Optimal Contracts of Energy Mix in a Retail Market under Asymmetric Information

Yue Chen[a], Wei Wei[a], Feng Liu[a], Miadreza Shafie-khah[b], Shengwei Mei[a], João P. S. Catalão[b,c,d]

[a]*The State Key Laboratory of Power Systems,*
Department of Electrical Engineering, Tsinghua University, 100084, Beijing, China.
[b]*C-MAST, University of Beira Interior, Covilhã 6201-001, Portugal.*
[c]*INESC TEC and the Faculty of Engineering of the University of Porto, Porto 4200-465, Portugal.*
[d]*INESC-ID, Instituto Superior Técnico, University of Lisbon 1049-001, Portugal.*

Abstract

Co-generation plants have become mainstream energy production facilities at the demand side owing to their high efficiency and flexibility in operation. During the transition to more integrated energy supply, trading of energy mix will become an important issue, and a retailer is expected to play an active role. This paper discusses the design of retailer's optimal contract with asymmetric information. Bilateral relationship between the retailer and consumers is characterized by a package contract based on publicly observable information. First, a mathematical model for the optimal contract design problem involving two energy markets is established. Then, the model is simplified by eliminating redundant constraints. Consumer behaviors behind each reduction step are revealed. Thereafter, the market equilibrium is characterized; its existence is proved and the impact on retailer's strategy is revealed. An illustrative example with locational marginal price based heat and power markets is presented. Case studies confirm the theoretical analysis and show that our model can promote retailer's profit. The impact of several factors, such as the preference difference, probability, energy conversion efficiency and reservation utility level, has been tested, providing more insights into the market behavior under asymmetric information.

Keywords: Asymmetric information game; energy mix; package contract; retail

[*]Corresponding author
Email address: `lfeng@tsinghua.edu.cn` (Feng Liu)

market.

Nomenclature

Indexes, Sets, and Symbols

i	Index of consumers
w	Index of buses in energy network 1
v	Index of nodes in energy network 2
$\mathscr{N}$	Set of consumer types
$\mathscr{C}$	Set of package contracts
$\mathscr{P}$	consumer's distribution
$\hat{\Xi}$	Optimal solution of retailer's contract design problem under symmetric information
Ξ^*	Optimal solution of retailer's decision-making problem under asymmetric information
$\Xi^{*'}$	Optimal solution of retailer's decision-making problem under countervailing incentive
φ_p, φ_h	Functions depict the relationship between energy consumption and energy prices
$\Gamma(.)$	Risk preference function
$U(.)$	Identical form in the consumer's utility function

Parameter

α_p, α_h	Preference coefficient for power/heat
η_{eh}	Transfer efficiency from electricity to heat
$\bar{U}$	Reserve utility
π_i	The probability of consumer with type i, especially π_L, π_H in the case studies
c_p,c_h	Cost coefficient in energy network 1/2 and $c_h = [c_h^1; c_h^2]$
F_p,B_p	Coefficient matrix in market-clearing problem 1
F_h,B_h	Coefficient matrix in market-clearing problem 2
b_p, d^p	Constant vector in market-clearing problem 1
b_h, d^h	Constant vector in market-clearing problem 2
A_i	Coefficient of the utility function of consumer i, especially A_L, A_H in the case studies
NU_i^j	Net utility of consumer i with contract j
IR_i	Information rent of consumer i, especially IR_l, IR_h in the case studies
$r1, \ldots, r_m$	Buses of energy system 1 from which energy system 2 buys energy

Decision Variable

x_p,x_h	Decision variables in the market-clearing problem in energy network 1 and 2, and $x_h = [x_h^1; x_h^2]$
λ^p	Marginal energy price in energy market 1, for example, the power market, and λ_w^p is the price for the retailer
λ^h	Marginal energy price in energy market 2, for example, the heat market, and λ_v^h is the price for the retailer
ω^p, ω^h	Dual variables of the inequalities in the market-clearing problems
$p_{i,d}, h_{i,d}$	Energy amount in package for consumer of type i, especially p_l, p_h in the case studies
S_i	Package price for consumer of type i, especially S_l, S_h in the case study
$\mathscr{C}_i$	Package contract i
p_u	Heat bought/generated in the heating system

Abbreviations

RCF	Retailer–consumer framework
LMEP	Locational marginal energy price
CHP	Combined heat–power
MCP	Market-clearing problem
HPM	Heat–power market
SI	Symmetric information
AI	Asymmetric information
SE	Separating equilibrium
IR	Information rent

1. Introduction

Global concerns about environmental pollution and energy shortage have drawn great attention to high-efficiency allocation of resources, leading to the prosperity of energy markets worldwide. In the past decades, the electricity market has continued to flourish in many countries, such as Australia and the US [1]. District heating markets have been liberalized in such countries as Finland and Sweden [2], where heating prices are deregulated. In Germany, green power marketing is promoted, which has successfully increased the share of renewable energy [3]. Generally, different energy systems and markets have been planned and operated individually. However, this situation is changing owing to the integration of energy networks. Most recently, inspired by high efficiency and low environmental impacts, co-generation plants, such as electric boilers, combined heat–power (CHP) units, and high-performance heat pumps, continue to flourish [4–6]. These energy conversion facilities create strong coupling across historically independent networks both in energy flow and market behavior[7, 8]. In the UK, there has been a trend toward "multi-utility" bundling [9], increasing the coupling of multiple energy markets. The multi-energy market is a prevalent trend and in the future, energy prices and consumption will probably be determined contractually with *energy mix*, which is a combination of multiple energy as a package to sell. In this context, a sophisticated market design for energy mix is necessary, albeit challenging, as there is still lack of in-depth understanding of its fundamental features, such as the incentive scheme, the equilibrium characterization, and the behavior patterns of market participants, to name a few. In multi-energy market researches, two main aspects are the market structure and the information structure. This study takes a first step to close the gap by considering a specific problem in which a retailer designs energy mix contracts for consumers in a *liberalized retail market* under *asymmetric information*.

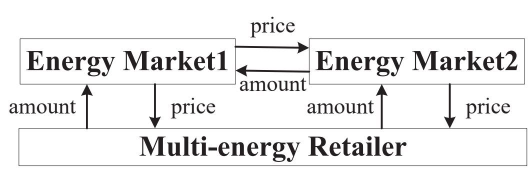

Figure 1: Structure of the multi-energy market.

The typical market structure of multi-energy trading is shown through Fig.1. First, the multi-energy retailer bids the desired amount of energy to the respective energy markets, and then receives the energy prices it has to pay. According to different market settings, the energy prices can be determined by a market clearing process [10] or just set as a fixed value. The marginal pricing method adopted in most market clearing procedures can better reflect the real-time needs of the system. Several existing works have studied the optimization of the multi-energy market. Reference [11] and [12] investigated the profitable operation of CHP plants in liberalized electricity markets. A decentralized multi-energy flow algorithm was proposed to optimize energy allocation in the carbon energy market [13]. A multi-lateral trading model for a gas–heat–power coupled system was proposed in [14], considering the strategic behaviors of different energy systems. The market power of a natural gas producer and its impact on the power system were analyzed in [15] and the optimal operation of gas-fired power plants in a competitive power market was considered in [16]. The above works only power market clearing is considered while the other forms of energy are treated as fixed source or demand. Nevertheless, it is worth noting that other energy markets can be cleared in a similar way, for example, a marginal price-based scheme for the heat-power market was considered in [17]. Reference [18] presented a strategic offering model for coupled gas and electricity markets, which considered clearing processes in both markets. In this paper, since energy mix is considered, it is more reasonable to take into account the market-clearing processes in all energy markets involved. Besides, electric boilers consume electricity whose price is time varying, so the marginal production cost of heat also varies across time periods. This is different from traditional boilers and CHPs which mainly consume coal and natural gas, whose prices are typically constant in a short period. In this regard, we envision a heat market in which the heat price is determined from an optimal thermal flow problem.

In an energy market, the retailer usually acts as an intermediary agent that purchases energy from energy distribution systems and resells it to end consumers by setting up bilateral contracts [19, 20]. A stochastic programming method was adopted in [21] to determine the optimal contract strategy of the retailer aiming at maximizing the retailer's profit while limiting its risk. A model based on a Stackelberg game was studied in [22] taking into account the response of consumers to the retail price. A stochastic bilevel model was presented in [23] to identify the optimal offering price of the energy hub manager. Decision-making conflict between the multi-energy players and the local energy system was discussed in [24]. In the abovementioned works, the pool price is set as a given value, without considering the impact of the retailer's decision on the market price. A bilevel approach was used in [25] to evaluate the multi-energy players' behaviors and the reaction of the local energy system with the market-clearing process modeled in the lower level. However, only power market clearing process was included, either.

It is worth nothing that previous works on multi-energy markets under a game theory framework usually assume a symmetric information structure, in which the utility function of each player is common knowledge. However, in reality, the information between multiple market participants is usually asymmetric. Even though a retailer can collect certain information or historical data to roughly estimate consumers' usage levels, their real willingness to pay is difficult to identify exactly before signing the contract. Such private information creates unclearness about consumers' true utility, particularly when different types of energy are involved simultaneously. On the other hand, strategic behaviors of some consumers could blur their true types. In such circumstances, consumers may deliberately misrepresent their energy consumption for a period to influence the decision of the retailer, for the sake of gaining additional profit via taking advantage of their better knowledge of their own preferences [26]. This situation is exactly the case of information asymmetry. This kind of information asymmetry cannot be eliminated via historical data analysis.

