Domestic Appliances Energy Optimization with Model Predictive Control

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Abstract

A vital element in making a sustainable world is correctly managing the energy in the domestic sector. Thus, this sector evidently stands as a key one for to be addressed in terms of climate change goals. Increasingly, people are aware to saving electricity by turning off the equipment that is not been used, or connect electrical loads just outside the on-peak hours. However, these few efforts are not enough to reduce the global energy consumption, which is increasing. Much of the reduction was due to technological improvements, however with the advancing of the years new types of control arise. Domestic appliances with the purpose of heating and cooling rely on thermostatic regulation technique. The study in this paper is focused on the subject of an alternative power management control for home appliances that require thermal regulation. In this paper a Model Predictive Control scheme is assessed and its performance studied and compared to the thermostat with the aim of minimizing the cooling energy consumption through the minimization of the energy cost while satisfying the adequate temperature range for the human comfort. In addition, the Model Predictive Control problem formulation is explored through tuning weights with the aim of reducing energetic consumption and cost. For this purpose, the typical consumption of a 24 h period of a summer day was simulated a three-level tariff scheme was used. The new contribution of the proposal is a modulation scheme of a two-level Model Predictive Control's control signal as an interface block between the Model Predictive Control output and the domestic appliance that functions as a two-state power switch, thus reducing the Model Predictive Control implementation costs in home appliances with thermal regulation requirements.

Keywords: Model predictive control; Discrete Control; Sampled Systems; Thermal Modelling; Energy management;

Nomenclature

A_w	The wall area.
A_t	The surface area of the tank.
$\boldsymbol{A}$	The state (or system) matrix.
$\boldsymbol{B}$	The input matrix.
$\boldsymbol{C}$	The output matrix.
C_{in}	The thermal capacitance of the indoor air.

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C_{RFi}	The heat storage capacity of the refrigerator.
C_{wa}	The thermal capacitance of the water.
C_{wl}	The thermal capacitance of the wall.
h_o	The combined convection and radiation heat transfer coefficient.
k	Sampling instant and the current control interval.
$k+i$	The time instant associated to the future state prediction for i=1...N.
m	The water mass.
N	The prediction horizon.
P	The control horizon.
Q_{wa}	The water specific heat
Q_{ac}	The cooling power input to the room.
Q_{eg}	The heating element electric rated power.
Q_s	The heat flow into an exterior surface of the house subjected to solar radiation
T_{in}	The temperature of the room.
T_{RF}	The fridge internal temperature.
T_{RFe}	The inlet temperature of the evaporator's refrigerant.
T_{out}	The ambient temperature.
T_s	The wall surface temperature.
T_{wa}	The water temperature variable.
T_{wa_inlet}	The incoming water temperature.
T_{wl}	The wall temperature.
$r(k+i)$	The set point reference.
R_{wd}	The thermal resistance of the windows.
R_{RFw}	The thermal resistance of the wall insulation.
R_{RFwe}	The thermal resistance of the wall between the cabinet and the evaporator.
R_{wl}	The thermal resistance of the wall.
$S(t)$	A binary variable that emulates the turn-on and turn-off of the thermostat.
U	The overall heat transfer coefficient for the WH wall.
$u(k)$	The present input.
$u(k+i\|k)$	Future control signals for i=0...P-1.
U_{min}	The lower limit of the control signal.
U_{max}	The upper limit of the control signal.
$x(k)$	The state vector.
$y(k)$	The system output.
$y(k+i\|k)$	The estimated outputs.
Y_{min}	The minimum limit of future outputs.
Y_{max}	The maximum limit of future outputs.
$\Lambda(k)$	The two level input vector.
φ_u	The control signal tracking error.
$\varphi_{\Delta u}$	The equivalent cost function term that minimizes control signal increments.
φ_y	Optimization of the error due to the output reference trajectory.
φ_ε	The constraint violation performance index.
Ψ	Input vector.
ω_i^y	Weighting factor that allocates more relevance to the term.
ω_i^u	Weighting factor that allocates more relevance to the term.
$\omega^{\Delta u}$	Weighting factor that penalizes high differences between successive estimated input signals u_k
ρ_ε	A constraint violation penalty weight.
ε_k	A slack variable at control interval k.

ζ	A dimensionless controller constant.
η	The electrical resistance heating element efficiency.

Table of abbreviations

AC	Air Conditioner
BTU	British Thermal Units
HVAC	Heating, Ventilation and Air Conditioning
MPC	Model Predictive Control
TH	Thermostat
WH	Water Heater
QP	Quadratic Programming
RF	Refrigerator
SISO	Single-Input and Single-Output

1. Introduction

The energy consumption in buildings is accountable for roughly 33% of the entire energy use, thus, contributing to the global CO_2 emissions [1]. The present environmental circumstances require firm investigation concerning the energy efficiency and possible energy savings in the building sector. Consequently, many new projects are supported by policy makers and researchers in order to improve the energy efficiency [2], to intensify the energy production from renewable resources and reduce the greenhouse gas emissions [3]. In the residential sector the energy efficiency and savings is gaining more and more importance, enthused either by economic concerns or environmental reasons [4].

The space heating to improve thermal comfort in dwellings and workplaces seems to be particularly relevant. For instance, as of 2008, circa 50% of the total energy demand for heat generation was utilized with the purpose of space heating [5]. Until now most of the efforts to lower the energy in building and residential have been concentrated on studying alternative materials that could reduce heat loss in the construction itself or by improving the operation of the domestic appliances. In this sense, a good example have is the paradigm shift from classical incandescent light bulb to led technology with significant energy savings since energy conversion efficiency is much higher. Also, over the years the manufacture of appliances has been modernizing different aspects of the domestic devices operation. At the moment, variable speed drives are common in vacuum cleaners, washing machines or air conditioning units (HVAC). Moreover, Modern HVAC systems are introducing variable speed compressors which set a new level of efficiency and comfort [6]. Despite these continuous improvements, most of domestic applications for regulating temperature are still based on conventional control techniques as is the case of the bang-bang control that has been around for decades. The thermostatic technique, extensively used in home HVAC system, space heater, water heater or washing machines, has shown several drawbacks [7]. The thermostat maintenance limits are the same which does not take into account the house thermal characteristics, the weather where the house is located or the HVAC system efficiency rating level. In addition, for example, there is no method to assess the rise in outdoor temperature that could prevent a continuous injection of heat inside the room by the thermostat in order to avoid the house to get overheated. In sum, the conventional thermostatic technology is built with standard rules that may be adequate to some houses and HVAC systems but not to others.

In the past, any usage of computational power was prohibitive. But to now, low power and powerful microcontroller units (MCUs) at derisory prices are revolutionizing the embedded systems market [8]. Plenty resources for floating point operations based mathematical calculations opens new possibilities to implement advanced energy management techniques from appliances point of view. This means that for optimizing temperature regulation in terms of energy savings at maximum comfort, dedicated thermal model can be acquired and personalized according to the house, weather, HVAC systems and area occupancy rates. Such amount of computational capacity could also enable the inclusion of novel energy usage related user behaviour prediction tools to precise load matching without compromising the comfort of the user [9].

This signifies that a growing number of electronic appliances and devices in a typical dwelling create space for efficiency increases on energy consumption and combined operations can be made to tackle energy waste in dwellings [10]. A possibility is implementing new tariff policies related to demand response programs that assist the customer with the alteration of their electricity consuming behaviours [11]. An alternative method consists in modernizing the control equipment specifically the domestic appliances with controlled temperature.

