolute value of each individual difference are shown at the bottom of each of the difference columns (*Diff.*). Table A.2 presents the final results of the application of the Sabine equation using an  $\alpha = 0.9$  in all openings to chapels or to the main altar area. As seen in Table A.2, this approach still does not give reasonable results (note that in this case, the Differences regarding the Volume of the Nave is the column to look to). Table A.3 displays the results of the application of the Sabine equation including the coupled spaces algorithm. In the column ABS(diff)%, the absolute differences (in percentage) of the  $RT_{SABINE}$  vs.  $RT_{REAL}$  are shown.

TABLE .1 - Calculation of RT using the Sabine (sab) and Eyring (eyr) equations with no coupled spaces effect considered. Differences in % using total volume (VT) or the volume of the nave (VN) in the calculations.

CHURCH	RTsab.V	RTsab.V	RTreal	Diff.-V	Diff.-VN	RTeyr.V	RTeyr.V	Diff.-VT	Diff.-VN
	T (s)	N (s)	(s)	T (%)	(%)	T (s)	N (s)	(%)	(%)
1 ALMANSIL	3.79	2.44	2.03	86	20	3.57	2.30	76	13
2 ARMAMAR	6.08	5.48	2.57	137	113	5.60	5.05	118	97
3 BASIL. ESTRELA lisboa	15.63	13.34	8.14	92	64	13.40	11.44	65	40
4 BRAVÃES	4.95	4.04	1.88	163	115	4.62	3.78	145	101
5 BUSTELO	4.59	3.66	4.07	13	-10	4.20	3.35	3	-18
6 CABEÇA SANTA	2.98	2.21	1.79	67	24	2.79	2.08	56	16
7 CAMINHA	4.15	3.31	2.85	45	16	3.82	3.05	34	7
8 CEDOFEITA. new porto	3.98	3.09	3.09	29	0	3.64	2.83	18	-8
9 CEDOFEITA. old porto	8.69	7.18	3.62	140	98	7.93	6.54	119	81
10 CETE	4.86	3.85	2.28	113	69	4.53	3.59	99	57
11 CLÉRIGOS	7.32	5.76	3.35	119	72	6.66	5.23	99	56
12 GOLEGÃ	6.82	5.98	3.62	88	65	6.24	5.47	72	51
13 LAPA	5.57	4.29	5.72	-3	-25	5.10	3.93	-11	-31
14 LEÇA DO BAILIO	9.23	8.59	4.37	111	96	8.32	7.74	90	77
15 LOUROSA	3.97	3.55	1.60	148	122	3.72	3.33	133	108
16 MÉRTOLA	5.04	5.04	4.56	11	11	4.71	4.71	3	3
17 MISERICÓRDIA évora	3.27	2.75	2.26	45	22	2.99	2.52	32	11
18 MOURA	7.71	6.98	6.57	17	6	7.06	6.39	8	-3
19 N. S. BOAVISTA porto	3.95	3.28	3.98	-1	-18	3.65	3.03	-8	-24
20 P. SOUSA	8.13	6.15	2.94	177	109	7.39	5.59	151	90
21 S. SACRAMENTO porto	4.67	3.36	5.02	-7	-33	4.29	3.08	-14	-39
22 S. CLARA porto	1.69	1.37	1.25	35	10	1.46	1.19	17	-5
23 S. B. CASTRIS	2.77	2.07	3.14	-12	-34	2.60	1.94	-17	-38
24 S. FRANCISCO évora	8.50	6.50	5.04	69	29	7.68	5.88	52	17
25 S. FRANCISCO porto	2.00	1.85	1.78	12	4	1.75	1.62	-2	-9
26 S. FRUTUOSO	2.29	1.94	1.20	92	62	2.18	1.84	82	54
27 S. GENS	3.48	2.91	1.53	127	90	3.29	2.75	115	80
28 S. P. FERREIRA	6.45	5.10	3.28	97	55	5.92	4.68	81	43
29 RATES	6.66	5.76	3.00	122	92	6.14	5.30	104	77
30 RORIZ	6.07	5.19	3.01	102	72	5.59	4.78	85	59
31 S. ROQUE lisboa	5.15	4.55	3.77	37	21	4.68	4.13	24	10
32 SÉ lamego	7.12	5.55	4.55	56	22	6.48	5.05	42	11
33 SÉ porto	9.37	6.90	3.59	161	92	8.43	6.20	134	73
34 SILVES	7.07	6.07	3.93	80	54	6.46	5.54	64	41
35 SEROA	4.26	4.26	4.57	-7	-7	3.94	3.94	-14	-14
36 SERRA PILAR v.n.gaia	8.12	7.30	7.83	4	-7	7.30	6.56	-7	-16
37 TIBÃES	5.32	3.35	2.72	96	23	4.89	3.08	80	13
38 VIANA DO ALENTEJO	4.47	4.21	3.05	47	38	4.16	3.91	36	28
39 VILA DO BISPO	2.11	1.56	1.78	19	-13	1.96	1.45	10	-19
40 VILA NG. DE AZEITÃO	3.06	2.41	2.31	32	4	2.86	2.25	24	-3
41 VOUZELA	2.93	2.22	1.45	102	53	2.73	2.07	89	43
	<b>AVGa</b>			<b>71</b>	<b>46</b>	<b>AVGabs</b>		<b>59</b>	<b>39</b>
	<b>bs</b>								

TABLE A.2 - Calculation of RT using the Sabine (sab) and Eyring (eyr) equations with coupled spaces effect considered with  $\alpha=0.9$  in all recesses. Differences in % using total volume (VT) or the volume of the nave (VN) in the calculations.