In practice, the adverse impact of neglecting information asymmetry has been observed in the wholesale electricity market in New England, US, promoting the revamp of market rules for the Day-Ahead Load Response Program (DALRP) [27]. In this program, the customer baseline, which is a 10-day rolling average of interval meter data from days with no load response event, is used as a benchmark. The consumers offer load reduction in the DALRP, which are cleared as measured against their customer baseline and paid at day-ahead locational marginal price (LMP). In real time, if consumers reduce usage by more than the amount cleared in the DALRP, they are paid for the excess at the real-time LMP, which is high than the day-ahead LMP in most cases. The designer of the DALRP anticipated that all consumers would make offers that can reflect the maximum reductions they could deliver in real time, at prices that reflected their opportunity costs. However, adverse consumer behaviors have been observed since August 2007. A majority of consumers submit load reduction offers at the minimum level allowed by the program rules, even though they are capable of delivering greater load reduction in real time, in order to earn the price difference between day-ahead LMP and real-time LMP. In this case, consumers' demand sensitivity to price is not transparent and information asymmetry exists. Some consequences of such adverse behaviors in the power system are discussed in [28]. In our case, when energy mix trading is considered with multiple market-clearing processes, such adverse behaviors may be more likely and influential on the market operation, which should not be neglected.

Generally, there are two types of asymmetric information problems: adverse selection, caused by ex-ante asymmetry, and moral hazard, caused by ex-post asymmetry [29]. In this study, we focus on adverse selection, which indicates that before trading, the player with private information has the incentive to misrepresent the information in order to hurt other players. A famous example is Akerlof's "lemon market" [30]. In economics, the problem of adverse selection has been fully considered under an information game framework for the labor market [31], insurance market [32], credit market [33], etc. However, there are few studies on multi-energy markets to date.

Energy, such as electricity, is a special product that cannot be stored on a large scale. In contrast to other markets, the energy market is restricted by various system and operational constraints. Reference [34] analyzes the statistics of the multiple price-list experiments and finds that households that consume less energy during peak hours are more likely to take part in time-of-use programs, which is evidence of adverse selection. Motivated by the debate about electricity market design, [35] empirically compares the effect of asymmetric demand information between sellers on two auction institutions. The importance of considering information asymmetry in the retailer's optimal pricing problem has been recognized [36]. The mathematical models developed therein not only consider load uncertainty due to inaccurate prediction but also the fact that consumers may take advantage of their private information on their own utility functions to choose contracts strategically, which in turn influences the profit of the retailer. The coordination of interaction between various types of consumers and a power company is investigated in [37] based on contract theory. The optimal contract design for plug-in hybrid electric vehicles (PHEVs) under asymmetric information is studied in [38]. These researches evidently show that the asymmetric information between a retailer and consumers in an energy market is a real problem that deserves research efforts. As multiple energy markets are coupled to each other, this issue turns to be more prominent. Our work makes the first step to understand such an important problem, by analyzing the strategic behaviors of the retailer and consumers under asymmetric information. It is expected to provide insights to facilitate the design of future multiple energy markets. On the other hand, the models and mathematical techniques used in deriving the theoretical results can also be applied to analyze other coupling markets, particularly under asymmetric information.

Table 1: Differences between this work and previous works.

		Information Structure	
		Symmetric information	Asymmetric information
Market Structure	No clearing process	Retailer's optimal strategy [21]-[24].	Retailer's optimal strategy [36]; Contract design [37, 38].
	Only power market clearing	Optimal operation [11, 12, 16]; Energy trading [13, 14, 25]; Market power analysis [15].	Strategic producers' bidding [39, 40].
	Multi market clearing	Strategic offering for gas-power market [18]; Energy trading for heat-power market [17].	The proposed model considers the retailer's optimal strategy under asymmetric information while taking into account the heat-power market clearing.

The work in this study differs from previous works both in market structure and information structure, which are summarized in Table I, and possesses three unique features, as follows.

1. **The "retailer–consumer" framework (RCF)-based pricing mechanism for transactive energy mix.** In this study, an RCF-based pricing mechanism for the multi-energy market is proposed. In practice, the consumer's real willingness to pay (or its preference) is usually private information and is not disclosed to the retailer. Hence, an asymmetric information game exists between the retailer and consumers. The RCF model deals with this problem by imposing incentive compatibility constraints and guiding the consumers' behaviors via observable contracts of energy mix. Unlike the frameworks of previous works, in which the

production price is a fixed value, for example, the "principal–agent" framework proposed in [29] and the contract design problem for PHEV owners in [38], our model considers the impact of consumption amount on energy prices by considering market-clearing processes as a sub-problem. To the best of our knowledge, this business mode is new in the literature of energy market research.

2. **Model reduction and consumer's behavior analyses.** To facilitate computation, several reduction techniques are proposed to simplify the RCF model by removing redundant constraints and identifying binding inequalities, resulting in a reduced RCF model that can be solved efficiently by commercial software. It is interesting that underlying the economic intuitions of consumers' behavior patterns for each reduction step can be unveiled with rigorous mathematical interpretation. We believe these results aid understanding of the interactions among multiple market participants, particularly under asymmetric information.

3. **Equilibrium characterization and retailer's behavior analysis.** Based on the reduced model, the necessary and sufficient condition for a separating equilibrium (SE) for a risk-neutral retailer is derived, and the distortion of the retailer's optimal contract strategy due to asymmetric information is analyzed. The underlying economic logic of the retailer is also characterized.

The rest of this paper is organized as follows. The mathematical formulation of the retailer's contract design problem based on RCF is described in Section 2. The reduction technique and underlying consumer behavior patterns are presented in Section 3. Based on the reduced RCF model, the market equilibrium is characterized with proofs and economic interpretations in Section 4. A heat–power market (HPM) RCF problem is presented as an illustrative example to demonstrate our proposed methodology in Section 5. Finally, we conclude with remarks in Section 6.

2. Problem Formulation

2.1. Market Settings and Notations

In this section, an RCF is formulated. For simplicity, the retailer in this study is assumed as the aggregator of one specific class of load, for example, office buildings.

A more general situation in which multiple classes of consumers are considered can be found in Appendix A. The main theoretical results in this study remain valid. Assume there are n types of consumer and denote $\mathscr{N} := \{1, 2, \cdots, n\}$ as the set of types. Without loss of generality, we assume there is only one (aggregate) consumer for each type. Then, consumer i represents all consumers of type-i (denoted as "she"). There are two types of energy, namely, electricity and heat. A multi-energy retailer (denoted as "he") wishes to sell a combination of the two types of energy to the consumers, by offering a series of energy "*package*" contracts. The package contract refers to a contract that offers a special package of mixed energy supply, which can be depicted by a tri-tuple (p_d, h_d, S), where p_d and h_d are the two energy amounts provided to the consumers, and S denotes the price of the package. Assume the retailer knows the number of consumer types, and hence, he needs to determine n contracts subject to n types of consumers. We use the subscript "i" to represent the specific contract for consumer i and denote this contract by $\mathscr{C}_i := (p_{i,d}, h_{i,d}, S_i)$. Then, $\mathscr{C} := \{\mathscr{C}_i, i \in \mathscr{N}\}$ is referred to as a *menu* of energy contracts. Assume the utilities of all consumers have an identical form, denoted by $U(p_{i,d}, h_{i,d})$. We further denote $A_i \cdot U(p_{i,d}, h_{i,d})$ as the utility function of consumer i, and $NU_i^j = A_i U(p_{j,d}, h_{j,d}) - S_j$ as the net utility of consumer i subject to contract $\mathscr{C}_j$. Here, A_i is the utility coefficient of consumer i. Since there are n consumers, we define $\mathscr{A} := \{A_i, i \in \mathscr{N}\}$. Generally, both A_i and $U(\cdot)$ are positive.

The information structure is set as: 1) the function $U(\cdot)$ is common knowledge; 2) the type of consumer i, A_i, is private knowledge that is only known to herself reflecting her willingness to pay.[1] The higher A_i is, the more consumer i is willing to pay for the same amount of energy $p_{i,d}$ and $h_{i,d}$.; 3) the retailer knows the number of consumer types as well as the distribution of the n type of consumers. The distribution is denoted by $\mathscr{P} := \{\pi_i \ (i \in \mathscr{N})\}$, and $\sum_{i \in \mathscr{N}} \pi_i = 1$. This information structure is *asymmetric* and an asymmetric information game exists among the retailer and consumers. Our main focus in this study is the influence of such information asymmetry on market equilibrium.

[1]Without confusion, sometimes we simply use "type A_i" to refer to the type of consumer whose utility function has a coefficient of A_i

The structure of the RCF is shown in Figure 2 and consists of two layers: 1) the market layer, which gives the energy prices by a market-clearing process; 2) the retailer layer, which describes the contract design problem under asymmetric information between the retailer and consumers. The retailer can access energy system 1 via bus w and energy system 2 via node v. Hence, he can buy energy from two energy markets at the local marginal prices on bus w and node v, denoted by λ_w^p and λ_v^h, respectively. Then he offers a contract menu $\{\mathscr{C}_i, i \in \mathscr{N}\}$ to the consumers, aiming to maximize his expected profit. Each consumer chooses which contract to sign to maximize her individual utility. Here, we assume that the consumer's "*reservation utility*" level is zero, which means that a consumer will accept the contract as long as she can obtain net utility greater than zero. The overall process is shown in Figure 3.

