A Fast Method for the Unit Scheduling Problem with Significant Renewable Power Generation

G.J. Osório[a], J.M. Lujano-Rojas[a], J.C.O. Matias[a], J.P.S. Catalão[a,b,c*]

[a] *University of Beira Interior, R. Fonte do Lameiro, Covilha, Portugal*
[b] *INESC-ID, R. Alves Redol, Lisbon, Portugal*
[c] *IST, University of Lisbon, Av. Rovisco Pais, Lisbon, Portugal*

Abstract

Optimal operation of power systems with high integration of renewable power sources has become difficult as a consequence of the random nature of some sources like wind energy and photovoltaic energy. Nowadays, this problem is solved using Monte Carlo Simulation (MCS) approach, which allows considering important statistical characteristics of wind and solar power production such as the correlation between consecutive observations, the diurnal profile of the forecasted power production, and the forecasting error. However, MCS method requires the analysis of a representative amount of trials, which is an intensive calculation task that increases considerably with the number of scenarios considered. In this paper, a model to the scheduling of power systems with significant renewable power generation based on scenario generation/reduction method, which establishes a proportional relationship between the number of scenarios and the computational time required to analyse them, is proposed. The methodology takes information from the analysis of each scenario separately to determine the probabilistic behaviour of each generator at each hour in the scheduling problem. Then, considering a determined significance level, the units to be committed are selected and the load dispatch is determined. The proposed technique was illustrated through a case study and the comparison with stochastic programming approach was carried out, concluding that the proposed methodology can provide an acceptable solution in a reduced computational time.

Keywords: Forecasting error, mixed-integer linear programming, stochastic unit commitment, renewable generation.

Nomenclature

m	Index for scenarios (m=1, 2,…, M).
n	Index for generators (n=1, 2,…, N).
d	Index for the interval in the discretization of PDF of load forecasting (d=1, 2,…, D).
j	Index for the interval in the discretization of PDF of wind forecasting (j=1, 2,…, J).
t	Index for time instant (t=1, 2,…, H).
z	Index for the interval in the discretization of start-up cost (z=1, 2,…, Z).
α	Significance level used to determine the confidence interval.
γ	Significance level used to determine the definitive unit scheduling (U_n^t).
ARN_m^t	Autoregressive time series for scenario m.
$\emptyset$	One-lag autocorrelation parameter.
ϵ	White noise of ARMA model.
$NTWPG^t$	Normalized total (forecasted) wind power generation at time t.
$TWPG^t$	Total (forecasted) wind power generation at time t (MW).
$NTWPG_m^t$	Normalized total (synthetically generated) wind production at time t for scenario m.
$TWPG_m^t$	Total (synthetically generated) wind power production at time t for scenario m (MW).
β	Limit to the outliers of the scenario generation process.
IFE_m	Vector that reflects the degree at which the hourly values of a determined scenario are within the corresponding forecasting error.

*Corresponding author at: University of Beira Interior, R. Fonte do Lameiro, 6201-001 Covilha, Portugal. Tel.: +351 275 329914; fax: +351 275 329972. *E-mail address*: catalao@ubi.pt (J.P.S. Catalão).

FE_m^t	Vector to represent if scenario m at time t is within the defined confidence interval according to the forecasting error.
$NP_r\{m\}$	Normalized probability of scenario m of wind power generation.
$P_r\{\cdot\}$	Probability of occurrence of a determined event.
$E\{\cdot\}$	Expected value of a determined variable.
$LB_{d,m}^t$	Binary variable to represent the selection of the d^{th} load interval of scenario m at time t.
LP_d^t	Probability of the d^{th} load interval at time t.
$WB_{j,m}^t$	Binary variable to represent the selection of the j^{th} wind power interval of scenario m at time t.
WP_j^t	Probability of the j^{th} wind power interval at time t.
R_m	Total generation cost of scenario m ($).
R	Total generation cost of the UC problem (h).
$FC_{n,m}^t$	Fuel consumption cost of unit n at time t for scenario m ($/h).
$SUC_{n,m}^t$	Start-up cost of unit n at time t for scenario m ($/h).
$SDC_{n,m}^t$	Shutdown cost of unit n at time t for scenario m ($/h).
$P_{n,m}^t$	Power generation of unit n at time t for scenario m (MW).
P_n^t	Power generation of unit n at time t (MW).
P_n^{max}	Maximum power generation of unit n (MW).
P_n^{min}	Minimum power generation of unit n (MW).
$MP_{n,m}^t$	Maximum available power of unit n at time t for scenario m (MW).
W_m^t	Aggregated wind generation for scenario m at time t (MW).
L_m^t	Load demand at time t for scenario m (MW).
SR	Required spinning reserve.
a_n, b_n	Parameters of the fuel consumption cost of unit n ($/h, $/MWh).
$v_{n,m}^t$	Binary variable to represent the commitment ($v_{n,m}^t$=1) or de-commitment ($v_{n,m}^t$=0) of unit n at time t for scenario m.
U_n^t	Definitive UC solution obtained from the proposed methodology, common to all scenarios considered.
K_n^z	Value of the interval z in the discretization of startup cost ($/h).
C_n	Shutdown cost of unit n ($/h).
UR_n	Ramp-up rate of unit n (MW/h).
DR_n	Ramp-down rate of unit n (MW/h).
SUR_n	Starting ramp rate of unit n (MW/h).
SDR_n	Shutdown ramp rate of unit n (MW/h).
UP_n	Amount of hours that generator n have to be initially committed in order to fulfill minimum up time constraint (h).
DW_n	Amount of hours that generator n have to be initially de-committed in order to fulfill minimum down time constraint (h).
MUT_n	Minimum up time of unit n (h).
MDT_n	Minimum down time of unit n (h).
OFF_n^t	Integer matrix that saves the cumulative account of the number of hours that generator n has been de-committed (h).
ON_n^t	Integer matrix that saves the cumulative account of the number of hours that generator n has been committed (h).
μ_{WFE}^t	Mean value of the discretized wind generation PDF at time t (MW).
μ_{LFE}^t	Mean value of the discretized load demand PDF at time t (MW).
μ_{TWPG}	Mean value of the time series $TWPG^t$ (MW).
σ_{WFE}^t	Standard deviation of the discretized wind generation PDF at time t (MW).
σ_{LFE}^t	Standard deviation of the discretized load demand PDF at time t (MW).
σ_{TWPG}	Standard deviation of the time series $TWPG^t$ (MW).
$VOLL$	Value of lost load ($/MWh).
$VRNS$	Value of reserve not supplied ($/MWh).
ENS_m^t	Energy not supplied of scenario m at time t (MWh).
RNS_m^t	Reserve not supplied of scenario m at time t (MWh).

1. Introduction

The constant increment in the price of fossil fuel and the environmental impact of human activities has been the most relevant factors in the development of wind energy and solar energy. However, the main barrier in the successful integration of this type of sources is related to their intrinsic variability, which under high penetration, it is reflected as the increment in the operational costs of the power system. In fact, according to the analysis of the Belgian power system [1], if the wind power production is underestimated, approximately a third of the expected cost savings could be lost. On the contrary, if the wind power production is overestimated, cost savings are lost due that it is necessary to use open cycle gas generators in order to compensate the forecasting error.

In order to reduce the impacts of the wind power forecasting error, several techniques have been proposed: the integration of energy storage systems (EES) [2], the analysis of the wind power aggregation [3], the incorporation of demand response programs [4], and the analysis of the optimal scheduling under uncertainty or stochastic unit commitment (UC) problem.

This paper focus on the development of a methodology to solve the unit commitment (UC) problem considering the uncertainty related to the wind power generation. In this context, Tuohy et al. [5] developed a stochastic programming (SP) approach based on scenario generation of wind power production, failure events, and load demand. The scenarios used were randomly generated to take into account the autocorrelation of the analysed time series (wind power generation, load demand, etc.) by means of an autoregressive moving average (ARMA) model. In this framework two stages are considered: in stage one “here-and-now” decisions are taken; while in stage two “wait-and-see” decisions are incorporated. In other words, “here-and-now” decisions are taken assuming perfect forecasting and “wait-and-see” decisions are taken in the light of the different sources of uncertainty. The incorporation of wind power generation by means of a representative amount of realistic scenarios can provide a reasoning manner to determine spinning reserve on an hourly basis [6]. However, this approach requires an important computational effort; according to the experiences of Ruiz et al. [7], the computational time could be until two or three orders of magnitude higher than those required for solving a deterministic UC problem. For this reason, improvements in the mathematical formulation of SP and decomposition techniques have been widely suggested in the literature.

Another approach proposed in the literature is based on chance-constrained programming (CCP). Ding et al. [8] have incorporated several uncertain variables, such as load demand, force outages, wind power, and energy prices in the UC problem using CCP. In this approach the stochastic constraints are substituted by their equivalent deterministic, in order to obtain a mathematical formulation that can be solved by using standard branch and bound algorithm. In a similar manner, Ji et al. [9] introduced a methodology based on CCP, where a combination of quantum-inspired binary gravitational search algorithm is used to determine the unit scheduling for several confidence levels and different forecasting errors.

Wang et al. [10] have developed a model that combines CCP and SP. Authors proposed a combined sample average approximation (SAA) algorithm that consists of three main processes: scenario generation, convergence analysis, and solution validation. The optimization problem is solved by using a mixed integer linear programming (MILP) formulation.

