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Impact of Energy Storage on Economic Dispatch of Distribution Systems: A Multi-Parametric Linear Programming Approach and Its Implications

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ABSTRACT As a main flexible resource, energy storage helps smooth the volatility of renewable generation and reshape the load profile. This paper aims to characterize the impact of energy storage unit on the economic operation of distribution systems in a geometric manner that is convenient for visualization. Posed as a multi-parametric linear programming problem, the optimal operation cost is explicitly expressed as a convex piecewise linear function in the MW/MWh parameter of the energy storage unit. Based on duality theory, a dual linear programming based algorithm is proposed to calculate an approximate optimal value function (OVF) and critical regions, circumventing the difficulty of degeneracy, a common challenge in the existing multi-parametric linear programming solvers. When the uncertainty of renewable generation is considered, the expected OVF can be readily established based on OVFs in the individual scenarios, which is scalable in the number of scenarios. The OVF delivers abundant sensitivity information that is useful in energy storage sizing. Leveraging the OVFs, a robust stochastic optimization model is proposed to determine the optimal MW-MWh size of the storage unit subject to a given budget, which gives rise to a simple linear program. Case study provides a clear sketch of the outcome of the proposed method, and suggests that the optimal energy-power ratio of an energy storage unit is between **5** and **6** from the economical perspective.

INDEX TERMS Distribution system, energy storage unit, multi-parametric programming, optimal value function approximation.

NOMENCLATURE

A. INDICES

i, j	Index of buses.
l	Index of lines.
t	Index of time periods.
$c(i)$	Set of downstream buses of bus i.
$(\cdot)_i$	Symbol associated with bus i.
$(\cdot)^l_{ij}$	Symbol associated with line l.

B. PARAMETERS

b^g_i	Generator cost coefficient.
c_t	Electricity price at the upstream grid.
p^d_{it}, q^d_{it}	Fixed active and reactive power demand.
r^l_{ij}, x^l_{ij}	Resistance and reactance of line l.
p^r_{it}	Renewable power output.
z^l_{ij}	Impedance of line l, $(z^l_{ij})^2 = (r^l_{ij})^2 + (x^l_{ij})^2$.
η^c_i, η^d_i	Charging/discharging efficiency of ESU.
P^S_{mi}	Maximum charging/discharging power of ESU.
E^S_{mi}	Energy capacity of ESU.

C. VARIABLES

E_{it}	State-of-charge of ESU.
p^g_{it}/p^n_{it}	Active power generation/injection at bus i.

p^c_{it}/p^d_{it}	Charging/discharging power of ESU.
P^l_{ijt}	Active power flow in line l.
q^g_{it}/q^n_{it}	Reactive power generation/injection at bus i.
Q^l_{ijt}	Reactive power flow in line l.
U_{it}	Squared voltage magnitude at bus i.

I. INTRODUCTION

THE penetration of renewable generation in distribution systems is growing rapidly [1]. The gap between peak and valley demands is also widened due to the development of industry and economy. The volatility of renewable generation and the increasing peak-valley gap imposes great challenges on the operation of distribution systems, where the dispatchable resource is rare. Deploying energy storage units (ESUs) is an effective mean to cope with these challenges [2]–[4]. Among existing work, storage planning and operation have attracted the most attention.

The planning problem entails the determination of the optimal site and size of ESU. In some instances, the location is given and the size is to be optimized. In either case, the biggest challenge for sizing ESU is the volatility of renewable power. To address this issue, a stochastic optimization (SO) model is suggested in [5], [6] for planning a stand-alone power supply system. Since the system is small, network power flow is neglected. A comprehensive model for expansion planning of distribution system, renewable plant, and ESU is proposed in [7] and reformulated as a mixed-integer linear program. The stochastic ESU planning approach is applied in the Western Electricity Coordinating Council (WECC) interconnected system [8], showing its ability in large-scale instances. A stochastic model predictive control method is developed in [9] to consider receding horizon operation of ESU and ultra-short-term wind power forecast. In this way, the dynamic nature of power system dispatch is captured. Nevertheless, the model entails the consideration of all horizons over the entire planning horizon in all scenarios at the same time, imposing a big challenge for computation.

For the purpose of scenario generation, SO approach requires the exact probability distribution of uncertain factors, which may be difficult to acquire. Without the information on the probability distribution, a robust optimization method is proposed in [10] to size ESU in power grids, in which the uncertainty set is the convex hull of several extreme scenarios restricting the range of uncertain data. The robust optimization (RO) model is further developed in [11] to coordinate the investment in transmission lines and ESUs and consider long-term and short-term uncertainties. Although RO approach requires very little distributional information on uncertainty and tackles the worst-case situation, it usually provides conservative results because the worst case rarely happens. An alternative approach is the point estimate method, which is used in the storage planning problem in distribution networks in [12]. Point estimate method needs a probabilistic renewable power forecast, which is comprised of a few scenarios together with their probabilities. So the performance relies on the accuracy of the input probability distribution. Distributionally robust optimization (DRO) is an emerging technique that inherits the advantages from both SO and RO. Uncertainty is described by a family of probability distributions around an empirical one. The performance under the worst-case distribution is guaranteed. A comprehensive tutorial on DRO can be found in [13]. This approach has been employed to sizing the ESU in stand-alone solar-powered charging stations [14] as well as in the energy hub connecting a power distribution grid and a district heating system [15], where network flow constraints are considered. It is shown that DRO offers less conservative planning strategies than the traditional RO, and the out-of-sample performance outperforms the traditional SO. Finally, the storage planning problem is considered in a competitive environment in [16]. The uncertainty is modeled via probability distribution, and the Nash equilibrium of storage planning strategies demonstrates that competition does not harm the expected effect of ESU.

Operation is another important issue of renewable energy and storage integrated power systems. Uncertainty handling remains one of the central targets of research. SO, RO, and DRO methods are utilized in the ESU scheduling problem under the framework of unit commitment [17]–[19], economic dispatch [20]–[24], and demand response [25], [26]. Recently, the potential benefits of ESU participating in energy and auxiliary service markets have been recognized. In [27], self-scheduling of a price-taker virtual power plant, which consists of wind power plants and ESU, is studied via SO. Uncertainties of wind generation and market prices are modeled by scenarios. The arbitrage strategy of ESU in day-ahead and real-time markets is investigated in [28]. Market prices are assumed to be uncertain and modeled by scenarios. It is found that compared with quantity-only bidding, simultaneous price and quantity bidding gives higher profits on average and also leads to a higher risk. To better simulate the market power of ESU, the strategic bidding of ESU is modeled via bilevel programs in [29]. The market clearing problem in the lower level gives the clearing price. By replacing the market clearing problem with KKT optimality condition and perform linearization, the final problem reduced to a mixed-integer linear program. In ref. [30], the lower level is decomposed into a day-ahead market and a balancing market, accounting for compensating real-time imbalance.

The grid contribution of ESU largely depends on its parameters, including the efficiency, maximum charging-discharging power (MW), and energy capacity (MWh). The impact of these parameters and other system configurations are investigated by numeric simulation, which requires solving an optimization problem (usually unit commitment or multi-period dispatch) repeatedly with different parameter sets. Evaluation of cost saving brought by ESU can be traced back to [31]. At that time, the optimization algorithm was less developed, and renewable generation was not considered.