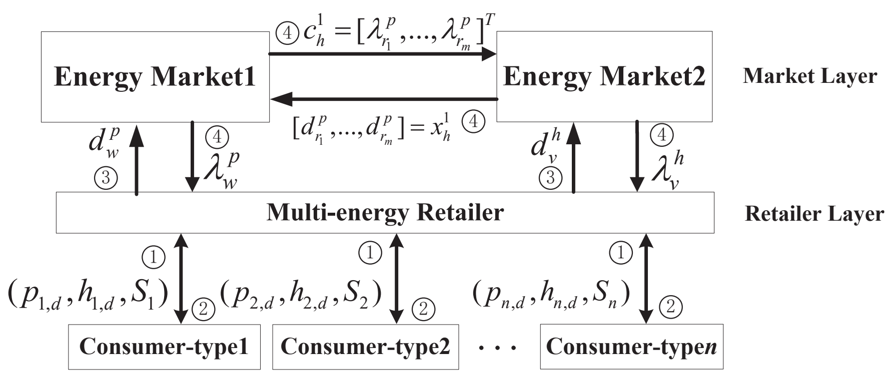

Figure 2: Structure of the "retailer–consumer" framework-based model in the multi-energy market.

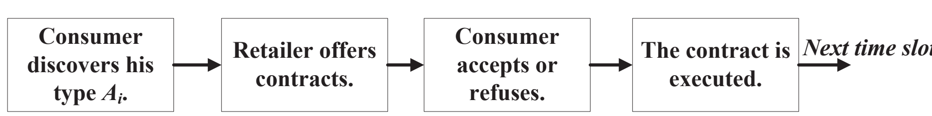

Figure 3: Time line of the "retailer–consumer" framework-based process.

Although currently there is no such contract of energy mix, our model offers an outlook for the future multi-energy market and an instrument to analyze market equilibrium under asymmetric information. Here, four steps are considered, as follows.

Step 1: The retailer finds out the possible utility coefficients of consumers $\mathscr{A} := \{A_i, i \in \mathscr{N}\}$ and the related distribution $\mathscr{P} := \{\pi_i \ (i \in \mathscr{N})\}$. This information can be acquired via statistics of historical data. Then, the retailer designs a set of energy package contracts $\mathscr{C}_i := (p_{i,d}, h_{i,d}, S_i)$ for consumers to choose.

Step 2: Knowing the selectable contracts $\mathscr{C}_i, i \in \mathscr{N}$, each consumer decides which contract to sign, aiming to maximize her net utility.

Step 3: After signing the contracts with consumers, the retailer reports his energy demand d_w^p, d_v^h to the energy markets (specifically, the power market and heat market in this study).

Step 4: The energy markets are cleared with the objective of minimizing operational costs. Then, the energy trading $x_h^1 = [d_{r_1}^p, ..., d_{r_m}^p]$ between the two energy markets as well as the marginal energy prices λ_w^p, λ_v^h for the retailer are given.

Remark 1: The energy conversion facility appears in the market layer of the model, such as the electric boiler, which can transform electricity into heat, in the illustrative case. It is worth noting that in current practice, the CHP unit appears at the consumer level (at the bottom of Figure 2) and directly provides electricity and heat to end users in most cases [41]. Since our study focuses on the market and retailer layers, the CHP unit is not included in the market model. In the future, large-capacity CHP units may participate in the HPM as an energy hub. This is another story that involves the modeling of energy hub behavior and the market mechanism design. Some pioneering works have been reported in [42, 43], such as the optimal bidding problem. However, the market setting is different from that proposed in this study, which employs an accurate power flow model and market-clearing scheme based on locational marginal energy price (LMEP). More dedicated research is still ongoing.

2.2. Energy Market-Clearing Problems

The energy prices λ_w^p and λ_v^h in Figure 2 are given by solving the market-clearing problems (MCPs) of energy markets 1 and 2 (EM1 and EM2, respectively). In EM1, MCP can be formulated as a constrained optimization problem in the form of

$$\text{MCP1:} \quad \min \quad c_p^T x_p \tag{1a}$$

$$\text{s.t.} \quad F_p x_p \le b_p \quad : \omega^p \tag{1b}$$

$$B_p x_p = d^p \quad : \lambda^p \tag{1c}$$

where x_p is the vector of decision variables in the energy market, for example, power injection in a power system or temperature setting in a heat system. The objective function (1a) is to minimize the total cost, where c_p is the cost coefficient of units, such as, power units or heat pumps. Inequality (1b) depicts the capacity limitations. Equation (1c) is the energy balance constraint and d^p refers to the demand vector. ω^p and λ^p are the dual variables and the LMEP in EM1 can be found in λ^p. A similar model (2) can be formulated considering the MCP for EM2.

$$\begin{aligned} \text{MCP2:} \quad \min \quad & c_h^T x_h && \text{(2a)} \\ \text{s.t.} \quad & F_h x_h \leq b_h \quad : \omega^h && \text{(2b)} \\ & B_h x_h = d^h \quad : \lambda^h && \text{(2c)} \end{aligned}$$

When the two energy markets are coupled, we say that one energy market can buy/sell energy from/to the other at the marginal prices, and λ^p and λ^h turn out to depend on each other. Demand change in either d^p or d^h influences the marginal energy prices in both markets. Without loss of generality, we assume that EM2 can buy energy from bus r_1,...,r_m in EM1 at LMEPs. The cost coefficient of EM2 can be divided into two parts, $c_h = [c_h^1; c_h^2]$, where c_h^1 refers to the costs of units connected with EM1 and c_h^2 refers to the costs of traditional units; and x_h can be divided into x_h^1 and x_h^2. We have $c_h^1 = [\lambda_{r_1}^p, ..., \lambda_{r_m}^p]^T$. The energy bought by EM2 is the demand in EM1, and we say that $[d_{r_1}^p, ..., d_{r_m}^p] = x_h^1$. Note that, if MCP1 and MCP2 are both linear programs, then each has a unique optimal solution in most cases. For simplicity, we use $\varphi_p(.)$ and $\varphi_h(.)$ to depict the relationship between LMEPs and the demand of the retailer, which is defined by $\lambda_w^p := \varphi_p(d_w^p, d_v^h)$ and $\lambda_v^h := \varphi_h(d_w^p, d_v^h)$. An example of such a market setting is the gas-power market in [18].

2.3. Retailer's pricing problem

The retailer is connected to energy system 1 at bus w and to energy system 2 at node v, so that it can buy energy from the two energy markets at the marginal prices λ_w^p and λ_v^h. Then, the retailer offers a menu of contracts $\mathscr{C} = \{\mathscr{C}_i, i \in \mathscr{N}\}$ to the consumers to earn a profit. Under *symmetric information*, the retailer knows the exact type of each consumer, and thus, can design a contract specialized for her. The RCF model under *symmetric information* is given as follows.

$$\max_{S_i,p_{i,d},h_{i,d}} f(\Xi) \quad = \sum_{i\in\mathscr{N}} \pi_i \Gamma(S_i - \lambda_w^p p_{i,d} - \lambda_v^h h_{i,d}) \tag{3a}$$

$$\text{s.t.} \quad A_i U(p_{i,d}, h_{i,d}) - S_i \geq 0, \forall i \in \mathscr{N} \tag{3b}$$

$$d_w^p = \textstyle\sum_i \pi_i p_{i,d},\ d_v^h = \sum_i \pi_i h_{i,d} \tag{3c}$$

$$\lambda_w^p = \varphi_p(d_w^p, d_v^h), \lambda_v^h = \varphi_h(d_w^p, d_v^h) \tag{3d}$$

Here, the objective function (3a) of the retailer is to maximize his expected profit, where $\Gamma(\cdot)$ is the risk preference function with $\Gamma'(\cdot) \geq 0$; and $\Gamma''(\cdot) \leq 0$ for the risk-averse type, $\Gamma''(\cdot) \geq 0$ for the risk-preferent type, and $\Gamma''(\cdot) = 0$ (or $\Gamma'(\cdot) = constant$) for the risk-neutral type. Constraint (3b) is the participation constraint, which ensures that contract $\mathscr{C}_i$ is always a feasible option for consumer i. The total demand d_w^p and d_v^h is the weighted sum of $p_{i,d}$ and $h_{i,d}$, as shown in (3c). λ_w^p and λ_v^h are the marginal energy prices given by MCP1 and MCP2.

The situation is different under *asymmetric information*, as the retailer does not know the exact type of each consumer and cannot sign specific contracts. An incentive compatibility constraint (4c) is included, which means that choosing contract $\mathscr{C}_i$ is the best choice for consumer i. Then, the RCF model under *asymmetric information* is given as follows.

$$\max_{S_i,p_{i,d},h_{i,d}} f(\Xi) \quad = \sum_{i\in\mathscr{N}} \pi_i \Gamma(S_i - \lambda_w^p p_{i,d} - \lambda_v^h h_{i,d}) \tag{4a}$$

$$\text{s.t.} \quad A_i U(p_{i,d}, h_{i,d}) - S_i \geq 0, \forall i \in \mathscr{N} \tag{4b}$$

$$i = \arg\max_{j\in\mathscr{N}} \{A_i U(p_{j,d}, h_{j,d}) - S_j\} \tag{4c}$$

$$d_w^p = \textstyle\sum_i \pi_i p_{i,d},\ d_v^h = \sum_i \pi_i h_{i,d} \tag{4d}$$

$$\lambda_w^p = \varphi_p(d_w^p, d_v^h), \lambda_v^h = \varphi_h(d_w^p, d_v^h) \tag{4e}$$

2.4. Definitions and Assumptions

In model (4), the retailer always expects that a consumer of type i would sign his matching contract $\mathscr{C}_i$. However, sometimes, consumer i strategically chooses to sign contract $\mathscr{C}_j$ instead of $\mathscr{C}_i$. When such behavior occurs, we say consumer i "*mimics*" consumer j. This occurs owing to asymmetric information by which the retailer is unaware of the exact type of the consumer, and the consumer can benefit from mimicking another type of consumers. Such benefit created by asymmetric information is referred to as *information rent* (IR). Denote NU_i^j as the net utility of consumer i signing contract $\mathscr{C}_j$, that is,

$$NU_i^j := A_i U(p_{j,d}, h_{j,d}) - S_j \tag{5}$$

Then, the definition of IR is given as below.

