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Hybrid Optimization Algorithm to Solve the Nonconvex Multiarea Economic Dispatch Problem

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***Abstract*—In this paper, multiarea economic dispatch (MAED) problems are solved by a novel straightforward process. The solved MAED problems include transmission losses, tie-line constraints, multiple fuels, valve-point effects, and prohibited operating zones in which small, medium, and large scale test systems are involved. The methodology of tackling the problems consists in a new hybrid combination of JAYA and TLBO algorithms simultaneously to take the advantages of both to solve even nonsmooth and nonconvex MAED problems. In addition, a new and simple process is used to tackle with the interaction between areas. The objective is to economically supply demanded loads in all areas while satisfying all of the constraints. Indeed, by combining JAYA and TLBO algorithms, the convergence speed and the robustness have been improved. The computational results on small, medium, and large-scale test systems indicate the effectiveness of our proposed algorithm in terms of accuracy, robustness, and convergence speed. The obtained results of the proposed JAYA–TLBO algorithm are compared with those obtained from ten well-known algorithms. The results depict the capability of the proposed JAYA–TLBO based approach to provide a better solution.**

***Index Terms*—JAYA–TLBO algorithm, multiarea economic dispatch (MAED), optimization, tie-line constraints.**

NOMENCLATURE

Indices

i, j	Generating unit indices.
k	Iteration index.
l	Prohibited operating zone index.
s	Decision variables index.
t	Candidate solution index.
M	Number of areas.

Constants

a_i, b_i, c_i, e_i, f_i	Cost coefficients of the ith generator.
$a_{ij}, b_{ij}, c_{ij}, e_{ij}, f_{ij}$	Cost coefficients of the jth generator in the ith area.
B_{qj}^i	Loss coefficient associated with the production of the qth and the jth generators in the ith area.
B_{oj}^i	Loss coefficient associated with the production of the jth generator in the ith area.
B_{00}^i	Loss coefficient parameter (MW) in the ith area.
DM_i	Difference between the teacher and ith solution.
L_i	Number of POZs for the ith generator.
N_g	Number of generating units.
N_{gi}	Number of generating units in the ith area.
$P_{gi\min}$	Lowest output power of the ith generator (MW).
$P_{gi\max}$	Highest output power of the ith generator (MW).
$P_{gi,l}^{\mathrm{Low}}, P_{gi,l}^{Up}$	Minimum and maximum boundary of the lth POZ for the ith generator, respectively.
$\mathrm{rand}(1, n)$	$(1 \times n)$ Vector consists of random numbers in the [0, 1] range.
$r_{1,s,t}, r_{2,s,t}, r_i$	Random numbers.
$T_{ij\min}$	Minimum capacity of the tie-line between the ith and jth areas.
$T_{ij\max}$	Maximum capacity of the tie-line between the ith and jth areas.
T_F	Random discrete number.
$X_{s,t,k}^{\mathrm{best}}$	Best population achieved until kth iteration.
$X_{s,t,k}^{\mathrm{worst}}$	Worst population achieved until kth iteration.

Variables

$F(P_g)$	Generating unit cost function.
$H(X)$	Objective function.
$\vec{P}_{Di}$	Demanded power for the ith area.
P_{gi}	Power output of the ith generating unit (MW).
$\vec{P}_g$	Power generation matrix.
$\vec{P}_{gi}$	Power generation vector for the ith area.
$\vec{P}_{Li}$	Transmission network losses in the ith area.

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$T_{i,j}$	Transmission power between the ith and jth areas.
$\vec{T}$	Exchanged power matrix.
$\vec{T_i}$	Exchanged power vector assigned to the ith area.
$P_{ge,i}$	Output power of generating unit i in the eth area (MW).
X	Decision variables.
$X_{s,t,k}$	Value of the sth variable for the tth candidate during the kth iteration.

I. Introduction

ECONOMIC dispatch is a highly paramount concept in the optimization and power system fields. The objective of economic dispatch is to allocate the demanded power to the committed generators and to minimize the cost function while satisfying all of the physical and operational constraints [1].

Inherently, the original economic dispatch problem is a second-order polynomial problem; however, a sinusoidal term has to be added to model the valve point loading effect [2].

Regarding literatures, the economic dispatch problems are solved by different mathematical techniques such as the lambda iteration [3], gradient method [4], quadratic programing [5], and linear programming [6]. Due to the nonconvexity and nonlinearity that arise from the valve point effect, utilizing mathematical methods is not recommended [7].

Although, in some studies, dynamic programming was used to solve the economic dispatch [2], but this method is not suggested because of dimensional sophistication [2]. In addition to mathematical techniques, some meta-heuristics methods were employed like genetic algorithm [8], [9], particle swarm optimization (PSO) [10]–[12], tabu search [13], simulated annealing [14], quasi-oppositional group search optimization [15], chaotic global best artificial bee colony [16], firefly algorithm [17], continuous quick group search optimizer (CQGSO) [18], fuzzy adaptive chaotic ant swarm optimization [19], and the augmented Lagrange Hopfield network [20].

Technically speaking, the multiarea economic dispatch (MAED) is an extension of the economic dispatch problem [1]. The cost function in the MAED is minimized with regard to all constraints. In fact, in a recent decade power system, integrated management attempted to increase the reliability and decrease the operational cost simultaneously (MAED). Actually, the economic dispatch problem in each area has to be solved and the power exchange among areas as a significant distinctive constraint must be determined. In previous reports, mathematical methods such as linear programming [21], the Dantzig–Wolfe decomposition principle [22], and the decomposition approach using expert systems [23] have been utilized to solve the MAED problem. The advantages of gradient-based methods include achieving the best global solution and less iteration [21]–[23]; however, these methods suffer from undesirable factors such as nonlinearity and discontinuity arising from the valve point effect and prohibited operating zones, respectively. In addition, as the dimensions of the problem increase, the complexity of the mathematical method increases [15]. Therefore, in order to overcome these problems in the literature, some meta-heuristics methods were proposed such as the PSO with reserve-constrained multiarea environmental/economic dispatch [24], the new nonlinear optimization neural network approach [25], the artificial bee colony optimization [26], the teaching–learning-based optimization (TLBO) [1], and the chaotic global best artificial bee colony [16]. However, since meta-heuristic algorithms have random behaviors, they cannot guarantee achieving an optimal solution. Thus, applying a powerful method is highly recommended; hence, some ideas are represented in recent literatures. One of the most significant ideas is combining different algorithms to take their advantages, simultaneously.

