Analytical Solution of Dynamic Economic Dispatch Considering Wind Generation

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***Abstract*—The variability of wind generation introduces uncertainty in the optimal scheduling of the system. Consequently, it is difficult for the system operator to determine the optimal amount of conventional generation that should be committed and its corresponding power production in order to reduce generation costs. Incorporation of forecasting error on the optimal unit scheduling has been extensively suggested in the literature. However, it strongly depends on the probability distribution adopted to represent wind power forecasting error. Cauchy distribution has demonstrated to be an adequate tool to represent forecasting error. In this paper, an analytical model to solve dynamic economic dispatch is presented. The proposed model is based on discretization of Cauchy distribution, so that its incorporation in the optimization problem is successfully done. This is illustrated by analyzing a representative case study and the results are compared to a Monte Carlo Simulation approach in order to show the accuracy of the proposed method.**

***Index Terms*—Economic dispatch, Cauchy distribution, wind generation, forecasting error**

I. Introduction

The variability of renewable power sources, and wind generation specifically, is one of the most important technical limitations to its full integration with the power system. Under high integration of wind generation, the uncertainty related to the forecasting error includes difficulties to determine the optimal amount of power generation that should be committed and the output power of these units in order to minimize the operating costs of the entire system. The problem of determining the optimal scheduling of conventional generation units of a determine power system provided of wind generation is currently solved by applying scenario generation/reduction-based techniques; a representative study was presented in [1], where a set of possible scenarios of wind power generation is generated by using an autoregressive moving average (ARMA) model, while the optimal scheduling for this scenario set is determined by solving the equivalent optimization problem by means of stochastic programming approach formulated as a mixed-integer linear programming (MILP) problem. However, this approach can be applied over a limited amount of scenarios, which could be a source of error; in order to overcome this problem, in [2] the incorporation of reserve requirements for each scenario into the stochastic programming approach was proposed; so that, the robustness of the obtained solution is increased. Other approaches based on Markov process to model wind power uncertainty have been developed. In [3], a model that combines a Markov process with dynamic programming was proposed; in order to overcome the problems related to the analysis of large scale systems, a unit aggregation method based on MILP was incorporated. In a similar way, in [4] Markov process is used to formulate scheduling problem based on MILP in terms of possible states of wind generation instead of scenarios; so that, the complexity of the optimization problem is reduced by interpreting the behavior of wind generation since a probabilistic point of view through the transition matrix of the Markov process. Other approach consists on the application of chance constrained programming; in this sense, in [5] a model that includes a risk constraint in the scheduling problem was presented, this technique allows incorporating several sources of uncertainty such as forecasting error of wind generation and demand changing the probabilistic constraints to determinate ones; so that, traditional optimization techniques can be implemented.

Techniques based on probabilistic analysis have been carried out by several authors, this approaches consist on the study of the optimization problem considering the mathematical expression of the probability distribution that models wind power forecasting error instead of the analysis of a determined set of scenarios generated from this probability distribution. A representative study of this type of methods was presented in [6], where wind power generation is represented by using a Weibull distribution (to model wind speed characteristics) evaluated on a power curve with linear behavior (to model wind farm power production), while economic dispatch (ED) problem is formulated in terms of fuel consumption cost and reserve costs. In a similar way, in [7] wind power generation is modeled as in [6], while the analysis of the optimization problem is carried out in terms of the wind power penetration factor, scale factor, and shape factor of Weibull distribution. In our previous work [8] forecasting error is modeled by using a beta distribution; so that, probability distribution of output power of conventional generators and generation cost can be estimated with a reasonable accuracy.

As a continuation of the research work presented in [8]; in this paper, wind power forecasting error is modeled by using Cauchy distribution, which has been suggested as a reasonable mathematical model to forecasting error since an operational point of view. The paper is organized as follow: section II describes the probabilistic model used to determine the power dispatch; then, in section III the proposed approach is illustrated through the analysis of a representative power system of 10 generators, and conclusions are presented in section IV.

II. Probabilistic Model for Power Dispatch

Our simplified analysis only considers generation system without taking into account transmission constraints. The system under analysis is shown in Fig. 1, where output power of generator j ($j = 1, \dots, J$) is represented by the probabilistic variable TU_j^t, FWF^t is the available wind generation represented by a Cauchy distribution, DWF^t is the distribution of dispatched wind generation (obtained from the solution of ED problem), and HL^t is hourly load at time t.