Definition 1 (information rent (IR))**.** For consumer i, the maximal net utility she can get from signing a contract is called her IR, which is defined by

$$IR_i := \max\{NU_i^j : \forall \mathscr{C}_j \in \mathscr{C}\} \tag{6}$$

At market equilibrium, the optimal package for consumer i is $\mathscr{C}_i$. Thus, (6) is equivalent to (7).

$$IR_i := NU_i^i = A_i(p_{i,d}, h_{i,d}) - S_i \tag{7}$$

Remark 2: In this study, "mimic" refers to behavior by which the consumer misrepresents her real utility and chooses the contract not designed for her. We are mainly interested in the common requirement among consumers, for example, both types need power and heat. Offering contracts together with a specific service is a way to cope with the problem of asymmetric information to some extent, like distinguishing an office building from a steel mill. However, when faced with two consumers of a similar service, such as two steel mills, this method may fail. Our study focuses on the contract design problem of the retailer who offers homogeneous packages, which consist of the same products but with different quantities and prices only. Specific services provided by the retailer are not considered in this study. A similar problem can be found in [34], where it can be concluded from statistics that households that consume less energy during peak hours would "mimic" households that consume more energy during peak hours and enroll in the time-of-use pricing programs to make profit. The mimic behavior between different types of PHEV owners in [38] is another example.

To simplify the analysis, without loss of generality, throughout the study, we make the following assumptions.

A1. $A_i > A_j > 0$ for any $i < j$ $(i, j \in \mathscr{N})$.

A2. The optimal solution $\Xi^* := \{(p^*_{i,d}, h^*_{i,d}, S^*_i),\ \forall i \in \mathscr{N}\}$ of model (4) exists, which implies that both the RCF problem and the MCP are always feasible.

A3. The marginal utility of consumers is nonnegative and diminishing in $p_{i,d}$ and $h_{i,d}$, which means that for all $i \in \mathscr{N}$,

$$\frac{\partial U(p_{i,d}, h_{i,d})}{\partial p_{i,d}} \geq 0 \quad ; \quad \frac{\partial U(p_{i,d}, h_{i,d})}{\partial h_{i,d}} \geq 0$$

and $\nabla^2 U(p_{i,d}, h_{i,d})$ is negative definite with

$$\frac{\partial^2 U(p_{i,d}, h_{i,d})}{\partial p_{i,d} \partial h_{i,d}} \quad = \quad \frac{\partial^2 U(p_{i,d}, h_{i,d})}{\partial h_{i,d} \partial p_{i,d}} \geq 0$$

3. Model Reduction and Underlying Behavior Patterns of Consumers

In this section, we simplify model (4) by eliminating redundant constraints and identifying binding constraints. Meanwhile, we characterize the behavior patterns of consumers behind each reduction step. All the proofs of the lemmas and theorem presented in this section are found in Appendix B.

3.1. Elimination of Redundant Constraints

As there is a finite number of contracts, by simple enumeration, the constraint (4c) can be unfolded into (8).

$$NU^i_i \geq \ NU^j_i, \ \ (\forall i, j \in \mathscr{N}) \tag{8}$$

Constraints in (8) can be further classified into two categories: constraints that involve *adjacent* types and those that involve *nonadjacent* types. The consumers are ordered by its utility coefficient A_i by the retailer. This can be undertaken using statistics of historical data. The adjacent and nonadjacent types are defined as below. It should be noted that the consumer does not need to know the exact information on which consumers are adjacent or nonadjacent.

Definition 2 (adjacent and nonadjacent types)**.** A type k is called an adjacent type of type i, if $k = i-1$ or $k = i+1$.

According to the definition, the adjacent-type constraints in (8) can be written as

$$NU_i^i \geq NU_i^{i+1}, \ \forall i \in \mathcal{N} \backslash \{n\} \tag{9a}$$

$$NU_i^i \geq NU_i^{i-1}, \ \forall i \in \mathcal{N} \backslash \{1\} \tag{9b}$$

Next, the following lemma shows that (8) can be simplified into (9), and describes the characteristics of consumers' behaviors from the perspective of the retailer.

Lemma 1. Assume Assumption A1 holds. Then, constraints (8) must be satisfied if (9) holds.

According to Lemma 1, constraint (4c) (or (8)) can be reduced to (9), so that the RCF problem (4) is simplified. From an economic point of view, this lemma essentially reveals a behavior pattern of consumers: if the contracts that the retailer offers can ensure that no consumer has incentive to mimic their adjacent type of consumers, then we can guarantee that the consumers have no motivation to mimic any nonadjacent types either. The implication of this behavior pattern is that, in order to have a consumer sign his matching contract, the retailer needs only prevent the consumer from mimicking her adjacent type of consumer. An example is given in Subsection 5.3.

Lemma 2. Assume Assumption A1 holds. Then, the constraints in (4b) for $i \in \mathcal{N} \backslash \{n\}$ must be satisfied if the following two conditions hold:

1. The constraints in (9a) are satisfied;
2. The constraint in (4b) for $i = n$ is satisfied.

With Lemma 2, all the constraints in (4b) except $i = n$ can be removed, which greatly simplifies the RCF model (4). Lemma 2 indicates that if the consumer with the least A_i, that is, A_n, chooses to accept the matching contract decided for her, that is, $\mathscr{C}_n$, then $\mathscr{C}_i$ is definitely a feasible option for consumer $i \in \mathcal{N} \backslash \{n\}$. This can be understood as follows. When (9a) are satisfied, the consumers would not like to mimic consumer with A_n, which implies $IR_i \geq NU_i^n$. Note that A_n is the minimum in $\mathscr{A}$, and we have $NU_i^n \geq IR_n$. Therefore, the retailer needs only $IR_n \geq 0$ to hold, and then, there must be $IR_i \geq 0$ for all $i \in \mathcal{N}$. This result means that each consumer has a nonnegative incentive to join the market.

3.2. Identification of Binding Constraints

Denote the optimal solution to the RCF problem (4) as $\Xi^* = \{(p_{i,d}^*, h_{i,d}^*, S_i^*),\ \forall i \in \mathcal{N}\}$. Then, we show that some constraints in (4b) and (9) must be binding, which can be used to simplify model (4) further.

Lemma 3. Assume Assumptions A1 and A2 hold. Then, at the optimum Ξ^* of the RCF model (4), the following must hold.

1. All constraints in (9a) are binding;
2. The constraint in (4b) for $i = n$ is binding.

The proof of Lemma 3 shows that, whenever assertions 1) and/or 2) are not satisfied, we can always find another better solution than the optimal solution Ξ^*, which is impossible. Therefore, inequalities (9a) and (4b) for $i = n$ must be binding, and hence, can be converted into equations.

Lemma 3 is in line with common sense, as we know that a consumer with a higher A_i has the incentive to mimic a lower adjacent type A_{i+1}, but not vice versa. Lemma 3 implies that

$$IR_i = \sum_{j=i}^{n-1} (A_j - A_{j+1}) U(p_{j+1}, h_{j+1}) \tag{10a}$$

and

$$IR_n = 0 \tag{10b}$$

This implication means that the IR of consumer n, that is, IR_n, is zero while other consumers can obtain positive IRs. Moreover, the higher A_i is, the more IR_i the consumer can obtain. As mentioned previously, under symmetric information, only the participantion constraints need to be considered. Hence, the IR of each consumer is zero. This essentially reveals that, under an asymmetric information circumstance, consumers may obtain positive IR by taking advantage of private information.

3.3. Equivalent Model Reduction

Since (9a) are all binding, we substitute them into (9b) to eliminate S_i in the constraints. Then, (9b) is converted into

$$(A_i - A_{i-1})[U(p^*_{i,d}, h^*_{i,d}) - U(p^*_{i-1,d}, h^*_{i-1,d})] \quad \geq \quad 0 \tag{11}$$

By Lemmas 1$\sim$3, we obtain an equivalent reduced model of (4), as stated in the following theorem.

Theorem 1 (Model Reduction)**.** Assume Assumptions A1 and A2 hold. Then, the RCF model (4) is equivalent to

$$\begin{aligned}
\max_{p_{i,d},h_{i,d}} f(\Xi) \qquad & = \sum_{i \in \mathcal{N}} \pi_i \Gamma \Big(A_i U(p_{i,d}, h_{i,d}) \\
& + \sum_{j=i}^{n-1} (A_{j+1} - A_j) U(p_{j+1,d}, h_{j+1,d}) \\
& - \lambda^p_w p_{i,d} - \lambda^h_v h_{i,d} \Big) \qquad \qquad (12a) \\
\text{s.t.} \qquad & (A_i - A_{i-1}) \cdot \big[U(p^*_{i,d}, h^*_{i,d}) \\
& \qquad - U(p^*_{i-1,d}, h^*_{i-1,d}) \big] \geq 0 \qquad (12b) \\
& d^p_w = \textstyle\sum_i \pi_i p_{i,d},\ d^h_v = \sum_i \pi_i h_{i,d} \qquad (12c) \\
& \lambda^p_w = \varphi_p(d^p_w, d^h_v), \lambda^h_v = \varphi_h(d^p_w, d^h_v) \qquad (12d)
\end{aligned}$$

The first and second terms in (12a) are obtained by substituting the binding constraints (9a) and (4b) with $i = n$, as stated in Lemma 3. (12b) is obtained by replacing (9b) with (11). It is interesting that in this model, the decision variables S_i have been eliminated, benefiting from the reduction techniques, which means that the retailer needs only decide the amount of energy mix. This feature greatly facilitates the analysis of equilibrium. The proof of Theorem 1 is straightforward by using Lemmas 1$\sim$3, and hence, is omitted here.