The most important objective in the ED problems is to satisfy power balance between generated power and demanded load. For this purpose, the demanded load must be supplied, while other constraints are met and the cost function is minimized. Indeed, due to exchanged power through tie-lines, fulfilling this constraint in the MAED problems is more complicated than ED. In fact, the interchanged power as a virtual load/generation plays an important role, meaning that the generators in one area alone are not responsible for providing demanded power for the assigned area. Therefore, in this paper, a new and simple process is used to model the effect of tie-lines not only to satisfy the power balance constraint but also to preserve the independency of each area.

Furthermore, a new hybrid JAYA and TLBO algorithm (JAYA–TLBO) to solve the MAED problem is represented. JAYA and TLBO algorithms are proposed by "R. Venkata Rao" [27], [28]. JAYA is a Sanskrit word, which means victory. In JAYA algorithm, individuals try to move toward the best solution and keep far from the worst one to achieve a better solution [26]. Besides, in TLBO algorithm, individuals are improved in teaching and learning procedures [28]. Although these algorithms are simple, their performance is not acceptable and their ability to find optimal solution is not guaranteed if the problem dimensions are increased. Noticeably, it is possible to combine JAYA with TLBO algorithm, which improves the convergence speed and robustness. For this purpose, in each iteration, the population has been modified with JAYA and TLBO algorithms, simultaneously, thereafter; the population with a better cost function is replaced with the old one. Finally, to evaluate the performance, the proposed method is applied to both original and MAED. The effectiveness and robustness of the JAYA–TLBO algorithm have been evaluated by the results.

The main objective of this paper is to present a robust, effective, fast, and efficient method to solve the nonlinear, nonconvex, and nonsmooth MAED problems. In order to reach this goal, the presented method has to find the proper output of each generator and, at the same time, proper exchanged power between areas in the small-, medium-, and large-scale systems. In addition, a new and simple process is utilized to supply the demanded load in each area such that the independency of each area is preserved. Finally, four different cases are studied and the results of the proposed method are compared with ten well-known algorithms. Furthermore, the improvement of the JAYA–TLBO algorithm in comparison with JAYA and TLBO individually is evaluated through a statistical method.

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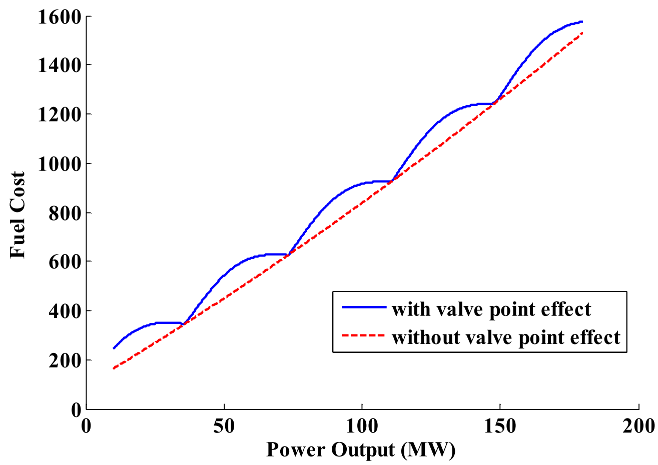

Fig. 1. Cost function with/without the valve point effect.

II. Multiarea Economic Dispatch

The original economic dispatch is one of the most important optimization problems in the power system domain. The aim of this problem is to determine a generation level that minimizes the fuel expenses while satisfying all constraints. A quadratic function must be used to model the economic dispatch. However, in huge generators, the valve point effect causes nonlinearity and nonconvexity of the cost function. Therefore, to model huge generators, a sinusoidal term has to be added to the cost function as the valve point effect

$$\begin{aligned} \min\ H(X) &= \sum_{i=1}^{N_g} F_i(P_{gi}) \\ F_i(P_{gi}) &= a_i \times P_{gi}^2 + b_i \times P_{gi} + c_i \\ &\quad + |e_i \times \sin(f_i \times (P_{gi\min} - P_{gi}))| \\ X &= [P_{g1}, P_{g2}, P_{g3}, \ldots, P_{gNg}]. \end{aligned} \tag{1}$$

Fig. 1 represents the power generator fuel curve with the four valve point effect.

MAED is a more generalized and sophisticated economic dispatch problem that contains some areas, each including its loads and generations. Not only the cost function is decreased by transmitting power from a lower cost area to a higher one but also the reliability and security of system are enhanced. Moreover, Fig. 2 shows the scheme of MAED with four areas, which are connected to each other.