CHURCH	RTsab.V	RTsab.V	RTreal	Diff.-VT	Diff.-VN	RTeyr.V	RTeyr.V	Diff.-VT	Diff.-V
	T (s)	N (s)		(s)	(%)	(%)	T (s)	N (s)	(%)
1 ALMANSIL	3.79	2.44	2.03	86	20	3.57	2.30	76	13
2 ARMAMAR	4.18	3.77	2.57	63	47	3.87	3.49	51	36
3 BASIL. ESTRELA lisboa	7.84	6.69	8.14	-4	-18	7.04	6.01	-14	-26
4 BRAVÃES	3.66	2.99	1.88	94	59	3.42	2.80	82	49
5 BUSTELO	3.37	2.69	4.07	-17	-34	3.06	2.44	-25	-40
6 CABEÇA SANTA	1.91	1.42	1.79	7	-21	1.77	1.32	-1	-26
7 CAMINHA	2.57	2.05	2.85	-10	-28	2.34	1.86	-18	-35
8 CEDOFEITA.new porto	3.52	2.73	3.09	14	-11	3.22	2.50	4	-19
9 CEDOFEITA.old porto	4.86	4.01	3.62	34	11	4.53	3.74	25	3
10 CETE	2.95	2.34	2.28	29	2	2.74	2.17	20	-5
11 CLÉRIGOS porto	4.29	3.37	3.35	28	1	3.91	3.08	17	-8
12 GOLEGÃ	2.73	2.39	3.62	-25	-34	2.48	2.17	-32	-40
13 LAPA porto	3.84	2.96	5.72	-33	-48	3.51	2.70	-39	-53
14 LEÇA DO BAILIO	6.34	5.90	4.37	45	35	5.80	5.39	33	23
15 LOUROSA	3.08	2.76	1.60	93	72	2.89	2.58	81	61
16 MÉRTOLA	5.04	5.04	4.56	11	11	4.71	4.71	3	3
17 MISERICÓRDIA évora	3.27	2.75	2.26	45	22	2.99	2.52	32	11
18 MOURA	4.72	4.27	6.57	-28	-35	4.38	3.97	-33	-40
19 N. S. BOAVISTA porto	2.18	1.81	3.98	-45	-54	1.98	1.64	-50	-59
20 PAÇO DE SOUSA	5.83	4.41	2.94	98	50	5.35	4.05	82	38
21 S. SACRAMENTO porto	3.32	2.38	5.02	-34	-53	3.03	2.17	-40	-57
22 S. CLARA porto	1.43	1.16	1.25	15	-7	1.21	0.98	-3	-21
23 S. B. CASTRIS	2.77	2.07	3.14	-12	-34	2.60	1.94	-17	-38
24 S. FRAN.CISCO évora	3.25	2.49	5.04	-36	-51	2.94	2.25	-42	-55
25 S. FRANCISCO porto	1.76	1.62	1.78	-1	-9	1.51	1.40	-15	-22
26 S. FRUTUOSO	2.29	1.94	1.20	92	62	2.18	1.84	82	54
27 S. GENS	2.58	2.16	1.53	69	41	2.44	2.04	59	33
28 S. P. FERREIRA	4.10	3.24	3.28	25	-1	3.78	2.99	15	-9
29 RATES	3.84	3.31	3.00	28	10	3.56	3.08	19	3
30 RORIZ	3.89	3.33	3.01	29	11	3.59	3.07	19	2
31 S. ROQUE lisboa	2.63	2.32	3.77	-30	-38	2.31	2.04	-39	-46
32 SÉ lamego	5.51	4.30	4.55	21	-6	5.03	3.93	11	-14
33 SE porto	6.29	4.63	3.59	75	29	5.74	4.22	60	18
34 SILVES	4.58	3.93	3.93	16	0	4.20	3.60	7	-8
35 SEROA	4.26	4.26	4.57	-7	-7	3.94	3.94	-14	-14
36 SERRA PILAR v.n.gaiã	3.45	3.10	7.83	-56	-60	3.07	2.76	-61	-65
37 TIBÃES	2.79	1.76	2.72	3	-35	2.53	1.59	-7	-42
38 VIANA DO ALENTEJO	3.22	3.03	3.05	6	-1	2.98	2.81	-2	-8
39 VILA DO BISPO	1.53	1.13	1.78	-14	-37	1.40	1.03	-21	-42
40 VILA N. DE AZEITÃO	2.23	1.75	2.31	-4	-24	2.06	1.62	-11	-30
41 VOUZELA	2.30	1.74	1.45	59	20	2.13	1.62	47	12
	<b>AVGabs</b>			<b>35</b>	<b>28</b>	<b>AVGabs</b>		<b>32</b>	<b>29</b>

TABLE A.3 - Calculation of RT using the Sabine (sab) equation with the coupled spaces effect considered (proposed algorithm). Differences in % using the final volume (VF) in the calculations.

