A Probabilistic Approach to Solve the Economic Dispatch Problem with Intermittent Renewable Energy Sources

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Abstract

In this paper, a methodology for solving the economic dispatch (ED) problem considering the uncertainty of wind power generation and generators reliability is presented. The corresponding probability distribution function (PDF) of available wind power generation is discretized and introduced in the optimization problem in order to probabilistically describe the power generation of each thermal unit, wind power curtailment, energy not supplied (ENS), excess of power generation, and total generation cost. The reliability of each unit is incorporated by estimating the joint PDF of power generation and failure events, while the PDF of ENS is incorporated by convoluting the PDF of ENS due to the forecasting error and any failure event. The performance of the proposed approach is analyzed by studying two power systems of 5 and 10 units. The proposed method is compared to Monte Carlo Simulation (MCS) approach, being able to reproduce the PDF in a reasonable manner, specifically when system reliability is not taken into account.
Keywords: Economic dispatch problem; greenhouse gas emissions; power system reliability; wind power forecasting error; probability distribution function.

Nomenclature

AWP^k	Discrete PDF of available wind power generation.
A_m, B_m, C_m	Parameters of cost curve of unit m.
DL_i^k	Value of the power consumed by the dump load at time k that corresponds to the sampling point i.
DL^k	Dump load at time k.
DR_m	Ramp down limit of unit m.
D^k	Load demand at time k.
$E_{(l,m)}$	Discrete PDF of power production when generators reliability is considered.
ENS_i^k	Value of energy not supplied at time k that corresponds to the sampling point i.
F_h^m	Discrete PDF of lack of power of unit m as a consequence of a failure event.
FOR_m	Forced outage rate of unit m.
F_b^e	CDF of power loss as a consequence of a failure in the generator system.
GHG_m	CO_2 emissions of unit m.
$NP_r\{\cdot\}$	Normalized probability of occurrence of a determined event.
P_h	Power value that corresponds to the discrete state h.
P_b	Power value that corresponds to the discrete state b.
$P_{m,i}^{k-1}$	Power production of unit m at time $k-1$ that corresponds to the sampling point i.
P^{max}	Maximum power value to be considered.
P^{min}	Minimum power value to be considered (assumed to be zero).
P_m^k	Discrete PDF of power production of unit m at time k.
P_m^{max}	Maximum output power of unit m.
P_m^{min}	Minimum output power of unit m.
$P_r\{\cdot\}$	Probability of occurrence of a determined event.
UR_m	Ramp up limit of unit m.
U_m, V_m	Parameters of the CO_2 emission curve of unit m.
W^k	Discrete PDF of wind power generation.

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W_{max}^{k}	Maximum value of available wind power generation at time k.
W_{min}^{k}	Minimum value of available wind power generation at time k.
X_m	Parameter of the CO_2 emission curve of unit m.
a_0 to a_3	Auxiliary variables.
awp_j^k	Value of available wind power generation of discrete state j at time k.
b_m	Discrete state that corresponds to the rated capacity of unit m.
s_r	Value that corresponds to the discrete state r.
w_j^k	Value of wind power generation of discrete state j at time k.
$z_{i,j}$	Total generation cost that corresponds to the sampling point i and the discrete state of available wind power generation j.
θ_i	Sampling point I of the interval $[\gamma, 1-\gamma]$.
h	Discrete state of power production ($h \epsilon [0, H]$).
B	Total number of bins of discrete PDF of power production.
H	Last state of h ($H = B - 1$).
I	Total number of sampling points of interval $[\gamma, 1-\gamma]$.
J	Total number of bins of the discrete PDF of wind power generation.
L	Last state of l ($L = (H+1)^2 = B^2$).
M	Total number of thermal units.
R	Last discrete state of beta PDF.
$VOLL$	Value of lost load.
$VOWE$	Value of wasted energy.
b	Discrete state of power production $b \epsilon [1, B]$.
i	Index of sampling point θ_i, $i \epsilon [1, I]$.
j	Discrete state of available wind power generation.
l	Discrete state of power production when generators reliability is considered.
m	Index for each generation unit.
r	Discrete state of beta PDF in the interval [0,1], $r \epsilon [0, R]$.
ΔP	Discretization step of the power values P_b.
$\Delta\theta$	Sampling increment of interval $[\gamma, 1-\gamma]$.
α, β	Parameters of continuous beta PDF.
γ	Significance level.
σ	Parameter of the discretization process.

Table of abbreviations

ESS	Energy Storage System
DRP	Demand Response Program
ARMA	Auto-Regressive Moving Average
UC	Unit Commitment
ED	Economic Dispatch
PDF	Probability Distribution Function
PSO	Particle Swarm Optimization
ENS	Energy Not Supplied
MCS	Monte Carlo Simulation

1. Introduction

Energy obtained from renewable energy sources has a key role to the sustainable development in the near future. Wind and solar energies have been continuously growing motivated by governmental incentives, the reduction in the operating and capital costs, and the increment in the revenue streams. Because of these conditions, the energetic policy is based on the increment of renewable power penetration. As a result, it is expected that in the year 2040, renewable generation is going to represent about 16% of total generation capacity in the United States.

Natural gas is going to be another important resource for power generation due to the expected reduction in market prices. In fact, it is likely that natural gas will become the main source of power generation in the United States in 2040, substituting the power capacity provided by coal-fired and nuclear power plants, sharing about 43% of the total generation capacity. This generation mix mainly composed by natural gas and renewable energies as the main power sources is going to lead to an important reduction in CO_2 emissions, reaching a decrease of about 11% from the emission levels of the year 2012 [1].

However, the variability related to the renewable energy sources and the difficulties related to storing energy represent important limitations in massive deployments of renewable sources to fully supply peak-load and base-load. To deal with the problems related to the stochastic nature of renewable energy sources, many approaches have been proposed, such as the analysis of geographic properties of aggregated wind power generation [2], the optimal management of energy storage systems (ESSs), implementation of demand response programs (DRPs) [3, 4], and improvements in scheduling techniques in order to incorporate the wind power uncertainty by means of their corresponding forecasting error.

Analyzing the geographic characteristics of the place to locate a determined wind farm in order to connect it with other ones and smooth the aggregated power production could require an additional investment that affects the profitability of the project [5]. Moreover, economic viability of a determined technology of ESS depends on the renewable penetration level and its variability, the regulatory environment, and the revenues in yearly bases [6]. The main barrier for the implementation of DRPs is related to the uncertainty in people's behavior when the electricity prices are dynamically changed. This uncertainty is reflected in the estimation of price elasticity, which is frequently used to decide the optimal use of demand response resources [7].

As a result, several approaches have been presented in the technical literature, such as stochastic programming, chance constrained programming, stochastic dynamic programming, robust optimization, and probabilistic approaches.

Stochastic programming approaches consist on carrying out the optimal management taking into account some possible situations or scenarios randomly generated. In our case, these scenarios represent the stochastic behavior of load demand, wind power generation and failure events.

In this regard, Tuohy et al. [8] introduced a methodology that employs scenarios randomly generated of load demand and wind power generation using an autoregressive moving average (ARMA) model combined with a reduction algorithm in order to select those representative scenarios. Then, power system management is carried out by solving a mixed integer programming optimization problem obtaining a feasible solution for the scenarios previously selected. However, in this approach a limited number of scenarios is analyzed, which represents an important source of error.