Hargreaves and Hobbs [11] have introduced a methodology based on stochastic dynamic programming (SDP) method. In this approach, the variables are discretized according to a determined increment so that the behaviour of each stochastic variable is represented by a finite number of levels. Load demand and wind power generation are represented by a Markov process, so that for a determined level, all possible combinations are considered.

Wang et al. [12] developed a scheduling model where the uncertainty of wind power generation is represented by means of a stochastic mathematical formulation, while the corresponding variability is taken into account by substitution of the classical hourly constraints by an enhanced version on a sub-hourly basis. To deal with the disadvantage of the computational time, an improved Benders decomposition algorithm is introduced.

Zhao and Guan [13] developed an approach that incorporates the advantages of SP and robust methodologies. In their mathematical formulation, weights are introduced in the objective function for the stochastic and robust component in order to represent the preferences of the system operator. This unified approach faces the problem of the computational efforts related to the analysis of a large amount of scenarios, while dealing with conservativeness of the solution obtained from robust formulation. The resulting solution offers low expected generation cost, while guaranteeing the robustness of the power system. The efficiency of the algorithm is improved through the application of Benders' decomposition.

In a similar manner, Jiang et al. [14] proposed a two-stage robust formulation incorporating network constraints. In this method the uncertainty is represented by means of a two-dimensional set, and the robust UC problem is solved by using Benders' decomposition.

Luh et al. [15] developed a formulation that incorporates wind power generation through a Markov process. Using historical data, state transition matrices are built and introduced in the UC problem in order to obtain a model based on states instead of scenarios. The optimization problem is formulated in a linear manner and solved by using branch and cut algorithm. Sturt and Strbac [16] proposed a mathematical formulation of stochastic UC problem that uses a structure based on quantiles to build the scenario tree, offering an important cost reduction compared to the results obtained from the application of deterministic methodologies.

Ji et al. [17] developed a methodology at which, from a large amount of scenarios generated, one representative is used. This scenario is chosen considering three main indexes: first index takes into account the power system security, second index takes into account generation costs, and the third index models the influence of the probability in the scheduling process. Once the representative scenario is chosen, the stochastic UC problem is solved by using gravitational search algorithm (GSA).

As can be seen from the literature review previously presented, the stochastic UC problem is a challenging problem that requires considerable computational effort and time. Moreover, MILP formulation has been widely accepted as a methodology to determine the unit scheduling. The relationship between the number of scenarios, duality gap, and computational time is a very interesting topic. In [18] and [19], test systems based on the real operation of the California Independent Systems Operator (CAISO) have been analysed under different operating conditions. From [18], authors concluded that a duality gap of about 0.5% offers a reasonable and feasible solution, reducing the computational time. Otherwise, the numbers of scenarios should be selected taking into account the computational burden. For these reasons, in reference [19], the parallel implementation of Lagrangian relaxation has been proposed. According to the reported results, the parallelization of the stochastic UC problem lead to a reduction in the computational time within one day period if the number of cores used equals the number of scenarios analysed. Regarding the selection of the number of scenarios, a similar conclusion to that reported in reference [18] was reached; the amount of scenarios to be used in practice should be selected according to the computational time and resources available.

The method proposed in this paper aims to establish a proportional relationship between the number of scenarios and the computational time required to analyse them. This is done through the analysis of each scenario separately. For example, according to the results reported in [7], the analysis of a single scenario takes 70 seconds in average, while the analysis of 12 scenarios (considering spinning reserve in all scenarios) takes 6300 seconds. If a proportional relationship could be established, the analysis of 12 scenarios would take 840 seconds, which represents a considerable reduction in the solution time. Based on this hypothesis, in this paper a new methodology that takes information from the analysis of each scenario separately is proposed. In more detail, the UC problem is deterministically analysed for each scenario. Then, this information is used to determine the probabilistic behaviour of each generator at each hour in the scheduling problem. Finally, based on this probabilistic analysis, unit scheduling and its corresponding economic dispatch (ED) are estimated.

The paper is organized as follow: Section 2 describes the scenario generation/reduction method used in this paper, section 3 describes the proposed approach for unit scheduling, section 4 describes the mathematical formulation of SP approach used as a point of reference, section 5 illustrates the proposed algorithm through a case study, and conclusions are presented in section 6.

2. Scenario Generation/Reduction Process

The representation of the stochastic characteristics of load demand and renewable power generation through some representative scenarios is a task that requires high accuracy due to its direct influence on the generation cost and power system operation. In this sense, several approaches have been proposed in the literature.

Pappala et al. [20] developed a methodology to scenario generation and reduction based on particle swarm optimization (PSO). Load demand and wind power generation are modelled as independent random variables with a Gaussian join probability distribution function (PDF). The scenario reduction process is based on the solution of an optimization problem using the PSO algorithm, where the search space is the set of all considered scenarios, while each scenario is represented as a particle and the reduced scenario tree is represented by a swarm. The objective function of the optimization problem is the distance between the scenarios. The main advantage of this approach is that it does not require the comparison between all the scenarios considered.

Morales et al. [21] proposed a methodology to the scenario generation of wind speed that consists on characterization and scenario generation processes. The characterization process consists on the normalization and fitting of the ARMA model of the time series of wind speed obtained from historical data, and the estimation of the corresponding spatial correlations through the variance-covariance matrix. While, the scenario generation process is carried out by using a white noise, the variance-covariance matrix previously estimated and the inverse probability transformation, in order to preserve the PDF.

Suomalainen et al. [22] developed a model able to represent the daily pattern of wind speed incorporating the low-frequency behaviour. This methodology consists of six steps: Evaluation of seasonality of the time series under analysis, adjustment of ARMA model, identification of day types in the time series, estimation of probability distribution matrix that corresponds to the day type, generation of daily profiles and hourly behaviour.

Haghi et al. [23] developed a method based on copula theory to the simulation of wind speed and power variations incorporating the temporal characteristics of these time series. This approach is able to consider the nonlinear temporal dependence and the non-Gaussian PDF.

Baringo and Conejo [24] proposed a methodology that uses duration curves of load demand and wind power generation in combination with k-means clustering algorithm in order to generate scenarios taking into account the correlation between load and wind power production.

Ma et al. [25] presented an approach that models the forecasting error through PDFs empirically determined, assuming that distribution of wind power variability could be modelled by using a *t* location-scale distribution. Scenarios are generated by means of inverse probability transformation using a multivariate normal distribution and its corresponding covariance matrix. Depending on the geographic conditions and the characteristics of the wind farm under analysis, other techniques such as Monte Carlo simulation correlated by using Cholesky factorization, Latin Hypercube Sampling correlated by using rank sorting, and copula theory could be employed [26].

The scenario generation and reduction method used in this paper consists on the generation of some hourly profiles in order to incorporate the correlated nature of wind power generation. Then, unexpected changes in the wind power production as a consequence of the forecasting error are simulated. Finally, the normalized probability of each scenario is estimated, which is later used during the stochastic UC solution. All these steps are described in the next sub-sections.

2.1 Generation of hourly profiles of renewable power generation

In the methodology used in this paper, the most important characteristics of the wind power time series, such as the correlation between consecutive observations, the forecasted wind power production and its corresponding error are taken into account. First, a set of scenarios is randomly generated considering the auto-correlated nature of the forecasted production and its hourly behaviour. Then, some of the scenarios previously generated are selected considering the estimated forecasting error. Finally, the best scenarios are chosen using the k-means clustering algorithm.

The scenarios generated have to incorporate the correlated behaviour of the forecasted production and its hourly profile. On the one hand, the auto-correlated nature of wind power is incorporated by creating a random series according to a first-order autoregressive Markov process, as is shown in equation (1):

$$ARN_m^t = \text{Ø}ARN_m^{t-1} + \epsilon, \tag{1}$$

where ϵ is represented by a Gaussian PDF with mean 0 and standard deviation equals to $\sqrt{1-\text{Ø}^2}$. On the other hand, the profile of the forecasted wind power production is incorporated by means of its normalization, as shown in equation (2):

$$NTWPG^t = (TWPG^t - \mu_{TWPG})/\sigma_{TWPG}. \tag{2}$$

A normalized time series that incorporates the auto-correlated nature and the hourly profile of the forecasted wind power production is obtained by the addition of the series previously presented in equations (1) and (2), so the resultant time series is shown in (3):

$$NTWPG_m^t = ARN_m^t + NTWPG^t. \tag{3}$$

Then, total wind power generation is obtained by application of the probability transformation presented in Fig. 1. This methodology is used by the software HOMER to the synthetic generation of wind speed time series [27].

"See Figure 1"

The outliers, which are defined as those scenarios with extremes and unlikely values, are located and deleted using the wind power forecasting error. Considering a determined significance level (α), the confidence intervals of each hour are estimated. Then, a vector of H elements (FE_m^t) is created for each scenario, this vector saves whether the scenario m is inside the confidence interval. In other words, considering the scenario m under analysis; if the value of $TWPG_m^t$ at time t is inside the confidence interval of this hour, the corresponding element of vector FE_m^t becomes 1, in other case it becomes 0. Then, the index IFE_m is defined as is shown in equation (4):

$$IFE_m = \left(\sum_{t=1}^{H} FE_m^t\right) / H. \tag{4}$$

This index reflects the degree in which the scenario m fulfils the forecasting error in each hour. A value of 1 means that scenario m is between the confidence interval in all hours; on the contrary, a value lower than 1 means that not all values of $TWPG_m^t$ are between the corresponding confidence intervals.

