

A Heuristic Methodology to Economic Dispatch Problem Incorporating Renewable Power Forecasting Error and System Reliability

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Abstract

With the constant increment of wind power generation driven by economic and environmental factors, the optimal utilization of generation resources has become a critical problem discussed by many authors. Within this topic, determination of optimal spinning reserve (SR) requirements is a key and complex issue due to the variable and unpredictable nature of renewable generation besides of generation unit reliability. Cost/benefit relationship has been suggested as a way to determine the optimal amount of power generation to be committed by taking into account renewable power forecasting error and system reliability. In this paper, a technique that combines an analytical convolution process with Monte Carlo Simulation (MCS) approach is proposed to efficiently build cost/benefit relationship. The proposed method uses discrete probability theory and identifies those cases at which convolution analysis can be used by recognizing those situations at which SR does not have any effect; while in the other cases MCS is applied. This approach allows improving significantly the computational efficiency. The proposed technique is illustrated by means of two case studies of 10 and 140 units, demonstrating the capabilities and flexibility of the proposed methodology.

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Keywords: Insular power systems, power system reliability, probabilistic economic dispatch, wind power forecasting error.

Nomenclature

s	Index for discretization levels of normal standard PDF $s \in [1, S]$
q	Index for discretization levels of wind power PDF $q \in [1, Q]$
p	Index for discretization levels of thermal power PDF $p \in [1, P]$
n	Index for generation units $n \in [1, N]$
m	Number of failure events of a determined unit $m \in [1, M]$
a	Index for discretization levels of FCC PDF $a \in [1, A]$
h	Index for discretization levels of initial power generation
t	Index for time step
M	Number of MCS trials
f_W	Discretized Weibull PDF
f_R	Discretized Wind power PDF
f_G	Discretized PDF of power generation loss (Convolution)
f_M	Discretized PDF of power generation loss (MCS)
$f_{IPG,n}$	Discretized PDF of initial power generation of unit n
f_C	Discretized PDF of FCC

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F_N	Standard normal CDF
F_W	Weibull CDF
F_G	Discretized CDF of power generation loss (Convolution)
$F_{IPG,n}$	Discretized CDF of initial power generation of unit n
ΔWP	Discretization step of wind power PDF
ΔTPG	Discretization step of thermal power PDF
ΔFCC	Discretization step FCC PDF
WS_s	Value of discretization levels of Weibull PDF
TPG_p	Value of discretization levels of thermal power PDF
FCC_a	Value of discretization levels of FCC PDF
WP_q	Value of discretization levels of wind power PDF
TPG^{max}	Maximum thermal power production
TPG^{min}	Minimum thermal power production
FCC^{max}	Maximum value of FCC
FCC^{min}	Minimum value of FCC
$WPG(\cdot)$	Function to estimate the power production of wind farm
v	Value of a determined wind speed velocity
R_p	Rated power of a single wind turbine of the wind farm
N_t	Number of wind turbines of the wind farm
α, β, σ	Parameters of the function $WPG(\cdot)$
v_i	Cut-in wind speed of wind turbine
v_r	Rated wind speed of wind turbine
v_o	Cut-off wind speed of wind turbine
FOR_n	Failure outage rate of unit n
$E\{\cdot\}$	Function to estimate expect value of a determined variable
λ	Intermediate discretization parameter
μ_s	Intermediate distribution of the transformation process
ϕ_s	Value of discretization levels of normal standard PDF
ω_s	Central value of discretized level WS_s
ξ_h	Discretization interval of CDF of initial power generation
$IPG_{n,h}$	Initial power generation of unit n and interval ξ_h
I_n	Intermediate variable for $E\{ENS_t\}$ and $E\{FCC_t\}$ calculation
Φ_h	Weight associated with the values $IPG_{n,h}$; $n \in [1, N]$
L_t	Load demand at time t
$G_{t,q,n,h}$	Thermal power of unit n at time t considering WP_q and $IPG_{n,h}$
$G_{min,n}$	Minimum generation of unit n
$G_{max,n}$	Maximum generation of unit n
UR_n	Ramp-up limit of unit n
DR_n	Ramp-down limit of unit n
$WPD_{t,q,h}$	Wind power dispatched at time t considering WP_q and $IPG_{n,h}$
$NL_{t,q,h}$	Net load at time t considering WP_q and $IPG_{n,h}$
$l_{t,q,h}$	Maximum power of thermal units at time t considering WP_q and $IPG_{n,h}$
$ENS_{t,q,h}$	ENS at time t considering WP_q and $IPG_{n,h}$
η_q	Weighted values of ENS according to wind generation
τ_h	Weighted values of ENS according to initial power generation
θ_h	Weighted values of FCC according to initial power generation
ENS_t	ENS at time t
FCC_t	FCC at time t
$VOLL$	Value of loss load
TGC	Total generation cost
FCC	Fuel consumption cost
ENS	Energy not supplied

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32 1. Introduction

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34 Besides of their remote and isolated location, insular systems have high operating costs due to their
35 fuel consumption and the costs related to its transportation. In most of cases, power generation is based on
36 steam turbines (STs), combined cycle gas turbines (CCGTs), diesel engines (DEs), open cycle gas
37 turbines (OCGTs), and renewable sources (REN); a representative example is the case of Canary Islands,
38 at which STs represent 22.4%, CCGTs represent 28.8%, DEs represent 17.7%, OCGTs represent 20%,
39 REN represent 10%, while cogeneration and other power sources represent 1% of the total installed
40 capacity [1]. However, most of these systems have good potential for exploitation of renewable energy
41 sources like solar and wind energy; a representative situation is the case of Cyprus, where grid parity for
42 installation of photovoltaic (PV) generation has been reached due to the high selling prices of energy and
43 the considerable reduction in the prices of PV panels [2]. Under these circumstances, it is expected a
44 strong growth of renewable generation in the next years; however, the variability and uncertainty related
45 to renewable generation is an important factor, which limits the integration of these sources to the grid.
46 Variability of renewable generation impacts spinning reserve (SR) requirements and the utilization of
47 renewable generation. In the case of mainland power grids; by one hand, primary reserves could increase
48 between 0.3% and 0.8%, while all other reserves could increase between 6% and 10% of the
49 corresponding installed wind power generation. On the other hand, conventional generators could reduce
50 their efficiency up to 9% [3]. A way for solving this problem consists on improving the flexibility of the
51 system by adding an energy storage system (ESS); however, its successful integration since an economic
52 point of view strongly depends on capacity tariffs, wind power potential and investments costs [4].
53 Another option consists on implementing a demand response (DR) program; a reference case is the
54 Canary Islands, where cost savings related to the implementation of DR program are estimated up to 30%
55 [5]. However, the success of any DR program depends on awareness and knowledge of the consumers as
56 well as the automation of household appliances. Other inexpensive option consists on improving the
57 quality of renewable power forecasting in order to reduce total generation costs, reduce renewable power
58 curtailment, and reduce the commitment of OCGTs [6]. Nevertheless, it is not possible predicting
59 renewable power generation perfectly; besides of this, improvements on forecasting tools has a limited
60 enhancement on power system performance [7]; so that, incorporation of mathematical models for

61 renewable power generation to solve economic dispatch (ED) and unit commitment (UC) problems to
62 estimate the amount of SR required is a critical necessity.

63 SR requirements could be determined by using a traditional approach based on the solution of
64 deterministic UC problem, solving stochastic UC problem taking into account failures and contingencies
65 of generating units; as well as, wind power forecasting error, incorporating a probabilistic constraint on
66 UC problem based on estimating the probability of load curtailment as a consequence of any contingency,
67 and analysing the cost/benefit relationship [8]. In the traditional approach, SR requirements are adjusted
68 so that the system be able to face the failure of the generation unit with highest capacity among the
69 committed generators plus a determined margin based on the amount of wind power forecasted [9]. Then,
70 deterministic UC problem constrained to the SR requirements aforementioned is solved typically using
71 mixed-integer linear programming (MILP) optimization approach, some formulations widely suggested in
72 the literature can be found in references [10-12]. Another way consists on represent the uncertain nature
73 of any contingency and wind power generation by means of a representative set of scenarios relaxing the
74 constraint related to the SR requirements in the mathematical formulation. As any potential contingency
75 and wind power generation condition is represented explicitly by means of the scenario set, the
76 corresponding amount of SR could be implicitly determined by solving stochastic UC problem.
77 Generation of the required scenario set could be carried out by using Monte Carlo Simulation (MCS)
78 approach in combination with a scenario reduction technique [13]. Several approaches have been
79 proposed in the literature for solving stochastic UC problem. In [14], an optimization framework based on
80 stochastic programming (SP) approach was proposed. Such a framework is composed of two stages: at
81 the first stage the commitment decisions are taken (before the uncertain parameters are realized), while at
82 second stage dispatch decisions are taken (once uncertain parameters are known); the goal of the method
83 consists on find the optimal commitment decisions common to all scenarios considered in order to
84 minimize the total operating cost; it is carried out by solving the equivalent deterministic optimization
85 problem of the stochastic model. According to the reported results, stochastic approach suggests an
86 adequate estimation of the SR requirements; however, the computational time required is high for a
87 limited amount of scenarios; so that, decomposition techniques, addition of SR requirements for each
88 scenario; as well as, an appropriate modelling of optimization problem (regarding the relaxation of the
89 integrality constraints and modelling unit failures as load increments [15]) are suggested. In [16] has been
90 proposed a long-term security constrained unit commitment (SCUC) using Lagrangian relaxation and