The value of ESU for managing wind power volatility is discussed in [32] with a unit commitment model with spinning and standing reserve. A stochastic unit commitment model is outlined in [33] to evaluate the short-term economic benefits of deploying ESU with different penetration levels of renewable generation, in which the uncertainty is better treated. In [34], a model predictive control scheme is presented for minimizing the operation cost of a distribution system, based on which the contribution of ESU on improving reliability and reducing the cost is quantified. Recently, flexibility is acknowledged as the key resource for coping with power fluctuation in different time scales. A dedicated flexibility metric is defined in [35], which can be calculated from a unit commitment problem. An optimization problem is proposed to maximize the flexibility metric. The effect of ESU on alleviating congestion in a distribution network is studied in [36], based on a multi-period dispatch model. In [37], a SO model is set forth to optimize the topology of the distribution network and ESU dispatch to evaluate system performance under $N-1$ contingency conditions.

Conceptually, evaluation of the economic impact of ESU requires expressing the optimal cost as a function in MW and MWh parameters of ESU, called the optimal value function (OVF). Solving the economic dispatch problem with changing parameter can provide the costs on the sampled parameters, but does not offer the subtle structure of the OVF and deeper insights on how such sensitivity information can help decision making in practice. This paper proposes a thorough mathematical method to quantify and visualize the economic impact of ESU parameters on distribution system operation. The contributions are twofold:

1) The distribution system operation is formulated as a multi-parametric linear programming (mp-LP) problem, where the parameter is MW/MWh configuration of ESU. Using duality theory, the OVF is shown to be convex and piecewise linear (PWL), and a dual LP based algorithm is developed to solve the mp-LP approximately; it circumvents the common difficulty of degeneracy. Compared with the traditional parameter sampling based approaches, the proposed method reveals the geometric structure of the OVF, and the sensitivity to parameter change is explicitly reflected by the coefficients constructed from dual variables. Such information can be easily visualized and could help engineers determine a proper energy-power ratio of energy storage.

2) The method is generalized to a stochastic setting with a large number of scenarios. An optimization model for storage sizing is established under the framework of DRO. Uncertain factors are modeled by scenarios associated with inexact probability values. Based on the obtained OVF under each scenario, a decomposition method is designed to retrieve the OVF of the DRO model in the worst-case distribution. Afterwards, the storage sizing strategy under a given budget can be retrieved from a simple LP. Although DRO can provide a single optimal solution without the need of an OVF, the parametric optimal solution and optimal value are extremely useful when engineers concern how the optimal solution is influenced by parameter perturbation, and also provide an alternative mean to solve bilevel decision-making problems in power markets.

The rest of this paper is organized as follows. The mp-LP model of distribution system operation is introduced in Sect. II, following which the algorithm for retrieving the deterministic OVFs is presented, and the properties of OVF is discussed. The extension in stochastic cases and the DRO method for storage sizing are presented in Sect. III; Case studies are conducted in Sect. IV; Finally, conclusions are summarized in Sect. V.

II. PROPOSED METHOD

We will first present an mp-LP model for quantifying the impact of ESU on economic dispatch of distribution system, and then we discuss the OVF and its property using compact form; Finally, we raise the algorithm for solving the mp-LP.

A. MULTI-PERIOD ECONOMIC DISPATCH AS AN MP-LP

The power flow in a distribution system with radial topology can be exactly described by the branch flow model (BFM) or Dist-Flow originally developed in [38]. By neglecting lossy terms, the BFM becomes linear [39]:

$$P^l_{ijt} + p^n_{jt} = \sum\nolimits_{k \in \pi(j)} P^l_{jkt}, \quad \forall j \tag{1a}$$

$$Q^l_{ijt} + q^n_{jt} = \sum\nolimits_{k \in \pi(j)} Q^l_{jkt}, \quad \forall j \tag{1b}$$

$$V_{jt} = V_{it} - \frac{r^l_{ij} P^l_{ijt} + x^l_{ij} Q^l_{ijt}}{V_{0t}}, \quad \forall (l, i, j) \tag{1c}$$

where p^n_{jt}/q^n_{jt} is the active/reactive power injection depending on the output of gas-fired units p^g_{jt}/q^g_{jt}, renewable generation p^r_{jt}, load p^d_{jt}/q^d_{jt}, and the output of ESU p^c_{jt}/p^d_{jt}, if there is any. For nodes without an ESU, p^c_{jt} and p^d_{jt} are set to 0. Equality (1a)/(1b) is active/reactive power balancing condition at each bus; (1c) describes voltage drop along each line. Model (1) will be referred to as linearized BFM in context and **Cons-BFM**$[t]$ in mathematical formula hereinafter. Compared with the traditional direct-current (DC) power flow model, model (1) considers reactive power and bus voltage, which are either neglected or fixed in the DC model. So the linearized BFM better captures the operating characteristics of the distribution system, although network loss is neglected.

The operation model the ESU can be described by

$$E_{jt+1} = E_{jt} + \eta^c_j p^c_{jt} - p^d_{jt}/\eta^d_j, \quad \forall j, \ \forall t \tag{2a}$$

$$0 \le p^c_{jt} \le P^S_{mj}, \ 0 \le p^d_{jt} \le P^S_{mj}, \quad \forall j, \ \forall t \tag{2b}$$

$$\alpha E^S_{mj} \le E_{jt} \le E^S_{mj}, \forall j, \quad \forall t \tag{2c}$$

where (2a) depicts the SoC dynamics; (2b) and (2c) set lower and upper bounds on charging/discharging power and SoC, where P^S_m and E_m are the main parameters whose impact will be investigated, α is a constant indicating the minimum SoC. Condition $p^c_t \cdot p^d_t = 0$ naturally holds if the locational marginal price at storage-connected bus is positive [40], which is met in the distribution system. ESU model (2) will be cited as **Cons-ESU** in mathematical formula later on. It couples with the linearized BFM (1) through (1a).

The objective of distribution system dispatch is to minimize the total operation cost

$$f_{DN} = \sum_{t} \xi_t P_t^{l_0} + \sum_{j,t} b_i^g p_{jt}^g \tag{3}$$

where $P_t^{l_0}$ stands for the active power in the line connect to the slack bus, so the first term is the purchase cost paid to the upstream power grid; the second term is the fuel cost of local generators.

The distribution system operation model is cast as

$$\begin{aligned} &\min\ f_{DN} \\ &\text{s.t. } \mathbf{Cons-BFM}[t], \quad \forall t \\ &\qquad \mathbf{Cons-ESU},\ \mathbf{Cons-BND} \end{aligned} \tag{4}$$

where **Cons-BND** denotes lower and upper bounds of decision variables except those included in (2).

A few discussions are given as follows.

1) In (3), the generator cost function is linear. There are two ways to consider a quadratic cost function. One option is to approximate it with a PWL function; one viable formulation is available in [41]. The other is to employ the multi-parametric quadratic programming theory. We recommend the first one because the objective function of a multi-parametric quadratic program must be strictly convex, which is a strong assumption.

2) In formulation (5), the location of the ESU is fixed. To examine how the site will influence system performance, we can change the location of ESU, calculate (9) for each case and compare the corresponding $v(\theta)$.

3) In many cases, a distribution system possesses only one large ESU, so the dimension of parameter space is two, and the OVF and critical regions can be visualized. The proposed method has no limitation in coping with multiple ESUs. In such circumstances, because the dimension of parameter is higher than 2, visualization of $v(\theta)$ is non-trivial. An example with two ESUs is given in the case study. In this case, we fix the ratio E_m/P_m^S, so that the dimension of θ remains two.