To overcome the aforementioned problem, Ruiz et al. [9] proposed the incorporation of spinning reserve requirements for each scenario, as well as the incorporation of extreme scenarios of failure events, such as single outage of the largest generation unit in order to provide a robust solution.

In other research work, Constantinescu et al. [10] paid special attention to the quality of scenarios used in stochastic programming optimization models. The authors have developed a model that joins a weather research and forecast model with a unit commitment (UC)/economic dispatch (ED) model in order to analyze the effects of wind power uncertainty on the scheduling problem. Among the most important findings, authors concluded that their proposed framework allows considering several aspects that cannot be analyzed by means of synthetically generated data; in special, the benefits obtained from the updating wind power forecasts during intra-day operations.

Bahmani-Firouzi et al. [11] have proposed a methodology at which scenarios are randomly generated by using a roulette wheel technique that uses the corresponding probability distribution function (PDF) of load demand and wind power generation. The stochastic optimization problem is solved by means of an improved multi-objective particle swarm optimization (PSO) algorithm.

Another optimization theory widely used is the chance constrained programming, at which the stochastic variables of the optimization problem are represented by using equivalent deterministic constraints. In this sense, Ding et al. [12] developed a model at which stochastic variables such as load demand, forced outage rates, energy prices, and wind power generation have been modeled, while the optimization problem is solved by implementing a standard branch and bound algorithm.

Hybrid techniques that combine stochastic programming with other optimization techniques have been recently developed. Zhao and Guan [13] have developed a methodology that takes advantage of stochastic programming in order to face the computational effort related to the analysis of a large amount of scenarios preserving the conservativeness of the solution obtained from the robust optimization technique.

Wang et al. [14] introduced a combined sample average approximation algorithm that combines a stochastic programming approach with chance-constrained programming approach in order to ensure using the wind power production at each time step.

Probabilistic approaches based on modeling stochastic variables as a Markov process have been recently introduced, as well. Hargreaves and Hobbs [15] introduced the stochastic dynamic approach, where the optimization problem has been formulated as a Bellman equation, while load demand and wind power generation have been modeled by using a Markov transition matrix. The optimization problem has been solved by employing a dynamic programming approach with unit aggregation in order to analyse systems with large number of units in a reasonable computational time.

Luh et al. [16] have developed a model where UC problem is based on states instead of scenarios. The corresponding states are determined after modeling the wind power generation as a Markov process through the transition matrix. Besides of this, the transition matrix is improved to take into account rare events in wind power production. The optimization problem is solved by using branch and cut algorithm.

Other probabilistic models based on the analytical representation of wind power production have been presented and analyzed. In this sense, Hetzer et al. [17] developed a general purposes ED model at which stochastic wind speed is represented as a Weibull PDF. Additionally, a factor to represent the overestimation and underestimation of the available wind power generation is incorporated in the objective function of the classical ED problem. On the one hand, the factor related to the overestimation represents the purchasing of power generation from a determined source (spinning reserve) to supply the required capacity. On the other hand, the factor related to the underestimation represents the cost of consuming the excess of power generation.

In a similar way, in references [18,19] authors have modeled wind speed variability by means of a Weibull PDF; however, some concepts frequently applied in stochastic programming frameworks such as those variables of first and second stages (here-and-now variables and wait-and-see variables) are incorporated. In reference [20], the probabilistic modeling presented in [18, 19] is improved by the incorporation of emissions due to oxides of nitrogen (NO_x), using an incomplete gamma function to represent the effects of wind power generation.

As can be noted from the analysis of the literature review previously described, the optimal scheduling considering uncertainty introduced by wind generation and failure events is a challenging task.

Many of the methodologies presented in the technical literature are based only in taking into account a limited number of scenarios, assuming the same probability of occurrence for all of them, which could be an important source of error. As a consequence, the obtained scheduling depends on the methodology used for scenario generation (ARMA, Markov process, etc.).

Regarding the probabilistic approaches previously explained, many of such approaches represent the effects of ramp constraints (limitation of power generation capacity) indirectly by means of penalty factors (cost of spinning reserve used to compensate wind power forecasting error). For these reasons, the development of a probabilistic model capable of considering all possible changes in wind power generation, as well as the effects of ramp constraints in the stochastic optimization problem, is required.

In this paper, wind power forecasting error is modeled as a beta PDF and incorporated in the ED problem in order to obtain PDF of power generation, energy not supplied (ENS), and generation cost, among others. Besides of this, the effects of ramp constraints on power production are modeled only considering some representative values of power production at previous time step, selected from the corresponding PDF. This simplification is carried out to make the optimization problem tractable. These are the main differences between the methodology presented in this paper and those previously proposed in references [17-20], as new contributions to earlier studies.

The rest of the paper is organized as follow: in section 2 the probabilistic ED problem is described and the proposed approach is carefully explained, i.e., the process required to obtain the discretized PDF of wind power forecasting error, the proposed approach to represent the power generation of each unit at time $k-1$, the methodology to solve the probabilistic ED problem under uncertainty, and the incorporation of generators reliability are fully described. In section 3, the performance of the proposed approach is illustrated by analyzing two case studies of 5 and 10 units. Finally, main conclusions are presented in section 4.

2 Probabilistic ED problem and proposed approach

The probabilistic ED problem consists on finding the optimal power generation of each unit committed, taking into account the uncertainty related with wind power forecasting error.

The system under analysis is shown in Fig. 1, where the aggregated wind power generation has been represented by only one wind farm. The power system is supposed to have a dump load, which is used to dissipate the energy surplus produced during those periods of low load. ENS is represented by a big unit capable of supplying any amount of power that cannot be provided by thermal units.

"See Figure 1"

The proposed approach consists of four main steps:

- Step 1: Discretization of the PDF of forecasted wind power generation.
- Step 2: Simplification of PDF of initial power production.
- Step 3: Incorporation of wind power forecasting error.
- Step 4: Incorporation of generators reliability.

In the first step, discretization of the PDF that represents the wind power forecasting error is carried out, assuming a beta PDF. In the second step, in order to make the optimization problem tractable, the PDF of power production at time instant $k-1$ is simplified, so that only some specific power production situations are taken into account. In the third step, the discretized PDF obtained in the first step is incorporated in the optimization problem by considering the simplification carried out in the second step. In the fourth step, generators reliability is incorporated by estimating the joint PDF of power production and failure events for each unit; while a convolution process is carried out between the PDF of ENS obtained in the third step from the incorporation of wind power forecasting error and from the reliability analysis of each unit.

2.1 Discretization of the PDF of forecasted wind power generation

The probabilistic representation of wind power forecasting error has been extensively analyzed in the literature, proposing many PDFs to its representation. The results obtained by De Giorgi et al. [21] from the implementation of forecasting tools based on ARMA models, artificial neural networks and adaptive neuro-fuzzy inference systems suggest a Gaussian PDF. Bludszuweit et al. [22], from the analysis of a measured time series of one year, have suggested using beta PDF in order to model those PDFs similar to a Gaussian PDF, and those particular PDFs with a tail. To accurately represent those situations at which power production and consequently forecasting error are zero due to wind speed is too low or too high to produce power from the wind farm. Hence, Tewari et al. [23] have proposed a mixed PDF.

Alternatively, Zhang et al. [24] have suggested employing the versatile PDF due to its analytical properties that facilitates the incorporation of wind power forecasting error in the ED problem. Recently, other methodologies based on copula theory [25] and Lévy alpha-stable PDF [26] have been suggested.