4. Optimal Contract Strategy and Underlying Behavior Patterns of the Retailer

In this section, we characterize the optimal contract strategy of the retailer under asymmetric information, and reveal the underlying behavior patterns of the retailer.

All the proofs of the lemmas and theorem presented in this section can be found in the Appendix C.

4.1. Existence of a Separating Equilibrium

In this subsection, we derive the condition that guarantees the existence of an SE. As the risk preference of the retailer involves high complexity in analyzing the equilibrium, here, we provide only the necessary and sufficient condition for a risk-neutral retailer. As for risk-averse/risk-preferent retailers, we empirically analyze the equilibrium using case studies.

When an SE exists, which means all the constraints in (12b) are strictly satisfied, we have the optimal conditions for $p_{i,d}$ by using the derivate of $p_{i,d}$, which is given by

$$\begin{aligned} 0 &= \left\{ \frac{\sum_{j=1}^{i-1} \left[\pi_j \Gamma'(V_j) \cdot (A_i - A_{i-1})\right]}{\pi_i \Gamma'(V_i)} + A_i \right\} \cdot \frac{\partial U_p}{\partial p_{i,d}} - \lambda_w^p \\ &\quad - \frac{\sum_{k=1}^{n} \pi_k \Gamma'(V_k) p_{k,d}}{\Gamma'(V_i)} \cdot \frac{\partial \phi_p}{\partial d_w^p} \\ &\quad - \frac{\sum_{k=1}^{n} \pi_k \Gamma'(V_k) h_{k,d}}{\Gamma'(V_i)} \cdot \frac{\partial \phi_h}{\partial d_w^p} \end{aligned} \tag{13}$$

where V_i are defined by

$$\begin{aligned} V_i &:= A_i U(p_{i,d}, h_{i,d}) + \sum_{j=i}^{n-1} (A_{j+1} - A_j) U(p_{j+1,d}, h_{j+1,d}) \\ &\quad - \lambda_w^p p_{i,d} - \lambda_v^h h_{i,d} \end{aligned}$$

The optimal conditions for $h_{i,d}$ are similar to (13).

Denote

$$\begin{aligned} C_i &:= \frac{\sum_{j=1}^{i-1} \left[\pi_j \Gamma'(V_j) \cdot (A_i - A_{i-1})\right]}{\pi_i \Gamma'(V_i)} + A_i \\ B_i &:= \frac{\sum_{k=1}^{n} \pi_k \Gamma'(V_k) p_{k,d}}{\Gamma'(V_i)} \\ D_i &:= \frac{\sum_{k=1}^{n} \pi_k \Gamma'(V_k) h_{k,d}}{\Gamma'(V_i)} \end{aligned}$$

Then, the optimal conditions can be abbreviated as

$$C_i \frac{\partial U}{\partial p_{i,d}} = \lambda_w^p + B_i \frac{\partial \varphi_p}{\partial d_w^p} + D_i \frac{\partial \varphi_h}{\partial d_w^p} \tag{14a}$$

$$C_i \frac{\partial U}{\partial h_{i,d}} = \lambda_v^h + D_i \frac{\partial \varphi_h}{\partial d_w^h} + B_i \frac{\partial \varphi_p}{\partial d_w^h} \tag{14b}$$

Next, we give the necessary and sufficient condition for the existence of an SE for a risk-neutral retailer. We start with the following lemma.

Lemma 4. Assume Assumption A3 holds for model (12). For a risk-neutral retailer (i.e., $\Gamma'(\cdot) = constant$), if $C_i > C_k$ ($\forall i, j \in \mathcal{N}$), then $p_{i,d} > p_{k,d}$ and $h_{i,d} > h_{k,d}$ must hold.

With this lemma, we can prove the following theorem.

Theorem 2 (Existence of a separating equilibrium)**.** Assume Assumption A3 holds for model (12). For a risk-neutral retailer, an SE exists if and only if $C_1 > C_2 > ... > C_{n-1} > C_n$.

Theorem 2 provides a necessary and sufficient condition for the existence of an SE of model (12), and hence, of the original model (4) if Assumptions A1 and A2 hold. The theorem indicates that if $C_1 > C_2 > ... > C_{n-1} > C_n$, a risk-neutral retailer will provide different package contracts for different consumers accordingly. For a risk-averse/risk-preferent retailer, the situation is more sophisticated. We can just solve problem (12) and then analyze the obtained contract strategies, which is demonstrated in the case studies.

4.2. Distortion of the Retailer's Strategy

In this subsection, we compare the optimal contract strategy of the retailer under symmetric and asymmetric information to show the impact of information. Denote $\hat{\Xi} := \{(\hat{p}_{i,d}, \hat{h}_{i,d}, \hat{S}_i),\ \forall i \in \mathcal{N}\}$ as the retailer's optimal contract strategy under *symmetric* information (model (3)), which is the solution of

$$A_i \frac{\partial U}{\partial p_{i,d}} = \lambda_w^p + B_i^{'} \frac{\partial \varphi_p}{\partial d_w^p} + D_i^{'} \frac{\partial \varphi_h}{\partial d_w^p} \tag{15a}$$

$$A_i \frac{\partial U}{\partial h_{i,d}} = \lambda_v^h + D_i^{'} \frac{\partial \varphi_h}{\partial d_w^h} + B_i^{'} \frac{\partial \varphi_p}{\partial d_w^h} \tag{15b}$$

where

$$
\begin{aligned}
B_i^{'} &:= \frac{\sum\limits_{k=1}^{n} \pi_k \Gamma'(V_k^{'}) p_{k,d}}{\Gamma'(V_i^{'})} \\
D_i^{'} &:= \frac{\sum\limits_{k=1}^{n} \pi_k \Gamma'(V_k^{'}) h_{k,d}}{\Gamma'(V_i^{'})} \\
V_i^{'} &:= A_i U(p_{i,d}, h_{i,d}) - \lambda_w^p p_{i,d} - \lambda_v^h h_{i,d}
\end{aligned}
$$

If we neglect the adjustment of energy price, then, because $C_i < A_i (i > 1)$ and $C_1 = A_1$, we have

1) $p_{1,d}^* = \hat{p}_{1,d}, h_{1,d}^* = \hat{h}_{1,d}$.

2) $p_{i,d}^* < \hat{p}_{i,d}, h_{i,d}^* < \hat{h}_{i,d}, \forall i \in \mathscr{N} \backslash \{1\}$.

The analysis indicates that the retailer provides the same package contract as under symmetric information for the consumer with the highest A_i (i.e., A_1). However, the retailer distorts downward the optimal packages for other types of consumers. At this time, the total amount of energy satisfies $d_w^{p*} < \hat{d}_w^p$ and $d_v^{h*} < \hat{d}_v^h$.

When taking into account the impacts of energy price, the analysis becomes more complicated. In our model setting, the change in energy amount exerts influence on energy prices, which in turn affect the optimal contract. As the two energy markets are coupled together, the variation trend of the energy price when both systems' consumption decreased is not explicit. We find the impact by simulation, as shown in Section 5.

Remark1: Potential limitations of the proposed method and possible remedies are summarized as follows.

1) **About the assumptions.** In Section 2, to simplify the analysis, three assumptions are made. Assumption A1 can be easily fulfilled by sorting the consumer types. Assumption A2 is made to ensure that the problem we discuss makes sense. Admittedly, it is possible that A2 does not meet. If the optimal solution of model (4) does not exist, Lemma 3 is no longer valid. In that situation, if we still follow Lemma 3 to simplify the model, then the feasible region of the obtained reduced model becomes smaller than the original model. Therefore, the optimal solution of the reduced model does not exist either. Assumption A3 requires the marginal utility of consumers should

be nonnegative, diminishing and concave, which is relatively mild for smart grid users, as suggested in [44]. Additionally, we assume that

$$\frac{\partial^2 U(p_{i,d}, h_{i,d})}{\partial p_{i,d} \partial h_{i,d}} = \frac{\partial^2 U(p_{i,d}, h_{i,d})}{\partial h_{i,d} \partial p_{i,d}} \geq 0$$

This means the marginal utility of power increases with the amount of heat and also the marginal utility of heat increases with the amount of power. The economic intuition behind is straightforward. When there is not enough heat, the amount of power added might be used for meeting some basic needs such as heating, and its marginal utility is low. When there is more heat, the amount of power added can be used for more advanced requirement, and thus, its marginal utility is higher. This phenomenon can be observed in most common utility functions, such as the Cobb-Douglas function and the CES utility function. Admittedly, if consumer's utility function fails to meet A3, the proof of Lemma 4 and Theorem 2 will no longer be valid. This may be a limitation of the proposed method.

2) **About the equilibrium analysis.** Only the existence of separating equilibrium for a risk-neutral retailer is proved. For a risk-aversive retailer, the situation is more sophisticated. However, the simplification techniques in Section 3 still work, and we can just solve problem (12) and then analyze the obtained contract strategies as demonstrated in the case studies.

3) **About the modeling.** To model the retailer's problem under asymmetric information, one important factor is the possible consumer types A_i and its related probability π_i. In practice, the exact value of π_i might be difficult to acquire. This might be another limitation of the proposed model. But fortunately, such information can be inferred from big data analysis, which is attracting increasing attention in recent years.