91 including mixed integer programming (MIP) to face coupling constraints over the analysed period. The
92 model was structured under dual relaxation where the large-scale system was decomposed into many
93 short-term SCUC sub-problems without constraints. A hybrid subgradient and Dantzig–Wolfe
94 decomposition approach was proposed for managing the Lagrangian multipliers in the large-scale dual
95 optimization. In [17] has been proposed an SCUC model where outages of generation units, transmission
96 lines, and load forecasting inaccuracies were modelled using MCS approach. For dual optimization,
97 coupling constraints among scenarios are relaxed and decomposed into deterministic long-term SCUC
98 sub-problems using Lagrangian relaxation combined with MIP model without resource constraints. In
99 proposed work it was referred that the computational efforts for solving scenario-based optimization
100 models is influenced by the number of scenarios to be considered to minimize the average generation
101 cost. In [18] has been proposed a fast SCUC where were included some components such as single-hour
102 UC with network, security, single-hour UC adjustment, UC, ED, and hourly network security checking,
103 including a MIP approach. It was referred that under increment of power systems, SCUC models face
104 serious problems such as modelling accuracy and computational burden. The proposed model was
105 performed and analysed considering some different power systems, namely, an 1168-bus system with 169
106 conventional units, a 4672-bus system with 676 conventional units, and two other large systems with
107 1352 and 2704 conventional units, considering or not fuel and emissions constraints and among others. In
108 [19] has been proposed a SCUC model considering the well-known features of wind power generation,
109 i.e. forecasted wind power generation. In the first step, wind power volatility is represented by scenarios.
110 Then, it is considered the initial dispatch in the sub-problem and generation re-dispatch in order to satisfy
111 the hourly volatility of wind power from the computed scenarios. In case of failure, Benders model cuts
112 are introduced in the main problem to revise the commitment solution. The aforementioned procedures
113 and interactions between the commitment problem and its feasibility continued till the forecasted wind
114 power generation be accommodated by re-dispatch procedure. In [20] has been solved a contingency-
115 constrained single-bus UC problem which incorporates a security criterion. The method was based on
116 robust optimization using umbrella contingency definition as a counterpart of the original problem,
117 resulting in a bi-level programming problem solved by its transformation to an equivalent single-level
118 MIP problem. In [21] has been carried out a comparison between scenario-based and interval
119 optimization approaches to stochastic SCUC, where the wind power generation and other uncertainties
120 were addressed for comparison of both models. Furthermore, MCS was included in scenario-based model

121 to compute the wind power generation uncertainty, while lower and upper bounds of the aforementioned
122 uncertainty were adopted in the interval optimization model. Meanwhile, the stochastic SCUC problem
123 was formulated as MIP problem and solved using the two aforementioned models. In [22] has been
124 proposed a probabilistic model of SCUC to minimize the energy cost, SR and possible loss of load. It was
125 also presented a new formulation for the expected energy not served considering the probability
126 distribution of forecast errors of wind and load, as well as outage replacement rates of generators;
127 furthermore the proposed model was solved using MILP. In [23] has been proposed a two-stage adaptive
128 robust optimization model for SCUC problem, where the first-stage UC decision and the second-stage
129 dispatch decision were robust against all uncertain nodal net injection realizations and the second-stage
130 fulfilled the adaptability uncertainty. The proposed model incorporated network, ramp-rate and
131 transmission security critical-constraints. In [24] has been proposed a model to solve SCUC based on the
132 concept of loadability set. It consists on projecting those feasible solutions onto the demand space in order
133 to refine SCUC. The main advantage of this approach is the reduction on the computational effort while
134 the accuracy of the obtained solution is preserved under light and heavy transmission congestions. In [25]
135 has been presented an optimization model based on chance-constrained programming using an estimation
136 of the confidence level of stochastic variables in order to supply demand taking into account several
137 simultaneous contingencies, while the mathematical structure of the optimization problem is formulated
138 as a MILP problem. In a similar way; in [26] effectiveness of chance-constrained programming to solve
139 SCUC was verified. In [27] has been proposed a methodology based on constrained ordinal optimization,
140 at which a sample of UC solutions are analysed in order to determine a reasonable solution considering a
141 set with large number of scenarios. The feasibility of each UC solution sampled is verified by using
142 analytical conditions.

143 Estimation of SR requirements based on probabilistic analysis consists on the application of a criterion
144 derived from the full capacity outage probability distribution on the UC solution. The computational
145 effort of this approach is low while the accuracy of the obtained solution is reasonable [28]. The analysis
146 of cost/benefit relationship consists of an auxiliary process carried out for each period of time (without
147 considering inter-temporal characteristics of UC problem) prior to the solution of UC problem. During the
148 auxiliary process, SR requirements are optimized taking into account the benefit related to SR margins
149 and the corresponding cost of provide it. Then, UC problem incorporating SR constraints is evaluated by
150 considering the inter-temporal characteristics of unit scheduling problem [29]. In [30], optimization of SR

151 requirements by the analysis of cost/benefit relationship was improved by adding a checking process
152 based on MCS approach.

153 In this paper, a special attention on the estimation of the cost/benefit relationship [29,30] is paid and a
154 hybrid approach has been developed and described. The proposed approach estimates the cost/benefit
155 relationship used to determine SR requirements by a hybrid technique that combines a convolution-based
156 approach and MCS method, in order to determine the optimal amount of power to be committed taking
157 into account wind power forecasting error and generation unit reliability. Forecasting error of wind speed
158 is represented by a discretized Weibull probability distribution function (PDF), which is evaluated on the
159 corresponding power curve of the wind farm in order to consider the non-linear characteristics of wind
160 generation, which is frequently modelled by a Gaussian PDF of net load, avoiding the estimation of some
161 possible curtailment of renewable generation.

162 The paper is organized as follow: section 2 briefly describes the cost/benefit relationship and how to
163 use it for determining the optimal capacity to be committed; section 3 describes the proposed
164 methodology including the model of wind power forecasting error and system reliability which is
165 modelled by a convolution process and MCS approach; in section 4 proposed methodology is illustrated
166 by the analysis of two case studies of 10 and 140 units; finally, conclusions are presented in section 5.

167

168 **2. Cost/benefit relationship for SR requirements determination**

169

170 Extensively analysed by Ortega-Vazquez and Kirschen, cost/benefit relationship optimizes the SR
171 requirements by balancing the cost related to SR and the economic benefit obtained from this reserve
172 margin, optimal SR is determined by finding the amount of power to be committed in order to minimize
173 generation cost according to (1),

174

$$175 \min E\{TGC\} = E\{FCC\} + VOLL \times E\{ENS\}. \quad (1)$$

176

177 Fig. 1 illustrates the behaviour of cost/benefit relationship, it is possible observing how as the committed
178 capacity increases (or in other words SR increases), the fuel consumption cost (FCC) increases while the
179 cost related to energy not supplied (ENS) decreases. Then, when the addition of both costs is carried out,
180 a local minimum on the generation cost can be identified and consequently the optimal SR required. In

181 [29], the analysis of cost/benefit relationship only considering generation unit reliability was carried out;
182 expected value of ENS was obtained by means of the capacity outage probability table [31] in order to
183 consider forced outage rate (FOR) of each unit. In [30], the methodology aforementioned was extended in
184 order to consider the influence of wind power generation, where forecasting error of net load is
185 represented as a Gaussian PDF under the assumption of large amount of wind turbines which are
186 geographically dispersed. In more detail, net load forecasting error is modelled by means of a discretized
187 Gaussian PDF of seven segments; then, the optimal SR is obtained for each interval analysing the
188 cost/benefit relationship according to [29]; finally, optimal SR required is determined as the summation of
189 the optimal SR of each interval weighted according to the probability of occurrence of the corresponding
190 interval.

191

192 “See Fig. 1”

193

194 Optimal SR requirements are determined for each time interval without taking into account the inter-
195 temporal characteristics of the optimization problem (ramp constraints of UC problem). Once the optimal
196 SR for each period of the scheduling horizon has been obtained, a traditional reserve constrained UC
197 problem is solved (incorporating ramp constraints of UC problem) in order to determine the optimal
198 scheduling and power dispatch [29].

199

200 **3. Proposed methodology**

201

202 As was stated before, this paper focus on the efficient evaluation of cost/benefit relationship in order to
203 determine the optimal SR requirement taking into account wind power forecasting error and generation
204 unit reliability. Building cost/benefit relationship requires solving ED problem under uncertain conditions
205 at which wind power generation and any failure event of generation system are the main sources of
206 uncertainty. In our previous work presented in [32], ED problem was extensively analysed and according
207 to the obtained results, the estimation of expected ENS by convolution could give overestimated results
208 since the effects of SR on ENS were not taken into account. However, when the load to be supplied is
209 higher than the total capacity committed there is no SR and expected ENS can be accurately determined
210 by a convolution process. This condition is particularly useful to build cost/benefit relationship.

211 In this paper, we have divided cost/benefit relationship in two different regions according to Fig. 2. In
212 Region I, the committed capacity is lower than the minimum net load to supplied; so that, SR does not
213 have any impact on system operation; while in Region II the committed capacity is high enough and SR
214 has a relevant influence of the operation of the system. Under this circumstance, those points of Region I
215 can be analysed by means of the convolution process; while those points of Region II can be analysed by
216 means of MCS method. This hybrid approach allows us carrying out an efficient analysis of cost/benefit
217 relationship due to the fact that the computational effort is concentrated in those points of high relevance
218 (Region II), while the others are analysed by analytical methods (Region I) in an efficiency and fast way.
219 Moreover, the methodology proposed in this paper uses a discretized Weibull PDF combined with the
220 power curve of the wind farm to model wind power uncertainty, which allows incorporating the non-
221 linear characteristics of wind generation on the cost/benefit relationship. On the other hand, power
222 production of thermal units at previous instant is modelled by using discretized PDF and incorporated on
223 the solution of ED problem, so that the inter-temporal characteristics are taken into account.