Hodge and Milligan [27] demonstrated the effectiveness of Cauchy PDF to represent the forecasting error over multiple timescales, analysing data from Electric Reliability Council of Texas (ERCOT).

In order to illustrate the methodology proposed in this paper to solve the probabilistic ED problem, beta PDF has been adopted. Assuming that the corresponding parameters are known, the discretization of this PDF is carried out by applying the methodology proposed by Punzo and Zini [28]. Fig. 2 shows the main characteristics of discretized beta PDF in interval [0,1], where the corresponding discretized PDF could be mathematically expressed in terms of discrete state r according to (1):

$$S = \{s_r, P_r\{s_r\}, r = 0,1,2, \dots, R\} \tag{1}$$

The value (s_r) that corresponds to each discrete state r is estimated by means of (2) in the interval [0,1].

$$s_r = \begin{cases} \max\left(\left\{\frac{r}{R} - \frac{\sigma}{R}, 0\right\}, \frac{r}{R} - \frac{\sigma}{R} + \frac{1}{R}\right), & r = 0, 1, 2, \dots, R-1 \\ \left[\frac{r}{J} - \frac{\sigma}{J}, 1\right], & r = R \end{cases} \tag{2}$$

"See Figure 2"

The corresponding probability value ($P_r\{s_r\}$) that corresponds to the discrete state r is calculated by using (3):

$$P_r\{s_r\} = \frac{(1+r)^{\alpha-1}(R+1-r)^{\beta-1}}{\sum_{a_0=0}^{R}(1+a_0)^{\alpha-1}(R+1-a_0)^{\beta-1}}; r = 0,1,2, \dots, R \tag{3}$$

In order to allocate the discretized PDF obtained from equations (1)-(3) in the range of interest of wind power generation $[W_{min}^k, W_{max}^k]$, a new discrete state (j) is introduced in terms of state r, which are related through the expression $j = r + 1$. The PDF of available wind power generation is estimated from discretized PDF in the interval [0,1] using (4):

$$AWP^k = \left\{awp_j^k = \left(W_{max}^k - W_{min}^k\right)s_{j-1} + W_{min}^k, j = 1,2, \dots, J\right\} \tag{4}$$

The notation to discretized PDF of wind power generation is presented in (5). Note that (4) represents the available wind power generation which is obtained from the forecasting process, while (5) represents the wind power produced, which is obtained from the solution of ED problem. This formulation allows considering the wind power curtailment from a probabilistic point of view.

$$W^k = \left\{w_j^k, P_r\{w_j^k\}, j = 1,2, \dots, J\right\} \tag{5}$$

2.2 Simplification of PDF of initial power production

The discretized PDF of power production at time $k-1$ is considered as input data available to solve the probabilistic ED problem. The incorporation of all possible combinations of power generation between the different units of the system lead to an infinity number of cases that should be evaluated, which make the optimization problem not tractable. In other words, if the discretized PDF of unit $m=1$ at time $k-1$ is divided into B bins, the number of combinations that results from considering the power generation of this unit and the possible power production of other units of the system ($m=2,\ldots,M$) could lead to a large amount of cases that should be evaluated. To deal with this problem, a simplification is introduced. Considering a determined significance level (γ), the interval $[\gamma, 1-\gamma]$ is swept with a determined step (sampling increment) $\Delta\theta$, obtaining I values. This is formulated in (6):

$$\theta = \{\theta_i \in [\gamma, 1-\gamma], i = 1,2,\cdots,I\} \tag{6}$$

Using the values defined in the set of equation (6), the discretized PDF of power generation at time $k-1$ and its corresponding CDF presented in Fig. 3, some selected power production values ($P_{m,i}^{k-1}$) can be selected by evaluating the inverse CDF of each element of set θ. Note that when θ_i=0.5, the power production at $k-1$ is the mean value of power production, which corresponds to the result obtained from the evaluation of the ED problem in the mean value of forecasted power generation. This methodology uses the concept of quantile to select and consider the power production values at time $k-1$. Another characteristic to take into account is when $\theta_i \rightarrow \gamma$ the low load conditions at $k-1$ are considered; on the contrary, when $\theta_i \rightarrow 1-\gamma$ the high conditions of load are considered. This allows considering the extreme conditions from low and high load conditions.

From the application of the methodology previously described, a similar table shown in Fig. 4 is obtained, where the power production at time $k-1$ according to sampling point i could be easily recognized. Something important to note is that the probabilities of occurrence of each column in Fig. 4 do not sum 1, due to that not all possible combinations are considered. To solve this problem, the corresponding probability ($P_r\{\cdot\}$) is substituted by the normalized probability ($NP_r\{\cdot\}$) of (7), whose sum is 1 for any amount of sampling points I.

$$NP_r\{P_m^{k-1} = P_{m,i}^{k-1}\} = \frac{\prod_{m=1}^{M}\left(P_r\{P_m^{k-1} = P_{m,i}^{k-1}\}\right)}{\sum_{i=1}^{I}\prod_{m=1}^{M}\left(P_r\{P_m^{k-1} = P_{m,i}^{k-1}\}\right)} \tag{7}$$

"See Figure 3"

"See Figure 4"

2.3 Incorporation of wind power forecasting error

Once the discretized PDF of available wind power generation and discretized PDF of power production at time $k-1$ are obtained, wind power forecasting error is incorporated in the probabilistic ED problem by following the algorithm described next:

- Step 1: Select the number of bins (B) to be considered in the discrete PDF of all variables of interest (power production of thermal units, wind power generation, energy not supplied and energy surplus). The maximum value of power (P^{max}) to be considered is chosen as well in this step, while the minimum value (P^{min}) is assumed to be zero. The corresponding bin is identified by the index $b \in [1, B]$.
- Step 2: Using the parameters selected in step 1, the increment of the discrete representation of power values (ΔP) is calculated by using (8):

$$\Delta P = \frac{P^{max} - P^{min}}{B-1} \tag{8}$$

  After this, the power value (P_b) that corresponds to discrete state b is obtained. This is implemented as a vector $P_b = P_1, P_2, \dots, P_b, \dots, P_B$, where $P_1 = P^{min} = 0$ and $P_B = P^{max}$. Then, any continuous power value obtained from the optimization process can be represented in a discrete manner, selecting the corresponding discrete state.
- Step 3: Create a table of B rows and M columns ($T_{(b,m)}$). This table is the discrete PDF of power generation of thermal units. All elements in this table are initialized as zero.
- Step 4: In this step, the first case of power generation at time $k-1$ (see Fig. 4) is selected. This is carried out by setting the index i equal to 1 ($i \leftarrow 1$).
- Step 5: The first discrete state of available wind power generation is selected. This is carried out by setting j equal to 1 ($j \leftarrow 1$).
- Step 6: Solve the classical ED problem for the corresponding combination (i, j). This is carried out by solving the optimization problem of (9)-(14) [29]:

$$z_{i,j} = \sum_{m=1}^{M} \left(A_m + B_m\left(P_{m,i}^k\right) + C_m\left(P_{m,i}^k\right)^2 \right) + VOWE\left(DL_i^k\right) + VOLL\left(ENS_i^k\right) \tag{9}$$

$$\sum_{m=1}^{M} P_{m,i}^k + w_j^k = D^k \tag{10}$$

$$P_{m,i}^k - P_{m,i}^{k-1} \le UR_m \tag{11}$$

$$P_{m,i}^{k-1} - P_{m,i}^{k} \leq DR_m \tag{12}$$

$$P_m^{min} \leq P_m^k \leq P_m^{max} \tag{13}$$

$$0 \leq w_j^k \leq awp_j^k. \tag{14}$$

- Step 7: From the solution of the optimization problem in step 6, variables w_j^k and $P_{m,i}^k$ are determined. Then, the corresponding probability values are calculated and allocated in the discrete PDF using the algorithm presented in Fig. 5. In similar manner, discretized PDF of ENS and generation cost are built.
- Step 8: If $j < J$, set $j \leftarrow j + 1$ and go back to step 6; else go to step 9.
- Step 9: If $i < I$, set $i \leftarrow i + 1$ and go back to step 5; else end.