B. GEOMETRIC DESCRIPTION OF THE IMPACT OF STORAGE

Define $P_m^S = [P_{m1}^S, P_{m2}^S, \cdots]^\top$, $E_m^S = [E_{m1}^S, E_{m2}^S, \cdots]^\top$. Let $\theta = [P_m^S, E_m^S]^\top$ be the vector of parameter and x the vector of all decision variables; then problem (4) has the following compact matrix form

$$\begin{aligned} v(\theta) = \ &\min\ c^\top x \\ &\text{s.t. } Ax \le b + B\theta \end{aligned} \tag{5}$$

where A, B are constant matrices and b, c constant vectors. We assume that the parameter θ varies in a hypercube

$$K = \{\,\theta \mid 0 \le \theta \le \theta_{\max}\,\} \tag{6}$$

which is called the parameter space. The task is to find an analytical form of the OVF $v(\theta)$ on the parameter space K.

For a fixed θ, suppose the set of active/inactive constraints at the optimal solution is

$$\begin{aligned} A'x &= b' + B'\theta \\ A^\star x &< b^\star + B^\star\theta \end{aligned}$$

Then, the optimal solution and optimal value are

$$x = A'^{-1}b' + A'^{-1}B'\theta,\ \theta \in CR \tag{7}$$

$$v(\theta) = c^\top A'^{-1}b' + c^\top A'^{-1}B'\theta,\ \theta \in CR \tag{8}$$

where CR is called a critical region in which the set of active constraints remains intact. Therefore, the analytical expression of $v(\theta)$ must take the following form

$$v(\theta) = \begin{cases} m_1 + n_1^\top\theta, & \theta \in CR_1 \\ m_2 + n_2^\top\theta, & \theta \in CR_2 \\ \quad\vdots & \\ m_N + n_N^\top\theta, & \theta \in CR_N \end{cases} \tag{9}$$

where $m_1, \cdots, m_N$ and $n_1, \cdots, n_N$ are constant scalars and vectors; $CR_1, \cdots, CR_N$ constitute a partition of K. Clearly, (9) is a PWL function.

C. AN APPROXIMATE ALGORITHM FOR SOLVING MP-LP

Existing parametric programming methods solve a mp-LP based on (7) and (8). However, they are ineffective because of degeneracy [42], which is a common issue in large-scale engineering problems. Degeneracy complicates the computation of critical regions. To overcome this difficulty, we develop a method that does not rely on the analysis of optimal basis invariant set. First, we disclose the convexity of OVF $v(\theta)$.

Claim: The OVF $v(\theta)$ is convex in θ.

To see this, for a fixed θ, the dual problem of LP (5) is

$$v(\theta) = \max_{\mu \in U} \mu^\top(b + B\theta), \quad U = \left\{\mu \,\middle|\, A^\top\mu = c, \mu \le 0\right\} \tag{10}$$

Because of strong duality, the optimal value of dual LP (10) is equal to $v(\theta)$ for any given θ. LP (10) shows that $v(\theta)$ is the point-wise maximum of affine functions $\mu^\top(b + B\theta)$ when the parameter μ takes all possible values in U, hence $v(\theta)$ is convex in θ, as point-wise maximum preserves convexity.

The form of LP (10) offers an alternative way to build an approximation of $v(\theta)$ without prior knowledge on critical regions. Let vert(U) be the set of vertices of set U. Since the optimum is finite, the optimal solution can always be found at some $\mu \in \text{vert}(U)$, although U may contain extreme rays. On this account, $v(\theta)$ can be expressed as

$$v(\theta) = \max_i \left\{\mu_i^\top b + \mu_i^\top B\theta\right\},\ \forall \mu_i \in \text{vert}(U) \tag{11}$$

Compare (9) and (11), coefficients m and n can be retrieved from dual variable via

$$m_i = \mu_i^\top b,\ n_i = \mu_i^\top B, \ \text{ for some } \gamma_i \in \text{vert}(\Gamma) \tag{12}$$

Nevertheless, only a few vertices produce valid pieces in (9) and (11), but they are not known in advance. We propose a heuristic method; the flow chart is summarized in Algorithm 1.

Algorithm 1 PWL Approximation of OVF

1: Discretize the parameter set K into uniformly distributed points, denoted by θ_i, $i = 1, \cdots, I$.
2: Solve problem (10) for each θ_i; the corresponding optimal solution is μ_i, $i = 1, \cdots, I$.
3: Eliminate duplicated elements in the series $\{\mu_i\}_{i=1}^{I}$, and compute coefficients m_i and n_i according to (12).
4: Remove redundant slice in $v(\theta)$ (see Remark 3).
5: Recover critical regions from $v(\theta)$ (see Remark 2).

Remark 1: The outcome of Algorithm 1 is an underestimator of the true OVF, because some μ_i in vert(U) may be missed; otherwise, the approximation is exact. The outcome of Algorithm 1 depends on the discretization for the parameter space K. We can compare the optimums offered by the PWL approximation and the original problem (5) on a set of sampled θ. If the maximum relative error is not satisfactory, then we increase the density of discretization.

Remark 2: After obtaining the PWL expression of $v(\theta)$, we can construct the critical region for each piece. Recall (9) and its point-wise maximum property, in CR_i, the value of $m_i + n_i^\top \theta$ must be greater than or equal to the values of other affine functions, yielding

$$CR_i = \left\{ \theta \middle| m_i + n_i^\top \theta \geq m_{[-i]} + n_{[-i]}^\top \theta \right\} \tag{13}$$

where the subscript $[-i]$ is a simple notation for $i = 1, \cdots, i-1, i+1, \cdots, N$, so there are actually $N-1$ inequalities in (13). Finally, redundant constraints should be removed.

Remark 3: In Algorithm 1, steps 4 and 5 entail redundancy elimination. The critical region has a polyhedral form of $\mathcal{H} = \{\theta | H\theta \leq h\}$; the epigraph of $v(\theta)$

$$\text{epi}[v(\theta)] = \left\{ (z, \theta) \middle| z \in \mathbb{R},\ z \geq m_i + n_i^\top \theta, i = 1, \cdots, N \right\}$$

is also a polyhedron sharing the same compact form, so we can address the problem using a unified method.

The i-th row of $\mathcal{H}$ is redundant if dropping off constraint $H_i\theta \leq h_i$ does not alter the feasible region defined by $\mathcal{H}$. According to Farkas lemma, the i-th row is redundant if it is a nonnegative combination of all remaining constraints in $\mathcal{H}$ plus a positive offset. So we test feasibility of constraint set

$$H_i = \sum_{j,j\neq i} \lambda_j H_j,\ h_i = \sum_{j,j\neq i} \lambda_j h_j + h_0, \quad \lambda \geq 0,\ h_0 > 0 \tag{14}$$

If (14) is feasible, $H_i\theta \leq h_i$ is redundant and should be removed from the critical region or the OVF.

III. IMPLICATIONS OF THE OVF

This section discusses some useful applications of the OVF obtained in Sect. II.

A. EXPECTED OVF CONSIDERING UNCERTAINTY

LP (5) is deterministic. We adopt a stochastic approach to consider uncertain renewable generation. Particularly, we have a collection of renewable generation scenarios and their probabilities ρ_s. Renewable output influences the right-hand coefficient b. The stochastic extension of problem (5) is

$$\begin{aligned} \mathbb{E}[v(\theta)] = \ & \min \sum_s \rho_s c^\top x^s \\ & \text{s.t. } Ax^s \leq b^s + B\theta,\ s = 1, \cdots, S \end{aligned} \tag{15}$$

where S is the number of scenarios. Uncertain purchase cost ξ_t can be taken into account by vector c^s associated with scenario s without changing the solution approach.