224

225 “See Fig. 2”

226

227 Next sub-sections describe the mathematical models of wind speed and wind power generation,
228 generation unit reliability, and their incorporation on ED problem in order to estimate generation costs.

229

230 **3.1 Probabilistic modelling of wind power forecasting error**

231

232 Wind speed has been probabilistically modelled by using a Weibull PDF since many years ago; Weibull
233 distribution only depends on two parameters which give it high flexibility, and these parameters could be
234 estimated in a simple way with high accuracy [33]. As power generation of conventional generators
235 depends on the behaviour of wind power production, the PDF of wind generation should be included on
236 ED optimization problem through a discretization process. In this sense, Barbiero [34] has developed a
237 general-purposes discretization method and it has been used here to represent Weibull PDF. In the
238 discretization process a support represented by \varnothing_s with S elements is built according to (2), the parameter
239 λ is set according to the preferences of the user ($\lambda = 3$ is suggested in [34]). Then, each value of the
240 support is evaluated on the cumulative distribution function (CDF) of a normalized Gaussian distribution

241 (Equation (3)) and the obtained values are evaluated on the inverse CDF of a Weibull distribution with
 242 the required parameters (Equation (4)); after that, the centres of each interval are determined (Equation
 243 (5)), and the corresponding discretized PDF is determined (Equations (6)-(8)),
 244

$$245 \quad \vartheta_s = -\lambda - \frac{2\lambda}{S-1} + \left(\frac{2\lambda}{S-1}\right)s; s = 1, 2, \dots, S \quad (2)$$

$$246 \quad \mu_s = F_N(\vartheta_s); s = 1, 2, \dots, S \quad (3)$$

$$247 \quad WS_s = F_W^{-1}(\mu_s); s = 1, 2, \dots, S \quad (4)$$

$$248 \quad \omega_s = \frac{WS_s + WS_{s+1}}{2}; s = 1, 2, \dots, S-1 \quad (5)$$

$$249 \quad f_W(WS_1) = F_W(\omega_1); s = 1 \quad (6)$$

$$250 \quad f_W(WS_s) = F_W(\omega_s) - F_W(\omega_{s-1}); s = 2, \dots, S-1 \quad (7)$$

$$251 \quad f_W(WS_S) = 1 - F_W(\omega_{S-1}); s = S. \quad (8)$$

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 259 Once discretized PDF of wind speed has been created (f_W), this is evaluated on the power curve of wind
 260 farm connected to the power system. In this paper, we have used the general-purposes power curve shown
 261 in Fig. 3, and mathematically described in (9)-(12) [35,36],
 262

263 “See Fig. 3”
 264

$$265 \quad WPG(v) = \begin{cases} 0 & ; 0 \leq v \leq v_i, v > v_o \\ \alpha + \beta v + \sigma v^2 & ; v_i \leq v \leq v_r \\ N_t R_p & ; v_r \leq v \leq v_o \end{cases} \quad (9)$$

$$266 \quad \alpha = \frac{1}{(v_i - v_r)^2} \left[v_i(v_i + v_r) - 4v_i v_r \left(\frac{v_i + v_r}{2v_r} \right)^3 \right] \quad (10)$$

268

$$\beta = \frac{1}{(v_i - v_r)^2} \left[4(v_i + v_r) \left(\frac{v_i + v_r}{2v_r} \right)^3 - (3v_i + v_r) \right] \quad (11)$$

270

$$\sigma = \frac{1}{(v_i - v_r)^2} \left[2 - 4 \left(\frac{v_i + v_r}{2v_r} \right)^3 \right]. \quad (12)$$

272

273 Wind power PDF is discretized in several intervals or states (Q) between 0 and the maximum capacity of
274 the wind farm ($N_t R_p$) with a step ($\Delta W P$) defined according to (13),

275

$$\Delta W P = \frac{N_t R_p}{Q - 1}. \quad (13)$$

277

278 Then, wind power generation PDF (f_R) can be obtained by applying the algorithm presented in Fig. 4. In
279 this algorithm, for a determined interval (s) of wind speed ($W S_s$) the corresponding wind power
280 generation ($W P G(W S_s)$) is determined; after that, the interval of wind generation (q) that corresponds to
281 the results obtained from the evaluation of the power curve is determined and the probability of
282 occurrence of the wind speed interval ($f_W(W S_s)$) is assigned to this value of power. This process is
283 repeated for all the intervals of discretized wind speed PDF; so that, wind power generation PDF is
284 obtained.

285

286 “See Fig. 4”

287

288 3.2 Probabilistic modelling of generation unit reliability

289

290 As stated in section 3, expected value of ENS of those points of Region I are determined by convolution.
291 In order to build PDF of ENS, several states (P) are established between TPG^{min} and TPG^{max} with a
292 step ($\Delta T P G$) defined according to (14),

293

$$\Delta T P G = \frac{TPG^{max} - TPG^{min}}{P - 1}, \quad (14)$$

295

296 ENS CDF can be obtained by applying (15) in a recursive way [31], starting from the CDF F_G shown in
 297 Fig. 5 (left side) and ending with the CDF presented in Fig. 5 (right side). Finally, ENS PDF can be
 298 obtained from Fig. 5 by subtraction of probability values, resulting in the PDF shown in Fig. 6. The term
 299 $G_{t,q,n,h}$ in (15) is the power generation of unit n when the interval of power generation q , and the interval
 300 of initial power generation h are considered; it is obtained from the solution of ED dispatch problem
 301 solved as previous step before the application of (15) (see Fig. 8),

302

$$303 \quad F_G(TPG_p) = (1 - FOR_n)F_G(TPG_p) + FOR_n F_G(TPG_p - G_{t,q,n,h}). \quad (15)$$

304

305 “See Fig. 5”

306

307 “See Fig. 6”

308

309 Now, the required expected value of ENS is estimated by means of (16),

310

$$311 \quad E\{ENS\} = \sum_{p=1}^P f_G(TPG_p) TPG_p. \quad (16)$$

312

313 Expected value of ENS of those points of Region II is obtained by using MCS method at which, failure
 314 events are modelled by using a Binomial PDF presented in (17) [37],

315

$$316 \quad f_M = \binom{M}{m} (1 - FOR_n)^m (FOR_n)^{M-m}. \quad (17)$$

317

318 **3.3 Incorporation of forecasting error and system reliability in the estimation of total generation** 319 **cost**

320

321 In order to build cost/benefit relationship, estimation of expected value of ENS and FCC taking into
 322 account system reliability and forecasting error is required. In this sense, a discretized PDF of FCC is
 323 built by considering values between FCC^{min} and FCC^{max} with step ΔFCC defined by (18),

324

325
$$\Delta FCC = \frac{FCC^{max} - FCC^{min}}{A - 1}. \quad (18)$$

326

327 In this paper, power generation at previous time step ($t - 1$) is incorporated by considering some selected
 328 scenarios chosen by using quintile concept from the analysis of discretized PDF of power generation. Fig
 329 7 presents CDF of power generation of unit n ($F_{IPG,n}$) at time step $t - 1$. Dividing the interval $[0,1]$ in
 330 several points (H) each of these points (ξ_h) can be evaluated in the CDF $F_{IPG,n}$ in an inverse way; so that,
 331 a correlated power production is estimated among all generation units. This approach allows us reducing
 332 the number of possible starting points of ED optimization problem. All chosen scenarios are finally saved
 333 in the matrix of initial power generation ($IPG_{n,h}$) to be used for determining the optimal power dispatch
 334 [32].

335

336 “See Fig. 7”

337

338 As the number of possible scenarios of initial power production has been reduced, the corresponding
 339 weight to represent the probability of occurrence of each scenario is estimated according (19) [32],

340

341
$$\Phi_h = \frac{\prod_{n=1}^N (f_{IPG,n}(IPG_{n,h}))}{\sum_{h=1}^H \prod_{n=1}^N (f_{IPG,n}(IPG_{n,h}))}; h = 1, 2, \dots, H. \quad (19)$$

342

343 For the actual time step t , expected value of ENS and FCC is estimated according to the algorithm shown
 344 in Fig. 8. In this algorithm each interval of wind power discretization (q) and initial power generation (h)
 345 is considered; so that, for each combination of discrete states, ED problem is solved and discretized PDF
 346 of FCC is built; then, the region of the cost/benefit relationship is determined by considering the value of
 347 the net load ($NL_{t,q,h}$). Note that each value of ENS and FCC is weighted according to the probability of
 348 occurrence of wind power generation (f_R) and initial power production (Φ_h). Finally, the required
 349 expected values are obtained by summation of all weighted values.