"See Figure 5"

2.4 Incorporation of generators reliability

For a determined unit m, the estimation of power production considering the failure events could be estimated by using the algorithm presented next. This algorithm has been adapted from the methodology proposed in [30] to the estimation of the joint PDF of power production and failure modes.

- Step 1: Using the discrete representation of any power value ($P_b \in [P^{min}, P^{max}]$), find the bin ($b_m$) that corresponds to the rated power of unit m (P_m^{max}). It can be carried out by adapting the algorithm presented in Fig. 5.
- Step 2: Create the state h ($h = 0,1,2,\ldots,H$) using the state b by means of expression $h + 1 = b$ to represent a determined state of power production and failure events. The value of power production of state h can be estimated as $P_h = P_{b-1}$. This change in states name is required to the estimation of joint PDF of power production and failure events.
- Step 3: In this step, the discrete PDF of failure events (F_h^m) of a determined unit m is represented by using (15):

$$F_h^m = \begin{cases} FOR_m\,, & h = 1 \\ 1 - FOR_m\,, & h = b_m \\ 0\,, & otherwise \end{cases} \tag{15}$$

- Step 4: Once the discrete PDF of power production (P_h) and discrete PDF of failure events (F_h^m) have been estimated, the discrete joint PDF of power production and failure events can be built. Power production and failure events are considered as two independent variables, so that the joint PDF can be obtained by multiplication of the probability occurrence of each event ($P_r\{P_m^k = P_h\}P_r\{F_h^m = P_h\}$). Joint PDF is represented by a table similar to that presented in Fig. 6.

"See Figure 6"

- Step 5: Create the discrete state l of power production when generators reliability is considered, $l = 0, 1, 2, \dots, L$, where $L = (H+1)^2 = B^2$. The corresponding power value associated with state l (P_l) is defined according to the (16):

$$P_l = l\left(\frac{\Delta P}{b_m - 1}\right) \tag{16}$$

- Step 6: In this step, the probability of state $l = 0$ ($P_{l=0}$) is estimated. This probability is calculated summing the elements (1, 1), the elements of row 1 from 2 until B, and the elements of column 1 from 2 until B of the table presented in Fig. 6.
- Step 7: The estimation of probabilities that corresponds to states $l = 1, 2, \dots, L$ is carried out by using the algorithm presented as follows:
  - Step 7.1: Create the table $E_{(l,m)}$ of B^2 rows and M columns. Initialize all its elements to zero.
  - Step 7.2: Set $a_1 \leftarrow 0$.
  - Step 7.3: Set $a_2 \leftarrow 0$.
  - Step 7.4: Calculate $a_3 = a_1 a_2$.
  - Step 7.5: If $a_3 > 0$, $E_{(a_3,m)} \leftarrow E_{(a_3,m)} + P_r\{P_m^k = P_h\} P_r\{F_h^m = P_h\}$, else go to step 7.6.
  - Step 7.6: If $a_2 < L$, set $a_2 \leftarrow a_2 + 1$ go to step 7.4, else go to 7.7.
  - Step 7.7: If $a_1 < L$, set $a_1 \leftarrow a_1 + 1$ go to step 7.3, else end.

The discrete PDF of power production incorporating the forecasting error of wind power generation and the generators reliability is represented by discrete states l, the power associated with corresponding state (P_l) and the probabilities of table $E_{(l,m)}$.

Regarding the ENS, the discrete PDF of this variable could be estimated by using the methodology explained in sub-section 2.3, representing ENS as a generation unit. The component of ENS due to generators reliability could be estimated by using the recursive expression of (17) [31]:

$$F_b^e(P_b) = (1 - FOR_m) F_b^e(P_b) + FOR_m F_b^e(P_b - P_m^{max}). \tag{17}$$

where F_b^e is the CDF of ENS due to any failure event in the generation system. From this result the required PDF could be easily estimated. Both of them are shown in Fig. 7.

Finally, discrete PDF of ENS taking into account wind power forecasting error and generators reliability is estimated as the convolution between the discrete PDF obtained from the procedure explained in sub-section 2.3 and that obtained from (17) and Fig. 7.

“See Figure 7”

3. Case studies

The proposed approach in this paper is illustrated by analyzing two case studies of 5 and 10 units provided of wind power generation. In order to evaluate the performance of the proposed methodology, the results obtained from the methodology explained in section 2 have been compared to those obtained from the application of Monte Carlo Simulation (MCS) methodology. In both cases, the number of trials considered in MCS was 50000. The test system based on MCS was built by considering three time instants. The first time instant corresponds to the actual conditions so that initial power generation was considered as a real value. In the second and third time instants, the conditions of available wind power generation were randomly generated, while the power generation of each unit and wind farm was obtained from the solution of the corresponding optimization problem by quadratic programming approach [29]. Then, using the results obtained from the second time instant, the PDF of initial power generation of each unit required by the proposed approach has been obtained. The results obtained from the third time instant are used to build the PDF of power generation of each unit, which is employed as a reference of comparison between the proposed methodology and the MCS methodology.

The number of bins considered to build the required PDFs was 1500 (B = 1500), the significance level considered was 0.05, the sampling increment used was 0.15, thus obtaining 7 sampling points (I =7). The discretization of PDF of available wind power generation was carried out by considering $\sigma = 0.01$ and $R = 3500$. The results obtained from the analysis of each case study are presented next in sub-sections 3.1 and 3.2. The proposed methodology has been implemented in MATLAB using a computer provided of i7-3630QM CPU at 2.40 GHz with 8 GB of memory and 64 bit operating system.

3.1 Analysis of 5-unit power system

The power system under analysis corresponds to a typical diesel-powered system of an island. The main characteristics such as rated capacity and generation costs are presented in Table 1. The minimum output power of each unit was assumed to be 50% of the corresponding rated power. This data was obtained from the analysis of the information provided by manufacturers.

"See Table 1"

The available wind power generation was modeled as a beta PDF with parameters $\alpha = 1.4$, $\beta = 3.1$, $W_{min}^{k} = 0$ kW, and $W_{max}^{k} = 600$ kW. The maximum power value considered was P^{max}=1000 kW. Finally, load demand was assumed to be a 892 kW at time k.

Fig. 8 shows the PDF of the wind farm (W^k) obtained from the solution of the optimization problem. It is observed how wind power generation is curtailed to about 424 kW due to the minimum output power of thermal units. This is an important problem in the integration of renewable energy sources to the grid that could be probabilistically analyzed by means of the approach proposed in this paper.