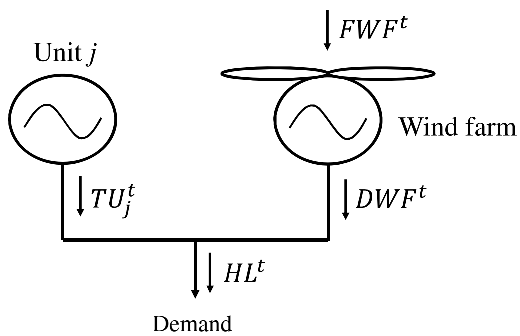

Figure 1. Scheme of the simplified power system under study.

The approach used in this paper consists of discretization process of Cauchy distribution and the solution of dynamic ED problem. Section II-A describes the discretization problem, while section II-B explains the solution of dynamic ED problem.

A. Discretization of the Cauchy Distribution

Cauchy distribution has been previously suggested as an accurate model of wind power forecasting error. In [9], an extensive analysis was carried out by using information from real installations. The analysis presented consists of a comprehensive comparison between Normal distribution, Beta distribution, Weibull distribution, and Cauchy distribution to model persistence forecast error. Fitting each of the probability distributions aforementioned by using maximum-likelihood estimator, the results shown that Cauchy distribution fit better than the others probability distributions in the majority of the cases (Cauchy distribution fits better than Beta distribution in 89% of the cases, and it fits better than Weibull distribution in 95% of the cases).

Cauchy distribution is incorporated on ED problem through a discretization process. Discretization process used in this paper was proposed by Barbiero [10]. The first step consists on adjusting the number of intervals (W) at which Cauchy distribution is discretized, each interval is represented by the index $w = 1, \dots, W$. The second step consists on defining a discrete representation of a normal distributed random variable (π_w) according to (1),

$$\pi_w = -\varepsilon - \frac{2\varepsilon}{W-1} + \left(\frac{2\varepsilon}{W-1}\right) w;\ w = 1, \dots, W; \quad (1)$$

where ε is a parameter to be adjusted by the used. In the third step, discretized representation of (1) is evaluated on the cumulative distribution function (CDF) of a normal distribution (F_N) according to (2),

$$\tau_w = F_N(\pi_w);\ w = 1, \dots, W; \quad (2)$$

In the fourth step, the probabilities obtained from the application of (2) (τ_w) are evaluated in the inverse CDF of Cauchy distribution according to (3),

$$FWF_w^t = F_C^{-1}(\tau_w);\ w = 1, \dots, W; \quad (3)$$

From this step, discretized representation of forecasting error (FWF_w^t) is obtained. In the final step, the probability ($P_r\{FWF^t = FWF_w^t\}$) that corresponds to each interval FWF_w^t is obtained by application of (4)-(7),

$$\eta_w = \frac{FWF_w^t + FWF_{w+1}^t}{2};\ w = 1, \dots, W-1; \quad (4)$$

$$P_r\{FWF^t = FWF_1^t\} = F_C(\eta_1)\ ;\ w = 1; \quad (5)$$

$$P_r\{FWF^t = FWF_w^t\} = F_C(\eta_w) - F_C(\eta_{w-1});\ w = 2, \dots, W-1; \quad (6)$$

$$P_r\{FWF^t = FWF_W^t\} = 1 - F_C(\eta_{W-1});\ w = W; \quad (7)$$

where η_w is an intermediate variable.

B. Dynamic Economic Dispatch under Uncertainty

In this sub-section is explained how ramp constraints are modeled and incorporated in the ED problem under uncertainty. In the methodology used in this paper, all stochastic variables are represented by discretized probability density functions (PDFs); in this sense, output power at time $t-1$ is represented by a discretized PDF for each generation unit j. In order to make the optimization problem mathematically tractable, power production at $t-1$ is simplified by using the quantile concept. First a determined amount of scenarios (S) of power production at $t-1$ is selected by the user, each of this scenarios is represented by the index $s = 1, \dots, S$. Then, a determined significance level (φ) is selected and the interval $[\varphi, 1-\varphi]$ is divided into several points (δ_s) with a determined step. After that, each point δ_s is evaluated in the discretized CDF of power production of each generation unit j and saved in the matrix of power production at time $t-1$ ($TU_{j,s}^{t-1}$) according to Fig. 2. $TU_{j,s}^{t-1}$ is a matrix of J rows and S columns, each column is a vector of possible output power of unit $j = 1, \dots, J$ at time $t-1$. Finally, a weight factor (V_s) for each of these columns is calculated according to (8),

$$V_s = \frac{\prod_j \left(P_r\{TU_j^{t-1} = TU_{j,s}^{t-1}\}\right)}{\sum_s \prod_j \left(P_r\{TU_j^{t-1} = TU_{j,s}^{t-1}\}\right)};\ s = 1, \dots, S; \quad (8)$$

where the factors $P_r\{TU_j^{t-1} = TU_{j,s}^{t-1}\}$ can be estimated by evaluating discretized PDF of unit j.