CHURCH	Vol.type	RTsab.V F (s)	RTreal (s)	Diff-VF (%)	ABS(diff) (%)	Diff (s)	ABS(diff) (s)	
1	ALMANSIL	V.nave	1.63	2.03	-20	20	-0.4	0.4
2	ARMAMAR	V.nave	4.26	2.57	66	66	1.7	1.7
3	BASIL. ESTRELA lisboa	V.nave	8.07	8.14	-1	1	-0.1	0.1
4	BRAVÃES	V.nave	2.93	1.88	56	56	1.0	1.0
5	BUSTELO	Vt-Vtr	4.17	4.07	3	3	0.1	0.1
6	CABEÇA SANTA	V.nave	1.72	1.79	-4	4	-0.1	0.1
7	CAMINHA	V.nave	2.84	2.85	0	0	0.0	0.0
8	CEDOFEITA.new porto	Vt-Vch.	3.92	3.09	27	27	0.8	0.8
9	CEDOFEITA.old porto	V.nave	3.95	3.62	9	9	0.3	0.3
10	CETE	V.nave	2.47	2.28	9	9	0.2	0.2
11	CLÉRIGOS porto	V.nave	3.38	3.35	1	1	0.0	0.0
12	GOLEGÃ	V.nave	2.11	3.62	-42	42	-1.5	1.5
13	LAPA porto	V.Total	5.57	5.72	-3	3	-0.1	0.1
14	LEÇA DO BAILIO	V.nave	4.87	4.37	11	11	0.5	0.5
15	LOUROSA	Vnave-Vla	1.81	1.60	13	13	0.2	0.2
16	MÉRTOLA	V.Total	5.04	4.56	11	11	0.5	0.5
17	MISERICÓRDIA évara	V.Total	3.27	2.26	45	45	1.0	1.0
18	MOURA	V.nave	3.88	6.57	-41	41	-2.7	2.7
19	N. S. BOAVISTA porto	V.Total	3.95	3.98	-1	1	0.0	0.0
20	PAÇO DE SOUSA	V.nave	2.88	2.94	-2	2	-0.1	0.1
21	SANT. SACRAMENTO porto	V.Total	4.67	5.02	-7	7	-0.3	0.3
22	S. CLARA porto	V.Total	1.69	1.25	35	35	0.4	0.4
23	S. B. CASTRIS	V.Total	2.77	3.14	-12	12	-0.4	0.4
24	S. FRANCISCO évara	V.nave	4.91	5.04	-3	3	-0.1	0.1
25	S. FRANCISCO porto	V.nave	1.64	1.78	-8	8	-0.1	0.1
26	S. FRUTUOSO	V.nave	1.51	1.20	26	26	0.3	0.3
27	S. GENS	V.nave	0.99	1.53	-35	35	-0.5	0.5
28	S. P. FERREIRA	V.nave	3.28	3.28	0	0	0.0	0.0
29	S. P. RATES	V.nave	3.37	3.00	12	12	0.4	0.4
30	RORIZ	V.nave	3.39	3.01	12	12	0.4	0.4
31	S. ROQUE lisboa	V.nave	4.50	3.77	19	19	0.7	0.7
32	SÉ lamego	V.nave	4.29	4.55	-6	6	-0.3	0.3
33	SÉ porto	V.nave	4.54	3.59	26	26	0.9	0.9
34	SILVES	V.nave	3.94	3.93	0	0	0.0	0.0
35	SEROA	V.Total	4.26	4.57	-7	7	-0.3	0.3
36	SERRA PILAR v.n.gaia	Vt-Vma	5.95	7.83	-24	24	-1.9	1.9
37	TIBÃES	V.nave	2.53	2.72	-7	7	-0.2	0.2
38	VIANA DO ALENTEJO	V.nave	2.89	3.05	-5	5	-0.2	0.2
39	VILA DO BISPO	Vt-Vch.	1.93	1.78	8	8	0.1	0.1
40	VILA. N. DE AZEITÃO	V.nave	1.75	2.31	-25	25	-0.6	0.6
41	VOUZELA	V.nave	1.85	1.45	28	28	0.4	0.4
<b>AVG</b>					<b>16</b>	<b>0.49</b>		