"See Figure 8"

Fig. 9 shows the PDF of power production of unit 1. Since this unit is one of the cheapest units in the system, it responds to the fluctuations of wind power generation. It is possible to observe how the probability of high wind power generation leads this unit to reduce its output power at its minimum value, while the probability of power production at high values is influenced by the PDF of available wind power generation. Similar results were obtained for the other units.

"See Figure 9"

Fig. 10 shows the PDF of total generation cost. As can be observed, this is highly influenced by the behavior of the available wind power generation and its forecasting error. The reader can note that this PDF has an inverse shape compared to the PDF of wind power generation, including the effects of wind power curtailment on the total generation cost.

"See Figure 10"

Through the analysis of Figs. 8-10, it is possible to observe the good performance of the proposed methodology compared to MCS. It could be quantized by means of a comparison between the expected values obtained from the application of each methodology, such as the comparison shown in Table 2.

"See Table 2"

The proposed methodology could be used to evaluate the greenhouse gas emissions of each thermal unit. To illustrate this application, the CO_2 emissions of each unit have been modeled by using the quadratic expression of (18):

$$GHG_m = U_m + X_m\left(P_{m,i}^k\right) + V_m\left(P_{m,i}^k\right)^2 \tag{18}$$

The corresponding parameters of (18) were obtained by fitting the experimental measurements presented in [32] related to the CO_2 emissions to (18). The obtained results are presented in Table 3.

"See Table 3"

Fig. 11 presents the PDF of CO_2 emissions of unit 1, which was obtained by evaluating the PDF power production of Fig. 9 in (18). Table 4 presents the expected value of CO_2 emissions for each unit.

It is possible to observe how the forecasting error of available wind power generation highly influences the emission probability of a determined amount of CO_2.

"See Figure 11"

"See Table 4"

3.2 Analysis of 10-unit power system

In this case study, the power system described in reference [33] has been adapted by adding the FOR values presented in Table 5, estimated according to the corresponding role of the unit (base-unit, cycling-unit, and peak-unit). Available wind power generation was modeled with the parameters $\alpha = 1.6$, $\beta = 6.3$, $W_{min}^{k} = 150$ MW, and $W_{max}^{k} = 500$ MW. The maximum power value considered was P^{max}=1700 MW. Finally, load demand was assumed to be a 1600 MW at time k. This system has been used to analyze the performance of the proposed approach when ramp constraints and failure events are considered. The obtained results are presented in sub-section 3.2.1 and 3.2.2.

"See Table 5"

3.2.1 Analysis of 10-unit system incorporating generators reliability

The methodology explained in sub-sections 2.3 and 2.4 has been used to the analysis of the system taking into account the reliability of generation units. The corresponding comparison with MCS methodology was carried out in Fig. 12, showing the PDF of power generation of unit 4. Reader can note how this unit has high probability of being committed at its maximum output power (130 MW). Due to its generation cost and technical characteristics, this unit does not respond the fluctuations of wind power neither to failure events of other units. Otherwise, there is a probability of 0.1 of being de-committed as a consequence of a failure event. According to these results, the proposed methodology offers a good performance.

"See Figure 12"

Fig. 13 shows the PDF of power generation of unit 6. As can be observed, this unit responds to any failure event of other units by increasing its power production, which produces important differences between the PDF obtained from the proposed methodology and the MCS approach.

Fig. 14 shows the PDF of generation cost related to the fuel consumption. As in the previous case study, this cost is strongly influenced by the PDF of available wind power forecasting error.

Fig. 15 presents the PDF of ENS, where important differences can be observed. The results obtained from the proposed methodology suggest higher values of ENS due that the increment in power production of those units able to provide spinning reserve is not considered.

"See Figure 13"

"See Figure 14"

"See Figure 15"

Table 6 shows the expected value of power production, ENS, and fuel consumption cost. It is possible to observe how the proposed methodology can reasonable model those units used to provide base-load, which are continuously operated at their maximum output power (units 1-4). However, the proposed methodology has difficulties to model the behavior of those units that increase their power production under any failure of the other units (units 5-10) that are used as cycling and peak units. This reasoning justifies the important differences in the estimation of the ENS observed.

"See Table 6"

3.2.2 Analysis of 10-unit system without incorporating generators reliability

In this sub-section, the results obtained from the analysis of the 10-unit system without considering the generators reliability are presented. Fig. 16 shows the PDF of wind power generation, which is totally accepted by the system without any curtailment. Fig. 17 shows the PDF of power production of unit 6. It is possible to observe how under these conditions (without considering unit reliability) the proposed approach can reproduce the PDF obtained from MCS. Fig. 18 shows the PDF of generation cost and the impact of forecasting error of wind power generation. The increment in generation cost is directly related to the decrement in wind power generation previously presented in Fig. 16.

"See Figure 16"

"See Figure 17"

"See Figure 18"

Table 7 summarizes the comparison between the expected value of power production and generation cost. As it can be observed, the proposed methodology presents good performance compared to those results obtained from the MCS approach. In the proposed methodology, the trade-off between the accuracy of obtained results and computational time is carried out by adjusting parameters I and R, which represents the total number of possible power production combinations at time $k-1$, and the amount of discretization levels of PDF of the available wind power generation.

These factors can be adjusted according to the computational resources available and the size of the system under analysis. Fig. 19 presents the behavior of computational time as a function of the factor R for two different values of parameter I. According to these results, the computational time has a linear behavior that facilitates the selection of the factor R taking into account the computational resources available.

"See Table 7"

"See Figure 19"

4 Conclusions

In this paper, a methodology for solving the classical ED problem incorporating the uncertainty of wind power generation and generators reliability was presented. In this approach, forecasting error of wind power generation was modeled as a discretized beta PDF, which allowed considering extreme conditions with its corresponding probability. Another important characteristic of the proposed methodology is that the power production of each unit at previous time instant was incorporated by means of simplified sampling of discretized PDF of power generation at this time step, which provided an efficient treatment of the problem. Finally, failure events of each unit were incorporated through the calculation of the joint PDF of power production and failure event, while ENS was probabilistically described through the convolution between the PDF of ENS related to wind power forecasting error and unit's failure. The proposed methodology was illustrated through the analysis of two power systems of 5 and 10 units, respectively, and the results were compared to those obtained from MCS. From this comparison it was possible to conclude that the proposed methodology could soundly describe the PDF of wind power generation, thermal power generation, ENS and generation cost when the failure events in any generation unit are not considered. As a future work, the methodology presented in this paper is going to be improved in order to consider the rise in the power production of those units used to provide spinning reserve when any failure occurs in the generation system, in order to avoid the overestimation in the expected value of ENS observed in the analysed cases. Other PDFs to model forecasting error, such as versatile PDF [24], Lévy alpha-stable PDF [26], and Cauchy PDF [27], will also be tested.

Acknowledgements

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Figure Captions

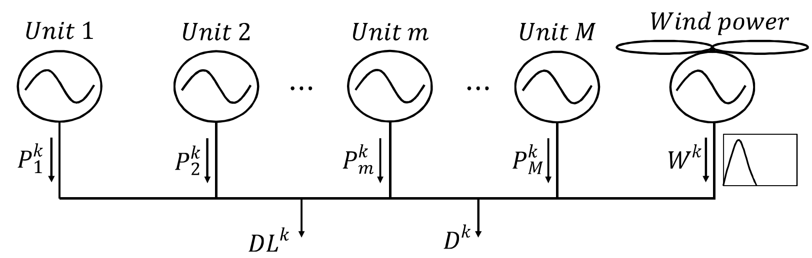

Figure 1
Power system under study.