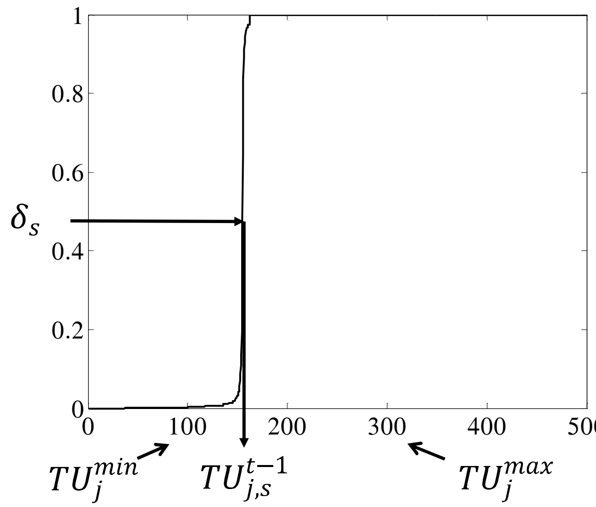

Figure 2. Discretized CDF of power production of unit j at time t-1.

Once TU_j^{t-1} has been described and the available wind power generation has been modeled through discretization process aforementioned, dynamic ED problem. Dynamic ED problem is shown in (9)-(14),

$$c_{s,w} = \sum_j \left\{\alpha_j + \beta_j\left(TU_{j,s}^t\right) + \gamma_j\left(TU_{j,s}^t\right)^2\right\} + VOLL(ENS_s^t); \quad (9)$$

$$\sum_j TU_{j,s}^t + DWF_w^t = HL^t\,; \quad (10)$$

$$TU_{j,s}^t - TU_{j,s}^{t-1} \le \theta_j; \quad (11)$$

$$TU_{j,s}^{t-1} - TU_{j,s}^t \le \lambda_j; \quad (12)$$

$$TU_j^{min} \le TU_{j,s}^t \le TU_j^{max}; \quad (13)$$

$$0 \le DWF_w^t \le FWF_w^t; \quad (14)$$

where α_j, β_j, and γ_j are coefficients of fuel consumption cost of unit j, $TU_{j,s}^t$ is power production of unit j that corresponds to the point s at time t, $VOLL$ is the value of lost load, ENS_s^t is the energy not supplied that corresponds to the point s at time t, θ_j and λ_j are ramp-up and ramp-down limits, respectively, TU_j^{min} and TU_j^{max} are minimum and maximum output power, while DWF_w^t and FWF_w^t are dispatched and forecasted (available) wind power generation; finally, $c_{s,w}$ is the generation cost that corresponds to the point s and interval w. For the solution of dynamic ED problem described in (9)-(14), for each point (s) of power generation at time $t-1$, the optimization problem is solved for all the intervals $w = 1, \ldots, W$ of wind generation and discretized PDF of each variable of interest is built; then, all the probabilities of discretized PDF are multiplied by the weight factor V_s and added to the result obtained for the previous point $s-1$; so that, the effects of ramp constraints and wind power uncertainty are represented in a single discretized PDF. In order to obtain discretized PDF of a determined variable of interest, for example generation cost (C^t), first a scale must be built, this is carried out by considering a determined maximum (C^{max}) and minimum (C^{min}) value for the variable, and a determined number of intervals (K) required to represent the variable with reasonable precision. The scale is composed by the succession of numbers from C^{min} until C^{max} in step $(C^{max} - C^{min})/(K-1)$. Then, the probability of occurrence of each value of the scale is represented by a vector with the same number of elements that the scale; so that, if a determined value needs to be represented, the corresponding bin is located in the scale and the probability is added to the corresponding position in the vector of probabilities. All this procedure is carefully described in [8].

III. Case Study

In order to illustrate the proposed methodology the analysis of a representative power system with 10 generators was carried out ($J = 10$). All information about the thermal units can be found in [11].The accuracy of the results obtained from the proposed methodology was measured by comparison with those obtained from Monte Carlo simulation (MCS) approach. As the proposed methodology in this paper requires the discretized PDF of power generation at time $t-1$ for each generator j, these were estimated by MCS approach. In this sense, three time intervals were defined: $t-2$, $t-1$, and t. At time $t-2$ the power production was assumed to be constant for each unit, at time $t-1$ random scenarios of wind power generation were generated and discretized PDF of each unit was obtained by MCS approach, these PDFs were used as an input to our proposed methodology; finally, at time $t-3$ random scenarios of wind power generation were generated and discretized PDF of each unit was obtained by MCS approach, these PDFs were used for comparative purposes in order to evaluate the accuracy of the proposed approach. Wind power generation was modeled by using a Cauchy distribution according to (15),