In order to minimize the cost function while satisfying the load demand and constraints, power generation and power transmission between all areas are determined in the MAED [1]. The mathematical model of the MAED that contains decision variables, cost function, and constraints is detailed as follows:

$$\begin{aligned} \min\ H(X) &= \sum_{i=1}^{M} \sum_{j=1}^{N_{gi}} F_{ij}(P_{gij}) \\ F_{ij}(P_{gij}) &= a_{ij} \times P_{gij}^2 + b_{ij} \times P_{gij} + c_{ij} \\ &\quad + |e_{ij} \times \sin(f_{ij} \times (P_{gij\min} - P_{gij}))| \end{aligned}$$

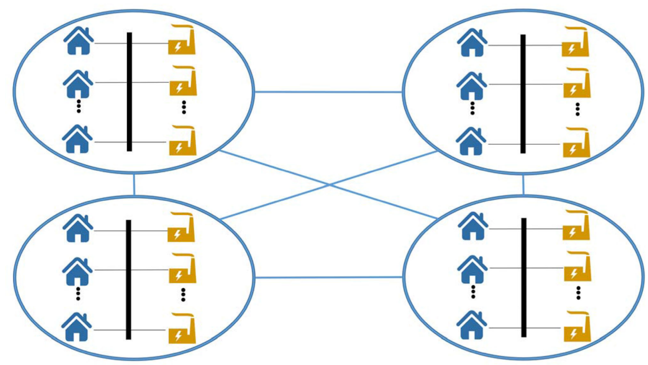

Fig. 2. Scheme of MAED in four areas.

$$\begin{aligned} X &= [\vec{P}_g, \vec{T}] \\ \vec{P}_g &= [\vec{P}_{g1}, \vec{P}_{g2}, \vec{P}_{g3}, \ldots, \vec{P}_{gM}] \\ \vec{P}_{gi} &= [P_{gi1}, P_{gi2}, P_{gi3}, \ldots, P_{giNg}] \\ &\qquad i = 1, 2, \ldots, M \\ \vec{T} &= [\vec{T}_1, \vec{T}_2, \ldots, \vec{T}_M] \\ [\vec{T}_1, \vec{T}_2, \ldots, \vec{T}_M] &= [[T_{1,1}, T_{1,2}, \ldots, T_{1,M}]\ , \\ &\qquad [T_{2,3}, T_{2,4}, \ldots, T_{2,M}], \\ &\qquad \ldots, [T_{M-1,M}]]\,. \end{aligned} \tag{2}$$

In order to generate power, in some generators, many types of fuels are used as sources (multifuel generators); therefore, the coefficients of the cost function are different [16]. Consequently, if the valve point effect, prohibited operating zones, and tie-line capacity constraints are applied to the model, the MAED problem becomes a complicated, nonlinear, and nonconvex problem. Therefore, a robust and effective optimization method is necessary to handle this problem [1].

A. Constraints

1) *Power generation constraint.* The power generation of each generator has a limitation, given as follows:

$$P_{gij\min} \le P_{gij} \le P_{gij\max}. \tag{3}$$

2) *Power balancing constraint.* Power generators have to provide the total load demand and the transmission network losses. Therefore, in a multiarea, the generated power in each area has the following characteristics:

$$\vec{P}_{gi} = \vec{P}_{Di} + \vec{P}_{Li} + \sum_{j=1, j\neq i}^{N} T_{ij} \quad i = 1, 2, \ldots, M. \tag{4}$$

The transmission network losses in the *i*th area are calculated as follows [16]:

$$\vec{P}_{Li} = \sum_{q=1}^{N_{gi}} \sum_{j=1}^{N_{gi}} P_{gij} B_{qj}^i P_{giq} + \sum_{j=1}^{N_{gi}} B_{0j}^i P_{gij} + B_{00}^i. \tag{5}$$

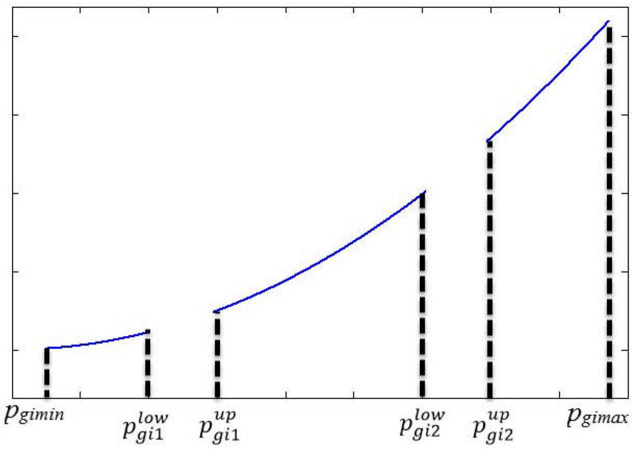

Fig. 3. Fuel curve of a power plant with two prohibited operating zones.

3) *Prohibited operating zone constraint.* Due to some practical restrictions, power generation in some intervals that may damage the generator is prohibited. Fig. 3 shows the fuel curve of a power generator with two prohibited operating zones. Furthermore, the following equation represents the prohibited operating zone constraint:

$$P_{gij} = \left\{\begin{array}{l} P_{gij\,\min} \leq P_{gij} \leq P^{\mathrm{Low}}_{gij,l-1} \\ P^{\mathrm{Up}}_{gij,l-1} \leq P_{gij} \leq P^{\mathrm{Low}}_{gij,l} \\ . \\ . \\ . \\ P^{\mathrm{Up}}_{gij,L_i} \leq P_{gij} \leq P_{gij\,\max} \end{array}\right\} \quad l = 2, 3, L_i.$$
$$i = 1, 2, \ldots, N_g \tag{6}$$

4) *Tie-line capacity constraint.* One of the most important distinctive constraints in MAED is the transmission tie-line capacity. Indeed, power exchange among areas must be set between the minimum and the maximum capacity of the transmission line, which is considered as follows:

$$-T_{ij\,\min} \leq T_{ij} \leq T_{ij\,\max}. \tag{7}$$

III. JAYA–TLBO Algorithm

A. JAYA Algorithm

JAYA algorithm is a new algorithm to find a better solution [27]. In order to find the better solution with JAYA algorithm, the individuals are moved toward the best solution and keep the worst one out simultaneously in each iteration.