Establishing a determined limit to the outliers (β), scenarios that correspond to the desired forecasting error could be selected. For example, if a value of β=0.8 is chosen, those scenarios with values of IFE_m equal or higher than β should be selected.

The set of scenarios to be used in the stochastic UC is selected by the application of k-means clustering algorithm on the dataset obtained by means of equations (1)-(4) and parameter β. The selection of the amount of scenarios to be considered depends on the number of clusters in the dataset and the available computational resources. The number of clusters could be determined by application of the Expectation-Maximization (EM) algorithm in combination with the Bayesian Information Criterion (BIC). This would represent an approximation of the lower limit for the amount of scenarios required. The methodology proposed in this paper aims to introduce a proportional relationship between the computational time and the number of scenarios (this aspect is going to be analysed through a case study in section 5), so that the upper limit for the amount of scenarios could be estimated by using the average computational time required to solve a single scenario. The impact of the computational time required to solve the ED problem could be neglected at this stage due to a linear programming problem that requires less computational effort compared to MILP problem of UC. Then, the amount of scenarios to be used should be higher than the number of clusters of the dataset and limited by that amount that corresponds to the available computational resources. Once the amount of scenarios has been determined, the clustering process is carried out initialized by means of k-means++ algorithm.

2.2 Simulation of sudden changes on renewable power generation

The autocorrelation and other characteristics of wind power production are considered by means of ARMA model, specifically in equations (1)-(3). However, spinning reserve requirements should be estimated considering any possible and unexpected change in wind generation as a consequence of the forecasting error and other climatic variables.

In this paper, this situation has been modelled by using integer and continuous random numbers. For each of the scenarios generated using the procedure described in section 2.1, a random number between 1 and H is generated. Then, for this hour, the sudden change in renewable power production is simulated by introducing a random number within the corresponding confidence interval of the forecasting error of this hour. This is illustrated in Fig. 2, where the hour 18 has been randomly chosen and the corresponding drop in the wind generation has been simulated for scenario m.

"See Figure 2"

2.3 Calculation of the normalized probability of each scenario

Once the scenarios are obtained by using the procedure explained in sub-sections 2.1 and 2.2, the normalized probability assigned to each scenario is estimated. The first step consists on discretizing the PDF of the forecasting error of load demand and wind power generation. Fig. 3 shows this discretization of wind generation for the case when seven segments (J=7) are chosen (load demand could be treated in a similar manner using the probability LP_d^t (d=*1*,…,D) instead of WP_j^t, and D=7) which is a typical value frequently used in power system reliability analysis [28]. From this discretization process, the probabilities LP_d^t and WP_j^t for their corresponding load and wind power intervals are obtained.

"See Figure 3"

In the second step, for a determined scenario m the status of the corresponding binary variables $WB_{j,m}^t$ and $LB_{d,m}^t$ are determined by taking into account the values of $TWPG_m^t$ and L_m^t, and their corresponding intervals in the discretized PDFs. Finally, the normalized probability of each scenario is calculated by using equation (5) [29]:

$$NP_r\{m\} = \frac{\prod_{t=1}^{H}\left(\sum_{d=1}^{D}\left(LB_{d,m}^t LP_d^t\right)\left(\sum_{j=1}^{J}\left(WB_{j,m}^t WP_j^t\right)\right)\right)}{\sum_{i=1}^{M}\left(\prod_{t=1}^{H}\left(\sum_{d=1}^{D}\left(LB_{d,i}^t LP_d^t\right)\left(\sum_{j=1}^{J}\left(WB_{j,m}^t WP_j^t\right)\right)\right)\right)}\,. \quad (5)$$

Note that equation (5) incorporates the variability related to the load demand; the set of scenarios related to this variable can be easily obtained by applying the procedure of section 2.1. The estimation for the normalized probability used in this paper corresponds to only one wind farm (aggregated wind generation); however, a more complete expression that incorporates the generation of several disaggregated wind farms can be found in reference [29].

3. Proposed approach for unit scheduling

The methodology proposed in this paper is based on the analysis of each scenario separately, so that the solution of successive deterministic UC problems is required. The deterministic UC problem has been extensively analysed in the literature and many methods have been proposed. Delarue et al. [30] have enhanced the traditional priority list method to the scheduling of systems with high integration of renewable sources, where net load has values considerably low. Carrion and Arroyo [31] proposed a MILP formulation widely used in the literature, while Morales-España et al. [32] have developed a novel formulation, incorporating start-up and shutdown trajectories of thermal generators, besides reducing the computational burden. Yuan et al. [33] have applied enhanced discrete evolution approach. Yuan et al. [34] have introduced second-order cone programming. Yu and Zhang [35] have combined Lagrangian relaxation and PSO algorithm. Roy and Sarkar [36] have applied quasi-oppositional teaching learning algorithm. Roy [37] proposed a method based on GSA. Dudek [38] has proposed a binary representation of start-up and shutdown times in order to be incorporated in a genetic algorithm (GA). Amjady and Ansari [39] developed a model based on Benders decomposition for hydrothermal unit commitment, and Rong et al. [40] proposed a methodology based on dynamic regrouping based sequential dynamic programming algorithm.

The method used in this paper for the solution of the UC problem was adapted from the MILP formulation proposed in reference [31]. As was stated before in the introduction section, the UC problem is solved separately for each scenario, so that the objective function to be minimized is the total generation cost for the corresponding scenario m, which is represented by equation (6). The power balance of the system is represented by equation (7) and the spinning reserve constraint is represented by equation (8). Fuel consumption cost is modelled by the linear relationship of equation (9); however, details about the linearization process frequently implemented to model quadratic cost functions could be found in [31]. Start-up and shutdown costs have been modelled using equations (10)-(13). Generation limits and ramping constraints are represented by equations (14)-(19). Finally, minimum up and down time constraints are presented in equations (20)-(27):

$$R_m = min \sum_{t=1}^{H} \sum_{n=1}^{N} \left(FC_{n,m}^t + SUC_{n,m}^t + SDC_{n,m}^t \right); \; m = 1, \dots, M, \tag{6}$$

$$\sum_{n=1}^{N} P_{n,m}^t + W_m^t = L_m^t; \; m = 1, \dots M; \; t = 1, \dots H, \tag{7}$$

$$\sum_{n=1}^{N} MP_{n,m}^t - \sum_{n=1}^{N} P_{n,m}^t \geq (SR) L_m^t; \; m = 1, \dots M; \; t = 1, \dots H, \tag{8}$$

$$FC_{n,m}^t = a_n v_{n,m}^t + b_n P_{n,m}^t; \; n = 1, \dots, N; m = 1, \dots M; \; t = 1, \dots H, \tag{9}$$

$$SUC_{n,m}^t \geq K_n^z \left[v_{n,m}^t - \sum_{q=1}^{z} v_{n,m}^{t-q} \right]; \; z = 1, \dots, Z; \; n = 1, \dots N; \; t = 1, \dots, H, \tag{10}$$

$$SUC_{n,m}^t \geq 0; \; n = 1, \dots, N; \; m = 1, \dots M; \; t = 1, \dots H, \tag{11}$$

$$SDC_{n,m}^t \geq C_n \left[v_{n,m}^{t-1} - v_{n,m}^t \right]; \; n = 1, \dots, N; \; m = 1, \dots M; \; t = 1, \dots H, \tag{12}$$

$$SDC_{n,m}^t \geq 0; \; n = 1, \dots, N; \; m = 1, \dots M; \; t = 1, \dots H, \tag{13}$$

$$P_n^{min} v_{n,m}^t \leq P_{n,m}^t \leq MP_{n,m}^t; \; n = 1, \dots, N; \; m = 1, \dots M; \; t = 1, \dots H, \tag{14}$$

$$0 \leq MP_{n,m}^t \leq P_n^{max} v_{n,m}^t; \; n = 1, \dots, N; \; m = 1, \dots M; \; t = 1, \dots H, \tag{15}$$

$$MP_{n,m}^{t} \le P_{n,m}^{t-1} + UR_n v_{n,m}^{t-1} + SUR_n\left[v_{n,m}^{t} - v_{n,m}^{t-1}\right] + P_n^{max}\left[1 - v_{n,m}^{t}\right];$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = 1, \dots H, \tag{16}$$

$$MP_{n,m}^{t} \le P_n^{max} v_{n,m}^{t+1} + SDR_n\left[v_{n,m}^{t} - v_{n,m}^{t+1}\right];$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = 1, \dots, H-1, \tag{17}$$

$$P_{n,m}^{t-1} - P_{n,m}^{t} \le DR_n v_{n,m}^{t} + SDR_n\left[v_{n,m}^{t-1} - v_{n,m}^{t}\right] + P_n^{max}\left[1 - v_{n,m}^{t-1}\right];$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = 1, \dots, H-1, \tag{18}$$

$$W_m^t \le TWPG_m^t;\ n = 1, \dots, N;\ m = 1, \dots M;\ t = 1, \dots, H-1, \tag{19}$$