Normally, in a usual dwelling, the equipment with the greater energy consumption and that tends to be the one that are constantly in operation throughout the day is the one that offers heating and cooling (i.e. water heater (WH), air conditioning (AC), and less significantly the refrigerator (RF)) [12]. By adopting energy efficiency measures there is room for real and tangible potential for energy savings that can reach up to 30% [13]. Consequently, one of the methods towards the goal of reducing the energetic demand is by modernizing the control technology that runs such types of home appliances. With the purpose of regulating the temperature the cooling and heating equipment utilize typical ON-OFF solutions. Given its low production price and that are so common the ON-OFF devices have proven to be the first choice by appliance manufacturers.

Currently, the Model Predictive Control (MPC) has been accepted by academics and industry as a very compelling method with solid theoretical foundations and proven capability to deal with a large number of control challenges[14]. This method is considered to be a broadly spread technology in industry intended for control design of very complex multivariable processes [15]. The MPC is a control method with the main goal to optimize a sequence of manipulated variable modifications influenced by a prediction horizon through the use of a process model with the purpose to optimize forecasts of process behaviour centred on a linear or quadratic objective, this objective being regulated by equality or inequality constraints [16]. On condition that the model is sufficiently precise and the constraints and performance index express correct and accurate performance objectives, then the MPC delivers a near-optimal execution. However, some cautions have to be taken since there is a necessity of a trade-off amongst complexity of the optimization and model accuracy – if the model is simpler, then easier will be to solve the optimization while if the model is more complex the optimization will require a higher computational power to optimize in a suitable time period [17].

As a part of MPC controller design process there is a need to set up the control objectives as a singular cost function. It is not likely to satisfy all the control objectives at the same time since usually the particular control actions have an increased significance over former actions. Consequently, performance compromises must to be reached among the contending control objectives. Thus, to the input and output variables of the process are associated specific weight factors, and as a result, enhancing the prioritization of the individual performance in order to carry out the control restricted by process constraints. Hence, the MPC utilizes distinct weights on tracing errors [18] and the attribution of control variables weight is called MPC tuning [19].

There are several benefits in using the MPC technique in building energy control such as several constraints from environmental, physical, and safety angles being able to be incorporated into the optimization problem; existing a methodical process to produce a dynamic building thermal model for the objective of predictive control; and the ability of the impact of disturbances, such as solar radiations and ambient temperature, to be handled in real-time [14]. On the other hand, the MPC has one major disadvantage. In order to solve an open-loop optimal problem a great volume of calculations needs to be performed at every sampling instant [16]. Therefore, the implementation of the controller demands for high computational capabilities in order to come up with a solution at every sampling instant [20].

This method has been subject of increasing popularity in different fields of application [21]. In the literature, various different MPC schemes have been developed aimed at the residential sector such as those that are utilized with the intention to improve the dwelling thermal comfort, decrease the peak load, and reduce the energetic expenses [22]. Generally, the MPC tool is intended for ventilation, heating, cooling and air conditioning (HVAC) equipment with the intention of minimizing the energetic expenses for more than just a simple reduction of the consumed energy [23]. In [24] is presented a MPC strategy with the purpose of improving the supply air temperature control of HVAC units by operating directly with the associated uncertainties and constraints. A MPC logic based on weather forecasts has been applied to the analysis of power management in a domestic off-grid system [25] and in a real-time building energy simulation [26]. Another implementation was the application of a MPC based thermal dynamics for office building energy management purposes [27] while in [28] a MPC strategy for energy efficient buildings with demand-response is proposed. A mixed-integer MPC for hybrid energy supply systems in buildings is analysed in [29].

In this paper a MPC scheme is proposed and its performance studied and compared to the thermostat with the purpose to minimize the cooling energy consumption through the minimization of the energy cost while satisfying the adequate temperature range for the human comfort. Three domestic appliances that are permanently connected to the grid are utilized for the MPC performance evaluation. These appliances convert electrical energy in thermal energy and regulated by a thermostat. The chosen appliances are the room temperature control by AC, the WH and RF. The novelty of the proposal is a power interface that modulates the MPC output command signal dynamic range into a discrete two level control signal. Thus, a linear power switch is not necessary between power and load. By proposing this modification in the MPC operation mode it is possible to limit the costs of its integration as a standard control solution for the studied domestic appliances.

The paper is organized as follows. The MPC formulation is depicted in Section II. Section III presents the domestic appliances under study and their thermal model and the testing methodology for MPC performance evaluation. Section IV contains the simulations and provides results discussion. Finally, the conclusions are summarized in Section V.

2. Model Predictive Control formulation

Any system to be controlled requires a sequence of control signals in the form of input vector $u \in \Box$ and the observation of the system response with the output vector $y \in \Box$. As the core of concept the MPC strategy incorporates a dynamic model of the system which is characterised by a state vector $x \in \Box$. Domestic loads are normally described by first-order differential equations which mean a linear state-space representation can be applied. The discrete-time MPC formulation requires the system to be represented by a discrete-time state-space equation.

$$x(k+1) = \boldsymbol{A}x(k) + \boldsymbol{B}u(k) \tag{1}$$

$$y(k) = \boldsymbol{C}x(k) \tag{2}$$

where the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are computed from the continuous –time state-space representation and $u(k)$ is the system input signal and $y(k)$ is the system output as the controlled variable at sampling instant k. The $u(k)$ control signal is normally identified as the manipulated variable. Based on current state, the dynamic model is used to forecast future system states $x(k+i|k)$ and outputs $y(k+i|k)$.

The control problem formulation methodology with MPC lies on time horizon prediction calculations. That is, the controller plays with future outputs and future control signals to keep to a feasible extent the system close to the set point reference. Finite control horizon for the future outputs is called prediction horizon N. The time frame implies that the set of future outputs are estimated at every sampling instant k. The estimated outputs are formulated as $y(k+i|k)$. Thus, k represents the current control interval and $k+i$ denotes the time instant associated to the future state prediction for $i=1...N$. Future manipulated variable $u(k+i|k)$, $i=0...P-1$, in which P is the control horizon, are estimated by a satisfying performance criterion algorithm. In other words, the input set calculated consist of the current input $u(k)$ and $P-1$ future inputs. These inputs are estimated in such way that calculated future outputs $y(k+i)$ allow the system to reach the set-point in an optimal form.

It should be noticed that despite a set of P inputs are determined at each sampling instant, just the first element is actually used for generating the control signal.

In order to satisfy the control objectives the calculation of future states, outputs, and manipulated variables are processed through a cost function which translates into a scalar cost value. By finding the minimum value of the cost function, the optimal $u(k)$ signal is extracted from the first element of the most suitable sequence of manipulated variables $u(k+i|k)$ according to the prediction timeframe used for the scenario.

From a control system point of view, the cost function consists of three standard terms, each one with a weight factor that penalizes the prediction variable effort. In other words, it establishes control objectives with an output penalty, an input penalty and an input rate penalty. Thus, the general expression for an objective function is:

$$\min_{U} \ \varphi(P,N) = \varphi_y + \varphi_u + \varphi_{\Delta u} + \varphi_\varepsilon \tag{3}$$

where φ_y optimizes the error due to the output reference trajectory, φ_u is the control signal tracking error, $\varphi_{\Delta u}$ minimizes control signal increments and φ_ε is associated to constraint violations. Since the cost function has a quadratic form, a quadratic programming *(QP)* solver generates an input vector Ψ solution as:

$$\Psi = \left[u(k|k)^T \ u(k+1|k)^T \dots u(k+P-1|k)^T \right] \tag{4}$$

In case of domestic appliances where only the temperature has to be controlled, a single-input and single-output (SISO) model is only necessary. Therefore, output variables number is limited to one. Then, the performance index for minimizing the tracking error is as follows:

$$\varphi_y = \sum_{i=1}^{N} \left\{ \omega_i^y \left[r(k+i|k) - y(k+i|k) \right] \right\}^2 \tag{5}$$

where $r(k+i|k)$ defines the set-point reference, $y(k+i|k)$ is the estimated output scaled by a weighting factor ω_i^y that allocates more relevance to the term.