5. An Illustrative Example

In this section, we use an HPM-based retailer's contract design problem to show the effectiveness of the proposed RCF. The HPM-RCF model might be implemented in countries or areas with long cold winters, where electricity and heat are two major sources of energy demand. The problem is depicted as a bi-level model. The upper level is formulated as (4) and the lower level as the HPM-clearing problem. A power market model with AC power flow, as adopted in [45], and a deregulated heat market model, as adopted in [17], are used here. To solve such a bilevel model, traditional methods replace the heat/power market clearing problem with its individual KKT optimality condition, introducing extensive non-convex complementarity constraints, and then solve the obtained mixed integer nonlinear program (MINLP),which can be solved by branch-and-bound method [46], and metaheuristic approaches including evolutionary algorithm [47–50], particle swarm method [51, 52] and the natural phenomena inspired algorithm [53].

The problem in this paper possesses some unique features. The upper level retailer's problem has several nonlinear constrains but controls only a few variables, and the lower level heat-power market clearing problem which is relatively large in size can be turned into convex optimization [17]. In this paper, the bilevel model is first simplified by eliminating redundant constraints and identifying binding constraints via the certified lemmas and theorems. After that, the reduced model can be separated into two parts: A nonlinear objective function and the market clearing problem. Then the pattern search algorithm [54], a derivative-free searching method, is adopted to solve the reduced problem and updates $p_{i,d}, h_{i,d}, \forall i \in \mathcal{N}$. To evaluate the objective function of the retailer, the heat-power market clearing problem which can be turned into linear programming is solved by CPLEX solver, and the energy price λ_w^p, λ_v^h are determined from the dual multiplier associated with nodal energy balancing constraints. This strategy takes full advantage of the computational superiority of convex optimization, and search optimum of the non-convexity objective function in a low dimensional space. The framework is shown in Fig.4.

In the convex optimization embedded pattern search algorithm for the reduced problem, the market clearing problem can be globally solved in less than one second without an initial guess; the main routine of pattern search algorithm evaluates the objective function at uniformly distributed samples. The sampled points are updated in each iteration, until an optimum is found [54]. Efficiency is guaranteed since the retailer controls only a few variables $p_{i,d}, h_{i,d}, \forall i \in \mathcal{N}$, and hence the dimension of searching space is low. In summary, in the proposed method, no initial point is

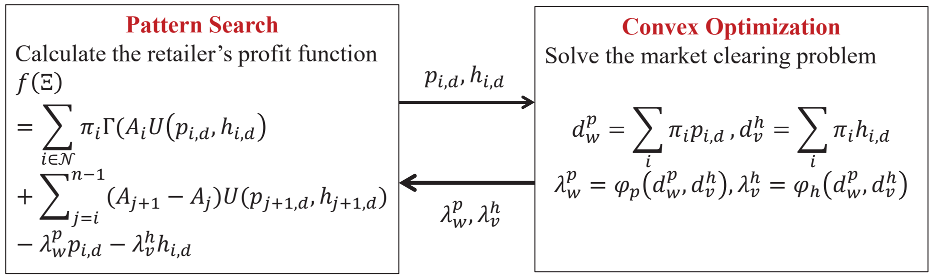

Figure 4: Procedure of solving the reduced model.

needed; convergence is guaranteed by the discussion in [55]. Efficiency is enhanced by performing the majority of computation (market clearing problem) via convex optimization.

Numerical experiments are presented on the IEEE 33-bus power system and 32-bus heat system to illustrate the efficacy of the proposed model and algorithm. The HPM-RCF model is solved by CPLEX12.6 on a laptop with Intel(R) Core(TM) i7 CPU with 2.00GHz and 8 GB of RAM.

5.1. Data

The topology of the IEEE 33-bus power system and 32-bus heat system is shown in Figure 5. The power system has five power units. The heat system buys electricity at buses 2 and 11 from the power system for the heat boilers. Two heat boilers are connected at nodes 18 and 32 in the heat system. The retailer can buy electricity from the power system at bus 3 and heat from the heat system at node 3. To simplify the illustration, we assume that there are two types of consumer (denoted as Type-H and Type-L) with probability of $\pi_H = \pi_L = 0.5$. The utility function is chosen as the sum of two isoelastic functions $U(p_{i,d}, h_{i,d}) = A_i(\sqrt{a_p p_{i,d}/N} + \sqrt{a_h h_{i,d}/N})$ (see [56]) with $N = 100$, $A_H = 0.623$ for Type-H, and A_L =0.492 for Type-L. It is easy to prove that the utility function satisfies assumptions A1 and A3. We let a_p =2.00 and a_h =2.93. We consider the market equilibrium of the retail spot market at 1 hour. The topology and parameters of the test system can be found in [57].

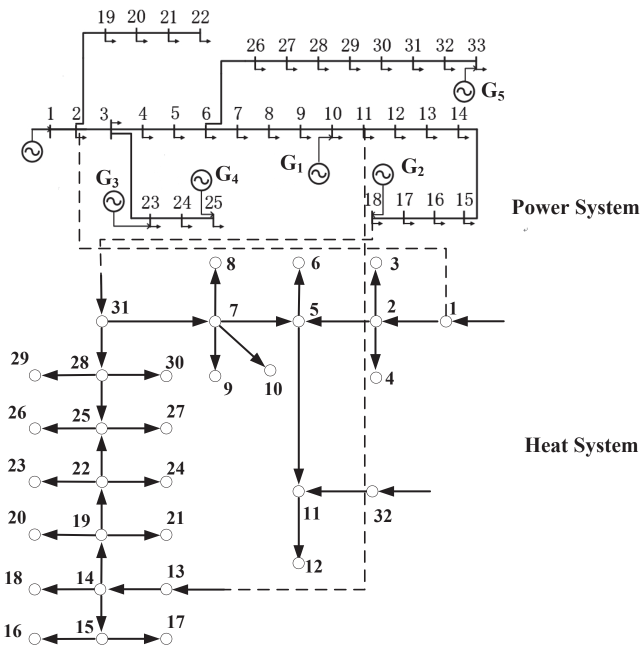

Figure 5: Topology of 33-node power system and 32-node heat system.

5.2. Benchmark Case

We apply the proposed model to the IEEE 33-bus power system and 32-bus heat system presented in the previous subsection. The optimal costs are \$3,737.0 for the power system, and \$1,294.2 for the heat system. At the market equilibrium, the heat system buys $p_{u1} = 3{,}033.6$ kW and $p_{u2} = 2{,}443.8$ kW from the power system and generates $p_{u3} = 3{,}076.0$ kW and $p_{u4} = 659.3$ kW via the heat boilers. The optimal package settings of the retailer and the IR of consumers under symmetric information (SI) and asymmetric information (AI) are shown in Table 2.

Some interesting observations are as follows.

1) The energy consumption and package price for Type-H are higher than those for Type-L, because a consumer with a higher utility is always willing to buy a larger package. This is in accordance with Lemma 4 for an SE.
2) The market equilibrium is distorted because of asymmetric information. Specifically, Type-H consumers benefit from an IR, based on their ability to mimic Type-L

Table 2: Optimal package settings of retailer and information rents of consumers

Type	risk-averse		risk-neutral		risk-preferent	
	SI	AI	SI	AI	SI	AI
p_h/kWh	520.0	595.1	520.0	600.0	520.0	598.8
h_h/kWh	617.2	674.7	617.2	674.7	617.2	678.5
S_h/\$	465.9	429.8	465.9	432.9	465.9	446.0
p_l/kWh	321.9	215.6	321.9	197.5	321.9	124.1
h_l/kWh	342.8	240.3	342.8	225.3	342.8	140.8
S_l/\$	280.5	232.5	280.5	224.0	280.5	177.3
IR_h/\$	0	62.2	0	59.9	0	47.4
IR_l/\$	0	0	0	0	0	0

consumers (whose IR is zero). The package for Type-L distorts downwards in order to reduce the IR of Type-H. We also find that the package for Type-H is higher than that under symmetric information. This finding is because market distortion results in an energy consumption decrease, reducing the energy price, which motivates the retailer to set a larger package contract.

3) Among the three types of retailers, the market distortion is smallest with a risk-averse type and largest with a risk-preferent type. A risk-averse retailer is reluctant to take a risk, and thus, he would rather pay more IR with less reduction of Type-L's package, while a risk-preferent retailer does the opposite.

5.3. More Insights into "Mimic" Behavior

First, we further discuss mimic behavior using the example of risk-neutral consumers in the benchmark case. The optimal contracts under symmetric and asymmetric information are shown in Table 2. Denote $(\hat{p}_h, \hat{h}_h, \hat{S}_h)$ and $(\hat{p}_l, \hat{h}_l, \hat{S}_l)$ as the optimal contract strategies under symmetric information for type-H and type-L, respectively, while (p_h^*, h_h^*, S_h^*) and (p_l^*, h_l^*, S_l^*) are the optimal contract strategies under asymmetric information for type-H and type-L, respectively. The net utilities of different types of consumers with different contracts are shown in Table 3.