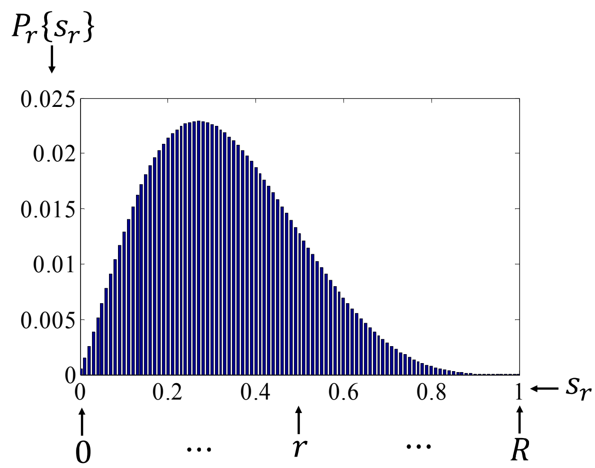

Figure 2
Characteristics of the discretized beta PDF.

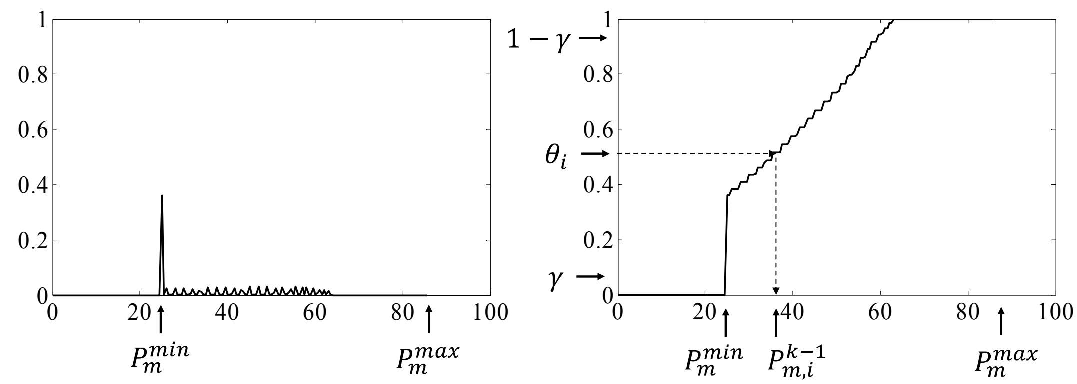

Figure 3
PDF of P_m^{k-1} (left side) and CDF of P_m^{k-1} (right side).

Sampling point

Unit	1	2	$\cdots$	i	$\cdots$	$\cdots$	I
1	$P_{1,1}^{k-1}$	$P_{1,2}^{k-1}$	$\cdots$	$P_{1,i}^{k-1}$	$\cdots$	$\cdots$	$P_{1,I}^{k-1}$
$\vdots$	$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\vdots$
m	$P_{m,1}^{k-1}$	$\cdots$	$\cdots$	$P_{m,i}^{k-1}$	$\cdots$	$\cdots$	$P_{m,I}^{k-1}$
$\vdots$	$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\vdots$
M	$P_{M,1}^{k-1}$	$P_{M,2}^{k-1}$	$\cdots$	$P_{M,i}^{k-1}$	$\cdots$	$\cdots$	$P_{M,I}^{k-1}$

Figure 4
Selected cases of power production at time $k-1$.

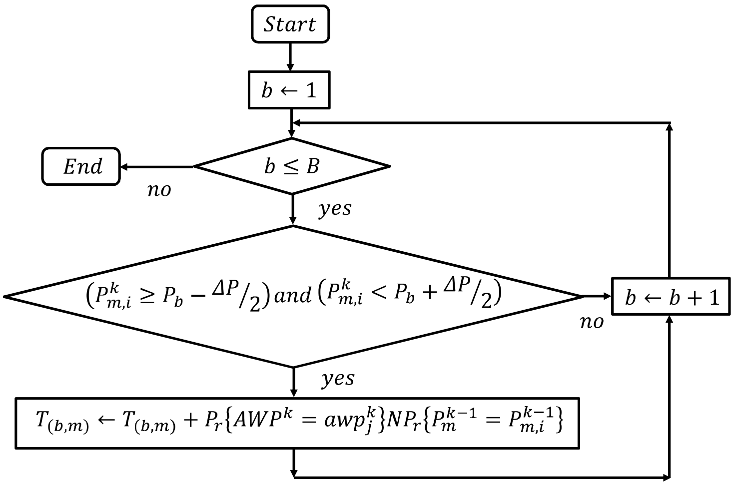

Figure 5
Allocation of power generation ($P_{m,i}^{k}$) in the PDF of P_m^k.

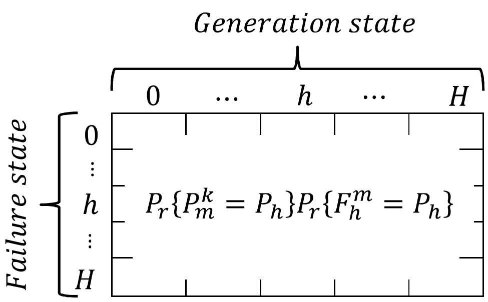

Figure 6
Illustration of the joint PDF of failure events and power production.

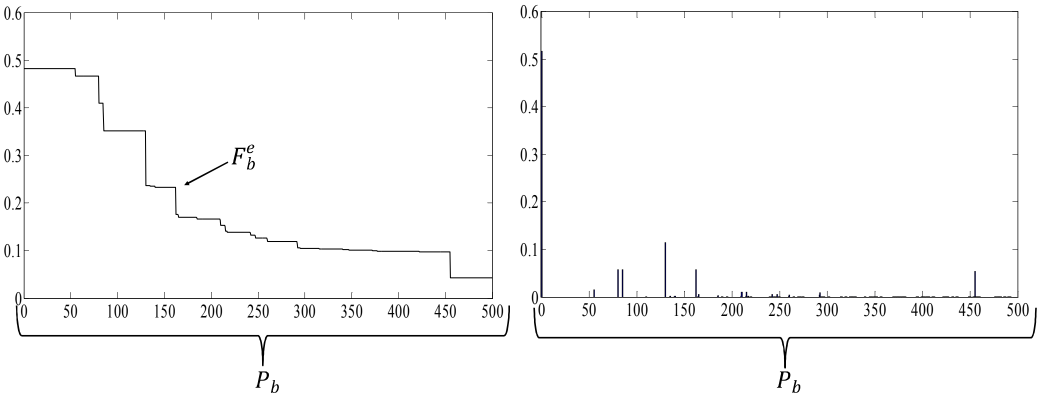

Figure 7
CDF of power generation loss (left side) and PDF of power loss (right side) due to failure events.

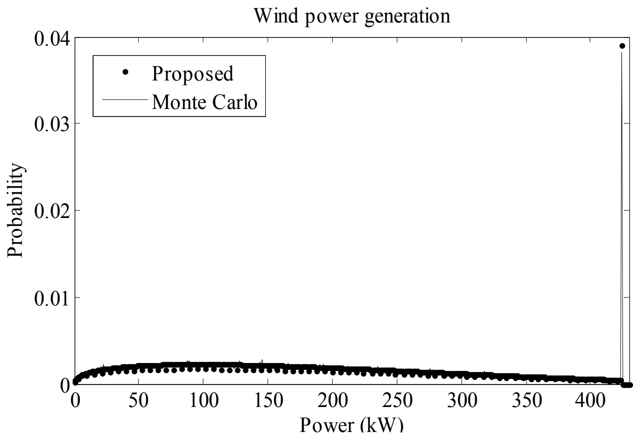

Figure 8
PDF of wind power generation (5-unit system).