$$UP_n = min\left\{H, \left[MUT_n - ON_n^0\right] v_{n,m}^0\right\};\ n = 1, \dots, N;\ m = 1, \dots M, \tag{20}$$

$$\sum_{t=1}^{UP_n}\left[1 - v_{n,m}^{t}\right] = 0;\ n = 1, \dots, N;\ m = 1, \dots M, \tag{21}$$

$$\sum_{q=t}^{t+MUT_n-1} v_{n,m}^{q} \ge MUT_n\left[v_{n,m}^{t} - v_{n,m}^{t-1}\right];$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = UP_n + 1, \dots, H - MUT_n + 1, \tag{22}$$

$$\sum_{q=t}^{H}\left\{v_{n,m}^{q} - \left[v_{n,m}^{t} - v_{n,m}^{t-1}\right]\right\} \ge 0;$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = H - MUT_n + 2, \dots, H, \tag{23}$$

$$DW_n = min\left\{H, \left[MDT_n - OFF_n^0\right]\left[1 - v_{n,m}^0\right]\right\};\ n = 1, \dots, N;\ m = 1, \dots M, \tag{24}$$

$$\sum_{t=1}^{DW_{n,m}} v_{n,m}^{t} = 0;\ n = 1, \dots, N;\ m = 1, \dots M, \tag{25}$$

$$\sum_{q=t}^{t+MDT_n-1}\left[1 - v_{n,m}^{q}\right] \ge MDT_n\left[v_{n,m}^{t-1} - v_{n,m}^{t}\right];$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = DW_n + 1, \dots, H - MDT_n + 1, \tag{26}$$

$$\sum_{q=t}^{H}\left\{1 - v_{n,m}^{q} - \left[v_{n,m}^{t-1} - v_{n,m}^{t}\right]\right\} \ge 0;$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = H - MDT_n + 2, \dots, H. \tag{27}$$

The proposed approach consists of building the PDF of the situation at which a determined generator (n) be committed or not at a determined time (t). Then, those generators with high probability of being committed are selected in order to determine a common scheduling for all scenarios considered. Finally, a repairing process is applied in order to obtain a feasible solution.

The PDF of committing a determined generator at a specific time is estimated using the normalized probability of equation (5). In other words, each of the scenarios generated according to the methodology presented in section 2 are supposed to be mutually exclusive, so that the required PDF can be estimated by the addition of the corresponding normalized probabilities. Then, the probability that a determined generator be committed or not could be estimated from the solution of the UC problem for each scenario and the corresponding normalized probability. The solution of the UC problem for each scenario is found using the MILP formulation described in equations (6)-(27). This idea is mathematically expressed in equation (28):

$$P_r\{U_n^t = 1\} = \sum_{m=1}^{M} NP_r\{m\} v_{n,m}^t \,;\; n = 1, \ldots, N. \tag{28}$$

Once the PDF has been estimated, those hours that have high probability of be committed are selected. For example, defining a determined significance level (ɣ), all those generators with probability of be committed equal or higher than ɣ could be selected. From this procedure, a binary matrix suggesting the commitment of a determined generator at a specific time is obtained. However, this solution could not fulfil minimum up and down time constraints. To overcome this problem, a minimum up and down time repairing process is applied. The complete algorithm to minimum up and down time repairing was developed by Dieu and Ongsakul [41] and it is briefly described as follows:

- Step 1: Update the matrices ON_n^t and OFF_n^t using equations (29) and (30).

$$ON_n^t = \begin{cases} ON_n^{t-1} + 1\,; & U_n^t = 1 \\ 0\,; & U_n^t = 0 \end{cases};\; n = 1, \ldots, N, \tag{29}$$

$$OFF_n^t = \begin{cases} OFF_n^{t-1} + 1\,; & U_n^t = 0 \\ 0\,; & U_n^t = 1 \end{cases};\; n = 1, \ldots, N, \tag{30}$$

- Step 2: Set t ← 1.
- Step 3: Set n ← 1.
- Step 4: If (U_n^t = 0) and (U_n^{t-1} = 1) and ($ON_n^{t-1} < MUT_n$). Then, U_n^t = 1.
- Step 5: If (U_n^t = 0) and (U_n^{t-1} = 1) and ($t + MDT_n - 1 \leq H$) and ($OFF_n^{t+MDT_n-1} < MDT_n$). Then, U_n^t = 1.
- Step 6: If (U_n^t = 0) and (U_n^{t-1} = 1) and ($t + MDT_n - 1 > H$) and ($\sum_{i=t}^{H} U_n^i > 0$). Then, U_n^t = 1.
- Step 7: Update the matrices ON_n^t and OFF_n^t.
- Step 8: If ($n < N$). Then, n ← n + 1 and go to Step 4.
- Step 9: If ($t < H$). Then, t ← t + 1 and go to Step 3. Otherwise, stop.

When the solution to the stochastic UC problem has been decided, the corresponding dispatch of each generator is determined. This task is carried out by solving the ED problem for each scenario using the solution of the UC problem previously estimated (U_n^t). The mathematical formulation for solving the ED problem is presented in equations (31)-(44) [6, 7, 31].

$$R_m = min\sum_{t=1}^{H}\sum_{n=1}^{N}\left(FC_{n,m}^t + SUC_n^t + SDC_n^t + VOLL \times ENS_m^t + VRNS \times RNS_m^t\right);\ m = 1, \dots, M, \quad (31)$$

$$\sum_{n=1}^{N} P_{n,m}^t + W_m^t + ENS_m^t = L_m^t;\ \ m = 1, \dots M;\ t = 1, \dots H, \quad (32)$$

$$\sum_{n=1}^{N} MP_{n,m}^t - \sum_{n=1}^{N} P_{n,m}^t + RNS_m^t \geq (SR)L_m^t;\ \ m = 1, \dots M;\ t = 1, \dots H, \quad (33)$$

$$FC_{n,m}^t = a_n U_n^t + b_n P_{n,m}^t;\ \ n = 1, \dots, N; m = 1, \dots M;\ t = 1, \dots H, \quad (34)$$

$$SUC_n^t \geq K_n^z\left[U_n^t - \sum_{q=1}^{z} U_n^{t-q}\right];\ \ z = 1, \dots, Z;\ n = 1, \dots N;\ t = 1, \dots, H, \quad (35)$$

$$SUC_n^t \geq 0;\ n = 1, \dots, N;\ t = 1, \dots H, \quad (36)$$

$$SDC_n^t \geq C_n[U_n^{t-1} - U_n^t];\ \ n = 1, \dots, N;\ t = 1, \dots H, \quad (37)$$

$$SDC_n^t \geq 0;\ n = 1, \dots, N;\ t = 1, \dots H, \quad (38)$$

$$P_n^{min} U_n^t \leq P_{n,m}^t \leq MP_{n,m}^t;\ n = 1, \dots, N;\ m = 1, \dots M;\ t = 1, \dots H, \quad (39)$$

$$0 \leq MP_{n,m}^t \leq P_n^{max} U_n^t;\ n = 1, \dots, N;\ m = 1, \dots M;\ t = 1, \dots H, \quad (40)$$

$$MP_{n,m}^t \leq P_{n,m}^{t-1} + UR_n U_n^{t-1} + SUR_n[U_n^t - U_n^{t-1}] + P_n^{max}[1 - U_n^t]\ ;$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = 1, \dots H, \quad (41)$$

$$MP_{n,m}^t \leq P_n^{max} U_n^{t+1} + SDR_n[U_n^t - U_n^{t+1}];$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = 1, \dots, H-1, \quad (42)$$

$$P_{n,m}^{t-1} - P_{n,m}^t \leq DR_n U_n^t + SDR_n[U_n^{t-1} - U_n^t] + P_n^{max}[1 - U_n^{t-1}]$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = 1, \dots, H-1, \quad (43)$$

$$W_m^t \leq TWPG_m^t;\ n = 1, \dots, N;\ m = 1, \dots M;\ t = 1, \dots, H-1. \quad (44)$$

Then, expected power production and expected generation cost are estimated by means of equations (45) and (46), respectively.

$$E\{P_n^t\} = \sum_{m=1}^{M} NP_r\{m\} P_{n,m}^t\ ;\ n = 1, \dots, N, \quad (45)$$

$$E\{R\} = \sum_{m=1}^{M} NP_r\{m\} R_m\ ;\ n = 1, \dots, N. \quad (46)$$

As the amount of power generation is limited through the significance level ɣ, it is likely that the spinning reserve requirement could not be achieved for some scenarios. This condition is probabilistically analysed by evaluating the probability of requiring additional generation to fulfil the reserve requirements. PDF of reserve not supplied (*RNS*) is built from the obtained results after solving the ED problem using equations (31)-(44); then, the expression ($P_r\{RNS = 0\}$) could be easily determined.

The proposed methodology is summarized in the flowchart shown in Fig. 4.