$$F_C(FWF^t) = \frac{1}{2} + tg^{-1}\left(\frac{FWF^t - a}{b}\right)\frac{1}{\pi}; \quad (15)$$

where parameters a and b were assumed to be 0 and 0.5, respectively; while the value of forecasted wind power generation with highest probability was assumed to be 200MW. Cauchy distribution with aforementioned parameters was divided into 50 intervals ($W = 50$), the parameter ε adjusted to 3, the significance level φ was adjusted to 1% and δ_s was obtained by evaluating the interval [0.01,0.99] with a step of 0.49; obtaining three points for the representation of power production at $t-1$ ($s = 1,2,3$). Discretized PDF of thermal power generation was created to represent values between 0MW and 500MW using 1500 bins, while discretized PDF of dispatched wind generation was created to represent values between 0MW and 318MW using 2000 bins; in similar way, discretized PDF of generation cost was created to represent values between 14,791$ and 38,365$ using 1500 bins. All these values can be adjusted by the user in a free manner according to magnitudes to be represented and required accuracy. Load was assumed to be $HL^t = 1{,}600MW$ Fig. 3 shows discretized PDF of available wind power generation (FWF^t) and dispatched wind generation (DWF^t); as can be observed wind power generation is completely integrated in the power system due to its relatively low value.

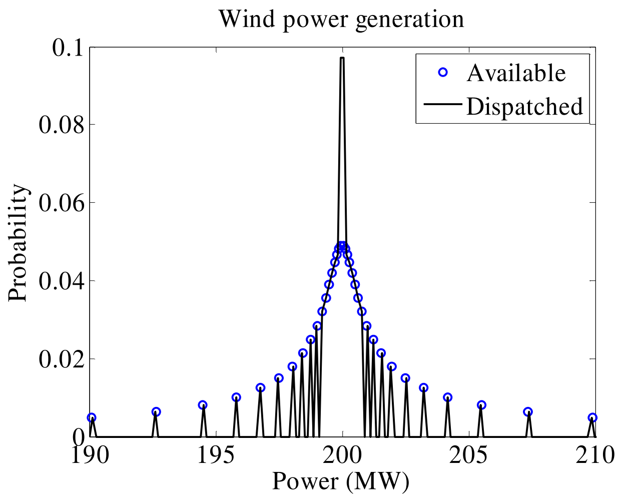

Figure 3. Available and dispatched wind generation.

Fig. 4 presents the comparison between the results obtained from the proposed method and MCS approach, as can be observed the proposed method can represent dispatched wind generation with a good accuracy.

Depending on the rated capacity, type and fuel consumption cost, as well as ramping capabilities, some generation units are able to respond to the fluctuations of wind generation in order to maintain power balance, while others maintain their power production in a stable level. Fig. 5 shows discretized PDF of power generation of unit 1, which maintains its power output at 455MW, this behavior can be found in other generation units of the system.

Fig. 6 presents discretized PDF of output power of generator 5 which regulates its power generation in a similar way according to the dispatched wind generation. As can be noted, proposed method presents a good agreement with the results obtained from MCS approach.

Fig. 7 presents discretized PDF of generation cost, which is highly influenced by wind power generation, showing a PDF with a similar shape that dispatched wind generation.

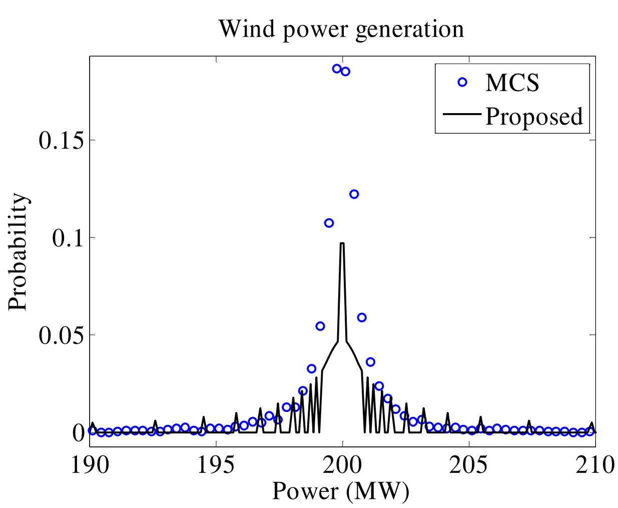

Figure 4. PDF of wind power generation.