Input signal tracking control objective is:

$$\varphi_u = \sum_{i=1}^{P} \left\{ \omega_i^u \left[u(k+i\,|\,k) - u_t(k+i\,|\,k) \right] \right\}^2 \tag{6}$$

where $u(k+i|k)$ is the control signal, $u_t(k+i-1|k)$ is the goal to be reached by the control signal. The difference error is multiplied by a weighting coefficient ω_i^u that gives more importance to this term.

Input signals wide variations are penalised to no allow abrupt changes on input variables. The equivalent cost function term is defined as:

$$\varphi_{\Delta u} = \sum_{i=1}^{P} \left\{ \omega_i^{\Delta u} \left[u(k+i\,|\,k) - u_k(k+i-1\,|\,k) \right] \right\}^2 \tag{7}$$

where $u(k+i-1|k)$ is the input signal from the previous sampling instant $k-1$ and $\omega^{\Delta u}$ penalizes high differences between successive estimated input signals u_k.

Constraint violation performance index is formulated as:

$$\varphi_\varepsilon = \rho_\varepsilon \, \varepsilon_k^2 \tag{8}$$

where ρ_ε is a constraint violation penalty weight and ε_k is a slack variable at control interval k.

Weights ω_i^y and ω_i^u must be tuned to guarantee the system performance desired. For instance, giving more importance to weight ω_i^y in preference to the weight ω_i^u, the controller goal is to estimate successive sets of future outputs that minimize the predicted divergences from the set point reference. On contrary, if ω_i^y is reduced, then the gap between the reference tracking to the plant output is going to rise.

MPC can be implemented considering constraints in the minimization problem. That is, fixing bounds in the amplitude and in the slew rate of the variables, the controller forces the system operation to respect physical operational limits.

Therefore, the formulation of a quadratic programming based constrained MPC is given by the Eq. 3 and the following constraint expressions:

$$Y_{min}(i) - \varepsilon_k \zeta_{min}^y(i) \le Y(k+i\,|\,k) \le Y_{max}(i) + \varepsilon_k \zeta_{max}^y(i),\ i = 1:N \tag{9}$$

$$U_{min}(i) - \varepsilon_k \zeta_{min}^u(i) \le U(k+i-1\,|\,k) \le U_{max}(i) + \varepsilon_k \zeta_{max}^u(i),\ i = 1:P \tag{10}$$

where Y_{min} and Y_{max} are the minimum and maximum limits of future outputs, the parameter ζ is a dimensionless controller constant and the lower and upper bounds for the control signal are represented by U_{min} and U_{max}, respectively.

3. Models and control problem formulation

Linear models are introduced here with the purpose to model the appliances. Then, a modulation scheme of a two-level MPC's control signal is proposed as the novelty of this study.

3.1 Thermal models for domestic appliances

A description of the thermal modelling approach for three domestic appliances which performs thermal regulation is made in this section.

a) Indoor environment temperature control

To create thermally comfortable indoor environments such as living rooms or bedrooms energy has to be used to add or remove heat. In this way the comfort level desired is set by setting a reference temperature and by measuring the space air temperature. Comfort level based on temperature is disturbed by the thermal mass of the space itself, the number of occupants that use the house division and the thermal exchange through the external walls from the external environment. Therefore, the temperature dynamics of a house division results from energy balances between the outside environment temperatures, the device (AC) that adds or remove heat from the division combined with the indoor thermal mass as depicted in Fig. 1.

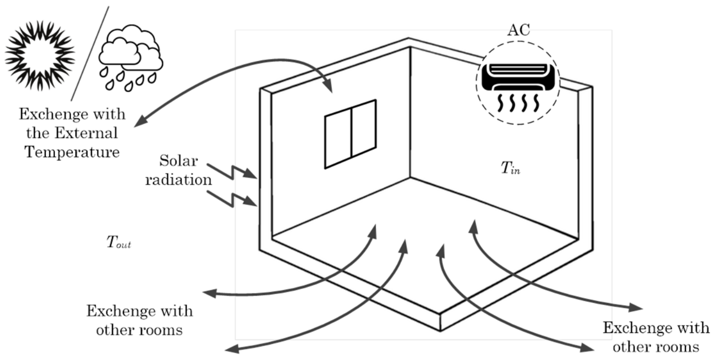

Fig. 1 – Indoor environment temperature control.

In order to evaluate and compare controller's performance a thermal mass model using a resistance-capacitance circuit analogy is employed. The model comprises the heat flow balance between the external wall and windows of a house division and the thermal capacitance regarding the indoor air. The following expressions were derived from [30]:

$$\frac{dT_{wl}}{dt} = \frac{Q_s}{C_{wl}} + \frac{T_{in} - T_{wl}}{R_{wl} C_{wl}} \tag{11}$$

$$\frac{dT_{in}}{dt} = \frac{Q_{ac} \times S(t)}{C_{in}} + \frac{T_{out} - T_{in}}{C_{in} R_{wd}} + \frac{T_{wl} - T_{in}}{C_{in} R_{wl}} \tag{12}$$

$$Q_s = \mathrm{A_w} h_o (T_{out} - T_s) \tag{13}$$

where Q_{ac} is the cooling power input to the room, T_{out} is the ambient temperature, T_{in} is the room's temperature, T_{wl} is the wall temperature, C_{wl} is the thermal capacitance of the wall, R_{wl} is the thermal resistance of the wall, R_{wd} is the thermal resistance of windows, C_{in} is the thermal capacitance of the indoor air, Q_s is the heat flow into an exterior surface of the house subjected to solar radiation, h_o is the combined convection and radiation heat transfer coefficient, A_w is the wall area, T_s is the wall surface temperature and $S(t)$ is a binary variable that emulate the turn-on and turn-off of the thermostat. The AC operation is a power switch block without internal loss. The value of the physical parameters is obtained from [31].

b) Water heater

Based on the energy flow inside it, the WH model utilised in this paper was generated to acquire the electric power demand of the WH. Generally, the electric energy consumed by the WH is utilised to satisfy the following goals: the compensation for the thermal losses from the WH tank to the ambient and the heating of the inlet cooled water that substitutes the heated water extracted from the tank. In the WH model used for this study the heated water is assumed to be entirely homogeneous inside the storing container. As a result, the temperature is also expected to be homogeneous in the tank and to be the only hot water temperature variable utilised in the WH model for this paper – T_{wa}. The model of WH follows the method presented in [32]:

$$\frac{dT_{wa}}{dt}=\frac{\rho Q_{wa}(T_{wa\_inlet}-T_{w})}{C_{wa}}-\frac{UA_t(T_{wa}-T_{amb})}{C_{wa}}+\frac{Q_{eg}\eta}{C_{wa}} \tag{14}$$

where ρ is the water density, Q_{wa} is the water specific heat, T_{wa_inlet} is the incoming water temperature, C_{wa} is the thermal capacitance of water, U is the overall heat transfer coefficient for the WH wall, A_t is the surface area of the tank, Q_{eg} is the heating element electric rated power and η the electrical resistance heating element efficiency. Parameters for model simulation were taken from [31].