"See Figure 4"

4. Stochastic programming approach for unit scheduling

SP approach has been suggested by many authors to solve unit scheduling problem under uncertainty. In order to evaluate the quality of the solution obtained from the proposed methodology in this paper, a SP optimization model with reserve requirements based on references [7, 31] was developed. The mathematical formulation of the SP approach is presented in equations (47)-(68).

$$min\left\{\sum_{m=1}^{M}\frac{1}{M}\left(\sum_{t=1}^{H}\sum_{n=1}^{N}\left(FC_{n,m}^{t}+SUC_{n}^{t}+SDC_{n}^{t}\right)\right)\right\} \tag{47}$$

$$\sum_{n=1}^{N}P_{n,m}^{t}+W_{m}^{t}=L_{m}^{t};\;\; m=1,\dots M;\; t=1,\dots H, \tag{48}$$

$$\sum_{n=1}^{N}MP_{n,m}^{t}-\sum_{n=1}^{N}P_{n,m}^{t}\geq(SR)L_{m}^{t};\;\; m=1,\dots M;\; t=1,\dots H, \tag{49}$$

$$FC_{n,m}^{t}=a_{n}U_{n}^{t}+b_{n}P_{n,m}^{t};\;\; n=1,\dots,N; m=1,\dots M;\; t=1,\dots H, \tag{50}$$

$$SUC_{n}^{t}\geq K_{n}^{z}\left[U_{n}^{t}-\sum_{q=1}^{z}U_{n}^{t-q}\right];\;\; z=1,\dots,Z;\; n=1,\dots N;\; t=1,\dots,H, \tag{51}$$

$$SUC_{n}^{t}\geq 0;\; n=1,\dots,N;\; t=1,\dots H, \tag{52}$$

$$SDC_{n}^{t}\geq C_{n}[U_{n}^{t-1}-U_{n}^{t}];\;\; n=1,\dots,N;\; t=1,\dots H, \tag{53}$$

$$SDC_{n}^{t}\geq 0;\; n=1,\dots,N;\; t=1,\dots H, \tag{54}$$

$$P_{n}^{min}U_{n}^{t}\leq P_{n,m}^{t}\leq MP_{n,m}^{t};\; n=1,\dots,N;\; m=1,\dots M;\; t=1,\dots H, \tag{55}$$

$$0\leq MP_{n,m}^{t}\leq P_{n}^{max}U_{n}^{t};\; n=1,\dots,N;\; m=1,\dots M;\; t=1,\dots H, \tag{56}$$

$$MP_{n,m}^{t}\leq P_{n,m}^{t-1}+UR_{n}U_{n}^{t-1}+SUR_{n}[U_{n}^{t}-U_{n}^{t-1}]+P_{n}^{max}[1-U_{n}^{t}]\,;$$

$$n=1,\dots,N;\; m=1,\dots M;\; t=1,\dots H, \tag{57}$$

$$MP_{n,m}^{t}\leq P_{n}^{max}U_{n}^{t+1}+SDR_{n}[U_{n}^{t}-U_{n}^{t+1}];$$

$$n=1,\dots,N;\; m=1,\dots M;\; t=1,\dots,H-1, \tag{58}$$

$$P_{n,m}^{t-1} - P_{n,m}^{t} \leq DR_n U_n^t + SDR_n[U_n^{t-1} - U_n^t] + P_n^{max}[1 - U_n^{t-1}]$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = 1, \dots, H-1, \qquad (59)$$

$$W_m^t \leq TWPG_m^t;\ n = 1, \dots, N;\ m = 1, \dots M;\ t = 1, \dots, H-1, \qquad (60)$$

$$UP_n = min\{H, [MUT_n - ON_n^0]U_n^0\}\ ;\ n = 1, \dots, N;\ m = 1, \dots M, \qquad (61)$$

$$\sum_{t=1}^{UP_n} [1 - U_n^t] = 0;\ n = 1, \dots, N;\ m = 1, \dots M, \qquad (62)$$

$$\sum_{q=t}^{t+MUT_n-1} U_n^q \geq MUT_n[U_n^t - U_n^{t-1}];$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = UP_n + 1, \dots, H - MUT_n + 1, \qquad (63)$$

$$\sum_{q=t}^{H} \{U_n^q - [U_n^t - U_n^{t-1}]\} \geq 0;$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = H - MUT_n + 2, \dots, H, \qquad (64)$$

$$DW_n = min\{H, [MDT_n - OFF_n^0][1 - U_n^0]\};\ n = 1, \dots, N, \qquad (65)$$

$$\sum_{t=1}^{DW_n} U_n^t = 0;\ n = 1, \dots, N, \qquad (66)$$

$$\sum_{q=t}^{t+MDT_n-1} [1 - U_n^q] \geq MDT_n[U_n^{t-1} - U_n^t];$$

$$n = 1, \dots, N;\ t = DW_n + 1, \dots, H - MDT_n + 1, \qquad (67)$$

$$\sum_{q=t}^{H} \{1 - U_n^q - [U_n^{t-1} - U_n^t]\} \geq 0;$$

$$n = 1, \dots, N;\ m = 1, \dots M;\ t = H - MDT_n + 2, \dots, H. \qquad (68)$$

5. Case Study

The proposed approach to the solution of the UC problem incorporating the uncertainty related to the wind power generation is illustrated by analysing a power system whose characteristics are presented in Tables 1 and 2, where the quadratic fuel consumption cost has been linearized according to the formulation presented in equations (9), (34), and (50). The forecasted load demand and wind power generation are shown in Table 3 [17, 31], and the required spinning reserve was assumed to be 0.1 (*SR*=0.1).

"See Table 1"

"See Table 2"

"See Table 3"

The average computational time to solve a single scenario is estimated in 6.931 seconds per scenario, while the number of clusters in the initial dataset was only one due to all the pre-processing process carried out in section 2.1. Under this context, 300 scenarios have been chosen in our illustrative case study (M = 300) according to the computational resources available. The computer employed has Intel (R) Core (TM) i7-3630QM CPU @ 2.40 GHz with 8.00 GB of memory and 64 Bit operating system. The expected time required for determining the unit scheduling is 2,079.379 seconds approximately.

The process explained in section 2, regarding scenario generation/reduction method, has been implemented in MATLAB programming language. Initially, 10,000 scenarios were randomly generated; then, considering a forecasting error of 20%, α = 0.01 and β = 1; 5,990 scenarios were selected. Next, 300 scenarios were selected from the application of k-means algorithm. The scenarios synthetically generated are shown in Fig. 5. While, Fig. 6 shows the results obtained from the estimation of the normalized probability ($NP_r\{m\}$) of each scenario *m* according to section 2.3.

"See Figure 5"

"See Figure 6"

The mathematical model of the proposed approach presented in section 3 was implemented in GAMS programming language considering duality gap equal to zero in order to obtain the optimal solution, while the optimization problem was solved by using branch and cut algorithm incorporated in CPLEX solver. Table 4 presents the estimated PDF of committing a determined unit at a specific time. It can be observed how those generators that are in base and cycling condition are committed in all the scenarios and consequently they have probability of being committed equal to 1. Moreover, peak units have probability lower than 1 in order to fulfil spinning reserve requirements. By selecting those generators with probability of being committed equal or higher than 1% (ɣ = 0.01 in $P_r\{U_n^t = 1\} \geq$ ɣ) and the application of the minimum up/down time repairing process, a solution to the UC problem was obtained, as it is shown in Table 5. This is how the decision of which unit should be committed is taken, using the probability of being committed or not. This procedure leads to a UC solution common to all scenarios.

"See Table 4"

"See Table 5"

Once a solution to the unit scheduling has been found, the expected power production was estimated through equation (45) and it is presented in Table 6. In a similar manner, the expected total generation cost is determined by using equation (46). The values of *VOLL* and *VRNS* were assumed to be artificially high. The SP formulation presented in section 4 was implemented in GAMS and used as point of comparison of the approach proposed in this paper, while the ED formulation presented in section 3 was used for the estimation of the expected generation cost. The expected generation cost obtained from the proposed approach was 518,507.516 $ in 2,233.337 seconds, while the equivalent result obtained from SP approach was 515,958.972 $ (duality gap equal to 0.0079%) in 11,120.16 seconds. As can be observed, from the application of the proposed approach an approximation to the optimal unit scheduling can be found in a reduced computational time; in this case, the difference in the expected generation cost is just 0.49%.

"See Table 6"

Table 7 presents the behaviour of the expected generation cost, the probability of requiring any additional reserve and the quality of the solution expressed as the comparison with the generation cost obtained from SP approach. From these results it is possible to observe how the quality of the solution decreases as the parameter γ increases; if a significance level of 1% for the reserve requirement is selected, the solution that corresponds to γ=0.01% could be selected. The significance level (γ) involved in the selection of the definitive unit commitment (U_n^t) defines the amount of power generation to be committed according to the corresponding probability required. Parameter γ has influence in the cost and the robustness of the scheduling, since for low values of parameter γ, more units will be committed and consequently the expected generation cost will be higher. On the contrary, as the value of the parameter γ increases, the probability of meeting the required reserve requirement is reduced. This parameter allows controlling the quality of the obtained solution. In general sense, it is possible concluding that the proposed methodology offers a satisfactory solution in a reduced computational time but it is not capable to guarantee the optimality of such solution, while the SP approach can guarantee the optimality of the solution but employing high computational resources.

"See Table 7"

The influence of the amount of scenarios on the computational time was analysed and compared to the SP approach. The results are shown in Fig. 7. During the evaluation of the proposed approach, duality gap was set to zero, while the evaluation of SP approach was carried out by considering duality gap equal to 0.01%.