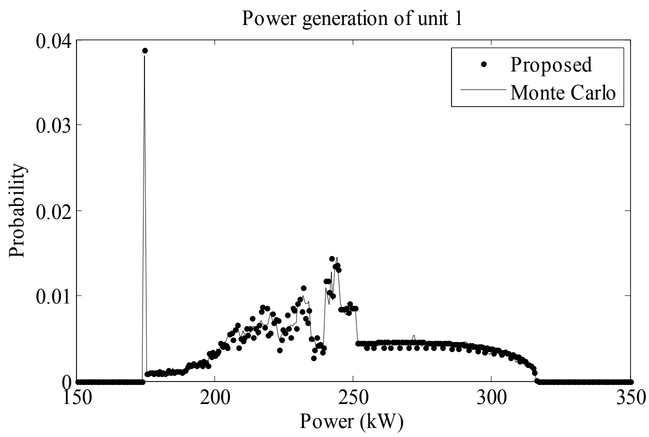

Figure 9
PDF of power generation of unit 1 (5-unit system).

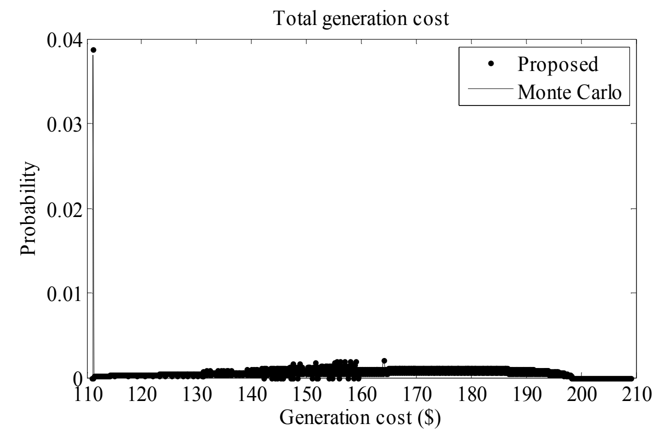

Figure 10
PDF of generation cost (5-unit system).

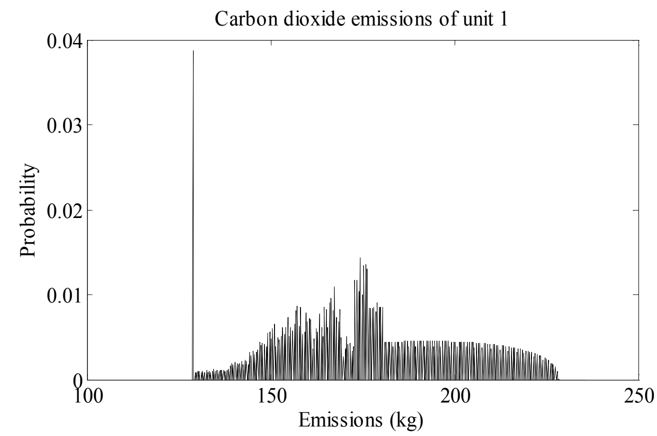

Figure 11
PDF of CO_2 emissions of unit 1 (5-unit system).

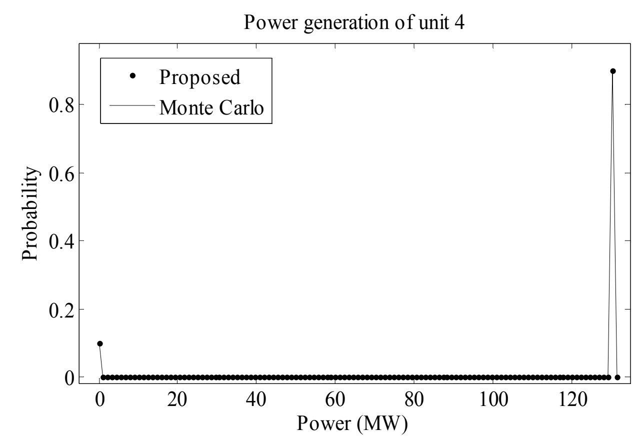

Figure 12
PDF of power generation of unit 4 (10-unit system).

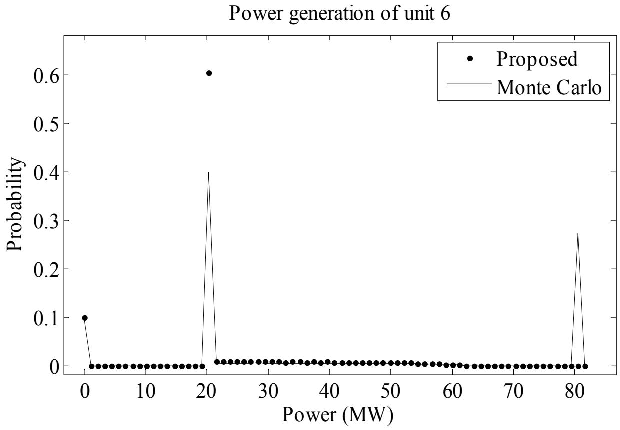

Figure 13
PDF of power generation of unit 6 (10-unit system).

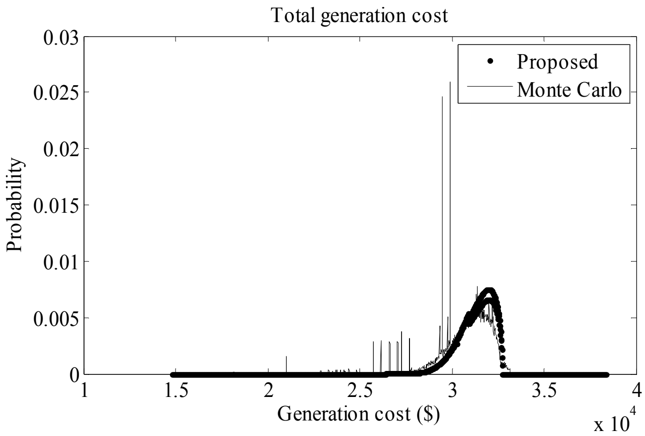

Figure 14
PDF of generation cost related to fuel consumption (10-unit system).

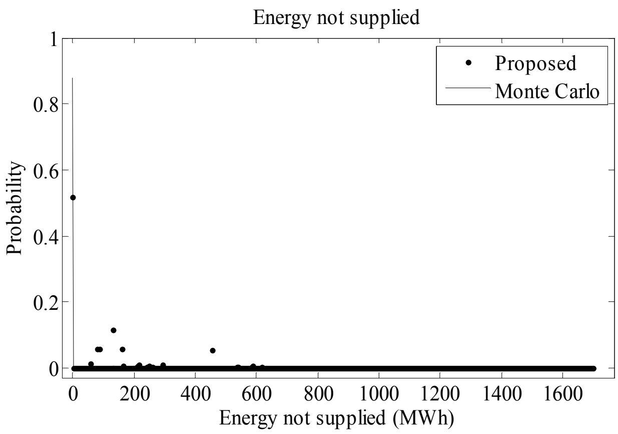

Figure 15
PDF of energy not supplied (10-unit system).