Problem (15) remains an mp-LP, therefore, we can apply Algorithm 1 to compute the expected OVF $\mathbb{E}[v(\theta)]$. However, the problem size grows quickly if S is large. Next, we discuss how to obtain $\mathbb{E}[v(\theta)]$ from the OVF in each scenario. Notice that in problem (15) the constraint in scenario s does not depend on variable x and parameter b in other scenarios. Define the OVF in scenario s by

$$v_s(\theta) = \min_x \left\{ c^\top x^s \middle| Ax^s \leq b^s + B\theta \right\}$$

where $v_s(\theta)$ has the same form as (9) and can be computed in parallel. The expected OVF under uncertainty is

$$\mathbb{E}[v(\theta)] = \sum_s \rho_s v_s(\theta) \tag{16}$$

The epigraph form (please refer to [43], Sect 3.1.7 for the definition of epigraph) of problem (16) is

$$\begin{aligned} & \min \sum_s \rho_s y_s \\ & \text{s.t. } y_s \geq m_k^s + (n_k^s)^\top \theta, \quad k = 1, \cdots, N_s,\ \forall s \end{aligned} \tag{17}$$

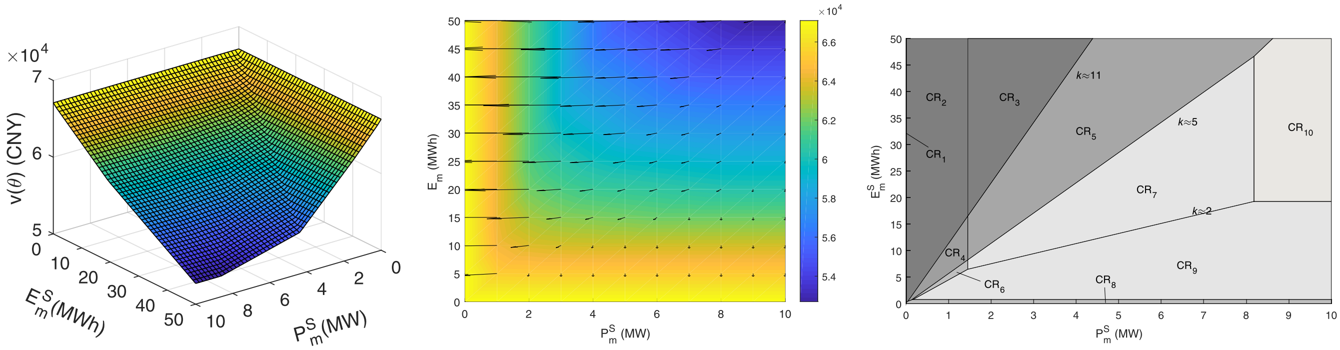
where N_s is the number of pieces in the OVF $v_s(\theta)$ in scenario s. If we regard problem (17) as an mp-LP in parameter θ, the expected OVF $\mathbb{E}[v(\theta)]$ can be solved by applying Algorithm 1, which is very efficient because problem (17) is much smaller than problem (15) in size. Through comparing $\mathbb{E}[v(\theta)]$ and individual $v_s(\theta)$, we can observe how uncertainty impacts the operation cost.

B. A DRO MODEL FOR STORAGE SIZING

The OVF $v_s(\theta)$ carries useful information. This section will discuss how the OVF can help determine the capacity of ESU. We consider the storage sizing problem with a given budget Γ. The total investment on the ESU depends on MW and MWh parameters. For simplicity, we assume the investment cost is a linear function in both parameters

$$\alpha_E^\top E_m^S + \alpha_p^\top P_m^S \tag{18}$$

The first MWh-related component reflects the cost of battery arrays, and the second MW-related components represents



(a) OVF (b) Contour plot (c) Critical regions

FIGURE 1. OVF and critical regions with 11 x 11 discretization.

the cost of power electronics convertors. We append the budget constraint in the parameter set K in (6), resulting in

$$K(\Gamma) = \{\ \theta \in K \mid \alpha_E^\top E_m^S + \alpha_p^\top P_m^S \le \Gamma\ \} \tag{19}$$

For the deterministic model (5), we can compute $v(\theta)$ over the budget-restricted set $K(\Gamma)$, denoted by $v_\Gamma(\theta)$. By optimizing $\min_\theta v_\Gamma(\theta)$, the optimal solution $\theta^\star$ gives the optimal size of ESU, including MW and MWh parameters at the same time. Apply this idea to the stochastic case (15), we should optimize the expected OVF $\mathbb{E}[v(\theta)]$.

Assume a collection of historical data with M samples is available. Then we can create a histogram with S bins as an estimation of the empirical distribution. Let $M_1, M_2, \cdots, M_S$ be the numbers of samples fall in each bin, where $\sum_{s=1}^S M_s = M$, then the empirical distribution is given by

$$\mathbb{P}_0 = [\rho_1^0, \cdots, \rho_S^0],\ \rho_s^0 = M_s/M,\ i = 1, \cdots, S$$

In general, the historical data available at hand is not adequate to fit the accurate distribution, so the probability ρ_s^0 is inexact, preventing the direct setup of problem (15).

Following the paradigm of DRO, we resort to ambiguity set of probability distribution in [44], which allows the probability ρ_s to vary around the empirical value ρ_s^0, giving rise to

$$\begin{aligned} D_\infty &= \left\{ \mathbb{P} \in \mathbb{R}_+^S \,\middle|\, \|\mathbb{P} - \mathbb{P}_0\|_\infty \le \gamma \right\} \\ &= \left\{ \rho \in \Delta_S \,\middle|\, \begin{array}{l} \gamma \ge \rho_s - \rho_s^0,\ s = 1, \cdots, S \\ \gamma \ge \rho_s^0 - \rho_s,\ s = 1, \cdots, S \end{array} \right\} \end{aligned} \tag{20}$$

where $\Delta_S = \{\rho \in [0, 1]^S : 1^\top \rho = 1\}$, $\rho = [\rho_1, \cdots, \rho_S]^\top$. Parameter γ reflects a confidence level on the distance between the empirical distribution and the true one. Apparently, more historical data can generate an empirical distribution $\mathbb{P}_0$ with higher accuracy. The proper value of γ can be determined from the following relation [44]

$$\Pr\{\|\mathbb{P} - \mathbb{P}_0\|_\infty \le \gamma\} \ge 1 - 2Se^{-2M\theta} \tag{21}$$

Therefore, if we want to maintain (20) with a confidence level of β, parameter γ should be selected as

$$\gamma = \frac{1}{2M} \ln \frac{2S}{1-\beta} \tag{22}$$

As the size of sampled data approaches infinity, γ decreases to 0, and the empirical distribution converges to the true one. If we have limited data, the ambiguity set D_∞ contains infinitely many probability distributions.

Let $v_\Gamma^s(\theta)$ denote the OVF $v_s(\theta)$ in scenario s over $K(\Gamma)$. In view of the distributional uncertainty described in (20), we consider the expected performance under the worst-case distribution, leading to the following DRO model

$$\max_{\rho \in D_\infty} \min_{\theta \in K(\Gamma)} \sum_s \rho_s v_\Gamma^s(\theta) \tag{23}$$

Formulation (23) means that once the optimal sizing strategy, which incurs a budget of Γ, is deployed, the expected operation cost in the worst-case scenario will be minimum.

To solve problem (23), we first write out the epigraph form of the inner minimization problem

$$\begin{aligned} \min_{\theta, y}\ & \sum_s \rho_s y_s \\ \text{s.t. } & y_s \ge m_k^s + (n_k^s)^\top \theta : \nu_k^s, \quad \forall k,\ \forall s \\ & \alpha^\top \theta \le \Gamma : \eta,\ \theta \ge 0 \end{aligned} \tag{24}$$

where m_k^s and n_k^s are scalar and vector associated with $v_\Gamma^s(\theta)$ in scenario s, the second constraint corresponds to the budget limit. Greek letters after the colon are dual variables.