"See Figure 7"

According to these results, it is possible observing the considerable increment in the computational time when a number of scenarios higher than 100 is chosen and the SP approach is implemented. However, for a reduced number of scenarios (50 scenarios or less) computational times are similar. On the contrary, the proposed approach presents a linear behaviour with the number of scenarios, which allows obtaining an important reduction in the computational time when a high amount of scenarios are employed.

The behaviour of the proposed methodology for two power systems of 50 and 100 generators was analysed. The characteristics of these systems were obtained by replication of the 10-units system presented in Tables 1 and 2, and multiplication of load demand and wind generation by the corresponding scaling factor, while the number of scenarios considered was 50. In these cases, duality gap used to analyse each scenario in the proposed approach was adjusted to 0.5% and the time limit of 28,800 seconds was assumed. When the 50-units system was analysed by using the SP approach, the expected generation cost was 2,556,389.49 $ in 28,806.73 seconds (duality gap equal to 0.2021%). Table 8 presents the behaviour of the proposed approach for several values of the parameter γ. The computational time required by the proposed approach was just 1,969.604 seconds.

"See Table 8"

When the 100-units system was analysed by using the SP approach, the expected generation cost was 5,116,542.844 $ in 28,813.99 seconds (duality gap equal to 0.32%). Table 9 presents the behaviour of the proposed approach for several values of the parameter γ. The computational time required by the proposed approach was just 4,411.592 seconds. From the results presented in Tables 8 and 9 it is possible observing the high error obtained in comparison to those obtained when the case of 10-units system was analysed (Table 7), at which the duality gap was adjusted to zero. Taking into account that these systems were analysed by adjusting the duality gap equal to 0.5%, it is possible concluding that the proposed approach is sensitive to the duality gap used to solve the scheduling of each scenario. In other words, the error obtained from the solution of each scenario is directly propagated to the estimated PDF of unit scheduling, which directly influences the quality of the obtained solution.

"See Table 9"

6. Conclusions

This paper presented a methodology for the solution of the UC problem to be applied in systems with high integration of renewable power sources. The proposed methodology consists of the generation of some representative scenarios, which are selected considering the auto-correlated nature, the hourly wind power forecasting and its corresponding error. In the next step, using the normalized probability of each scenario, the PDF of a determined generator to be committed or not is determined by solving each scenario separately using MILP formulation. Finally, according to a determined probability level (ɣ), those hours with probability of committing a determined unit equal or higher than ɣ are selected and the minimum up/down time repair is applied in order to obtain a feasible solution. The proposed methodology was illustrated through a case study and the comparison with SP approach was carried out, concluding that the proposed approach can provide a satisfactory solution in a reduced computational time.

Acknowledgements

This work was supported by FEDER funds (European Union) through COMPETE and by Portuguese funds through FCT, under Projects FCOMP-01-0124-FEDER-020282 (PTDC/EEA-EEL/118519/2010) and UID/CEC/50021/2013. Also, the research leading to these results has received funding from the EU Seventh Framework Programme FP7/2007-2013 under grant agreement no. 309048.

References

[1] Luickx PJ, Delarue ED, D'haeseleer WD. Impact of large amounts of wind power on the operation of an electricity generation system: Belgian case study. Renewable and Sustainable Energy Reviews 2010;14(7): 2019-2028.

[2] Cau G, Cocco D, Petrollese M, Kaer SK, Milan C. Energy management strategy based on short-term generation scheduling for a renewable microgrid using a hydrogen storage system. Energy Conversion and Management 2014;87:820-831.

[3] Santos-Alamillos FJ, Pozo-Vázquez D, Ruiz-Arias JA, Lara-Fanego V, Tovar-Pescador J. A methodology for evaluating the spatial variability of wind energy resources: Application to assess the potential contribution of wind energy to baseload power. Renewable Energy 2014;69:147-156.

[4] Mazidi M, Zakariazadeh A, Jadid S, Siano P. Integrated scheduling of renewable generation and demand response programs in a microgrid. Energy Conversion and Management 2014;86:1118-1127.

[5] Tuohy A, Meibom P, Denny E, O'Malley M. Unit commitment for systems with significant wind penetration. IEEE Transactions on Power Systems 2009;24(2):592-601.

[6] Wang J, Botterud A, Bessa R, Keko H, Carvalho L, Issicaba D, Sumaili J, Miranda V. Wind power forecasting uncertainty and unit commitment. Applied Energy 2011;88:4014-4023.

[7] Ruiz PA, Philbrick CR, Zak E, Cheung KW, Sauer PW. Uncertainty management in the unit commitment problem. IEEE Transactions on Power Systems 2009;24(2):642-651.

[8] Ding X, Lee W-J, Jianxue W, Liu L. Studies on stochastic unit commitment formulation with flexible generating units. Electric Power Systems Research 2010;80(1):130-141.

[9] Ji B, Yuan X, Li X, Huang Y, Li W. Application of quantum-inspired binary gravitational search algorithm for thermal unit commitment with wind power integration. Energy Conversion and Management 2014;87:589-598.

[10] Wang Q, Guan Y, Wang J. A chance-constrained two-stage stochastic program for unit commitment with uncertain wind power output. IEEE Transactions on Power Systems 2012;27(1):206-215.

[11] Hargreaves JJ, Hobbs BF. Commitment and dispatch with uncertain wind generation by dynamic programming. IEEE Transactions on Sustainable Energy 2012;3(4):724-734.

[12] Wang J, Wang J, Liu C, Ruiz JP. Stochastic unit commitment with sub-hourly dispatch constraints. Applied Energy 2013;105:418-422.

[13] Zhao C, Guan Y. Unified stochastic and robust unit commitment. IEEE Transactions on Power Systems 2013;28(3):3353-3361.

[14] Jiang R, Zhang M, Li G, Guan Y. Two-stage network constrained robust unit commitment problem. European Journal of Operational Research 2014;234(3):751-762.

[15] Luh PB, Yu Y, Zhang B, Litvinov E, Zheng T, Zhao F, Zhao J, Wang C. Grid integration of intermittent wind generation: a Markovian approach. IEEE Transactions on Smart Grid 2014;5(2):732-741.

[16] Sturt A, Strbac G. Efficient stochastic scheduling for simulation of wind-integrated power systems. IEEE Transactions on Power Systems 2012;27(1):323-334.

[17] Ji B, Yuan X, Chen Z, Tian H. Improved gravitational search algorithm for unit commitment considering uncertainty of wind power. Energy 2014;67:52-62.

[18] Yao Y, Meyers C, Smith S, Edmunds T. Large-scale stochastic optimization for unit commitment and economic dispatch. Lawrence Livermore National Laboratory; 2012.

[19] Papavasiliou A, Oren SS, Rountree B. Applying high performance computing to transmission-constrained stochastic unit commitment for renewable energy integration. IEEE Transactions on Power Systems 2014;PP(99): 1-12.

[20] Pappala VS, Erlich I, Rohrig K, Dobschinski J. A stochastic model for the optimal operation of a wind-thermal power system. IEEE Transactions on Power Systems 2009;24(2):940-950.

[21] Morales JM, Mínguez R, Conejo AJ. A methodology to generate statistically dependent wind speed scenarios. Applied Energy 2010;87(3):843-855.

[22] Suomalainen K, Silva CA, Ferrão SP, Connors S. Synthetic wind speed scenarios including diurnal effects: Implications for wind power dimensioning. Energy 2012;37(1):41-50.

[23] Haghi HV, Bina MT, Golkar MA. Nonlinear modeling of temporal wind power variations. IEEE Transactions on Sustainable Energy 2013;4(4):838-847.

[24] Baringo L, Conejo AJ. Correlated wind-power production and electric load scenarios for investment decisions. Applied Energy 2013;101:475-482.

[25] Ma X-Y, Sun Y-Z, Fang H-L. Scenario generation of wind power based on statistical uncertainty and variability. IEEE Transactions on Sustainable Energy 2013;4(4):894-904.

[26] Díaz G, Casielles PG, Coto J. Simulation of spatially correlated wind power in small geographic areas-Sampling methods and evaluation. International Journal of Electrical Power and Energy Systems 2014;63:513-522.

[27] Farret FA, Simões. Integration of alternative sources of energy. Wiley; 2006.

[28] Billinton R, Allan RN. Reliability evaluation of power systems. New York: Plenum; 1996.

[29] Aghaei J, Niknam T, Azizipanah-Abarghooee R, Arroyo JM. Scenario-based dynamic economic emission dispatch considering load and wind power uncertainties. Electrical Power and Energy Systems 2013;47:351-367.

[30] Delarue E, Cattrysse D, D'haeseleer W. Enhanced priority list unit commitment method for power systems with a high share of renewables. Electric Power Systems Research 2013;105:115-123.

[31] Carrión M, Arroyo JM. A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem. IEEE Transactions on Power Systems 2006;21(3):1371-1378.

[32] Morales-España G, Latorre JM, Ramos A. Tight and compact MILP formulation of start-up and shut-down ramping in unit commitment. IEEE Transactions on Power Systems 2013;28(2):1288-1296.

[33] Yuan X, Su A, Nie H, Yuan Y, Wang. Application of enhanced discrete evolution approach to unit commitment problem. Energy Conversion and Management 2009;50:2449-2456.

[34] Yuan X, Tian H, Zhang S, Ji B, Hou Y. Second-order cone programming for solving unit commitment strategy of thermal generators. Energy Conversion and Management 2013;76:20-25.