c) Refrigerator

RFs are very familiar home appliances, existent in virtually every house which signifies that most people utilize them every single day. The fridge internal temperature is represented by T_{RF} and is intended to describe the system behaviour and to be suitable for control. T_{RF} is elevated by the ambient temperature ($T_{out} \geq 0°C$) and lowered by the inlet temperature of the evaporator's refrigerant ($T_{RFe} < 0°C$). The heat transfers are delimited by the thermal resistance of the wall between the cabinet and the evaporator (R_{RFwe}) and the thermal resistance of the wall insulation (R_{RFw}). Besides, the behaviour of the RF is additionally influenced by its heat storage capacity (C_{RFi}). The thermal dynamic behaviour is approximated with the equations (15-16) where for the warm-up (15) and the cool-down (16) phases are represented as follows [33]:

$$\frac{dT_{FRi}}{dt} = -\frac{T_{FRi}}{C_{FRi}R_{FRw}} + \frac{T_{out}}{C_{FRi}R_{FRw}} \tag{15}$$

$$\frac{dT_{FRi}}{dt} = -\frac{T_{FRi}}{C_{FRi}\left(R_{FRw}R_{FRwe}\right)} + \frac{T_{out}}{C_{FRi}R_{FRw}} + \frac{T_{FRe}}{C_{FRi}R_{FRwe}} \tag{16}$$

3.2 MPC design for domestic load control

a) Proposed architecture

The implementation of the MPC is represented in Fig. 2. Typically, the MPC composition is made of the blocks prior to the 2-level signal modulator block. The proposed enhanced version of the MPC has an additional signal processing block represented by the grey area in Fig. 2.

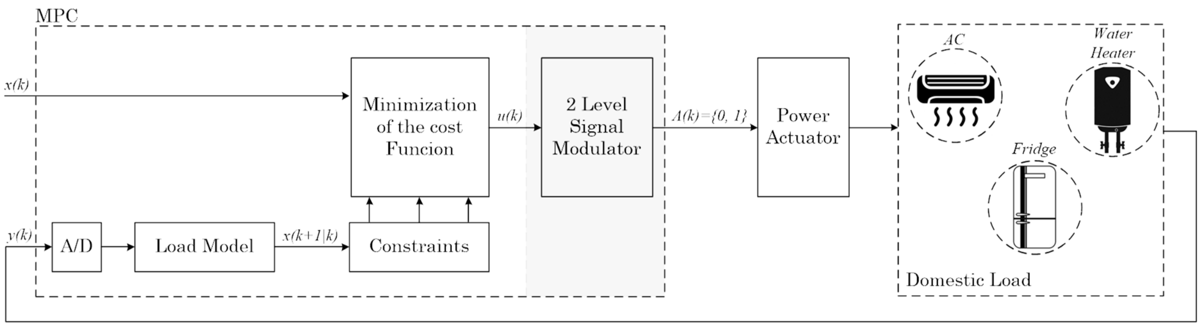

Fig. 2 – The general view of the 2-level control signal operated MPC.

b) Two level signal modulator

Any signal to be translated to actuator device is subject to the physical constraints of the device itself in terms of output. A conventional actuator can have a linear response; however, it is limited by lower and upper bounds. From a classic design view point the manipulated variable response is constrained according to the actuator physical constraints. In order for the adoption of MPC as an alternative controller to be affordable all potential costs have to be minimised. Consequently, it should be noticed the actuator itself is an expensive part. Thus, linear power management requires an adequate power switch. For instance, solid state relays that are good choices still have prohibitive costs for the simple domestic appliance control. Rather than using a linear power switch, in this paper a two-level control signal interface is proposed that modulates the limited continuous set of manipulated variables to a discrete set of integers. The two-level input vector *Λ(k)* is given by:

$$\Lambda(k) = \{0,1\} \tag{17}$$

In order to code *u(k)* signal dynamic range into a 2-discrete power control signal, a simple comparison operator is proposed which has the following operating behaviour:

$$\Lambda(k) = \begin{cases} 1 & \dfrac{U_{\max}}{2} < u(k) \le U_{\max} \\ 0 & 0 < u(k) \le \dfrac{U_{\max}}{2} \end{cases} \tag{18}$$

The manipulated variable dynamic range is divided in two parts. All *u(k)* values below half of its dynamic range is processed by the 2-level modulation scheme as 0. Thus, the power delivered to the domestic load is zero. On the other hand, if the optimized *u(k)* signal is on the top half of the dynamic range the domestic load receives full power. In Fig. 3 is shown the use of the proposed two level power control modulation scheme.

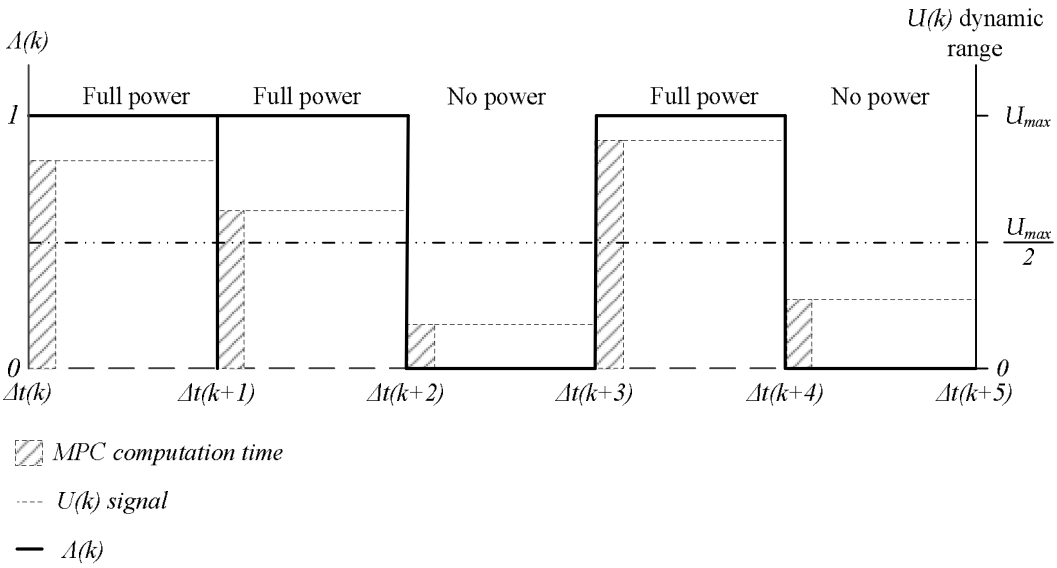

Fig. 3 – Two level modulation operation technique.

4. Simulation and Results

The testing framework consists of three domestic appliances commonly found in residences whose function is to provide heating and cooling services: WH, RF and AC. This set of loads where chosen since their use has a daily frequency or in many cases even hourly, while other appliances, even though consume more energy, are not utilised so frequently.

The conventional thermostatic control serves as a reference to the MPC evaluation. A daily three level tariff scheme was utilised for this study. The assessment of the energy cost is based on the prices practiced in the Canadian residential market and is utilized throughout a period of 24 hours. The MPC controller is explored with two different weighting sets in order to evaluate the impact on electric bill reduction goal. As the calculation time horizon, *P* control moves number is set to 4 and 12 is the set of *N* predicted outputs.