The dual problem of LP (24) reads

$$\begin{aligned} \max_{\nu, \eta}\ & \Gamma \eta + \sum_s \sum_k m_k^s \nu_k^s \\ \text{s.t. } & \sum\nolimits_k \nu_k^s = \rho_s,\ \forall s,\ \nu \ge 0 \\ & \sum\nolimits_k \nu_k^s n_k^s + \alpha \eta \le 0,\ \eta \le 0 \end{aligned} \tag{25}$$

Substituting (25) into (23), we obtain

$$\begin{aligned} \max_{\rho, \nu, \eta}\ & \Gamma \eta + \sum_s \sum_k m_k^s \nu_k^s \\ \text{s.t. } & \sum\nolimits_k \nu_k^s = \rho_s, \quad \forall s \\ & \sum\nolimits_k \nu_k^s n_k^s + \alpha \eta \le 0 \\ & \nu \ge 0, \quad \eta \le 0,\ \rho \in D_\infty \end{aligned} \tag{26}$$

Problem (26) remains an LP.

Finally, we would like to clarify that a detailed investment problem may have to consider practical factors such as multi-stage investment and net present values. The purpose of this paper is to pose a tool for quantifying and visualizing the impact of storage parameters as well as its implications which are not restricted to storage sizing. Most existing evaluation methods, if not all, may not offer such abundant implications.

The parameterized optimal solution (7) and optimal value (8) are also desired in electricity market problems. For example, in a capacity market, a merchant energy storage unit first submits how much capacity it would like to offer, and then the market is cleared according to an economic dispatch problem in the form of (5). The bidding strategy and the market clearing results are contained in the parameter θ and the optimal solution x, respectively. The profit of the merchant storage unit $F(x)$, which is a function of x and thus influenced by θ, is to be maximized. This problem gives rise to a bilevel program. The traditional method is to replace the market clearing problem with its KKT optimality conditions and further linearize the complementarity and slackness condition, resulting in a mixed-integer linear program. As the size of the market clearing problem grows with the scale of the power system, solving the bilevel decision-making problem may take some time. The parametric perspective in this paper offers an alternative way to solve the strategic bidding problem in the capacity market. As disclosed in (7), the optimal solution x can be explicitly expressed as a piecewise linear function in θ, so the analytical expression of $F(x(\theta))$ in θ can be retrieved. As a result, the optimal bidding strategy can be searched in the parameter set K whose dimension is much lower than x, if the market involves only one or just a few periods.

If multiple energy storage units or multiple time periods are involved, the dimension of parameter θ in the lower-level problem grows higher, complicating the solution of mp-LP. This is an intrinsic difficulty of multi-parametric programs. Another issue arises in the nodal price based markets. Nodal price is the dual variable of the economic dispatch problem, which is not included in the analysis in Section II. To overcome this difficulty, a strictly convex quadratic programming model is used in [45]. The parametric analysis is based on the KKT optimality condition, which simultaneously offers the analytical expressions of the primal variable, dual variable, and optimal value. As the parametric program can be solved offline, such a methodology has potential in online applications and is a promising direction in power market studies.

IV. CASE STUDIES

A. SYSTEM CONFIGURATION

The proposed method is tested on a modified IEEE 33-bus system shown in Fig. 2. The peak demand is 9.39 MW. Wind and solar generation with a total capacity of 2.8 MW is integrated at bus 3; real weather data in Qinghai Province, China are used to produce renewable power output. An ESU with $\eta^c = \eta^d = 0.95$ is placed at bus 3. A gas-fired unit with a capacity of 8MW connects to bus 6; the marginal production cost is 1000 CYN/MWh. The time-of-use electricity price at the slack bus is 800 CNY/MWh from 7 a.m. to 8 p.m. and 400 CNY/MWh in the remaining periods of the day. All experiments are implemented on a laptop with Intel i57267U CPU and 8 GB memory. Optimization problems are coded in MATLAB environment with YALMIP interface [46] and solved by CPLEX 12.8.

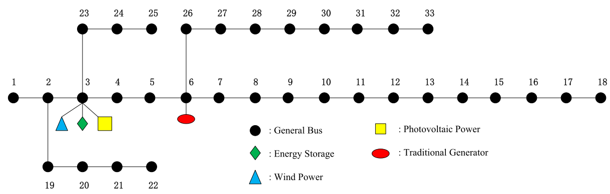


FIGURE 2. **Topology of the 33-bus system.**

B. VISUALIZATION OF THE OVF

Using the load and renewable generation profiles in a typical day, the OVF is constructed by Algorithm 1. The parameter set is $K = \{(P_m^S, E_m^S)|0 \le P_m^S \le 10\text{MW}, 0 \le E_m^S \le 50\text{MWh}\}$, and the initial discretization has 11×11 uniformly distributed points in K. The obtained OVF is

$$v(\theta) = \begin{cases} 67091 - 15654.4P_m^S & [P_m^S, E_m^S] \in \text{CR}_1 \\ 66996 - 2789.17P_m^S & [P_m^S, E_m^S] \in \text{CR}_2 \\ 66681 - 2571.56P_m^S & [P_m^S, E_m^S] \in \text{CR}_3 \\ 66996 - 857.19P_m^S - 169.47E_m^S & [P_m^S, E_m^S] \in \text{CR}_4 \\ 66681 - 639.57P_m^S - 169.47E_m^S & [P_m^S, E_m^S] \in \text{CR}_5 \\ 66996 - 380.83P_m^S - 253.04E_m^S & [P_m^S, E_m^S] \in \text{CR}_6 \\ 66681 - 163.21P_m^S - 253.04E_m^S & [P_m^S, E_m^S] \in \text{CR}_7 \\ 67091 \qquad\qquad - 452.21E_m^S & [P_m^S, E_m^S] \in \text{CR}_8 \\ 67002 \qquad\qquad - 338.95E_m^S & [P_m^S, E_m^S] \in \text{CR}_9 \\ 65346 \qquad\qquad - 253.04E_m^S & [P_m^S, E_m^S] \in \text{CR}_{10} \end{cases}$$

The OVF and critical regions are displayed in Fig. 1; the gray scale is associated with the slope of the OVF. To validate this OVF, we discretize K into 101×101 uniformly distributed points, and solve problem (5) for each point; the corresponding optimum $v_p(\theta)$ is exact. Among the 101×101 samples, the maximum relative error $|v_p(\theta) - v(\theta)|/v_p(\theta) = 0.0011$, because of missing some vertex μ in (11). Nevertheless, the uniform discretization is hopeful to allocate most active vertices that corresponds to a slice in the PWL function $v(\theta)$, so the relative error is generally very small. A simple way to improve accuracy is to employ more samples in a finer initial discretization.

C. IMPLICATIONS ON MARGINAL COST REDUCTION

The OVF $v(\theta)$ contains abundant sensitivity information. Take CR_8 for example, in this critical region, $v(\theta)$ is independent of P_m^S, because $E_m^S \approx 0$ and there is nowhere to store energy. In addition, $\partial v(\theta)/\partial E_m^S$ achieves the maximum among all the 10 critical regions, implying that increasing E_m^S brings the largest marginal benefit, which is 452.21CNY/MWh. Similarly, if P_m^S is very small, such as in CR_1, $v(\theta)$ is independent of E_m, due to the lack of adequate charging and discharging ability. The marginal benefit of increasing P_m^S is 15654.4 CNY/MW, which is the largest among all the critical regions. By and large, when either P_m^S or E_m^S is very small, improving P_m^S brings larger benefit. If P_m^S and E_m^S research some certain values, such as in CR_5 and CR_7, both of their values affect $v(\theta)$; however, the marginal benefit decreases.