[35] Yu X, Zhang X. Unit commitment using Lagrangian relaxation and particle swarm optimization. Electrical Power and Energy Systems 2014;61:510-522.

[36] Roy PK, Sarkar R. Solution of unit commitment problem using quasi-oppositional teaching learning based algorithm. Electrical Power and Energy Systems 2014;60:96-106.

[37] Roy PK. Solution of unit commitment problem using gravitational search algorithm. Electrical Power and Energy Systems 2013;53:85-94.

[38] Dudek G. Genetic algorithm with binary representation of generating unit start-up and shut-down times for the unit commitment problem. Expert Systems with Applications 2013;40:6080-6086.

[39] Amjady N, Ansari MR. Hydrothermal unit commitment with AC constraints by a new solution method based on benders decomposition. Energy Conversion and Management 2013;65:57-65.

[40] Rong A, Hakonen H, Lahdelma R. A dynamic regrouping based sequential dynamic programming algorithm for unit commitment of combined heat and power systems. Energy Conversion and Management 2009;50(4):1108-1115.

[41] Dieu VN, Ongsakul W. Ramp rate constrained unit commitment by improved priority list and augmented Lagrange Hopfield network. Electric Power Systems Research 2008;78(3):291-301.

Figure Captions

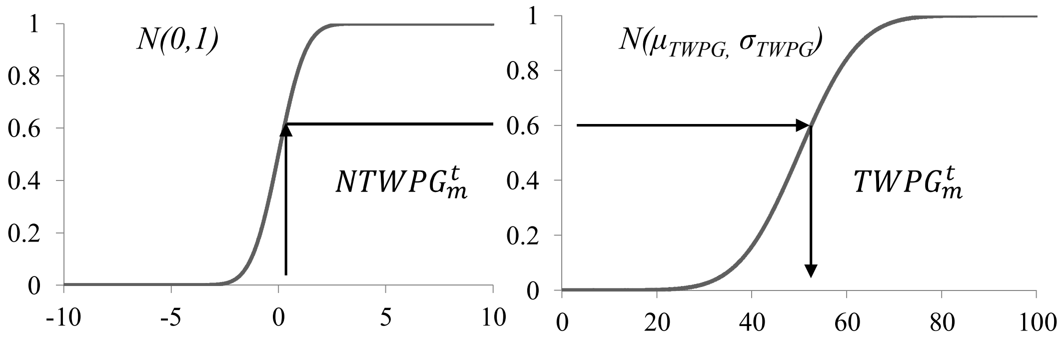

Figure 1
Probability transformation

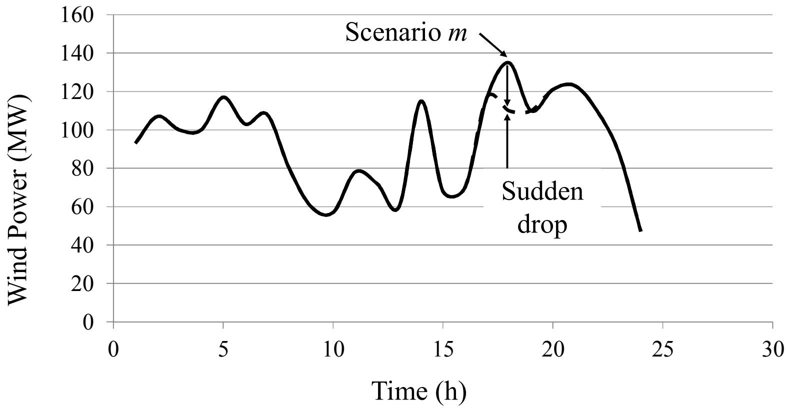

Figure 2
Simulation of sudden changes on wind power generation

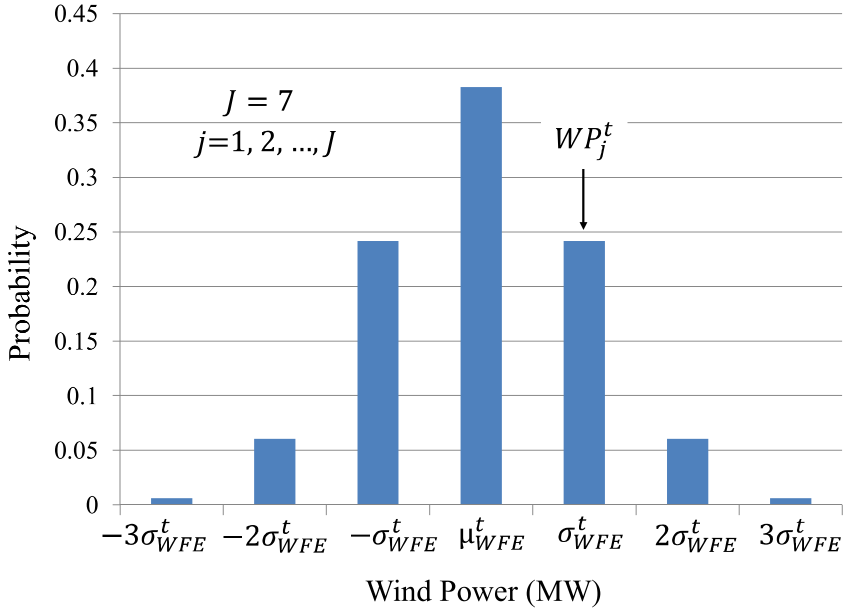

Figure 3
Discretization of PDF of wind generation

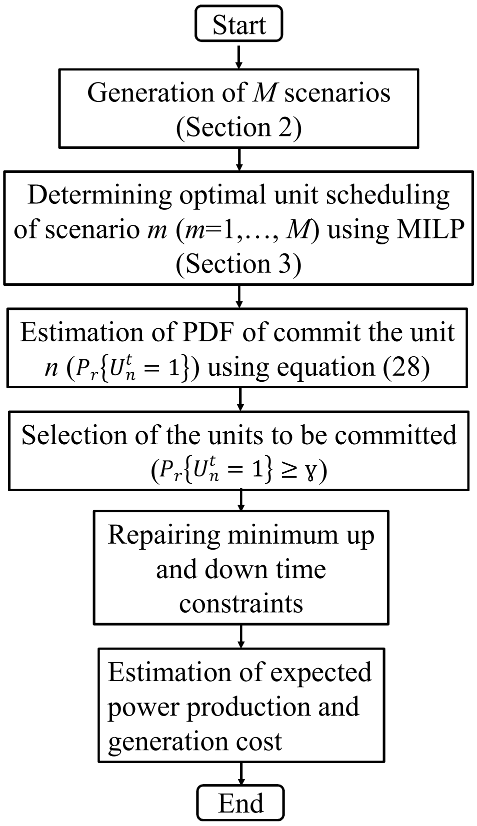

Figure 4
Flowchart of the proposed methodology

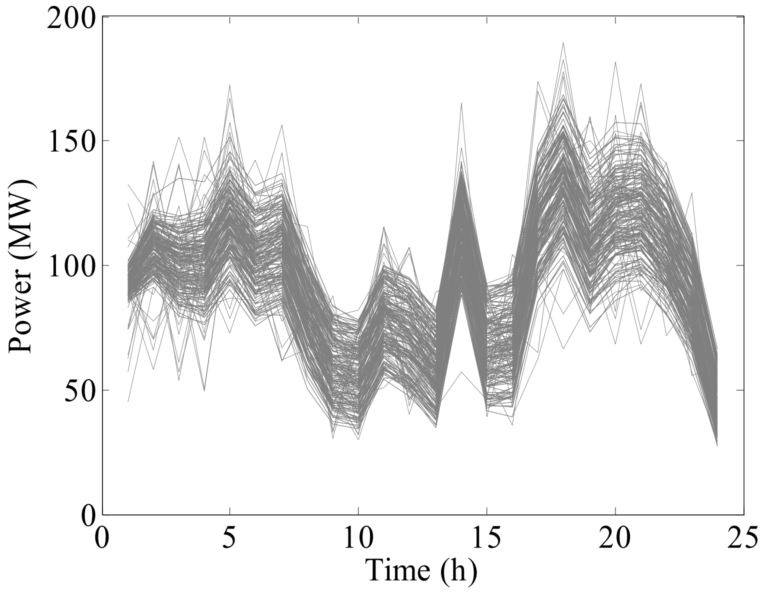

Figure 5
Scenarios of wind generation

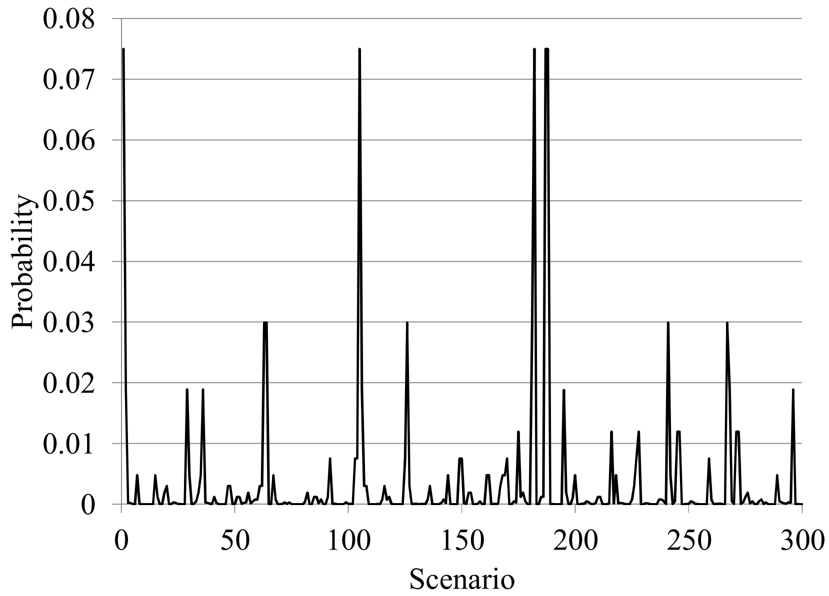

Figure 6
Normalized probability of scenarios of wind generation

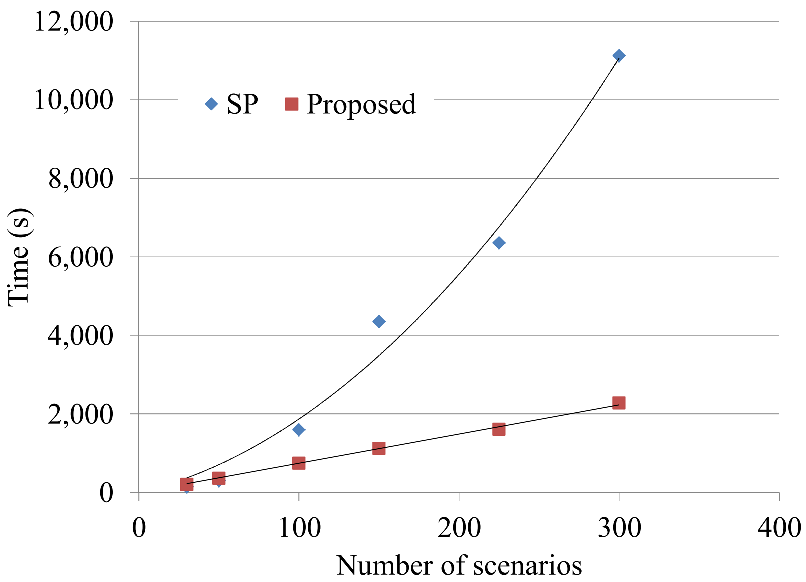

Figure 7
Comparison of computational time

Table Captions

Table 2

Description of the 10-unit power system

n	P_n^{min} (MW)	P_n^{max}(MW)	a_n ($/h)	b_n ($/MWh)	*DR* (MW/h)	*UR* (MW/h)
1	150	455	959.82	16.480	130	130
2	150	455	944.05	17.448	130	130
3	20	130	690.80	16.900	90	90
4	20	130	670.30	16.817	60	60
5	25	162	421.52	20.444	60	60
6	20	80	354.410	22.972	40	40
7	25	85	477.860	27.827	40	40
8	10	55	656.370	26.188	40	40
9	10	55	663.050	27.414	40	40
10	10	55	668.480	27.902	40	40

Table 2

Description of the 10-unit power system (continued)

n	P_n^0(MW)	IS_n (h)	MUT_n (h)	MDT_n (h)	CSC_n ($)	HSC_n ($)	CST_n (h)
1	455	8	8	8	9,000	4,500	5
2	163	8	8	8	10,000	5,000	5
3	0	-6	6	6	1,800	900	4
4	0	-5	5	5	1,120	560	4
5	0	-5	5	5	1,100	550	4
6	0	-3	3	3	340	170	2
7	0	-3	3	3	340	170	2
8	0	-3	3	3	520	260	0
9	0	-3	3	3	520	260	0
10	0	-1	1	1	60	30	0

IS_n: Initial state of unit n

CSC_n: Cold startup cost of unit n

HSC_n: Hot startup cost of unit n

CST_n: Cold startup time of unit n

Table 3
Forecasted wind generation and load demand

Time (h)	Wind (MW)	Load (MW)	Time (h)	Wind (MW)	Load (MW)
1	93	700	13	60	1,400
2	107	750	14	115	1,300
3	100	850	15	68	1,200
4	100	950	16	70	1,050
5	117	1,000	17	117	1,000
6	103	1,100	18	135	1,100
7	108	1,150	19	110	1,200
8	80	1,200	20	121	1,400
9	60	1,300	21	123	1,300
10	57	1,400	22	110	1,100
11	78	1,450	23	88	900
12	72	1,500	24	47	800

Table 4
Estimated PDF of unit scheduling

Unit	Time (h)																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3	0	0	0	0	0	0	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.90	0.03	0.03	0.03	0.03	0.04	0.04	0.03	0.01	0.01
4	0	0	0	0	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0	0
5	0	0	0	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0