This algorithm has some significant advantages such as high speed, low computational efforts, and high performance [27]. Moreover, there are no controlling parameters in this algorithm and the mentioned feature causes easy tuning implementation [27]. In this paper, to modify this algorithm, all moving possible states are considered to escape from local minimums and find a better solution rapidly. The following equations represent the modified algorithm:

$$\begin{aligned} X^{\mathrm{new}}_{1,s,t,k} &= X_{s,t,k} + r_{1,s,t}(X_{s,t,\mathrm{best}} - |X_{s,t,k}|) \\ &\quad - r_{2,s,t}(X_{s,t,\mathrm{worst}} - |X_{s,t,k}|) \\ X^{\mathrm{new}}_{2,s,t,k} &= X_{s,t,k} + r_{1,s,t}(X_{s,t,\mathrm{best}} - |X_{s,t,k}|) \\ &\quad + r_{2,s,t}(X_{s,t,\mathrm{worst}} - |X_{s,t,k}|) \\ X^{\mathrm{new}}_{3,s,t,k} &= X_{s,t,k} - r_{1,s,t}(X_{s,t,\mathrm{best}} - |X_{s,t,k}|) \\ &\quad - r_{2,s,t}(X_{s,t,\mathrm{worst}} - |X_{s,t,k}|) \\ X^{\mathrm{new}}_{4,s,t,k} &= X_{s,t,k} - r_{1,s,t}(X_{s,t,\mathrm{best}} - |X_{s,t,k}|) \\ &\quad + r_{2,s,t}(X_{s,t,\mathrm{worst}} - |X_{s,t,k}|). \end{aligned} \tag{8}$$

B. TLBO Algorithm

TLBO algorithm is as simple as JAYA algorithm and has the same high convergence speed and effective performance. TLBO algorithm procedures comprise teaching and learning phases; in the teaching phase, the best obtained solution is considered as a teacher and other solutions move toward it. However, in the learning phase, individuals will be modified by interacting between themselves. The TLBO algorithm is formulated as follows.

Teaching phase:

$$X^{\mathrm{new}}_{s,t,k} = X_{s,t,k} + \mathrm{DM}_i. \tag{9}$$

The DM_i can be calculated as follows:

$$\mathrm{DM}_i = r_i \times (X^{\mathrm{best}}_{s,t,k} - T_F). \tag{10}$$

T_F is only 1 or 2 $\{1, 2\}$, and r_i is between [0, 1].

Learning phase: In this step, two solutions (X_i, X_j), where $i \neq j$, are selected as learners. X^{new}_i is calculated as follows:

$$X^{\mathrm{new}}_i = X_i + \mathrm{rand} \times (\mathrm{absolute}(X_i - X_j)). \tag{11}$$

If the cost function of X^{new}_i provides a fewer value, X^{new}_i is replaced by X_i.

Although each of JAYA and TLBO algorithms is simple, they may converge to the local minimum by increasing dimensions. To overcome this drawback, combining JAYA and TLBO algorithms is suggested. In this method, the population is modified by JAYA and TLBO algorithms, simultaneously. If the new achieved solution of JAYA or TLBO is better than that of the previous one, it takes the place of the old solution. Otherwise, the existing solution is memorized. To implement the JAYA–TLBO algorithm in the MAED problem, the steps followed in Fig. 4 should be taken.

Besides the hybrid JAYA–TLBO algorithm, we propose a new and simple process to tackle the power balance constraint. In fact, fulfilling the equality constraint, here, the power balance constraint, as an importa.

Step 1: Select an area randomly.

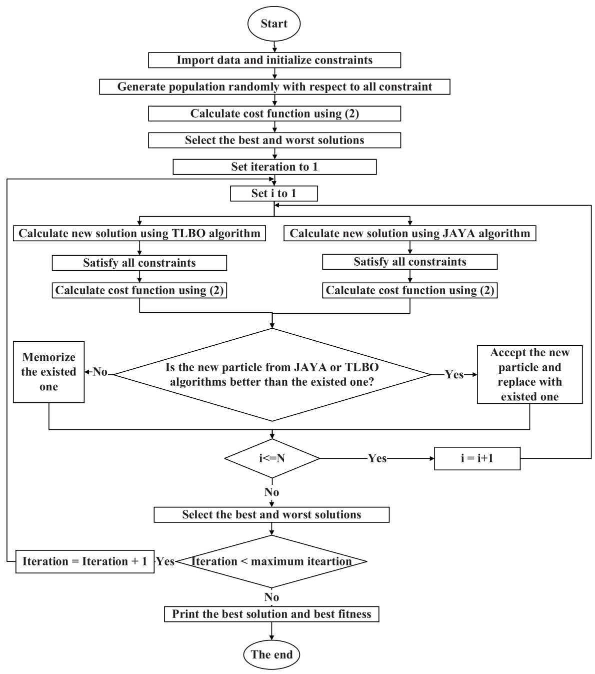

Fig. 4. Flowchart of the JAYA–TLBO algorithm.

Step 2: Specify the new demanded load based on the input and output powers to the selected area

$$P_{Li}^{\text{new}} = P_{Li}^{\text{old}} + \sum_{j=1}^{M} T_{ij}. \quad (12)$$

Step 3: Identify s_i

$$s_i = \sum_{j=1}^{N_{gi}} P_{gij}^{\max} - P_{Li}^{\text{new}}. \quad (13)$$

Step 4: Determine the area capability constraint (ACC).

In this step, the power of connected tie-lines is changed in such a way that the summation of them is less than s_i.

In addition to the tie-line capacity and ACC, another constraint is needed to apply the power adjustability to the system. This constraint implies that an area is not able to transmit power more than its net power, which is the difference between the generated and demanded power. Therefore, according to the conditions of an area, there are two modes for power transmission, given as follows.

1) If the generated power in one area is less than the demanded power $((\sum_{j=1}^{N_{gi}} P_{gi}^{\max} - P_{Li}) < 0)$.

For this case, the net transmitted power by connected tie-lines must be between $\sum_{j=1}^{N_{gi}} T_{ij\min}$ and $(\sum_{j=1}^{N_{gi}} P_{gi}^{\max} - P_{Li})$ when the tie-line capacity constraint for each tie-line has to be satisfied.

2) If the generated power in one area is more than the demanded power $((\sum_{j=1}^{N_{gi}} P_{gi}^{\max} - P_{Li}) > 0)$.