Sometimes, the energy-power ratio $\kappa = E_m^S/P_m^S$ is kept constant in engineering practice. From the cost reduction perspective, critical values of κ are represented by the slopes of the solid lines passing the origin, which are $\kappa \approx 11$, $\kappa \approx 5$, and $\kappa \approx 2$ in Fig. 1. When κ varies in either [2, 5] or [5, 11], the marginal benefit is constant.

D. IMPACT OF UNCERTAINTY AND DISCRETIZATION

We test the proposed method with fewer initial discretization points. Specifically, we use 9×9 and 6×9 lattices, and the results are shown in Fig. 3. Compared with those in Fig. 1, most critical regions remain unchanged, except for some small ones; the maximum relative errors in these two cases are 0.0011 and 0.0013, respectively, which is accurate enough for assessment or planning related applications. This observation demonstrates that the discretization needs not to be very dense.

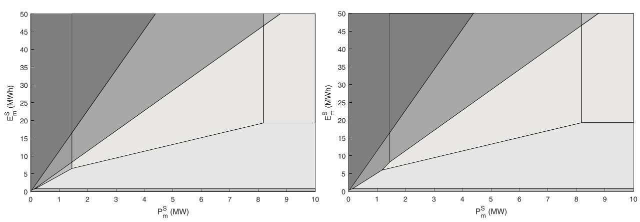


FIGURE 3. Critical regions of OVF with 9 x 9 (left) and 6 x 9 (right) grid.

To investigate the impact of renewable generation uncertainty, using wind and solar output data in spring, summer, autumn, and winter, we create 20 scenarios (5 for each season) with equal probability of 0.05. The OVF in each scenario is obtained, and then mp-LP (17) is solved by Algorithm 1 with 21×21 and 11×11 lattice sampling. The OVFs and critical regions are plotted in Fig. 4. The maximum relative errors are 6.6×10^{-4} in both discretization patterns. Since renewable generation and load in multiple seasons are considered, the critical regions show different shapes compared with those in Fig. 1. Nevertheless, the critical values of κ are still between 2 and 12.

E. IMPLICATION ON STORAGE SIZING

Taking into account the capital cost of ESU, an additional inequality appears in the budget-constrained parameter

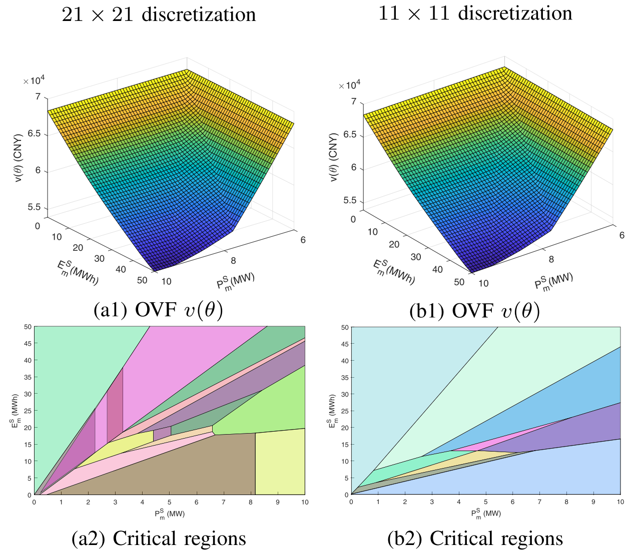


FIGURE 4. OVF and critical region with 20 scenarios and different discretization.

set $K(\Gamma)$, as in (19), where $\alpha_E = 10^6$CNY/MWh and $\alpha_p = 1.5 \times 10^6$CNY/MW. Wind and solar data in 100 days are used in this test. Considering distributional uncertainty, the expected daily operation cost in the worst distribution $\{\rho_s\}_{s=1}^{100}$ can be found by solving LP (26). Given the worst distribution, we solve LP (24) to obtain the optimal MW/MWh parameter. Such experiments are performed with different values of Γ. The case without ESU is the baseline, and the optimum is 81010 CNY. We have found two worst distributions, one for $\Gamma = 10, 20, 30$ million CNY and the other for $\Gamma = 40, 50$ million CNY. The minimum of OVF on the edge induced by the budget constraint interprets the optimal sizing strategy, which is shown through Fig. 5 (two OVFs correspond to two worst distributions). Increase the value of Γ, the average daily operation cost decreases correspondingly. Assume the lifespan of ESU is 12 years, the net profit, which is the total saved cost during the service time minus the budget Γ, is exhibited in Table 1. For any fixed Γ, we can seek the optimum of $v(\theta)$ with respect to P_m^S and E_m^S, denoted by $E_m^{S*}/P_m^{S*} = \kappa$ the optimal energy-power ratio. Results obtained from Fig. 5 are provided in Table 1, indicating that the optimal energy-power ratio is approximately 5.7. Larger ESU costs more budget, but the marginal utility decreases. On this account, the optimal investment on ESU is 40 million CNY. Excessive investment leads to a lower net profit, which can be observed in the last row of Table 1.

We examine the impact of the gap between peak and valley prices under the budget $\Gamma = 4 \times 10^7$CNY and the valley price of 400 CNY/MWh. Results are given in Table 2, showing that the peak price does not significantly influence the energy-power ratio κ, which varies between $5 \sim 6$ in all the instances. When the peak price is lower than 800CNY/MWh, investing on ESU does not bring economical benefits; when the peak price is higher than 800CNY/MWh, the sizing

TABLE 1. Storage sizing strategy under different budgets.

Γ (10^6CNY)	P_m^S (MW)	E_m^S (MWh)	κ (hour)	Average cost (10^4CNY/day)	Net Profit (10^6CNY)
0	0	0	–	8.101	0
10	1.44	7.85	5.45	7.834	1.677
20	2.78	15.83	5.69	7.578	2.916
30	4.17	23.75	5.69	7.329	3.818
40	5.56	31.67	5.70	7.087	4.431
50	6.94	39.58	5.70	6.870	3.935

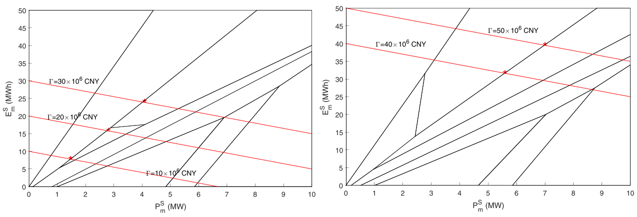


FIGURE 5. Optimal sizing strategy under different budgets.

TABLE 2. Storage sizing strategy under different peak prices.

Peak price (10^6CNY)	P_m^S (MW)	E_m^S (MWh)	κ (hour)	Average cost (10^4CNY/day)	Net Profit (10^6CNY)
600	6.01	30.97	5.15	6.116	-15.94
700	5.62	31.56	5.61	6.611	-6.28
800	5.55	31.66	5.70	7.087	4.43
900	5.55	31.66	5.70	7.564	15.17
1000	5.55	31.66	5.70	8.042	25.93

strategy is barely affected by the peak-valley price gap, because we assume electricity cannot be sold back to the transmission grid, prohibiting the additional income from arbitrage.