350

351 “See Fig. 8”

352

353 **4. Case studies**

354

355 The methodology described in previous sections to build cost/benefit relationship is illustrated by
356 analysing two power systems of 10 units and 140 units provided with wind power generation. As the
357 proposed methodology models initial power generation of thermal units as a discretized PDF, all of them
358 have been obtained by MCS method. To carry out this task, three time steps have been considered, at $t -$
359 2 the value of initial power generation has been assumed as a constant value for all the units; while for
360 $t - 1$ and t , discretized PDF of initial power generation is obtained by solving ED problem. Then,
361 discretized PDF at time $t - 1$ is used as an input in our proposed approach and the results obtained at
362 time t are used to carry out a comparative analysis between the proposed methodology and MCS
363 approach. At time t , wind speed is modeled as a Weibull PDF with the same characteristics for all case
364 studies, changing the parameters of the wind farm according to the respective case under analysis.

365 Fig. 9 shows discretized PDF of wind speed at time t with shape factor equal to 2 and scale factor equal
366 to 7, discretization was carried out by applying the procedure previously described in sub-section 3.1
367 using equations (2)-(8) and $\lambda=5$. Regarding the wind farm model, in all cases a single wind turbine was
368 assumed to have $v_i=3$ m/s, $v_r=12$ m/s, $v_o=25$ m/s, and $R_p=2$ MW. The value of N_t was defined by the
369 particular case study under analysis.

370

371 “See Fig. 9”

372

373 **4.1 10-units power system**

374

375 In this sub-section results obtained from the analysis of a small capacity system are presented. Table 1
376 shows the characteristics of thermal generators under analysis [38].

377

378 “See Table 1”

379

380 First, a comparison between the results obtained from convolution process and MCS method is carried
381 out in order to evaluate their quality and efficiency. For this purpose only the commitment of the units 1-6
382 was considered. As was stated before, three different time steps are considered, at time $t - 2$ thermal

383 units generate 1,412 MW, at time $t - 1$ load demand is 1,600 MW, and at time t load demand is 1,750
384 MW. Regarding wind speed forecasting error, at time $t - 1$ it was modeled as a Weibull PDF with shape
385 factor equal to 10 and scale factor equal to 3. The number of wind turbines of wind farm was assumed to
386 be 50 ($N_t=50$). Fig. 10 shows discretized PDF of wind power generation at time t obtained by evaluating
387 equations (9)-(13) and the algorithm shown in Fig. 4, assuming $Q=24$.

388

389 “See Fig. 10”

390

391 4.1.1 Analysis of the Region I points

392

393 In order to analyse the quality of the results obtained from the convolution process, the first 6 generators
394 are committed. It is important to note that, as the committed capacity cannot supply the load demand, SR
395 does not have any influence on the analysis; so that, convolution process could be directly applied.
396 Discretization of conventional power production was carried out by assuming $TPG^{min}=0$ MW,
397 $TPG^{max}=1,500$ MW, and $P=150$. Discretization of wind generation was carried out by assuming $Q=24$.
398 Regarding the initial power production, the number of scenarios considered was 5 ($H=5$); MCS approach
399 was adopted as a reference considering 10,000 trials ($M=10,000$). Table 2 shows the results from the
400 comparative analysis between convolution method and MCS approach; as can be observed, an error of
401 7.39% in the estimation of FCC and 0.73% in the estimation of ENS was found in a reduced
402 computational time.

403

404 “See Table 2”

405

406 4.1.2 Analysis of the Regions I/II points

407

408 As the analysis of cost/benefit relationship progress, some points could belong to Region I or II
409 depending on the discretized value of wind power generation. In order to analyse this condition, number
410 of wind turbines has been increased to 250 ($N_t=250$), load demand at $t - 1$ was changed to 1,400 MW
411 and 1,500 MW in t . Discretized PDF was built by considering $TPG^{min}=0$ MW, $TPG^{max}=1,700$ MW, and
412 $P=100$. As in the sub-section 4.1.1, the number of MCS trials was adjusted to 10,000 ($M=10,000$) for the

413 reference analysis; however, the number of MCS trials considered for our proposed approach was
414 adjusted to 500 ($M=500$) and the number of discrete levels of wind power generation to 3 ($Q=3$) in order
415 to reduce the computational efforts. The number of scenarios considered to represent the initial power
416 generation was 5 ($H=5$). According to the results reported in Table 3, an error of 4.65% in the estimation
417 of FCC and 3.07% in the estimation of ENS was found, reducing the computational effort.

418

419

“See Table 3”

420

421 **4.1.3 Building cost/benefit relationship (10-Units system)**

422

423 In this sub-section, cost/benefit relationship is analysed by taking into account all the units of the system
424 under the operating conditions described in sub-section 4.1.2 and $VOLL=1,000$ \$/MWh. Fig. 11 shows the
425 most relevant part of cost/benefit relationship, the black line is the relationship obtained by evaluating
426 5,000 trials in 2,384.8 s (expected cost 60,114.91 \$); while, those grey ones were obtained by evaluating
427 different sets between 10 and 1,000 trials, obtaining computational times between 9.3 s and 477.8 s. The
428 optimal amount of power to be committed depends on the amount of MCS considered; in this case,
429 optimal capacity is around 1,607 MW ($E\{TGC\}$ equal to 32,149.236 \$) for low number of MCS
430 ($M \rightarrow 10$); however, this value increases to 1,662 MW ($E\{TGC\}$ equal to 58,307.81 \$) as the number of
431 MCS trials increases ($M \rightarrow 1,000$).

432

433

“See Fig. 11”

434

435 Table 4 presents the frequency at which the analysis is in Region I or II and the corresponding
436 computational time. As can be observed, when the committed capacity gets its minimum value, the
437 analysis is carried out by convolution in a reduced computational time; then, as the committed capacity
438 increases, the relevance of SR increases and at some intervals MCS approach is applied, incrementing the
439 calculation effort; finally, as the total capacity of the system is reached, the mathematical efforts are
440 incremented due to in each MCS trial an optimization problem (ED) committing all units has to be
441 solved. In general sense, proposed methodology only applies MCS approach in those cases at which SR

442 has an important influence on the results; while in the rest of cases convolution process is used in order to
443 improve the computational efficiency.

444

445 “See Table 4”

446

447 **4.2 140-units power system**

448

449 The behaviour of the proposed methodology analysing large scale systems is investigated by means of the
450 systems described in Table 5 [39]. Wind power generation has been modelled by means of a wind farm
451 with 9,000 turbines ($N_t=9,000$). Load demand at $t - 2$ was assumed to be 60,000 MW, at time $t - 1$ was
452 assumed to be 49,342 MW, and at t was assumed to be 51,000 MW. Discretized PDF was built by
453 considering $TPG^{min}=0$ MW, $TPG^{max}=65,000$ MW, and $P=100$. The number of MCS trials was adjusted
454 to 1,000 ($M=1,000$) for the reference analysis; however, the number of MCS trials considered for our
455 proposed approach was adjusted to 100 ($M=100$) and the number of discrete levels of wind power
456 considered was 3 ($Q=3$); while, the number of scenarios considered to represent the initial power
457 generation was 3 ($H=3$).

458

459 “See Table 5”

460

461 This large amount of MCS trails for our proposed methodology was considered in order to have a good
462 reference of comparison; the cost/benefit relationship obtained is shown in Fig. 12, where the optimal
463 capacity to be committed is 56,360 MW; this curve was built in a computational time of 22,638.097 s.

464

465 “See Fig. 12”

466

467 Fig. 13 presents the region analysis for this case study, where it is possible observing a similar behaviour
468 for the 10-units case study; when low capacity is committed most of the calculations are carried out by
469 convolution; however, as the committed capacity is increased, application of MCS is required in order to
470 consider the effects of SR.

471

472

“See Fig. 13”

473

474 Fig. 14 presents the computational time as a function of the committed capacity (expressed in percent of
475 the total computational time 22,638.097 s), where the increment in the calculation effort as a consequence
476 of MCS approach is observed.

477

478

“See Fig. 14”

479

480 The computational time could be reduced by using MCS approach with a lower set of trials, a lower
481 amount of discretized levels to represent wind power generation and initial power production. For
482 example, if the number of MCS is reduced to 10 ($M=10$), the number of discrete states of wind generation
483 is adjusted to 2 ($Q=2$), and only one state of initial power generation ($H=1$) is considered by adjusting
484 $\xi_{h=1}=0.5$; the computational time could be reduced to 564.998 s, obtaining a similar result regarding to
485 optimal committed capacity; although, the estimation of total generation cost could be highly distorted; in
486 our case this is estimated as 1,607,668.225 \$; while for our reference solution it was estimated as
487 1,444,166.681\$. Generally speaking, the amount of MCS trials should be selected according to the
488 computational resources and calculation time available; however, low amount of MCS trails directly
489 impacts on the estimation of $E\{TGC\}$. The proposed approach was implemented in MATLAB
490 programming language using a computer provided of i7-3630QM CPU at 2.40 GHz with 8 GB of
491 memory and 64 bit operating system.

492

493 5. Conclusions

494

495 In this paper, the cost/benefit relationship has been exhaustively analysed and a methodology to build this
496 curve based on discrete probability theory and MCS approach was developed and tested. The proposed
497 method aims to significantly improve the efficiency of the calculation required to build the cost/benefit
498 relationship used for optimal SR requirements determination by identifying those cases at which a
499 convolution analysis can be used; while in the rest of the cases, those at which calculation of expected
500 ENS is influenced by SR, are estimated by means of MCS. Accordingly, the computational efficiency is
501 considerably improved, while a reasonable result is reached. Two case studies of 10 and 140 units were

502 analysed under wind power penetration, demonstrating the flexibility of the proposed technique reached
503 by adjusting the number of discrete states and MCS trials.

504

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506

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511

512 **References**

513

514 [1] Boletín mensual diciembre 2013, Red Eléctrica de España, 2014.