In this case, the net transmitted power by the connected tie-lines must be between $(\min\{(\sum_{j=1}^{N_{gi}} P_{gi\max} - P_{Li}), \sum_{j=1}^{M} T_{ij\max}\}$ and $\sum_{j=1}^{M} T_{ij\min}$ when the tie-line capacity constraint for each tie-line has to be satisfied.

Step 5: Equality constraint of load and power generation must be satisfied.

Step 6: Calculate the cost function in the flowchart (see Fig. 4).

If all areas are selected the process is completed. Otherwise select another area randomly and go to step 2.

It should be noted that the output power of generators is the control variable. Hence, it is possible to fix them to the boundary limits.

TABLE I
COMPARISON OF METHODS FOR SYSTEM A

Method	Best ($/h)	Mean ($/h)	Worst ($/h)
JAYA-TLBO	**121409.11**	**121411.66**	**121416.19**
JAYA	121806.95	121949.27	122170.61
TLBO	122241.02	123227.04	124154.75
PSO [30,31]	123930.45	124155.00	124312.63
MPSO [30,31]	122252.27	-	-
HGA [30,31]	121418.27	121784.04	-
DE [30,31]	121416.29	121422.72	121431.47
HDE [30,31]	121698.51	122304.30	-
CQGSO [18]	121412.55	121423.52	121438.69

IV. Case Study and Results

The new proposed algorithm is applied to both original and MAED problems to evaluate the performance of this algorithm. In this paper, the study cases are four systems. The first case (case A) contains forty generators in one area, while the second case (case B) contains six generators in two areas. The third case (case C) contains ten generators in three areas, and the last case (case D) contains forty generators in four areas, which is known as a highly complicated system. All simulations are run by using MATLAB 8.3 on a laptop (2.6 GHz, 8 GB RAM).

A. Forty Generators in One Area With 10 500 MW Load Demand

This complicated system contains 40 generators and a lot of local minimums. Therefore, many methods are impractical and unable to find the best solution; the system details are comprehensively represented in [29].

Table I compares the simulation results with the presented results in the literature.

Achieving optimal solution is not guaranteed in this case, due to the large number of decision variables.

According to Table I, the best, mean, and worst obtained solutions by the JAYA–TLBO algorithm are close to each other, which guarantees the robustness of the proposed method. Also, the worst solution of the JAYA–TLBO algorithm is less than the best solution of most of the previous algorithms. Hence, the method is robust enough to handle a complex problem. It is notable that this system includes nonconvex parameters, which make it difficult to find the optimal solution.

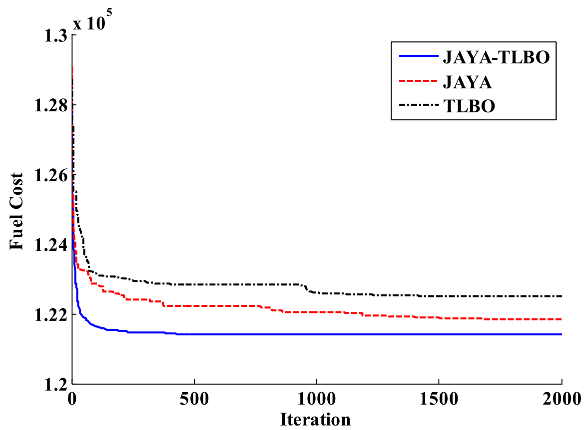

Fig. 5. Convergence curve from both JAYA and JAYA–TLBO algorithms for system A.

TABLE II
RESULTS OF SYSTEM B

Power Generation (MW)	JAYA-TLBO	JAYA	TLBO [1]	DE [1]	GBABC [16]
P11	498	498	500	500	499.99
P12	200	200	200	200	200
P13	150	150	150	150	149.99
P21	180	170	204.327	204.334	204.34
P22	194.3867	183.56	154.709	154.704	154.69
P23	54.17328	75	67.5795	67.5770	67.57
T12	80.77	80.77	--	--	82.77
PL	13.55	13.56	13.61	13.59	13.6157
Cost ($/h)	**12252.71**	**12253.71**	**12255.39**	**12255.42**	**12255.38**
Time(sec)	**1.2259**	**1.72**	**5.0734**	**5.9219**	**0.8800**

Fig. 5 shows the convergence curve of the JAYA, TLBO, and JAYA–TLBO algorithms for case A.

As shown in Fig. 5, the JAYA–TLBO algorithm is not only faster but can also find the more economical solution than that obtained by JAYA or TLBO algorithm. The two significant issues of the convergence curve to examine the effectiveness of an algorithm are speed and accuracy. In fact, the performance of an algorithm is perfect if the convergence slope reduces rapidly and monotonically.

According to Fig. 5, the JAYA and TLBO algorithms independently converge to the local minimums, but the JAYA–TLBO algorithm can easily pass through the local minimums rapidly and monotonically.

B. Six Generators in Two Areas

This system includes six generators in two different areas with 1263 MW as the total load demand. Each area has three generators, where the load demand is distributed as 758.7 (60%) and 505.2 MW (40%) for the first and second areas, respectively.

In this case, the prohibited operating zone and losses are also considered. Other details and information are explained in [1].

The power transmission capacity from the first area to the second area and vice versa is 100 MW. The JAYA–TLBO algorithm is applied to this system, and the results demonstrate the robustness and effectiveness of the proposed method. Table II shows the results for system B, which represents a lower cost in comparison to that of other algorithms.