F. TWO STORAGE UNITS

In this case, two ESUs connect to the 33-bus system at bus 3 and bus 6, respectively, and the peak demand is 11.27MW. The energy-power ratio is kept at $\kappa = 6$, so the capacity cost of ESU is $\alpha_1 = \alpha_2 = 1.25 \times 10^6$ CNY/MWh, and the parameter $\theta = [E_m^1, E_m^2]$ has a dimension of 2. The parameter set becomes $K(\Gamma) = \{\theta \in K | \alpha_1 E_m^1 + \alpha_2 E_m^2 \leq \Gamma\}$. If the same 100 scenarios are used, the optimal choice offered by problem (26) is to invest only one ESU at bus 3, so we select the top 3 worst-case scenarios, each with a probability of 1/3. The OVF and critical regions are plotted in Fig. 6. It can be observed that the daily operation cost decreases with the increase of E_m^1 and E_m^2; when the budget is sufficiently large, investing on the ESU at bus 3 is more effective, because the marginal cost reduction brought by the ESU at bus 6 decreases quickly. The optimal sizing strategies corresponding to different budgets are obtained by solving model (26). Results are presented in Table 3. Without ESU, the optimal daily average operation cost is 82412 CNY.

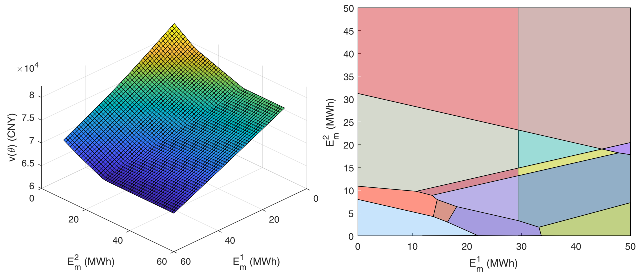


FIGURE 6. Optimal sizing strategy under different budgets.

TABLE 3. Storage sizing strategy under different peak prices.

Γ (10^6CNY)	E_m^1 (MWh)	E_m^2 (MWh)	Average cost (10^4CNY/day)	Net Profit (10^6CNY)
0	0	0	8.241	0
10	0.01	7.99	7.963	2.203
20	11.00	5.00	7.724	2.675
30	21.99	2.01	7.499	2.482
40	28.38	3.62	7.284	1.934

According to the last column of Table 3, the optimal investment decision is to build ESUs with capacities of 11MWh at bus 3 and 5MWh at bus 6, respectively, spending 20 million CNY. Across the ESU lifespan of 12 years, the net profit is 2.675 million CNY.

V. CONCLUSION

This paper proposes a multi-parametric linear programming model to quantify the economic impact of energy storage unit on distribution system operation. The duality-inspired method to solve the multi-parametric model has two appealing features. First, it does not rely on the analysis of optimal basis invariant sets, and thus is not influenced by model degeneracy, a well-known difficulty in multi-parametric linear programming. Second, its accuracy is adjustable; when a certain error tolerance is allowed, the computation entails just a moderate number of linear programs in the dual form. Computational experiments demonstrate that the proposed method can solve problems with sizes of practical interests. The obtained optimal value function carries abundant sensitivity information that provides references for energy storage sizing. By and large, when either charging capacity or energy capacity is very small, investment on charging capacity brings larger benefit; from the economical perspective, the optimal energy-power ratio of energy storage is between 5 and 6. Quantitative MW/MWh size can be obtained from the optimal value function.

REFERENCES

[1] T. Adefarati and R. C. Bansal, "Integration of renewable distributed generators into the distribution system: A review," *IET Renew. Power Gener.*, vol. 10, no. 7, pp. 873–884, Aug. 2016.

[2] S. R. Deeba, A. Thomas, T. K. Saha, D. Chakraborty, and R. Sharma, "Evaluation of technical and financial benefits of battery-based energy storage systems in distribution networks," *IET Renew. Power Gener.*, vol. 10, no. 8, pp. 1149–1160, Sep. 2016.

[3] H. Saboori, R. Hemmati, S. M. S. Ghiasi, and S. Dehghan, ''Energy storage planning in electric power distribution networks—A state-of-the-art review,'' *Renew. Sustain. Energy Rev.*, vol. 79, pp. 1108–1121, Nov. 2017.

[4] C. Opathella, A. Elkasrawy, A. A. Mohamed, and B. Venkatesh, ''Optimal scheduling of merchant-owned energy storage systems with multiple ancillary services,'' *IEEE Open Access J. Power Energy*, vol. 7, pp. 31–40, Nov. 2020.

[5] E. Hajipour, M. Bozorg, and M. Fotuhi-Firuzabad, ''Stochastic capacity expansion planning of remote microgrids with wind farms and energy storage,'' *IEEE Trans. Sustain. Energy*, vol. 6, no. 2, pp. 491–498, Apr. 2015.

[6] H. Alharbi and K. Bhattacharya, ''Stochastic optimal planning of battery energy storage systems for isolated microgrids,'' *IEEE Trans. Sustain. Energy*, vol. 9, no. 1, pp. 211–227, Jan. 2018.

[7] M. Asensio, P. M. de Quevedo, G. Munoz-Delgado, and J. Contreras, ''Joint distribution network and renewable energy expansion planning considering demand response and energy storage—Part I: Stochastic programming model,'' *IEEE Trans. Smart Grid*, vol. 9, no. 2, pp. 655–666, Mar. 2018.

[8] R. Fernandez-Blanco, Y. Dvorkin, B. Xu, Y. Wang, and D. S. Kirschen, ''Optimal energy storage siting and sizing: A WECC case study,'' *IEEE Trans. Sustain. Energy*, vol. 8, no. 2, pp. 733–743, Apr. 2017.

[9] K. Baker, G. Hug, and X. Li, ''Energy storage sizing taking into account forecast uncertainties and receding horizon operation,'' *IEEE Trans. Sustain. Energy*, vol. 8, no. 1, pp. 331–340, Jan. 2017.

[10] R. A. Jabr, I. Dzafic, and B. C. Pal, ''Robust optimization of storage investment on transmission networks,'' *IEEE Trans. Power Syst.*, vol. 30, no. 1, pp. 531–539, Jan. 2015.

[11] X. Zhang and A. J. Conejo, ''Coordinated investment in transmission and storage systems representing long- and short-term uncertainty,'' *IEEE Trans. Power Syst.*, vol. 33, no. 6, pp. 7143–7151, Nov. 2018.

[12] M. Sedghi, A. Ahmadian, and M. Aliakbar-Golkar, ''Optimal storage planning in active distribution network considering uncertainty of wind power distributed generation,'' *IEEE Trans. Power Syst.*, vol. 31, no. 1, pp. 304–316, Jan. 2016.

[13] W. Wei and J. Wang, *Modeling and Optimization of Interdependent Energy Infrastructures*. Cham, Switzerland: Springer, 2020.

[14] R. Xie, W. Wei, M. E. Khodayar, J. Wang, and S. Mei, ''Planning fully renewable powered charging stations on highways: A data-driven robust optimization approach,'' *IEEE Trans. Transport. Electrific.*, vol. 4, no. 3, pp. 817–830, Sep. 2018.

[15] Y. Cao, W. Wei, J. Wang, S. Mei, M. Shafie-Khah, and J. P. S. Catalao, ''Capacity planning of energy hub in multi-carrier energy networks: A data-driven robust stochastic programming approach,'' *IEEE Trans. Sustain. Energy*, vol. 11, no. 1, pp. 3–14, Jan. 2020.

[16] Y. Zheng, D. J. Hill, and Z. Y. Dong, ''Multi-agent optimal allocation of energy storage systems in distribution systems,'' *IEEE Trans. Sustain. Energy*, vol. 8, no. 4, pp. 1715–1725, Oct. 2017.

[17] D. Pozo, J. Contreras, and E. E. Sauma, ''Unit commitment with ideal and generic energy storage units,'' *IEEE Trans. Power Syst.*, vol. 29, no. 6, pp. 2974–2984, Nov. 2014.