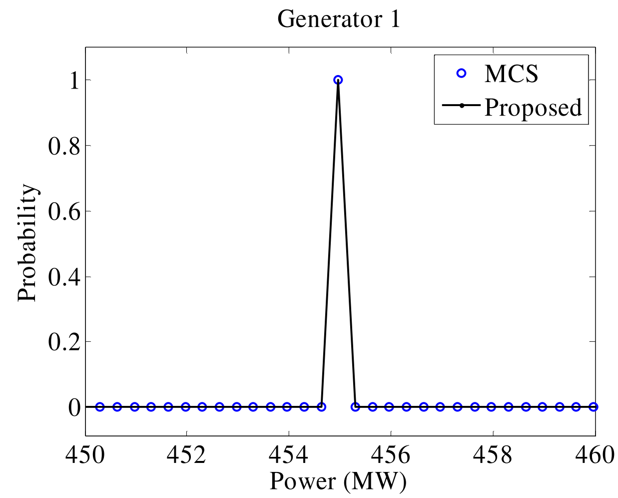

Figure 5. PDF of power production of unit 1.

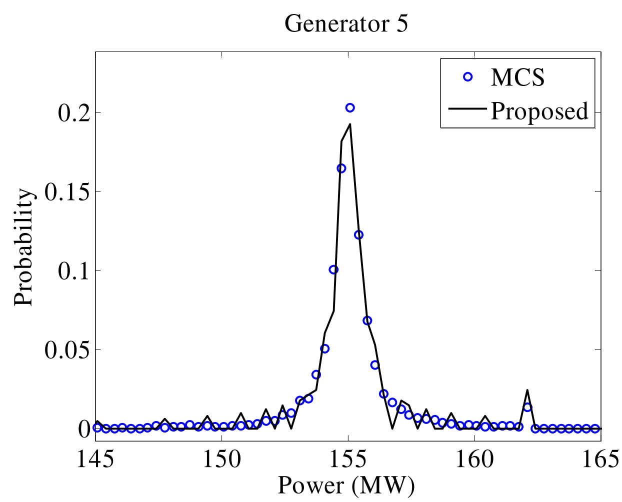

Figure 6. PDF of power production of unit 5.

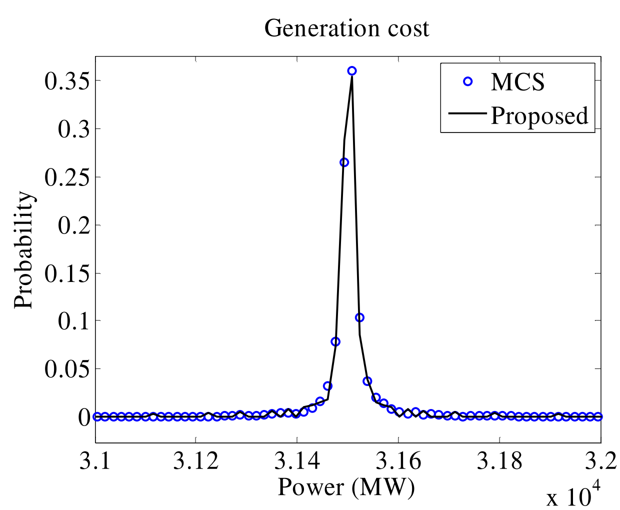

Figure 7. PDF of generation cost.

The capabilities of the proposed method to represent behavior of power dispatch according to a determined wind generation can be evaluated by comparing the expected value of all the variables of interest. For our case study these results are presented in Table I, where a good agreement can be easily recognized.

TABLE I. EXPECTED VALUE OF POWER GENERATION AND COST

n	MCS	Proposed
1	454.969980	454.969980
2	454.969980	454.969980
3	130.086724	130.086724
4	130.086724	130.086724
5	155.009006	154.569683
6	20.055904	20.368411
7	25.016678	25.016678
8	10.006671	10.080719
9	10.006671	10.024427
10	10.006671	10.006671
$E\{DFT^t\}$	199.948766	200.000000
$E\{C^t\}$	31503.692155	31503.361480

IV. CONCLUSIONS

In this paper, a probabilistic model to solve the ED problem incorporating ramp constraints of thermal units and wind power forecasting error represented by a Cauchy distribution was presented. The proposed approach discretized Cauchy distribution into several intervals which were incorporated into the optimization problem in order to find discretized PDF of dispatched wind generation, thermal power production and generation cost. The proposed methodology was illustrated by analyzing a power system of 10 units and the obtained results were compared to those obtained from MCS approach. A good agreement between both methods was observed.

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