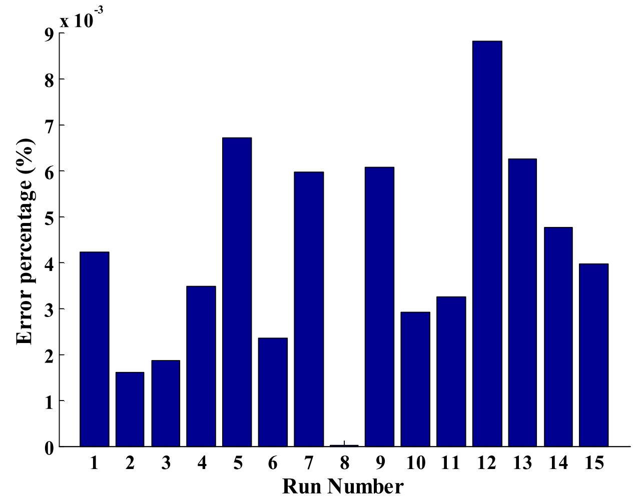

Fig. 6. Error percentage for system B.

It should be noted that the simulation time is normalized by the following equation:

$$\text{Time (s)} = \frac{\text{CPU speed (GHz)}}{3\ \text{(GHz)}} \times \text{Simulation time (s)}. \tag{14}$$

The results for system B make it the best solution compared to other algorithms. As shown in Table II, the JAYA–TLBO algorithm in comparison with other algorithms not only supplies load demand with a minimum cost but also has acceptable simulation (execution) time.

One of the most important issues in an optimization method is robustness. In fact, an algorithm is robust if the error percentage in each run is low and acceptable. Error percentage can be calculated as follows:

$$\text{Error percentage} = \frac{|\text{final value} - \text{optimal value}|}{\text{optimal value}} \times 100\%. \tag{15}$$

Fig. 6 shows the error percentage for system B by 15 independent runs and also illustrates the final error value for each trial run. As shown in the figure, the error percentage of the last iteration of 15 independent runs is less than 0.009%, which is very small and shows the robustness and effectiveness of the proposed algorithm. Another important feature to examine an algorithm is the error reduction speed curve for each run.

Fig. 7 shows the error reduction speed curve for each independent run. It should be noted that due to low dimension in this case, error percentage of first iteration is small.

As shown in Figs. 6 and 7, the lower cost is found robustly by the JAYA–TLBO algorithm, which demonstrates the high quality and speed of the proposed algorithm.

C. Ten Power Generators in Three Areas

In this part, the JAYA–TLBO algorithm is applied to the MAED problem with ten generators in three areas. The total load demand is 2700 MW, and the power transmission losses

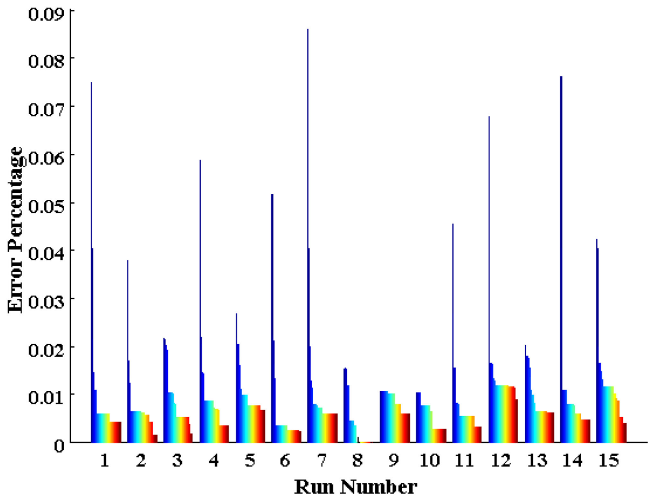

Fig. 7. Error reduction speed curve for each independent run in system B.

TABLE III
COMPARISON OF THE JAYA–TLBO ALGORITHM WITH OTHER METHODS FOR SYSTEM C

Power Generation (MW)	JAYA-TLBO	JAYA	TLBO [1]	DE [1]	Fuel Type
P11	217.07	221.21	224.30	225.44	2
P12	212.65	211.29	210.66	210.16	1
P13	499.96	500.00	491.60	491.28	2
P14	238.17	235.33	240.62	240.89	3
P21	251.00	257.82	249.56	251.00	1
P22	233.18	241.00	235.89	238.86	3
P23	263.14	259.65	263.74	264.09	1
P31	233.45	236.76	237.13	236.99	3
P32	334.82	317.85	332.59	326.53	1
P33	252.25	254.73	249.46	250.33	1
PL	35.69	35.60	35.69	35.62	
Cost ($/h)	**654.83**	**655.70**	**655.38**	**654.02**	
Time(sec)	**3.37**	**4.36**	**61.67**	**65.03**	

are also considered for this system. In this system, three types of fuel are considered.

The first area includes four generators with 1350 MW load demand (50% of the total load). The second and third areas contain three generators, and their load demand is 675 MW for each area (25% of the total load demand). The tie-line capacity from each area to another is 100 MW. In this case, the valve point effect is taken into account, which causes complexity, nonlinearity, and nonconvexity. The rest of the information is represented in [32]. Table III shows the compression among JAYA, TLBO, DE, and hybrid JAYA-TLBO algorithms. The obtained results illustrate the high performance of the algorithm while satisfying all constraints.

According to Table III, the proposed method finds the best solution with an acceptable accuracy. So, the hybrid JAYA–TLBO algorithm can be used for complex problems, in which achieving the best global cost is important.

The execution time in meta-heuristic methods is the main drawback, so these methods in real-time applications are not suitable. As shown in Table III, the JAYA–TLBO algorithm

TABLE IV
DISTRIBUTION OF 40 POWER THERMAL GENERATING UNITS FOR SYSTEM D

Units	**Generated Power (MW)**						
U1~7	110.82	111.02	97.401	179.73	87.906	139.99	259.59
U8~14	284.60	284.59	130	94.007	94.016	304.52	394.27
U15~21	394.28	394.27	489.29	489.28	534.73	511.30	523.27
U22~28	523.28	523.29	523.28	523.28	523.29	10.000	10.003
U29~35	10	88.105	190	189.99	190	164.79	164.80
U36~40	164.80	89.14	89.12	102.51	511.28		
TL1~6	199.98	-7.82	-81.47	-199.99	-99.99	-100	

TABLE V
COMPARISON OF THE JAYA–TLBO ALGORITHM WITH OTHER METHODS FOR SYSTEM D

Method	JAYA-TLBO	TLBO	DE [1]	EP [1]	RCGA [1]
Best Cost ($/h)	**121694.4**	**122200.3**	**121794.8**	**123591.9**	**128046.5**

yields a better solution in minimum time. Furthermore, the JAYA–TLBO algorithm succeeds escaping from local minimums to achieve the best solution. It should be noted that JAYA, TLBO, and also JAYA–TLBO algorithms perform by the vector-evaluated behavior, which causes high speed and is clear in the results.