4.1 Case Study

In this section the characteristics of the domestic loads used in the simulation are described.

a) Air conditioner

The acclimatization of the room is provided by AC system having a cooling capacity of 8900 BTUs (2.608kW). Heat exchange with the external environment through the external wall of the room, it is the main factor of disturbance to maintain the internal temperature in thermal comfort level desired. In order to test both control strategies, the rate of heat loss/generation through the external wall of the room is modelled using a temperature based time series with significant wide thermal amplitude variation upon 24 hours corresponding to a summer day. The TH device is configured with a setting of +/1º referred to a temperature of 23ºC. Constraints on MPC operation are valid for the same temperature range while the main focus is to minimise the power consumption. Thus, the temperature control objective for the desired temperature range is defined with soft constraints between 22.5ºC and 23.5ºC. The purpose of softening the constraints is to permit the controlled variables to violate their constraints by modest amounts, while in the case of hard constraints no violations are allowed [34]. The main simulation parameters are the following: the sampling time ΔT is 15min and prediction horizon size is 48.

b) Water heater

The WH unit heats the water to be used on personal hygiene activities by the house habitants. Hot water consumption has a peak-hour at early on the morning and at evening before the sleeping period. Thus, temperature regulation system must preserve the water hot enough during those peak-periods.

The heating element inside of WH is rated at 4.5kW and 184 L is the reservoir capacity of the unit. The TH set point (SP) is set to 55ºC with a hysteric range of +/-1.5 ºC. The same temperate fluctuation band is adopted for MPC configuration. WH external wall temperature is fixed at 23 ºC. The main simulation parameters are: the sampling time ΔT is 5 min and prediction horizon is 144.

c) Refrigerator

The temperature of the interior is normally regulated by thermostatic relay. Opening the RF's door increases the energy consumption to recover the previous internal temperature setting. The conventional control is compared to MPC alternative considering a RF with a compressor's electric motor rated at 0.23kW.

The MPC system is set up to preserve the internal temperature between 3.9ºC and 5.1ºC. Disturbing events are recreated with two door opening closing sequences, which are simulated at 10-11 pm and at 14-15pm respectively. The main simulation parameters are the following: the sampling time ΔT is 15min and prediction horizon size is 48.

4.2 Transient response characterization of the controllers

This section presents the essential outcomes by comparing two sets of MPC weights on controller performance versus the thermostatic relay response.

a) Air conditioner controlled temperature

The performance of the MPC technique is presented in Fig. 4, where two different weights set are employed to tune the controller and compared to the thermostatic approach. Data shown is related to the AC energy consumption, room internal temperature and environment temperature. As expected the room temperature when regulated by the thermostat shows a maximum and minimum deviation about the set-point, dictated by thermostatic hysteric characteristic. On the other hand, room temperature profile is more erratic with the AC unit actuated by a MPC type controller. By applying different weighting set to the MPC controller, it can be seen that one of MPC weight set overpass the higher limit of the temperature regulation range, although the deviation is very small. In terms of temperature variation, the MPC shows lower amplitude.

b) Water heater

In Fig. 5 one of the MPC weight set clearly worse the MPC performance since the temperature evolution does not respect the input constraint. In fact, temperature constraint violation can surpass 1ºC. In another period of the day, the same weight set denotes again some visible deviation.

c) Refrigerator

The simulation in Fig. 6 points out that the TH controller confines easily the successive disturbances impact, due to its hysteric nature. That is, in the first disturbance event which consists of opening the fridge's door several times in a short amount of time, the controller performs a sequence of opening and closing of the switch associated to the TH. As for the next disturbance with the door kept open for a longer time, the refrigerator consumes additional energy to overcome internal cold air loss. In this simulation scenario both MPC weight sets lead to similar regulation responses. In addition, in both tuning sets when the second disturbance arrives, the performance response is insufficient, allowing the temperature rise observed in Fig. 6.

4.3 Energy consumption and electric bill savings

Tables I, II and III gather economic and electric nature data to characterize energy usage efficiency as function of the controller type employed. The energy costs associated to each time frame tariff of the day are also illustrated. One can see at Table I and Table II that MPC weight set 2 enables higher energy consumption reduction in relation to MPC weight set 1, despite its poorer performance in regulating the temperature according to the output constraint. Consequently, the second controller tuning set presents the lowest energy bill.

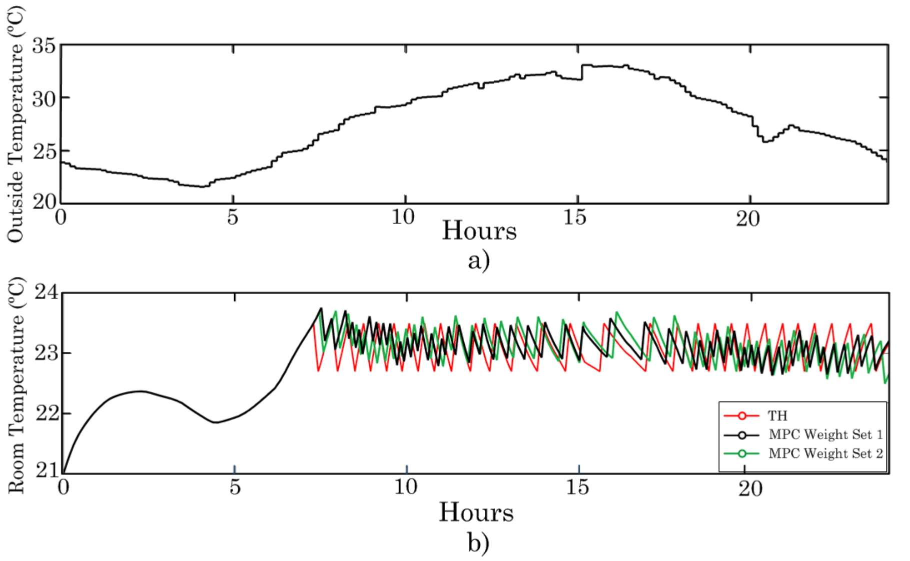

Fig. 4 – Air conditioner operation: a) Ambient temperature b) TH and MPC responses.

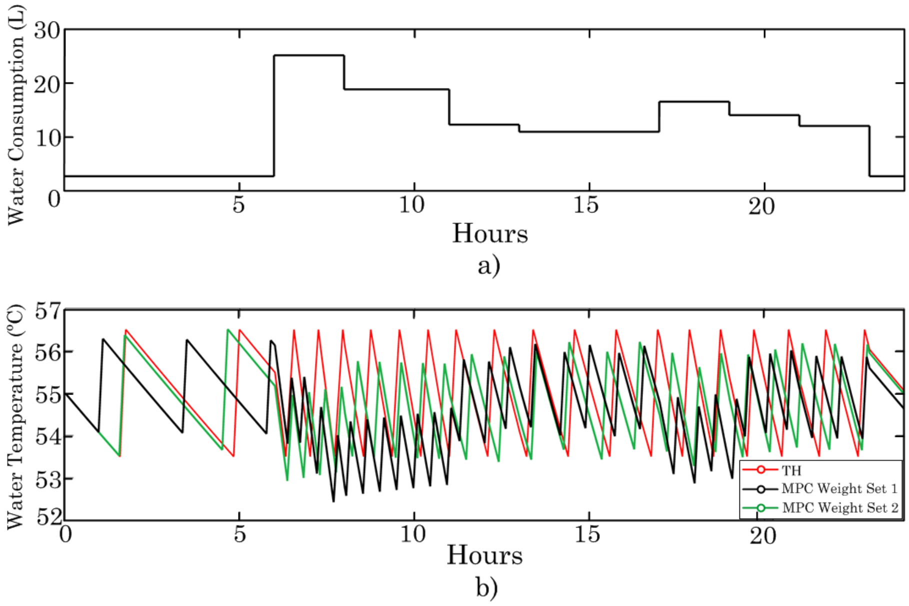

Fig. 5 – Water heater operation: a) Water consumption b) TH and MPC responses.