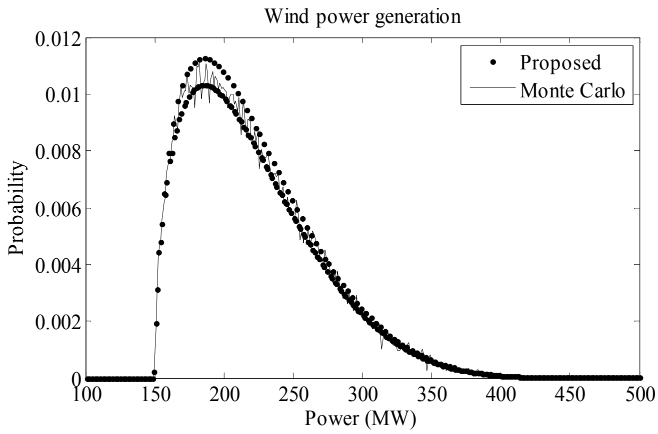

Figure 16
PDF of wind power generation (10-unit system).

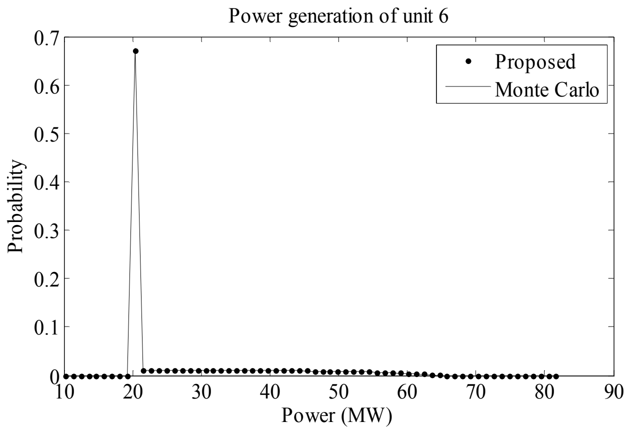

Figure 17
PDF of power generation of unit 6 (10-unit system).

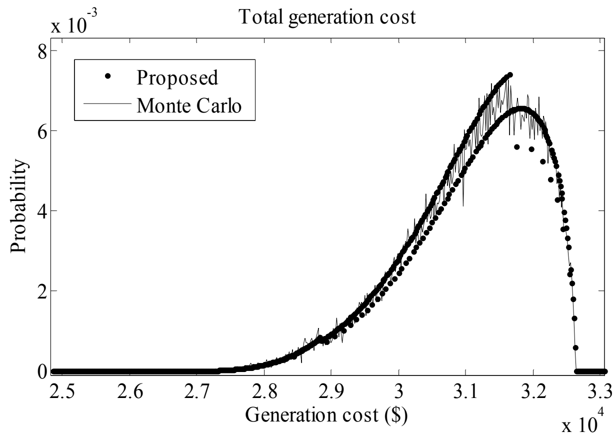

Figure 18
PDF of generation cost (10-unit system).

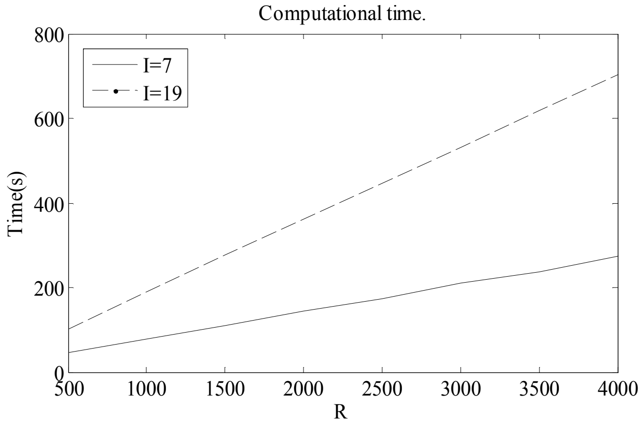

Figure 19
Behavior of the computational time.

Table Captions

Table 1
Description of 5-unit system.

m	P_m^{max} (kW)	A_m (\$/h)	B_m (\$/kWh)	C_m (\$/kW^{2}h)
1	350	10.3904	0.1472992	0.00012224
2	300	8.6332	0.1534112	0.00012224
3	125	3.5908	0.1842768	0.00009168
4	100	3.2852	0.1815264	0.00012224
5	60	2.2156	0.2270608	-0.0003056

Table 2
Expected value comparison between MCS and proposed method (5-unit system).

Comparison	MCS	Proposed
Wind farm (kW)	185.788939	184.988933
Unit 1 (kW)	243.892262	244.178446
Unit 2 (kW)	218.904510	219.191016
Unit 3 (kW)	107.157612	107.361965
Unit 4 (kW)	88.606244	88.752082
Unit 5 (kW)	47.612342	47.486873
Total cost ($)	159.023486	159.189200
Time (s)	787.462000	149.976000

Table 3
CO_2 emission model (5-unit system).

m	P_m^{max} (kW)	U_m (kg/h)	X_m (kg/kWh)	V_m (kg/kW^{2}h)
1	350	28.062	0.5075	0.0004
2	300	24.104	0.5626	0.0002
3	125	16.244	0.4506	0.001
4	100	11.148	0.5544	0.0006
5	60	9.163	0.6201	-0.0014

Table 4
Expected value of CO_2 emissions (5-unit system).

m	CO_2 emissions (kg)
1	176.313261
2	157.271476
3	76.610344
4	65.232853
5	35.148854

Table 5
Description of 10-unit system.

m	P_m^{max} (MW)	FOR_m
1	455	0.05
2	455	0.05
3	130	0.1
4	130	0.1
5	162	0.1
6	80	0.1
7	85	0.1
8	55	0.01
9	55	0.01
10	55	0.01

Table 6
Expected value comparison between MCS and proposed method considering system reliability.

Comparison	MCS	Proposed
Unit 1 (MW)	432.286025	432.031354
Unit 2 (MW)	432.248759	431.392345
Unit 3 (MW)	117.740821	117.378252
Unit 4 (MW)	117.866024	117.378252
Unit 5 (MW)	125.958954	115.711446
Unit 6 (MW)	39.962974	23.878370
Unit 7 (MW)	31.075342	22.455013
Unit 8 (MW)	22.839449	10.104748
Unit 9 (MW)	20.058911	10.104736
Unit 10 (MW)	15.733109	10.104736
ENS (MWh)	24.049636	106.069599
Fuel cost ($)	30736.441648	31313.845805
Time (s)	1194.889000	228.262000

Table 7
Expected value comparison between MCS and proposed method without considering system reliability.

Comparison	MCS	Proposed
Wind (MW)	220.910611	220.877230
Unit 1 (MW)	454.769847	454.769847
Unit 2 (MW)	454.075239	454.097234
Unit 3 (MW)	130.420280	130.420280
Unit 4 (MW)	130.420280	130.420280
Unit 5 (MW)	128.632910	128.303661
Unit 6 (MW)	26.490740	26.665282
Unit 7 (MW)	24.949967	24.949967
Unit 8 (MW)	10.206805	10.206805
Unit 9 (MW)	10.206805	10.206805
Unit 10 (MW)	10.206805	10.206805
Total cost ($)	31087.151684	31087.762562
Time (s)	1222.937000	235.965000