D. Forty Generators in Four Areas

As mentioned previously, two different states are considered for this system: forty generators in one area (original economic dispatch) and forty generators in four areas (MAED).

Forty generators in one area is a complicated problem and many methods were unable to find the best solution. Besides, the complexity of the system is increased when different areas are added. Therefore, solving such a problem requires a powerful and robust optimization method to overcome the mentioned drawbacks and satisfy constraints.

In this section, the JAYA–TLBO algorithm is applied to 40 generators in the four areas case, which is known as a complicated problem with many nonlinear parameters. It means that the proposed algorithm should be modified to distribute the population in all searching spaces while individuals are able to leave the local minimums and the best solution is found. All required information and data for this system are represented in [29].

Table IV provides the distribution of the generated power obtained by JAYA–TLBO, and Table V provides a comparison of the JAYA–TLBO algorithm with other methods.

Actually, in order to obtain a better solution for the MAED problem, not only cheap power plants should generate power but also accurate power exchange between areas is necessary.

The aforementioned methods cannot yield the optimal solution because of high dimensions and multiarea of this case, but the result of the JAYA–TLBO algorithm is better. Fig. 8 shows the convergence curve of JAYA, TLBO, and JAYA–TLBO algorithms. Furthermore, Fig. 9 illustrates the final error value for each trial run.

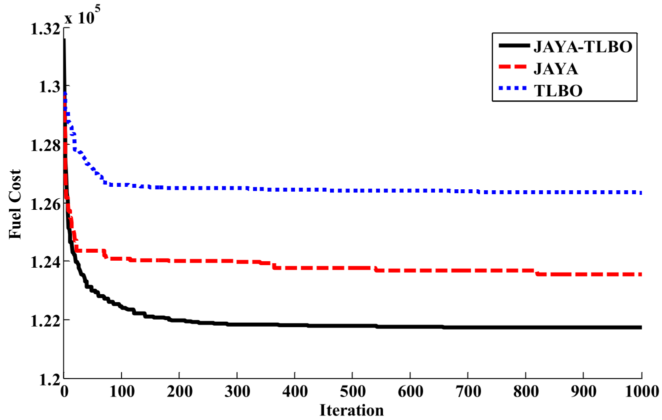

Fig. 8. JAYA and the JAYA–TLBO convergence curves of system D.

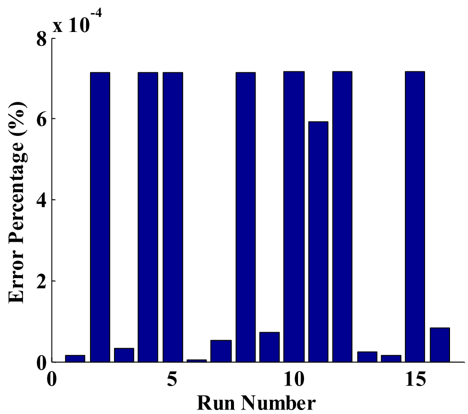

Fig. 9. Error percentage for system D.

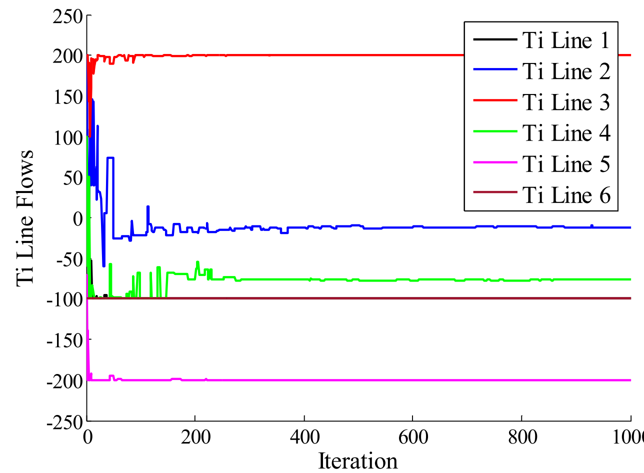

Fig. 10. Iterative evolution of the tie-line flow convergence curves for system D.

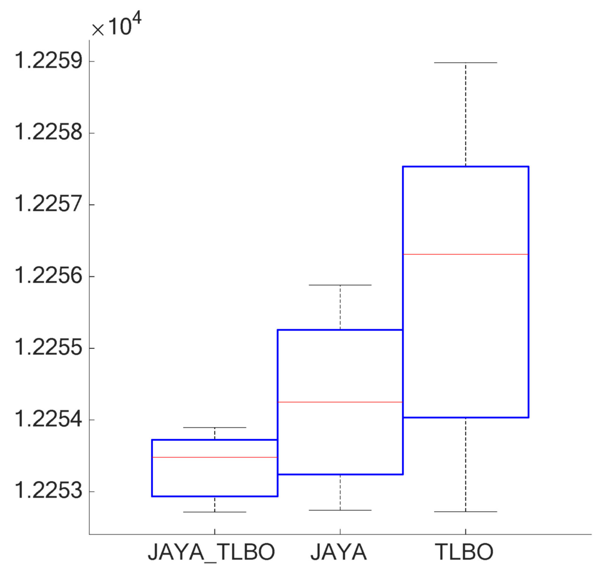

Fig. 11. Box plot of JAYA, TLBO, and JAYA–TLBO for case B.