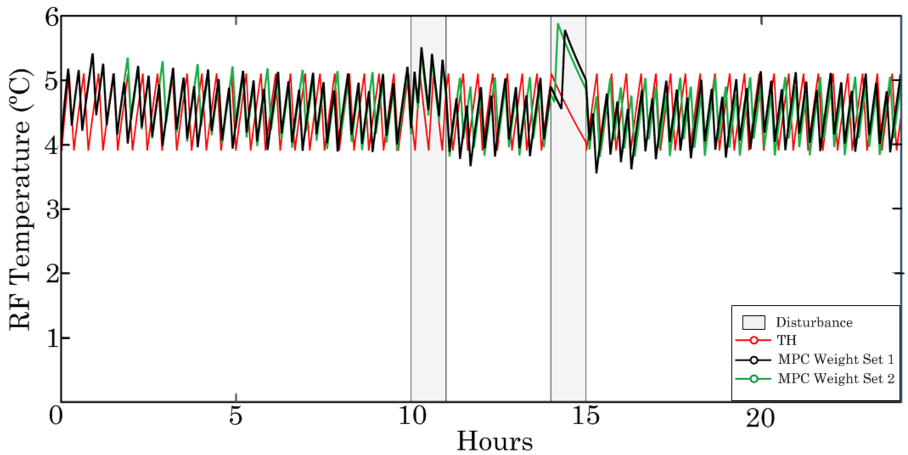

Fig. 6 – Refrigerator operation: TH and MPC responses.

Table 1 - Air Conditioner

	Thermostat		MPC Weight Set 1		MPC Weight Set 2	
	Energy (kWh)	*Cost ($)*	*Energy (kWh)*	*Cost ($)*	*Energy (kWh)*	*Cost ($)*
Off-Peak	5.005	0.310	5.065	0.314	5.012	0.311
Mid-Peak	8.417	0.774	8.546	0.786	8.441	0.777
On-Peak	12.934	1.397	12.661	1.367	12.661	1.367
Total	26.356	2.482	26.272	2.468	26.114	2.455

Table 2 - Water Heater

	Thermostat		MPC Weight Set 1		MPC Weight Set 2	
	Energy (kWh)	*Cost ($)*	*Energy (kWh)*	*Cost ($)*	*Energy (kWh)*	*Cost ($)*
Off-Peak	5.919	0.367	6.278	0.389	6.480	0.402
Mid-Peak	6.740	0.620	6.975	0.642	6.480	0.596
On-Peak	4.939	0.533	4.185	0.452	4.320	0.467
Total	17.598	1.521	17.437	1.483	17.280	1.465

Table 3 - Refrigerator

	Thermostat		MPC Weight Set 1		MPC Weight Set 2	
	Energy (kWh)	*Cost ($)*	*Energy (kWh)*	*Cost ($)*	*Energy (kWh)*	*Cost ($)*
Off-Peak	0.824	0.051	0.828	0.051	0.840	0.057
Mid-Peak	0.504	0.046	0.483	0.044	0.473	0.045
On-Peak	0.549	0.059	0.552	0.060	0.550	0.064
Total	1.878	0.157	1.863	0.155	1.863	0.156

For these two domestic appliances, in both tuning sets loaded on the MPC controller, the cost of the energy consumed is lower than the appliance controlled TH. As there are three distinct tariffs during the 24h time frame, the goal is to diminish the energy cost for the period of ON peak hours. This is true for the AC and WH appliances.

On the other hand, the same tuning set 2 in the case of the refrigerator the electricity bill is slightly higher, as can be verified in Table III. Nonetheless, the energy cost computed continues to be lower than the conventional solution based on TH control. It is now evident that to reduce the energy consumption using a MPC scheme type, the controller parameters values choice must be selected through a tuning procedure to achieve a good performance. However, to achieve this goal a penalizing effect may prevent to fulfil the constraint conditions.

Finally, in Fig. 7, Fig. 8 and Fig. 9 the total energy costs relationship to energy consumption profile are shown for each appliance. In the case of both AC and WH the electricity bill reduction is aligned with the energy usage linearly which does not occur in the case of the RF.

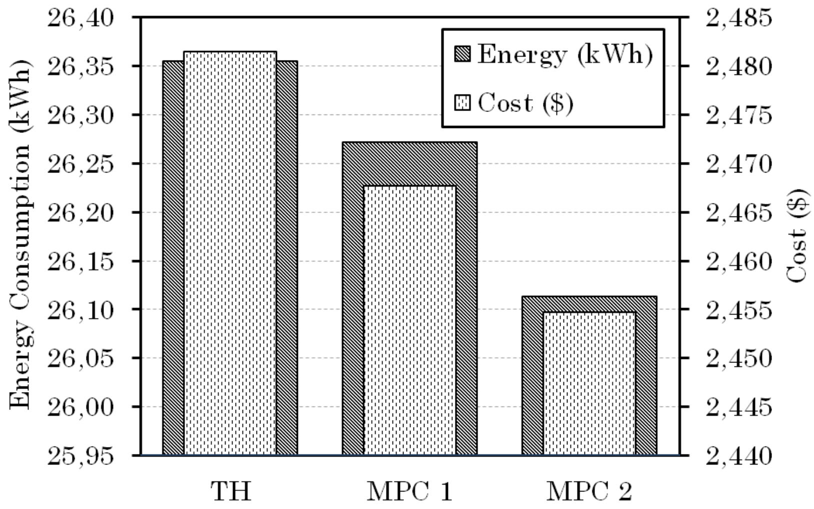

Fig. 7 – AC: Energy consumption vs energy costs.

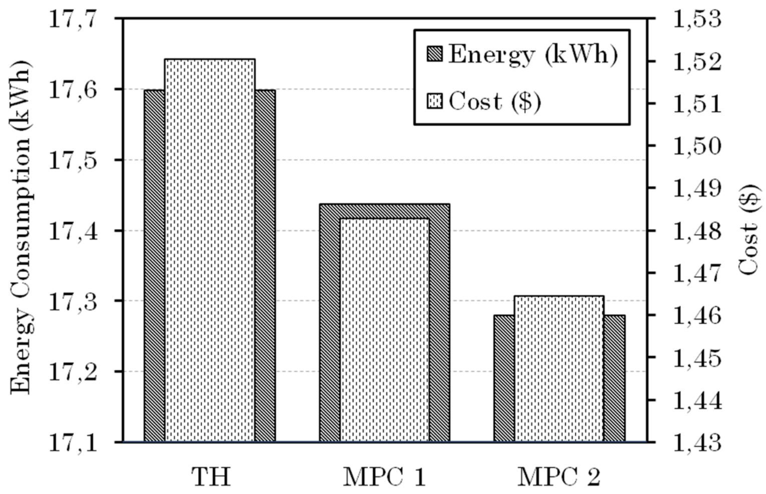

Fig. 8 – WH: Energy consumption vs energy costs.