515 [2] Fokaides PA, Kylili A. Towards grid parity in insular energy systems: The case of photovoltaics (PV)
516 in Cyprus. *Energy Policy* 2014;65:223-28.

517 [3] Brouwer AS, Van den Broek M, Seebregts A, Faaij A. Impacts of large-scale intermittent renewable
518 energy sources on electricity systems, and how these can be modelled. *Renew Sustainable Energy Rev*
519 2014;33:443-66.

520 [4] Papaefthymiou SV, Papathanassiou SA. Optimum sizing of wind-pumped-storage hybrid power
521 stations in island systems. *Renew Energy* 2014;64:187-96.

522 [5] Dietrich K, Latorre JM, Olmos L, Ramos A. Demand response in an isolated system with high wind
523 integration. *IEEE Trans Power Syst* 2012;27:20-29.

524 [6] Mc Garrigle EV, Leahy PG. Quantifying the value of improved wind energy forecasts in a pool-based
525 electricity market. *Renew Energy* 2015;80:517-24.

526 [7] Martínez-Anido CB, Florita A, Hodge B-M. The impact of improved solar forecasts on bulk power
527 system operations in ISO-NE. Golden, Colorado: National Renewable Energy Laboratory; 2014.

528 [8] Ortega-Vazquez MA, Kirschen DS. Should the spinning reserve procurement in systems with wind
529 power generation be deterministic or probabilistic? In: *Proceedings IEEE-SUPERGEN conference on*
530 *Sustainable Power Generation and Supply*. Nanjing, China; 2009.

531

532 [9] Tuohy A, Meibom P, Denny E, O'Malley M. Unit commitment for systems with significant wind
533 penetration. IEEE Trans Power Syst 2009;24:592-601.

534 [10] Carrión M, Arroyo JM. A computationally efficient mixed-integer linear formulation for the thermal
535 unit commitment problem. IEEE Trans Power Syst 2006;21:1371-78.

536 [11] Frangioni A, Gentile C, Lacalandra F. Tighter approximated MILP formulations for unit
537 commitment problems. IEEE Trans Power Syst 2009;24:105-13.

538 [12] Morales España G, Latorre JM, Ramos A. Tight and compact MILP formulation of strat-up and shut-
539 down ramping in unit commitment. IEEE Trans Power Syst 2013;28:1288-96.

540 [13] Wu L, Shahidehpour M, Li T. Stochastic security-constrained unit commitment. IEEE Trans Power
541 Syst 2007;22:800-11.

542 [14] Ruiz PA, Philbrick CR, Zak E, Cheung KW, Sauer PW. Uncertainty management in the unit
543 commitment problem. IEEE Trans Power Syst 2009;24:642-51.

544 [15] Ruiz PA, Philbrick CR, Sauer PW. Modeling approaches for computational cost reduction in
545 stochastic unit commitment formulations. IEEE Trans Power Syst 2010;25:588-89.

546 [16] Fu Y, Shahidehpour M, Li Z. Long-term security-constrained unit commitment: hybrid Dantzig-
547 Wolfe decomposition and subgradient approach. IEEE Trans Power Syst 2005;20:2093-106.

548 [17] Wu L, Shahidehpour M, Li T. Stochastic security-constrained unit commitment. IEEE Trans Power
549 Syst 2007;22:800-11.

550 [18] Fu Y, Shahidehpour M. Fast SCUC for large-scale power systems. IEEE Trans Power Syst
551 2007;22:2144-51.

552 [19] Wang J, Shahidehpour M, Li Z. Security-constrained unit commitment with volatile wind power
553 generation. IEEE Trans Power Syst 2008;23:1319-27.

554 [20] Street A, Oliveira F, Arroyo J. Contingency-constrained unit commitment with $n - K$ security
555 criterion: A robust optimization approach. IEEE Trans Power Syst 2011;26:1581-90.

556 [21] Wu L, Shahidehpour M, Li Z. Comparison of scenario-based and interval optimization approaches to
557 stochastic SCUC. IEEE Trans Power Syst 2012;27:913-21.

558 [22] Liu G, Tomsovic K. Quantifying spinning reserve in systems with significant wind power
559 penetration. IEEE Trans Power Syst 2012;27:2385-93.

560

561 [23] Bertsimas D, Litvinov E, Sun XA, Zhao J, Zheng T. Adaptive robust optimization for the security
562 constrained unit commitment problem. *IEEE Trans Power Syst* 2013;28:52-63.

563 [24] Kalantari A, Restrepo JF, Galiana FD. Security-constrained unit commitment with uncertain wind
564 generation: The loadability set approach. *IEEE Trans Power Syst* 2013;28:1787-96.

565 [25] Pozo D, Contreras J. A chance-constrained unit commitment with an $n - K$ security criterion and
566 significant wind generation. *IEEE Trans Power Syst* 2013;28:2842-51.

567 [26] Ding X, Lee W-J, Jianxue W, Liu L. Studies on stochastic unit commitment formulation with
568 flexible generating units. *Electr Pow Syst Res* 2010;80:130-41.

569 [27] Wu H, Shahidehpour M. Stochastic SCUC solution with variable wind energy using constrained
570 ordinal optimization. *IEEE Trans Sust Energy* 2014;5:379-88.

571 [28] Chattopadhyay D, Baldick R. Unit commitment with probabilistic reserve. In: *Proceedings IEEE*
572 *conference on Power Engineering Society Winter Meeting*. New York, USA; 2002.

573 [29] Ortega-Vazquez MA, Kirschen DS. Optimizing the spinning reserve requirements using a
574 cost/benefit analysis. *IEEE Trans Power Syst* 2007;22:24-33.

575 [30] Ortega-Vazquez MA, Kirschen DS. Estimating the spinning reserve requirements in systems with
576 significant wind power generation penetration. *IEEE Trans Power Syst* 2009;24:114-24.

577 [31] Billinton A, Allan RN. *Reliability evaluation of power systems*. 2th ed. New York and London:
578 Plenum Press; 1996.

579 [32] Osório GJ, Lujano-Rojas JM, Matias JCO, Catalão JPS. A probabilistic approach to solve the
580 economic dispatch problem with intermittent renewable energy sources. *Energy* 2015;82:949-59.

581 [33] Carta JA, Ramírez P, Velázquez S. A review of wind speed probability distributions used in wind
582 energy analysis Case studies in the Canary Islands. *Renew Sustainable Energy Rev* 2009;13:933-55.

583 [34] Barbiero A. A general discretization procedure for reliability computation in complex stress-strength
584 models. *Math Comput Simulat* 2012;82:1667-76.

585 [35] Dialynas EN, Machias AV. Reliability modelling interactive techniques of power systems including
586 wind generating units. *Archiv für Elektrotechnik* 1989;72:33-41.

587 [36] Giorsetto P, Utsurogi KF. Development of a new procedure for reliability modelling of wind turbine
588 generators. *IEEE Trans Power App Syst* 1983;PAS-102:134-143.

589

590 [37] Karaki SH, Chedid RB, Ramadan R. Probabilistic performance assessment of wind energy
591 conversion systems. IEEE Trans Energy Convers 1999;14:217-224.
592 [38] Senjyu T, Miyagi T, Yousuf SA, Urasaki N, Funabashi T. A technique for unit commitment with
593 energy storage system. Int J Elec Power 2007;29:91-98.
594 [39] Coelho LDS, Bora TC, Mariani VC. Differential evolution based on truncated Lévy-type flights and
595 population diversity measure to solve economic load dispatch problems. Int J Elec Power 2014;57:178-
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643 Figures
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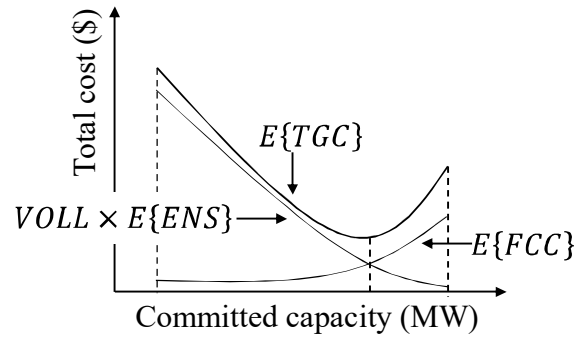


Fig. 1: Cost/Benefit relationship for a determined time step.

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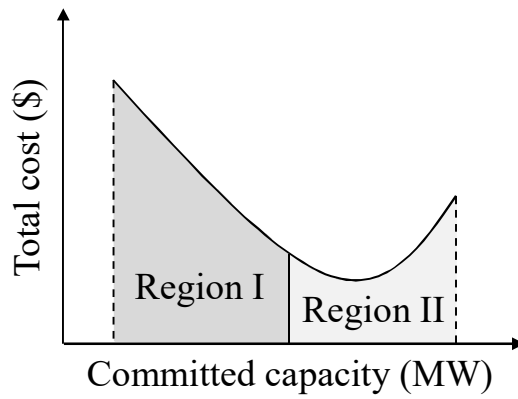


Fig. 2: Regions of analysis of cost/benefit relationship for a determined time step.

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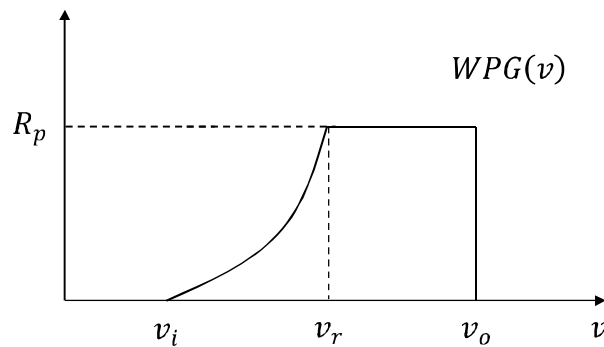


Fig. 3: Power curve of a single wind turbine.