Table 3: Net utility of consumers with optimal contracts under symmetric and asymmetric information

	symmetric information		asymmetric information	
NU_i^j/\$	$(\hat{p}_h, \hat{h}_h, \hat{S}_h)$	$(\hat{p}_l, \hat{h}_l, \hat{S}_l)$	(p_h^*, h_h^*, S_h^*)	(p_l^*, h_l^*, S_l^*)
A_H	0.0	**75.0**	**59.9**	59.9
A_L	-97.9	**0.0**	-43.7	**0.0**

Under symmetric information, the retailer knows the exact utility coefficient A_i of each consumer. Thus, the retailer can design contracts to target consumers, and specifically, gives contract $(\hat{p}_h, \hat{h}_h, \hat{S}_h)$ to the consumer with A_H and contract $(\hat{p}_l, \hat{h}_l, \hat{S}_l)$ to the consumer with A_L. However, under asymmetric information, the situation appears to be much more complex. As the retailer does not know the exact type of each consumer, he offers a series of contracts for consumers' selection. If the retailer still offers the contracts designed for the symmetric situation, which are $(\hat{p}_h, \hat{h}_h, \hat{S}_h)$ and $(\hat{p}_l, \hat{h}_l, \hat{S}_l)$, then from Table 3, we find as follows. Since $75.0 = NU_H^{\hat{l}} > NU_H^{\hat{h}} = 0.0$, the optimal strategy for the consumer with A_H is to "mimic" the type with A_L and choose $(\hat{p}_l, \hat{h}_l, \hat{S}_l)$. Hence, the contracts $(\hat{p}_h, \hat{h}_h, \hat{S}_h)$,$(\hat{p}_l, \hat{h}_l, \hat{S}_l)$ cannot distinguish between the two consumers, which is unexpected for the retailer. Following the model in this study, the optimal contracts under asymmetric information are (p_h^*, h_h^*, S_h^*) and (p_l^*, h_l^*, S_l^*). From Table 3, we have $59.9 = NU_H^{h^*} = NU_H^{l^*} = 59.9$ and $0.0 = NU_L^{l^*} > NU_L^{h^*} = -43.7$, and thus, each type of consumer chooses the contract specifically designed for her without motivation to mimic the other type. Under symmetric information, the retailer's profit is \$397.03. Under asymmetric information, if it still follows the model under symmetric information to design the optimal contracts, its profit will become \$304.95. If it follows the model proposed in this paper, the profit obtained is \$ 328.10, increasing by about 7.6%. The proposed model can successfully distinguish different types of consumers and increase the retailers profit.

Then, we use an example with four types of consumer to illustrate Lemma 1 better. We choose $A_1 = 0.505$, $A_2 = 0.472$, $A_3 = 0.429$, and $A_4 = 0.343$ with $A_1 > A_2 > A_3 > A_4$. The form of the utility function and coefficient a_p, a_h are the same as the benchmark case. The optimal contracts designed for each type are marked as $(p_i^*, h_i^*, S_i^*), \forall i$. The net utilities of each consumer when choosing different contracts are shown in Table 4.

Table 4: Net utility of different types of consumer with different contracts

NU_i^j/\$	A_1	A_2	A_3	A_4
(p_1^*, h_1^*, S_1^*)	**89.87**	64.24	30.84	-35.58
(p_2^*, h_2^*, S_2^*)	89.87	**66.19**	35.34	-26.01
(p_3^*, h_3^*, S_3^*)	88.03	66.19	**37.75**	-18.82
(p_4^*, h_4^*, S_4^*)	71.30	56.73	37.75	**0.00**

From Table 4, we find that the optimal contract for A_i is (p_i^*, h_i^*, S_i^*). Take consumer with A_1 as an example. The contract designed for her adjacent type is (p_2^*, h_2^*, S_2^*) and the contracts designed for her nonadjacent types are (p_3^*, h_3^*, S_3^*) and (p_4^*, h_4^*, S_4^*). When the retailer designs the contract, according to Lemma 1, he need only prevent A_1 from mimicking A_2, that is, to ensure $NU_1^1 > NU_1^2$. Then, because in Table 4, we have $89.87 = NU_1^2 > NU_1^3 = 88.03$ and $89.87 = NU_1^2 > NU_1^4 = 71.30$, if A_1 has no incentive to mimic her adjacent type ($NU_1^1 > NU_1^2$), then she has no motivation to mimic a nonadjacent type either. For A_2, because $66.19 = NU_2^3 > NU_2^4 = 56.73$, a similar conclusion holds.

5.4. Impact of Type Difference of Consumers

In this subsection, we examine the impact of type difference of consumers on the optimal strategies. To this end, we fix A_L to 0.492, and change A_H from 0.5 to 0.75. Other parameters remain the same as the benchmark case. Contract price, consumer's utility, and retailer's profit under different A_H values are shown in Figure 6. The change of the optimal package is shown in Figure 7.

From Figure 6, it is observed that, with the increase of the values of A_H, the difference between the contract prices of different types of consumers increases. The IR for Type-H increases, while that for Type-L remains zero. This result is because, when A_H is close to A_L, the preferences of Type-H and Type-L are similar, and hence, their optimal contracts are also alike. An extreme case occurs when A_H equals A_L. In such a case, there is actually only one type of consumer in the market and the information

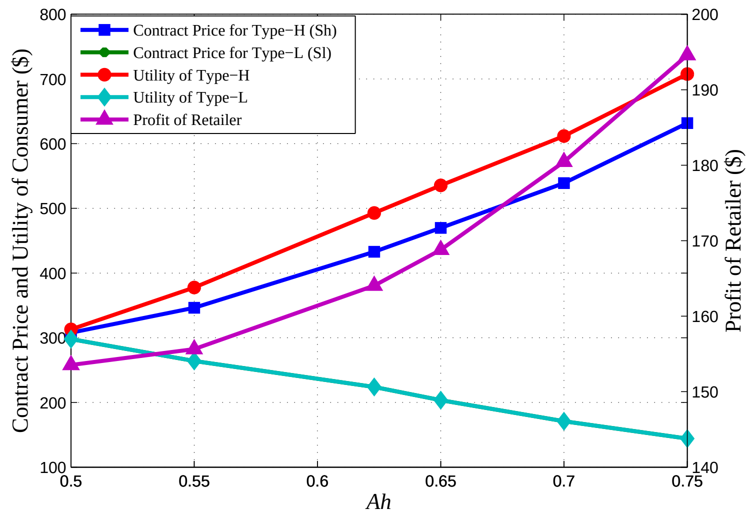

Figure 6: Changes of contract price, consumer's utility, and retailer's profit with different A_h.

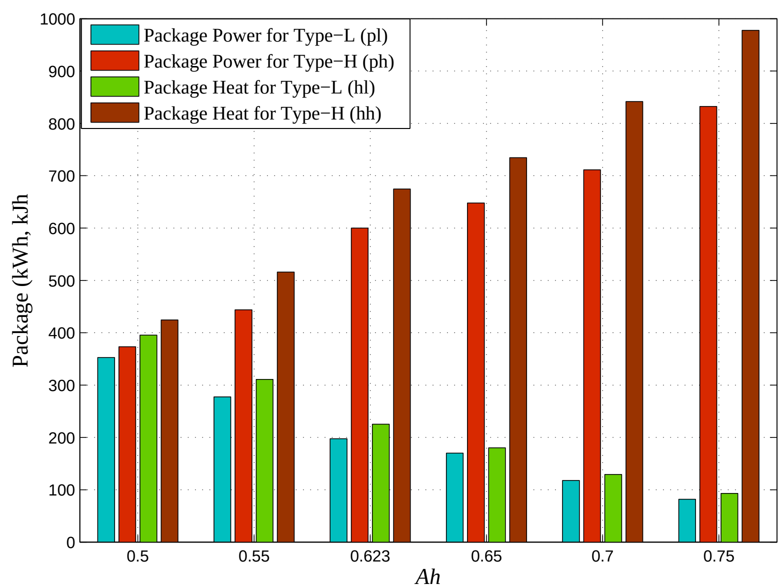

Figure 7: Changes of packages with different A_h.

asymmetry vanishes.

When A_H deviates from A_L, the Type-H consumer will be more prone to mimic the Type-L consumer. As a consequence, more IR should be provided to distinguish them. The retailer's profit increases with A_H, because a consumer with higher preference tends to accept a contract with more energy consumption and higher price, as shown in Figure 7. Thus, the retailer can benefit from such a behavior pattern. Furthermore, in Figure 7, the package for the Type-L consumer declines with an increasing A_H. The reason is that a consumer with higher preference can create more profit for the retailer. Therefore, the retailer would rather sacrifice the Type-L consumer to maintain the optimality of the Type-H consumer's package. Electricity and heat offered in one package show simultaneous growth, confirming the claim in Lemma 4.

5.5. *Impact of Type Probabilities*

The probability of different types of consumer, namely, π_i, is another important factor that influences the retailer's optimal contract strategy. To demonstrate this feature, we change π_H from 0.3 to 0.7 with $\pi_L = 1 - \pi_H$, while other parameters remain the same as the benchmark. Contract price, consumer's utility, and retailer's profit under different A_H are shown in Figure 8. The change of optimal package is shown in Figure 9.

In Figure 8, the utility of the Type-H consumer remains nearly unchanged and the IR gradually decreases. For the Type-L consumer, her utility decreases and the IR remains zero. The reason is that, an increasing π_H implies the Type-H consumer makes up a greater percentage of consumers, which deserves more concern from the retailer. Hence, the retailer tends to offer a higher Type-H contract price to earn more profit. Nonetheless, the retailer has to ensure that the Type-H consumer would not choose the contract designed for the Type-L consumer. To this end, the retailer chooses to reduce the package for Type-L, as shown in Figure 9. As an extreme case, say when π_H is high enough, the retailer would like to completely give up the Type-L consumer and design the package for Type-H only.