According to Fig. 8, the JAYA and TLBO algorithms stick to the local minimums, but the JAYA–TLBO algorithm passes the local minimums easily and the best solution is found rapidly.

The better solution is found in less iteration by the JAYA–TLBO algorithm compared to that by JAYA or TLBO algorithm independently. In fact, JAYA and TLBO algorithms are not capable enough to achieve an optimal solution in a complex system, because these methods stick to local minimums. Nevertheless, a strategy combining these algorithms improves the speed and effectiveness of results.

As mentioned previously, the operation cost decreases when power exchange is determined exactly, which is important in the MAED problem. Also, the convergence speed of the exact transferred power is important. Fig. 10 shows the iterative tie-line power transmission convergence curve in system D. According to Fig. 10, the power transmission between areas converges rapidly to the final value.

V. Sensitivity Analysis

In order to analyze the sensitivity of the proposed method, it is applied to the mentioned cases for 50 independent runs and compared to those obtained from JAYA and TLBO individually.

The "box plot" as a well-known statistical technique is used to visually summarize and compare the obtained results. The "box plot" helps to identify the hidden patterns in a group of numbers [34]. Figs. 11–13 depict the box plot for the JAYA–TLBO, JAYA, and TLBO algorithms for cases B, C, and D, respectively.

As it is clear, the mean of the obtained results from JAYA–TLBO (see red lines in Figs. 11–13) is less than those obtained from JAYA and TLBO individually. In addition, the obtained results from 50 independent runs of the proposed method are more compressed than those from JAYA and TLBO (see blue boxes in Figs. 11–13).

Another interesting question is "What is the significance of difference between the results that obtained from the proposed method and those obtained from JAYA and TLBO?". In order to verify this, we use the paired *t*-test for comparison between the results of JAYA–TLBO and other methods (JAYA and TLBO). In addition, the *p*-values are given in Tables VI–VIII. As it is

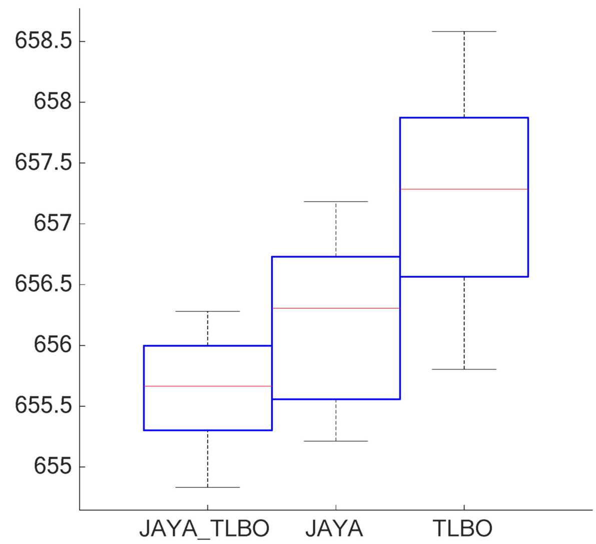

Fig. 12. Box plot of JAYA, TLBO, and JAYA–TLBO for case C.

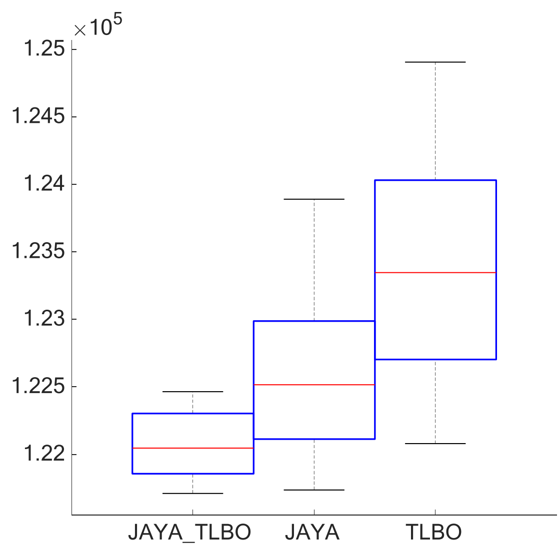

Fig. 13. Box plot of JAYA, TLBO, and JAYA–TLBO for case D.

TABLE VI
PAIRED *t*-TEST FOR CASE B

	JAYA	TLBO
JAYA_TLBO	0.043	0.025

TABLE VII
PAIRED *t*-TEST FOR CASE C

	JAYA	TLBO
JAYA_TLBO	0.029	0.012

TABLE VIII
PAIRED *t*-TEST FOR CASE D

	JAYA	TLBO
JAYA_TLBO	0.01	0.0078

clear, the obtained *p*-values are less than 0.043, which suggests that the performance improvement of the JAYA–TLBO method over the JAYA and TBLO methods independently is statistically significant.

VI. CONCLUSION

The MAED problem as an important issue in the modern power networks was studied in this paper. In the MAED problems, identifying the proper generated power of each unit and the exchanged power through tie-lines that connect the areas is of great essence. The JAYA–TLBO algorithm was used to handle such a complex problem, while a new and simple process was used to satisfy the power balance constraint. The proposed method guarantees the independency of all areas, and, at the same time, the demanded load will be supplied economically. Although JAYA and TLBO are not capable enough to obtain an optimal solution of a complex system, the combination of them leads to a robust method known as the JAYA–TLBO method to solve complicated problems. To evaluate the capability and effectiveness of this method in terms of accuracy, robustness, and convergence speed, it was applied to the MAED problems with different complexities. The proposed method does not require any controlling parameter. Even in large-scale systems, this algorithm is easily implemented for both constrained and unconstrained optimization problems. The results depict that the proposed JAYA–TLBO algorithm can obtain a better solution robustly for highly complicated problems. As the penetration of renewal generation is expected to increase, so the future research will focus on uncertainty and its effect on the MAED operation and planning.

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