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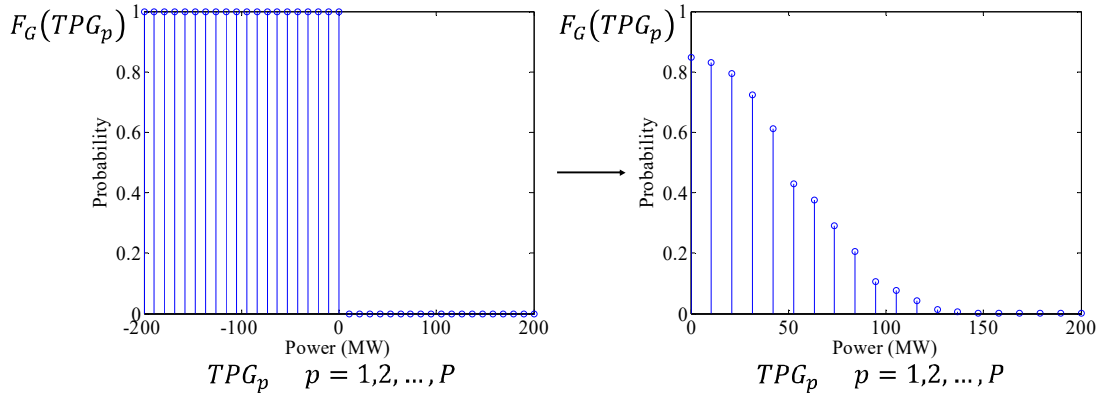
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 $f_R(WP_q) = 0; q = 1, 2, \dots, Q$ 
 $WP_q = 0, \Delta WP, \dots, N_t R_p - \Delta WP, N_t R_p$ 
for  $s = 1, 2, \dots, S$ 
  if  $(WPG(W S_s) \leq WP_1 - \Delta WP/2)$ 
     $f_R(WP_1) = f_R(WP_1) + f_W(W S_s)$ 
  elseif  $(WPG(W S_s) \geq WP_Q + \Delta WP/2)$ 
     $f_R(WP_Q) = f_R(WP_Q) + f_W(W S_s)$ 
  else
    for  $q = 1, 2, \dots, Q$ 
      if  $(WPG(W S_s) \geq WP_q - \Delta WP/2)$  and  $(WPG(W S_s) < WP_q + \Delta WP/2)$ 
         $f_R(WP_q) = f_R(WP_q) + f_W(W S_s)$ 
      end
    end
  end
end
end

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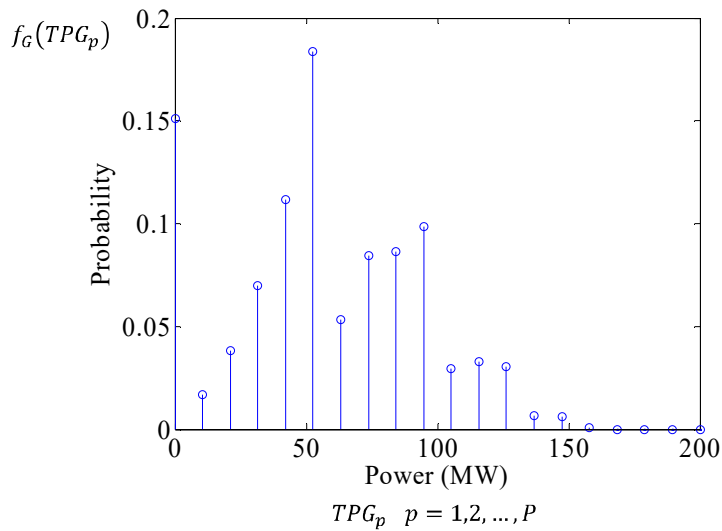
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Fig. 4: Algorithm for discretization of wind power generation.



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Fig. 5: Determination of ENS probability distribution by convolution.



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Fig. 6: Probability distribution of ENS.

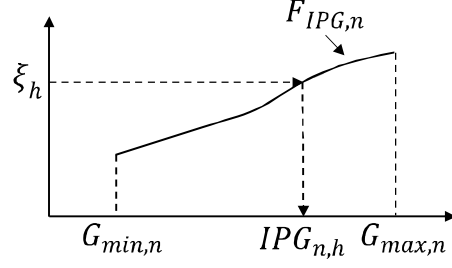


Fig. 7: Analysis of CDF of power generation of unit n .

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```

for  $h = 1, 2, \dots, H$ 
   $I_n = IPG_{n,h}; n = 1, 2, \dots, N$ 
   $f_c(FCC_a) = 0; a = 1, 2, \dots, A$ 
  for  $q = 1, 2, \dots, Q$ 
     $K_{t,q,h} = \min \sum_{n=1}^N \{ \gamma_n + \delta_n(G_{t,q,n,h}) + \varepsilon_n(G_{t,q,n,h})^2 \}$ 
     $G_{t-1,q,n,h} = I_n; n = 1, 2, \dots, N$ 
     $\sum_{n=1}^N \{ G_{t,q,n,h} \} + WPD_{t,q,h} = L_t$ 
     $G_{t,q,n,h} - G_{t-1,q,n,h} \leq UR_n; n = 1, 2, \dots, N$ 
     $G_{t-1,q,n,h} - G_{t,q,n,h} \leq DR_n; n = 1, 2, \dots, N$ 
     $G_{min,n} \leq G_{t,q,n,h} \leq G_{max,n}; n = 1, 2, \dots, N$ 
     $0 \leq WPD_{t,q,h} \leq WP_q$ 
     $FCC_a = FCC^{min}, FCC^{min} + \Delta FCC, \dots, FCC^{max} - \Delta FCC, FCC^{max}$ 
    if  $(K_{t,q,h} \leq FCC_1 - \Delta FCC/2)$ 
       $f_c(FCC_1) = f_c(FCC_1) + f_R(WP_q)$ 
    elseif  $(K_{t,q,h} \geq FCC_A + \Delta FCC/2)$ 
       $f_c(FCC_A) = f_c(FCC_A) + f_R(WP_q)$ 
    else
      for  $a = 1, 2, \dots, A$ 
        if  $(K_{t,q,h} \geq FCC_a - \Delta FCC/2) \text{ and } (K_{t,q,h} < FCC_a + \Delta FCC/2)$ 
           $f_c(FCC_a) = f_c(FCC_a) + f_R(WP_q)$ 
        end
      end
    end
     $NL_{t,q,h} = L_t - WPD_{t,q,h}$ 
     $l_{t,q,h} = \sum_{n=1}^N \{ \min(G_{t-1,q,n,h} + UR_n, G_{n,max}) \}$ 
    if  $(NL_{t,q,h} > l_{t,q,h})$ 
      Determine  $E\{ENS_{t,q,h}\}$  by convolution
    else
      Determine  $E\{ENS_{t,q,h}\}$  by MCS approach
    end
     $\eta_q = E\{ENS_{t,q,h}\} \times f_R(WP_q)$ 
  end
   $\tau_h = (\sum_{q=1}^Q \eta_q) \times \Phi_h$ 
   $\theta_h = (\sum_{a=1}^A FCC_a \times f_c(FCC_a)) \times \Phi_h$ 
end
 $E\{ENS_t\} = \sum_{h=1}^H \tau_h$ 
 $E\{FCC_t\} = \sum_{h=1}^H \theta_h$ 

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Fig. 8: Algorithm for incorporation of forecasting error and system reliability.

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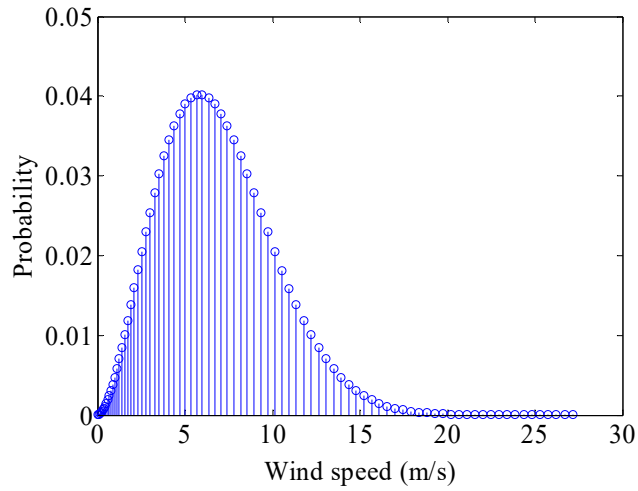


Fig. 9: Discretized wind speed PDF.

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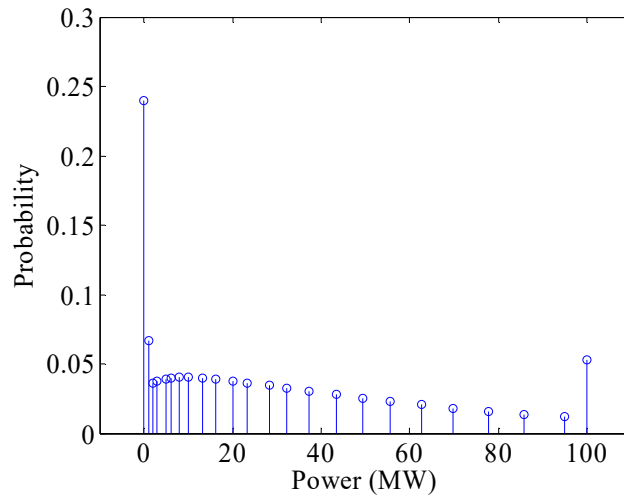


Fig. 10: Discretized wind power PDF.