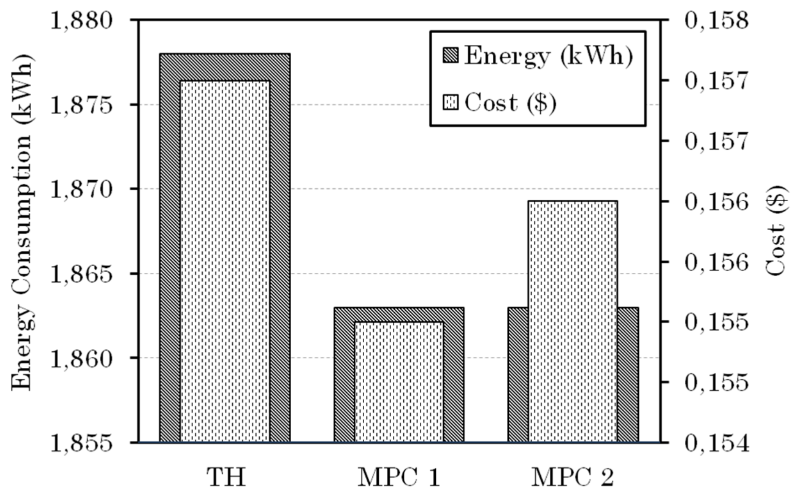

Fig. 9 – RF: Energy consumption vs energy costs.

4.4 Varying the MPC weights

In this section, several simulations are made by adjusting different MPC weights for several possible scenarios and estimate its consequences with the purpose of minimizing the amount of energy that is spent on the various appliances used within dwellings. For every electrical load the energy, the temperature variation and the cost are assessed and several results are presented by varying the different MPC weight combination. The weight tuning allows finding a set of coefficients to maximize the MPC performance regarding the minimization of the energy consumption. All the combinations of the weights ω^y and ω^u were tested between 0 and 1 with 0.05 of resolution.

a) Air conditioner

Several results were obtained regarding the AC operation by running the model and adjusting the MPC weights vs the energy output, energy cost and temperature. Depictions of the energy profile of the AC run with the MPC can be observed in Fig. 10. In this case the variated arrangement concerning the weights only produces values near the frontier. However, by the reason of existing 3 distinct electricity tariffs throughout 24h, the attempt is to decrease as much as possible the cost of the energy throughout the ON peak hours. The specific arrangement of weights shows advantages for the dwelling owner through the reduction of the energy consumption in the considered period of one day as can be observed the Fig. 11. Hence, the goal of keeping the temperature between the limits is also achieved as depicted by the Fig. 12.

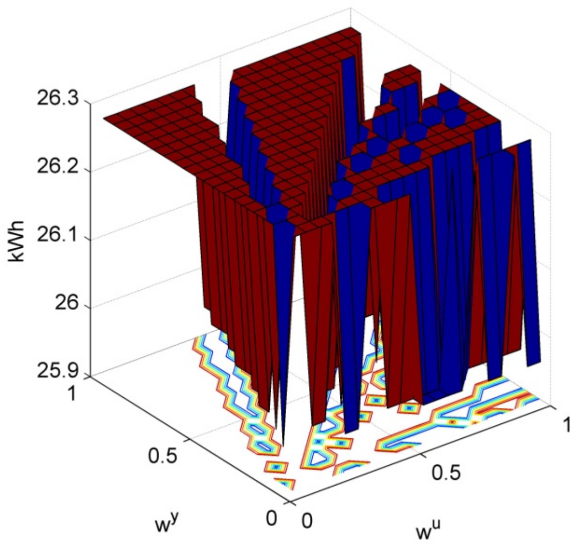

Fig. 10 – AC: Energy output vs weights tuning.

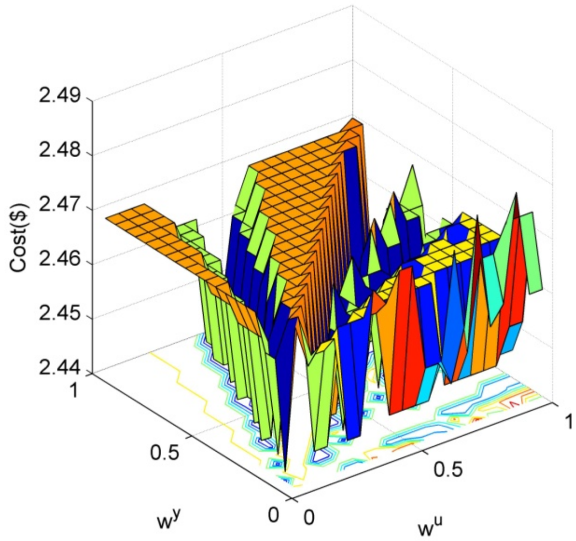

Fig. 11 – AC: Energy cost vs weights tuning.

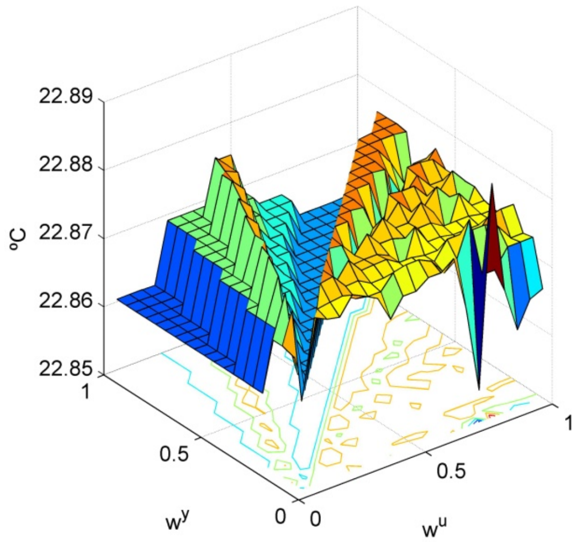

Fig. 12 – AC: temperature vs weights tuning.

b) Water heater

Four levels of energy consumption exist in the simulation made for the energy output vs weights tuning and are shown in Fig. 13. In the case of a lower limit of energy consumption the weights' arrangements demand for the weight tuning of the model output to be adjusted with a decreased value despite the fact that no restrictions exist for the tuning of manipulated variable weight. However, in Fig. 14 the same tendency as the previous figure is not depicted, in this case the observed peak in the energy cost is a consequence of the price tariff effect. The requirement of the regulation is fulfilled as can be observed in Fig. 15.

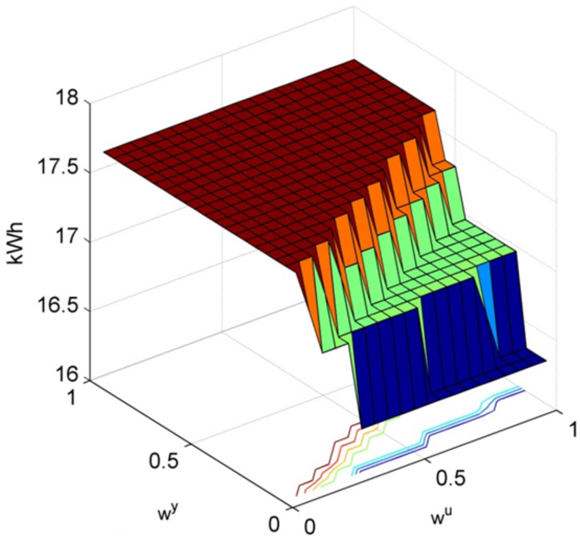

Fig. 13 – WH: Energy output vs weights tuning.