[18] N. Li, C. Uckun, E. M. Constantinescu, J. R. Birge, K. W. Hedman, and A. Botterud, ''Flexible operation of batteries in power system scheduling with renewable energy,'' *IEEE Trans. Sustain. Energy*, vol. 7, no. 2, pp. 685–696, Apr. 2016.

[19] A. Lorca and X. A. Sun, ''Multistage robust unit commitment with dynamic uncertainty sets and energy storage,'' *IEEE Trans. Power Syst.*, vol. 32, no. 3, pp. 1678–1688, May 2017.

[20] N. G. Cobos, J. M. Arroyo, N. Alguacil, and J. Wang, ''Robust energy and reserve scheduling considering bulk energy storage units and wind uncertainty,'' *IEEE Trans. Power Syst.*, vol. 33, no. 5, pp. 5206–5216, Sep. 2018.

[21] C. Duan, L. Jiang, W. Fang, J. Liu, and S. Liu, ''Data-driven distributionally robust energy-reserve-storage dispatch,'' *IEEE Trans. Ind. Informat.*, vol. 14, no. 7, pp. 2826–2836, Jul. 2018.

[22] H. Farzin, M. Fotuhi-Firuzabad, and M. Moeini-Aghtaie, ''A stochastic multi-objective framework for optimal scheduling of energy storage systems in microgrids,'' *IEEE Trans. Smart Grid*, vol. 8, no. 1, pp. 117–127, Jan. 2017.

[23] A. J. Lamadrid *et al.*, ''Stochastically optimized, carbon-reducing dispatch of storage, generation, and loads,'' *IEEE Trans. Power Syst.*, vol. 30, no. 2, pp. 1064–1075, Mar. 2015.

[24] R. A. Jabr, S. Karaki, and J. A. Korbane, ''Robust multi-period OPF with storage and renewables,'' *IEEE Trans. Power Syst.*, vol. 30, no. 5, pp. 2790–2799, Sep. 2015.

[25] C. Zhang, Y. Xu, Z. Y. Dong, and J. Ma, ''Robust operation of microgrids via two-stage coordinated energy storage and direct load control,'' *IEEE Trans. Power Syst.*, vol. 32, no. 4, pp. 2858–2868, Jul. 2017.

[26] N. Good and P. Mancarella, ''Flexibility in multi-energy communities with electrical and thermal storage: A stochastic, robust approach for multi-service demand response,'' *IEEE Trans. Smart Grid*, vol. 10, no. 1, pp. 503–513, Jan. 2019.

[27] A. Jamali *et al.*, ''Self-scheduling approach to coordinating wind power producers with energy storage and demand response,'' *IEEE Trans. Sustain. Energy*, vol. 11, no. 3, pp. 1210–1219, Jul. 2020.

[28] D. Krishnamurthy, C. Uckun, Z. Zhou, P. R. Thimmapuram, and A. Botterud, ''Energy storage arbitrage under day-ahead and real-time price uncertainty,'' *IEEE Trans. Power Syst.*, vol. 33, no. 1, pp. 84–93, Jan. 2018.

[29] H. Cui, F. Li, X. Fang, H. Chen, and H. Wang, ''Bilevel arbitrage potential evaluation for grid-scale energy storage considering wind power and LMP smoothing effect,'' *IEEE Trans. Sustain. Energy*, vol. 9, no. 2, pp. 707–718, Apr. 2018.

[30] E. Nasrolahpour, J. Kazempour, H. Zareipour, and W. D. Rosehart, ''A bilevel model for participation of a storage system in energy and reserve markets,'' *IEEE Trans. Sustain. Energy*, vol. 9, no. 2, pp. 582–598, Apr. 2018.

[31] J. T. Alt, M. D. Anderson, and R. G. Jungst, ''Assessment of utility side cost savings from battery energy storage,'' *IEEE Trans. Power Syst.*, vol. 12, no. 3, pp. 1112–1120, Aug. 1997.

[32] M. Black and G. Strbac, ''Value of bulk energy storage for managing wind power fluctuations,'' *IEEE Trans. Energy Convers.*, vol. 22, no. 1, pp. 197–205, Mar. 2007.

[33] N. Li and K. W. Hedman, ''Economic assessment of energy storage in systems with high levels of renewable resources,'' *IEEE Trans. Sustain. Energy*, vol. 6, no. 3, pp. 1103–1111, Jul. 2015.

[34] Y. Xu and C. Singh, ''Adequacy and economy analysis of distribution systems integrated with electric energy storage and renewable energy resources,'' *IEEE Trans. Power Syst.*, vol. 27, no. 4, pp. 2332–2341, Nov. 2012.

[35] A. Nikoobakht, J. Aghaei, M. Shafie-Khah, and J. P. S. Catalao, ''Assessing increased flexibility of energy storage and demand response to accommodate a high penetration of renewable energy sources,'' *IEEE Trans. Sustain. Energy*, vol. 10, no. 2, pp. 659–669, Apr. 2019.

[36] B. Bocker, S. Kippelt, C. Weber, and C. Rehtanz, ''Storage valuation in congested grids,'' *IEEE Trans. Smart Grid*, vol. 9, no. 6, pp. 6742–6751, Nov. 2018.

[37] P. M. de Quevedo, J. Contreras, M. J. Rider, and J. Allahdadian, ''Contingency assessment and network reconfiguration in distribution grids including wind power and energy storage,'' *IEEE Trans. Sustain. Energy*, vol. 6, no. 4, pp. 1524–1533, Oct. 2015.

[38] M. E. Baran and F. F. Wu, ''Network reconfiguration in distribution systems for loss reduction and load balancing,'' *IEEE Trans. Power Del.*, vol. 4, no. 2, pp. 1401–1407, Apr. 1989.

[39] H.-G. Yeh, D. F. Gayme, and S. H. Low, ''Adaptive VAR control for distribution circuits with photovoltaic generators,'' *IEEE Trans. Power Syst.*, vol. 27, no. 3, pp. 1656–1663, Aug. 2012.

[40] Z. Li *et al.*, ''Extended sufficient conditions for exact relaxation of the complementarity constraints in storage-concerned economic dispatch,'' *CSEE J. Power Energy Syst.*, vol. 4, no. 4, pp. 504–512, Dec. 2018.

[41] L. Wu, ''A tighter piecewise linear approximation of quadratic cost curves for unit commitment problems,'' *IEEE Trans. Power Syst.*, vol. 26, no. 4, pp. 2581–2583, Nov. 2011.

[42] F. Borrelli, A. Bemporad, and M. Morari, ''Geometric algorithm for multiparametric linear programming,'' *J. Optim. Theory Appl.*, vol. 118, no. 3, pp. 515–540, Sep. 2003.

[43] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY, USA: Cambridge Univ. Press, 2004.

[44] C. Zhao and Y. Guan, ''Data-driven stochastic unit commitment for integrating wind generation,'' *IEEE Trans. Power Syst.*, vol. 31, no. 4, pp. 2587–2596, Jul. 2016.

[45] S. Taheri, V. Kekatos, and H. Veeramachaneni, ''Strategic investment in energy markets: A multiparametric programming approach,'' 2020, *arXiv:2004.06483*. [Online]. Available: http://arxiv.org/abs/2004.06483

[46] J. Löfberg, ''YALMIP: A toolbox for modeling and optimization in MATLAB,'' in *Proc. IEEE Int. Symp. Comput. Aided Control Syst. Design*, Taipei, Taiwan, Sep. 2004, pp. 284–289.

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