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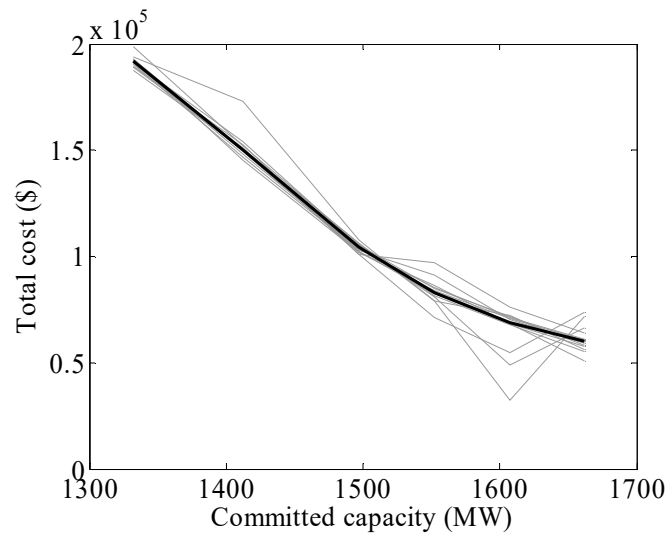


Fig. 11: Cost/benefit relationship for 10-units power system.

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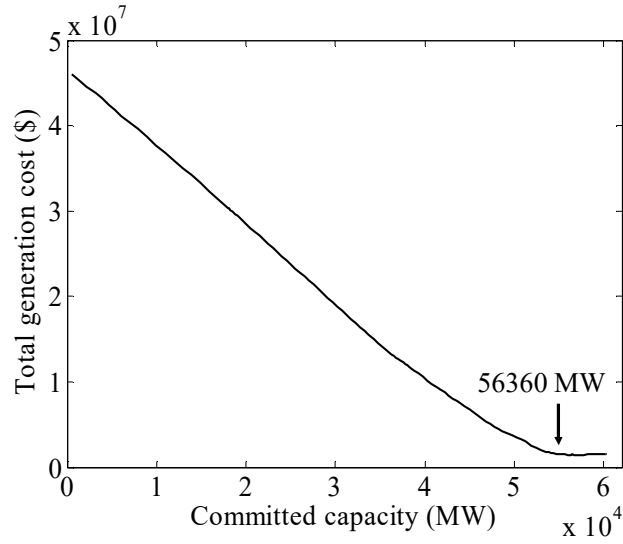


Fig. 12: Cost/benefit relationship for 140-units power system.

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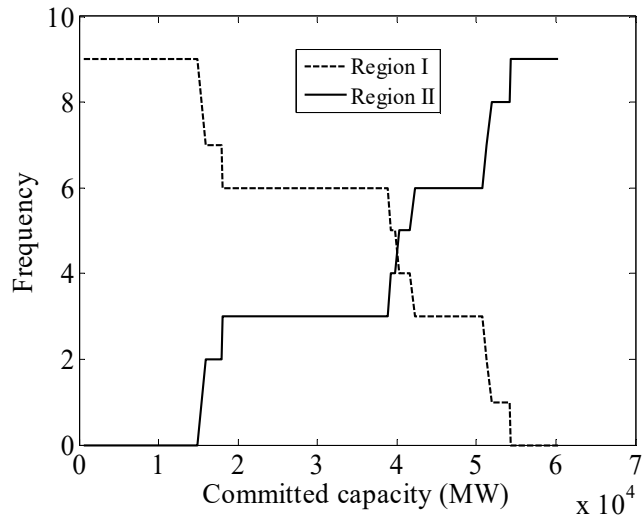


Fig. 13: Region analysis of 140-Units system.

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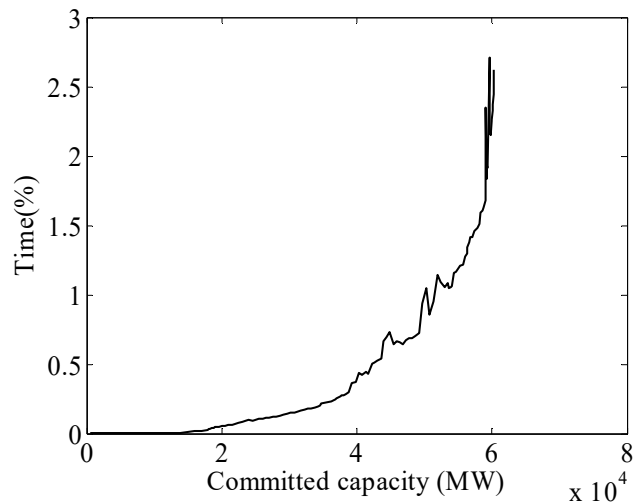


Fig. 14: Computational time analysis for 140-Units system.

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Tables

Table 1: Data of 10-units power system.

n	$G_{min,n}$ (MW)	$G_{max,n}$ (MW)	γ_n (\$/h)	δ_n (\$/MW)	ϵ_n (\$/MW ²)	DR_n (MW/h)	UR_n (MW/h)	FOR_n
1	150	455	1000	16.19	0.00048	130	130	0.05
2	150	455	970	17.26	0.00031	130	130	0.05
3	25	162	450	19.7	0.00398	90	90	0.1
4	20	130	680	16.5	0.00211	60	60	0.1
5	20	130	700	16.6	0.002	60	60	0.1
6	20	80	370	22.26	0.00712	40	40	0.1
7	25	85	480	27.74	0.00079	40	40	0.1
8	10	55	660	25.92	0.00413	40	40	0.0001
9	10	55	665	27.27	0.00222	40	40	0.0001
10	10	55	670	27.79	0.00173	40	40	0.0001

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Table 2: Comparison between MCS and convolution methods (Region I).

Variable	MCS	Convolution	Time (s)
$E\{FCC_t\}$	27,048.784	29,048.000	142.726
$E\{ENS_t\}$	409.991	412.999	12.875

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712

Table 3: Comparison between MCS and convolution/MCS methods (Regions I/II).

Variable	MCS	Convolution	Time (s)
$E\{FCC_t\}$	26,136.951	27,352.252	144.633
$E\{ENS_t\}$	117.155	120.749	25.173

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Table 4: Region analysis and computational time (10-Units).

Unit (MW)	Region I	Region II	Time (s)
455 (1)	15	0	0.219
910 (1-2)	10	5	5.875
1072 (1-3)	10	5	8.344
1202 (1-4)	6	9	16.236
1332 (1-5)	5	10	20.923
1412 (1-6)	5	10	23.938
1497 (1-7)	5	10	26.674
1552 (1-8)	1	14	39.111
1607 (1-9)	1	14	44.127
1662 (1-10)	0	15	51.878

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Table 5: Data of 140-units power system.

ID	$G_{min,n}$ (MW)	$G_{max,n}$ (MW)	γ_n (\$/h)	δ_n (\$/MW)	ϵ_n (\$/MW ²)	DR_n (MW/h)	UR_n (MW/h)	FOR_n
NUCLEAR#01	360	580	226.799	2.842	0.000064	18	18	0.04
NUCLEAR#02	415	645	382.932	2.946	0.000252	18	18	0.04
NUCLEAR#03	795	984	156.987	3.096	0.000022	36	36	0.04
NUCLEAR#04	795	978	154.484	3.04	0.000022	36	36	0.04
NUCLEAR#05	578	682	332.834	1.709	0.000203	204	138	0.04
NUCLEAR#06	615	720	326.599	1.668	0.000198	216	144	0.04
NUCLEAR#07	612	718	345.306	1.789	0.000215	216	144	0.04
NUCLEAR#08	612	720	350.372	1.815	0.000218	216	144	0.04
NUCLEAR#09	758	964	370.377	2.726	0.000193	48	48	0.04
NUCLEAR#10	755	958	367.067	2.732	0.000197	48	48	0.04
NUCLEAR#11	750	1007	124.875	2.651	0.000324	54	36	0.04
NUCLEAR#12	750	1006	130.785	2.798	0.000344	54	36	0.04
NUCLEAR#13	713	1013	878.746	1.595	0.00069	30	30	0.04
NUCLEAR#14	718	1020	827.959	1.503	0.00065	30	30	0.04
NUCLEAR#15	791	954	432.007	2.425	0.000233	30	30	0.04
NUCLEAR#16	786	952	445.606	2.499	0.000239	30	30	0.04
NUCLEAR#17	795	1006	467.223	2.674	0.000261	36	36	0.04
NUCLEAR#18	795	1013	475.94	2.692	0.000259	36	36	0.04
NUCLEAR#19	795	1021	899.462	1.633	0.000707	36	36	0.04
NUCLEAR#20	795	1015	1000.367	1.816	0.000786	36	36	0.04