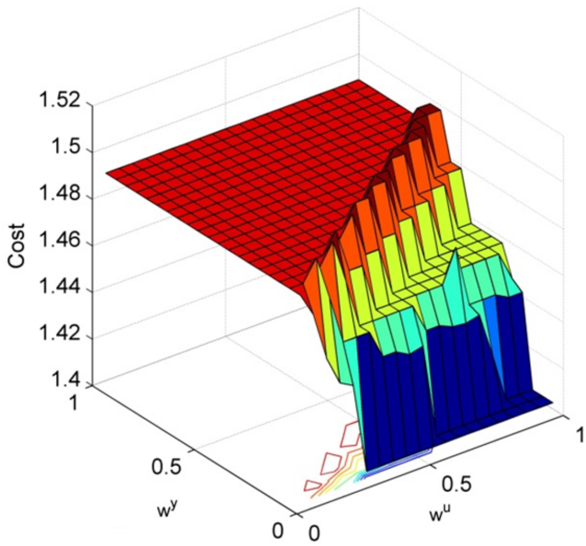

Fig. 14 – WH: Energy cost vs weights tuning.

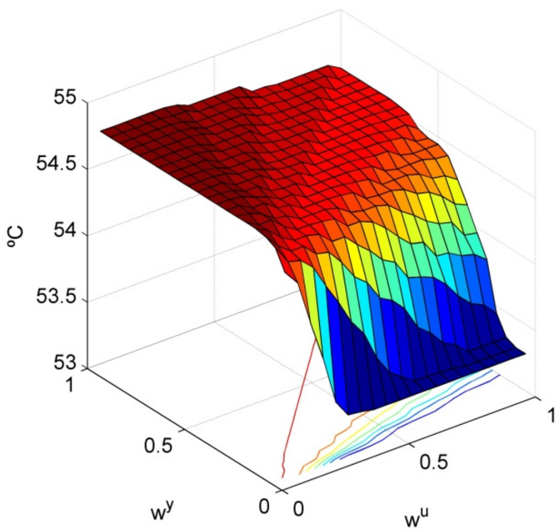

Fig. 15 – WH: Water temperature vs weights tuning.

c) Refrigerator

Contrasting with the aforementioned home appliances, the results of the simulation in the case of the refrigerator indicate that the lowest level of energy is obtained by setting to 0 the weight of the output of the system when two weights are only acceptable on the manipulated value side. Such results can be observed in Fig. 16. For energy cost vs tuning of weights in the case of the refrigerator the highest level of the energy cost coincides with the highest level of the energy output as can be observed in Fig. 17. Through the use of the MPC it can be observed that in the preceding cases the condition limits were followed. However, this did not occur in the inside temperature case as observed in Fig. 18.

In the case of the refrigerator can be noticed that having a goal to reach for the energy consumption minimum bears a negative consequence in the aim of keeping the temperature in the desired defined limits.

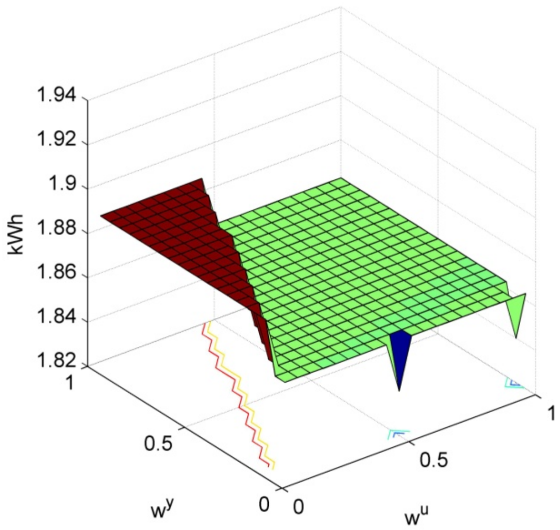

Fig. 16 – Refrigerator: Energy output vs weights tuning.

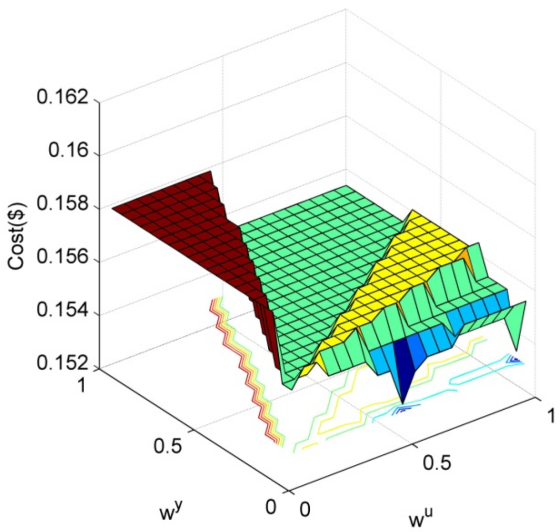

Fig. 17 – Refrigerator: Energy cost vs weights tuning.

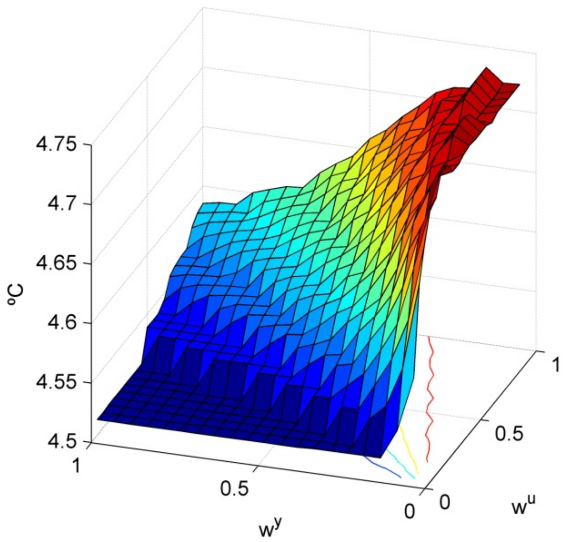

Fig. 18 – Refrigerator: Inside temperature vs tuning of weights.

5. Conclusion

This paper presented a study concerning the adoption of an alternative control strategy in thermostat operated cooling and heating domestic equipment in the households. The MPC technique was investigated to assess its capability to improve energy consumption efficiency with the goal of reducing electric bill. Rather than using a linear power switch, in this paper is proposed a two-level control signal interface that modulate the bounded continuous set of manipulated variables to a discrete set of integers, as a new contribution to earlier studies. Three typical domestic loads were utilized as case studies. The MPC performance was explored by tuning the controller with two different weight sets and compared to a thermostat control. The simulation results made clear that there was reduction on the consumed energy when the thermostatic regulation was replaced by the MPC. Therefore, the MPC based thermal regulation had a positive impact of circa 2% on the energy bill reduction. Also, the two MPC weight sets have proven that it is necessary to adjust the controller weights in order to maximize the potential of energy cost savings. The results of the simulation by varying the MPC weights indicated that the studied appliances need a particular tuning of weights with the purpose to decrease the consumption of the energy to the lowest possible limit. The results indicate that by involving a multi-tariff structure the costs' curve is considerably modified when a comparison is made to the curve of the energy. Thus, it is enough to follow the energy curve in order to estimate the tuning of the MPC weights. However, using the MPC does not always have positive results. In the case of the RF it can be noticed that having a goal to reach for the energy consumption minimum bears a negative consequence in the aim of keeping the temperature in the desired interval for the human comfort.

Acknowledgements

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