6	0	0	0	0	0	0	0	0	1.00	1.00	1.00	1.00	1.00	0.74	0.10	0.00	0	0	0.96	0.98	0.98	0.04	0.00	0
7	0	0	0	0	0	0	0	0	0.00	0.99	0.99	0.99	0.99	0.00	0	0	0	0	0.00	0.00	0.00	0	0	0
8	0	0	0	0	0	0	0	0	0	0.65	0.98	1.00	0.28	0	0	0	0	0	0	1.00	0.93	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0.01	1.00	0	0	0	0	0	0	0	0.98	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0.03	0	0	0	0	0	0	0	0.96	0	0	0	0

Table 5
Definitive decision of the unit scheduling

Unit	Time (h)																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
4	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
5	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
6	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	1	1	1	1	0	0
7	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	1	1	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0

Table 6
Expected power production over the horizon of scheduling

Unit	Time (h)																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	452.8	391.3	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0	455.0
2	151.0	243.1	288.1	360.8	392.6	427.7	422.5	404.2	453.9	455.0	455.0	455.0	455.0	418.6	365.3	235.4	150.2	216.3	326.0	455.0	396.6	271.8	267.8	289.7
3	0	0	0	0	0	0	0	80.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	125.0	115.1	129.9	124.6	130.0	130.0	129.8	80.0	0
4	0	0	0	0	0	80.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	130.0	80.0	0	0
5	0	0	0	25.0	25.6	25.4	26.8	43.9	44.9	112.9	131.1	162.0	106.7	25.0	25.0	25.0	25.0	25.0	25.0	50.1	25.0	25.0	0	0
6	0	0	0	0	0	0	0	0	20.0	20.0	20.0	36.1	20.4	20.0	20.0	0	0	0	20.0	20.0	20.0	20.0	0	0
7	0	0	0	0	0	0	0	0	0	25.0	25.0	25.0	25.0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	10.0	10.0	10.2	10.0	0	0	0	0	0	0	10.0	10.0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	10.0	10.0	0	0	0	0	0	0	0	10.0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	10.0	0	0	0	0	0	0	0	10.0	0	0	0	0

Table 7
Behaviour of generation cost for several values of ɣ (10-unit system)

ɣ	$E\{R\}$	$P_r\{RNS = 0\}$	Error (%)
0.0001	523,426.6	0.999722	1.447321
0.001	523,426.6	0.999722	1.447321
0.01	518,507.5	0.998472	0.493943
0.02	517,168.9	0.986806	0.234495
0.03	517,168.9	0.986806	0.234495
0.04	515,820.4	0.986111	-0.02686
0.05	512,424.4	0.949722	-0.68505
0.06	512,424.4	0.949722	-0.68505
0.07	512,424.4	0.949722	-0.68505
0.08	512,424.4	0.949722	-0.68505
0.09	512,424.4	0.949722	-0.68505
0.1	511,989.9	0.949583	-0.76927

Table 8
Behaviour of generation cost for several values of ɣ (50-unit system)

ɣ	$E\{R\}$	$P_r\{RNS = 0\}$	Error (%)
0.0001	2,680,318	1	4.847809
0.001	2,663,395	1	4.18582
0.01	2,633,176	1	3.003711
0.02	2,625,136	1	2.689185
0.03	2,617,850	1	2.404209
0.04	2,604,744	1	1.891506
0.05	2,600,582	1	1.728718
0.06	2,596,948	1	1.586557
0.07	2,595,354	1	1.524217
0.08	2,593,443	1	1.449456
0.09	2,587,955	1	1.234767
0.1	2,586,222	1	1.166966
0.2	2,574,595	0.999167	0.712145
0.3	2,565,285	0.9975	0.347964
0.4	2,548,150	0.795833	-0.32231
0.5	2,538,647	0.63	-0.69405

Table 9
Behaviour of generation cost for several values of ɣ (100-unit system)

ɣ	$E\{R\}$	$P_r\{RNS = 0\}$	Error (%)
0.0001	5,368,030	1	4.915172
0.001	5,321,134	1	3.998619
0.01	5,269,032	1	2.980316
0.02	5,258,307	1	2.770699
0.03	5,243,674	1	2.484709
0.04	5,214,841	1	1.921191
0.05	5,212,409	1	1.873658
0.06	5,203,711	1	1.703644
0.07	5,200,415	1	1.639233
0.08	5,194,545	1	1.524501
0.09	5,187,589	0.999167	1.388561
0.1	5,187,589	0.999167	1.388561
0.2	5,164,497	0.999167	0.937246
0.3	5,152,418	0.999167	0.701155
0.4	5,114,534	0.904167	-0.03926
0.5	5,084,910	0.665833	-0.61824