COAL#01	71	119	1220.645	61.242	0.032888	120	30	0.03
COAL#02	120	189	1315.118	41.095	0.00828	120	30	0.03
COAL#03	125	190	874.288	46.31	0.003849	60	60	0.03
COAL#04	125	190	874.288	46.31	0.003849	60	60	0.03
COAL#05	90	190	1976.469	54.242	0.042468	150	150	0.03
COAL#06	90	190	1338.087	61.215	0.014992	150	150	0.03
COAL#07	280	490	1818.299	11.791	0.007039	300	180	0.03
COAL#08	280	490	1133.978	15.055	0.003079	300	180	0.03
COAL#09	260	496	1320.636	13.226	0.005063	510	300	0.03
COAL#10	260	496	1320.636	13.226	0.005063	510	300	0.03
COAL#11	260	496	1320.636	13.226	0.005063	510	300	0.03
COAL#12	260	496	1106.539	14.498	0.003552	510	300	0.03
COAL#13	260	506	1176.504	14.651	0.003901	600	600	0.03
COAL#14	260	509	1176.504	14.651	0.003901	600	600	0.03
COAL#15	260	506	1176.504	14.651	0.003901	600	600	0.03
COAL#16	260	505	1176.504	14.651	0.003901	600	600	0.03
COAL#17	260	506	1017.406	15.669	0.002393	600	600	0.03
COAL#18	260	506	1017.406	15.669	0.002393	600	600	0.03
COAL#19	260	505	1229.131	14.656	0.003684	600	600	0.03
COAL#20	260	505	1229.131	14.656	0.003684	600	600	0.03
COAL#21	260	505	1229.131	14.656	0.003684	600	600	0.03
COAL#22	260	505	1229.131	14.656	0.003684	600	600	0.03
COAL#23	260	505	1267.894	14.378	0.004004	600	600	0.03
COAL#24	260	505	1229.131	14.656	0.003684	600	600	0.03
COAL#25	280	537	975.926	16.261	0.001619	300	300	0.03
COAL#26	280	537	1532.093	13.362	0.005093	300	300	0.03
COAL#27	280	549	641.989	17.203	0.000993	360	360	0.03
COAL#28	280	549	641.989	17.203	0.000993	360	360	0.03
COAL#29	260	501	911.533	15.274	0.002473	180	180	0.03
COAL#30	260	501	910.533	15.212	0.002547	180	180	0.03
COAL#31	260	506	1074.81	15.033	0.003542	600	600	0.03
COAL#32	260	506	1074.81	15.033	0.003542	600	600	0.03
COAL#33	260	506	1074.81	15.033	0.003542	600	600	0.03
COAL#34	260	506	1074.81	15.033	0.003542	600	600	0.03
COAL#35	260	500	1278.46	13.992	0.003132	660	660	0.03
COAL#36	260	500	861.742	15.679	0.001323	900	900	0.03
COAL#37	120	241	408.834	16.542	0.00295	180	180	0.03
COAL#38	120	241	408.834	16.542	0.00295	180	180	0.03
COAL#39	423	774	1288.815	16.518	0.000991	600	600	0.03
COAL#40	423	769	1436.251	15.815	0.001581	600	600	0.03
LNG_CC#01	160	250	3427.912	56.613	0.024493	702	702	0.07
LNG_CC#02	160	250	3751.772	54.451	0.029156	702	702	0.07
LNG_CC#03	160	250	3918.78	54.736	0.024667	702	702	0.07
LNG_CC#04	160	250	3379.58	58.034	0.016517	702	702	0.07
LNG_CC#05	160	250	3345.296	55.981	0.026584	702	702	0.07
LNG_CC#06	160	250	3138.754	61.52	0.00754	702	702	0.07
LNG_CC#07	160	250	3453.05	58.635	0.01643	702	702	0.07
LNG_CC#08	160	250	5119.3	44.647	0.045934	702	702	0.07
LNG_CC#09	165	504	1898.415	71.584	0.000044	1350	1350	0.07
LNG_CC#10	165	504	1898.415	71.584	0.000044	1350	1350	0.07
LNG_CC#11	165	504	1898.415	71.584	0.000044	1350	1350	0.07
LNG_CC#12	165	504	1898.415	71.584	0.000044	1350	1350	0.07
LNG_CC#13	180	471	2473.39	85.12	0.002528	1350	1350	0.07
LNG_CC#14	180	561	2781.705	87.682	0.000131	720	720	0.07
LNG_CC#15	103	341	5515.508	69.532	0.010372	720	720	0.07
LNG_CC#16	198	617	3478.3	78.339	0.007627	2700	2700	0.07
LNG_CC#17	100	312	6240.909	58.172	0.012464	1500	1500	0.07
LNG_CC#18	153	471	9960.11	46.636	0.039441	1656	1656	0.07
LNG_CC#19	163	500	3671.997	76.947	0.007278	2160	2160	0.07
LNG_CC#20	95	302	1837.383	80.761	0.000044	900	900	0.07
LNG_CC#21	160	511	3108.395	70.136	0.000044	1200	1200	0.07
LNG_CC#22	160	511	3108.395	70.136	0.000044	1200	1200	0.07
LNG_CC#23	196	490	7095.484	49.84	0.018827	1014	1014	0.07
LNG_CC#24	196	490	3392.732	65.404	0.010852	1014	1014	0.07
LNG_CC#25	196	490	7095.484	49.84	0.018827	1014	1014	0.07
LNG_CC#26	196	490	7095.484	49.84	0.018827	1014	1014	0.07
LNG_CC#27	130	432	4288.32	66.465	0.03456	1350	1350	0.07
LNG_CC#28	130	432	13,813.00	22.941	0.08154	1350	1350	0.07
LNG_CC#29	137	455	4435.493	64.314	0.023534	1350	1350	0.07
LNG_CC#30	137	455	9750.75	45.017	0.035475	1350	1350	0.07
LNG_CC#31	195	541	1042.366	70.644	0.000915	780	780	0.07
LNG_CC#32	175	536	1159.895	70.959	0.000044	1650	1650	0.07

LNG CC#33	175	540	1159.895	70.959	0.000044	1650	1650	0.07
LNG CC#34	175	538	1303.99	70.302	0.001307	1650	1650	0.07
LNG CC#35	175	540	1156.193	70.662	0.000392	1650	1650	0.07
LNG CC#36	330	574	2118.968	71.101	0.000087	1620	1620	0.07
LNG CC#37	160	531	779.519	37.854	0.000521	1482	1482	0.07
LNG CC#38	160	531	829.888	37.768	0.000498	1482	1482	0.07
LNG CC#39	200	542	2333.69	67.983	0.001046	1668	1668	0.07
LNG CC#40	56	132	2028.945	77.838	0.13205	120	120	0.07
LNG CC#41	115	245	4412.017	63.671	0.096968	180	180	0.07
LNG CC#42	115	245	2982.219	79.458	0.054868	180	120	0.07
LNG CC#43	115	245	2982.219	79.458	0.054868	180	120	0.07
LNG CC#44	207	307	3174.939	93.966	0.014382	180	120	0.07
LNG CC#45	207	307	3218.359	94.723	0.013161	180	120	0.07
LNG CC#46	175	345	3723.822	66.919	0.016033	318	318	0.07
LNG CC#47	175	345	3551.405	68.185	0.013653	318	318	0.07
LNG CC#48	175	345	4322.615	60.821	0.028148	318	318	0.07
LNG CC#49	175	345	3493.739	68.551	0.01347	318	318	0.07
LNG#01	3	19	669.988	75.464	0.90236	210	210	0.0002
LNG#02	3	28	134.544	129.544	0.110295	366	366	0.0002
OIL#01	94	203	1269.132	89.83	0.014355	120	120	0.0001
OIL#02	94	203	1269.132	89.83	0.014355	120	120	0.0001
OIL#03	94	203	1269.132	89.83	0.014355	120	120	0.0001
OIL#04	244	379	4965.124	64.125	0.030266	480	480	0.0001
OIL#05	244	379	4965.124	64.125	0.030266	480	480	0.0001
OIL#06	244	379	4965.124	64.125	0.030266	480	480	0.0001
OIL#07	95	190	2243.185	76.129	0.024027	240	240	0.0001
OIL#08	95	189	2290.381	81.805	0.00158	240	240	0.0001
OIL#09	116	194	1681.533	81.14	0.022095	120	120	0.0001
OIL#10	175	321	6743.302	46.665	0.07681	180	180	0.0001
OIL#11	2	19	394.398	78.412	0.953443	90	90	0.0001
OIL#12	4	59	1243.165	112.088	0.000044	90	90	0.0001
OIL#13	15	83	1454.74	90.871	0.072468	300	300	0.0001
OIL#14	9	53	1011.051	97.116	0.000448	162	162	0.0001
OIL#15	12	37	909.269	83.244	0.599112	114	114	0.0001
OIL#16	10	34	689.378	95.665	0.244706	120	120	0.0001
OIL#17	112	373	1443.792	91.202	0.000042	1080	1080	0.0001
OIL#18	4	20	535.553	104.501	0.085145	60	60	0.0001
OIL#19	5	38	617.734	83.015	0.524718	66	66	0.0001
OIL#20	5	19	90.966	127.795	0.176515	6	12	0.0001
OIL#21	50	98	974.447	77.929	0.063414	300	300	0.0001
OIL#22	5	10	263.81	92.779	2.740485	6	6	0.0001
OIL#23	42	74	1335.594	80.95	0.112438	60	60	0.0001
OIL#24	42	74	1033.871	89.073	0.041529	60	60	0.0001
OIL#25	41	105	1391.325	161.288	0.000911	528	528	0.0001
OIL#26	17	51	4477.11	161.829	0.005245	300	300	0.0001
OIL#27	7	19	57.794	84.972	0.234787	30	18	0.0001

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