The profit of the retailer rises with increasing π_H, because a consumer with higher preference would like to consume more and pay more. In Figure 9, the package for

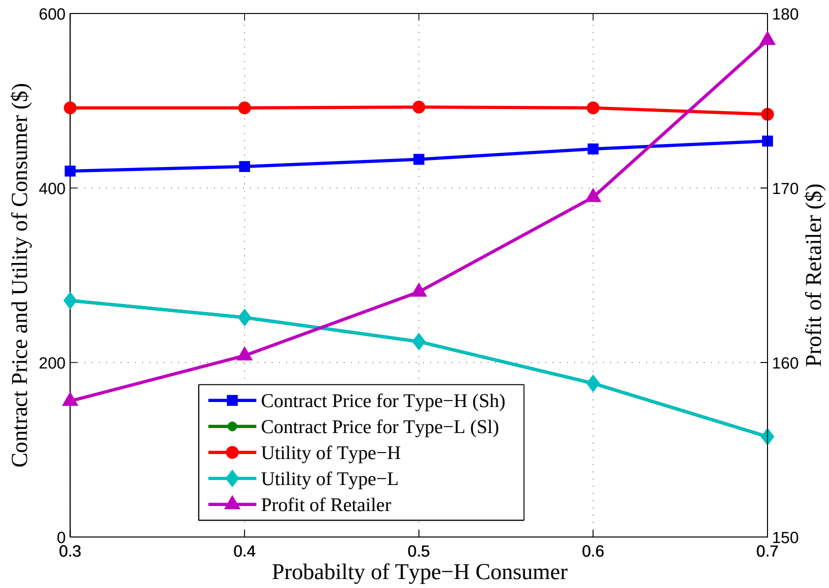

Figure 8: Changes of contract price, consumer's utility, and retailer's profit with different π_H.

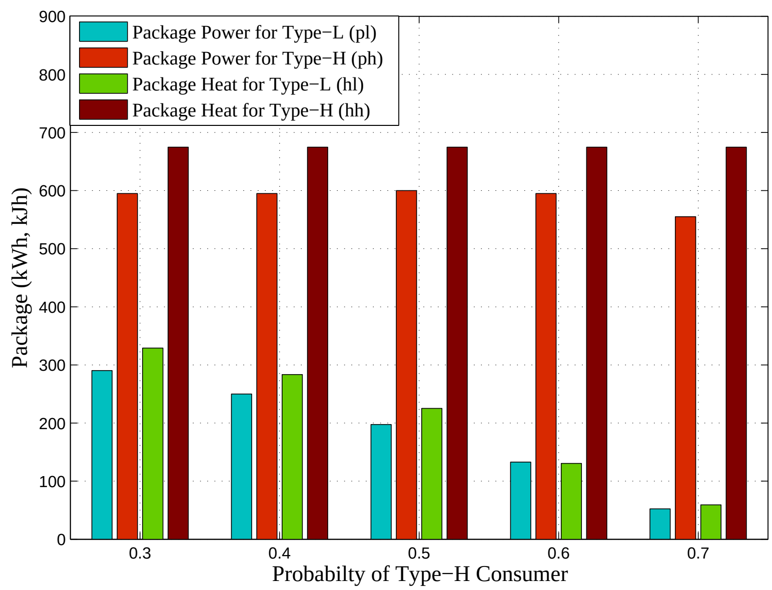

Figure 9: Change of packages with different π_H.

Type-H remains nearly the same, which can be inferred from (14a) and (14b), as C_h is independent of π_H. The decrease of C_l can account for the reduction in the package designed for Type-L.

5.6. *Impact of Power-to-Heat Efficiency*

Power-to-heat efficiency is another factor that may influence the contract strategy of the retailer. Here, we change the efficiency η_{eh} from 0.75 to 0.95. The changes of energy prices are shown in Figure 10; the changes of contract price, utility, and retailer's profit in Figure 11; and the change of packages in Figure 12.

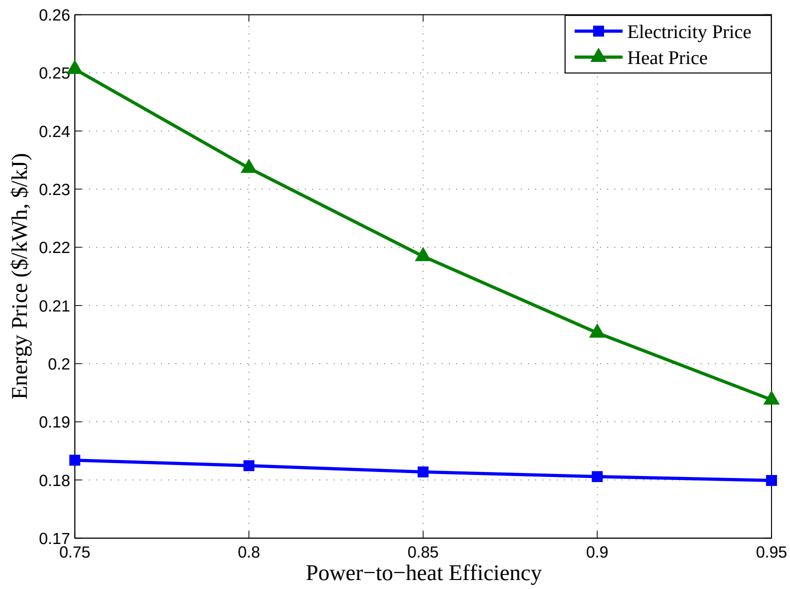

Figure 10: Energy prices under different η_{eh}.

With the increasing power-to-heat efficiency, both energy prices decrease. However, the reduction in the heat price is more remarkable than that in the power price. This outcome occurs because, when efficiency increases, the demand for using power to produce heat decreases accordingly and the cost for the heat system declines, motivating the increase in heat consumption. On the other hand, the growth in heat consumption results in more demand for power, which in turn partly counteracts the power

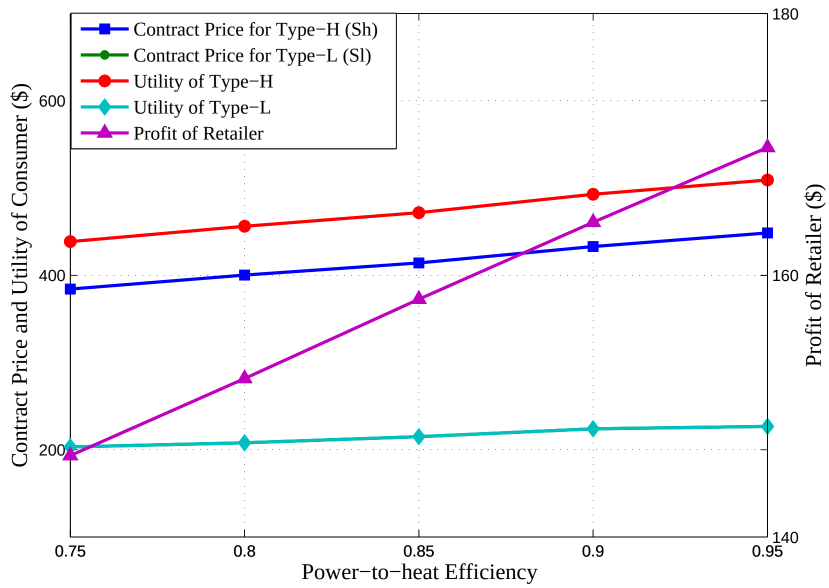

Figure 11: Contract price, utility, and retailer's profit under different η_{eh}.

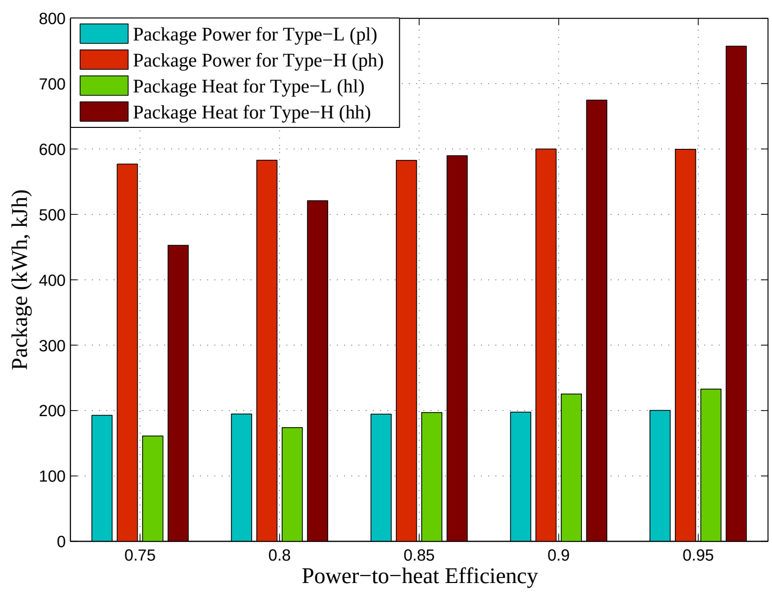

Figure 12: Packages under different η_{eh}.

reduction by efficiency. Thus, the change in power price is moderate. The change of packages in Figure 12 confirms this fact.

In Figure 11, the contract price, consumer's utility, and retailer's profit increase with efficiency. As mentioned earlier in this section, higher efficiency leads to lower energy prices and more energy consumption. Thus, the retailer can ultimately benefit from efficiency promotion.

5.7. Impact of Reservation Utility

In the above settings, we assume that both consumers' reservation utilities are zero, which means the consumer is willing to accept the contract as long as her net utility is no less than zero. However, in a more realistic situation, a consumer has positive reservation utility. Furthermore, it is reasonable to assume that the consumer with higher A_i has higher aspiration and requires higher reservation utility. Suppose the reservation utility for consumer 1 is $\bar{U}$. We increase $\bar{U}$ from 0 to \$140. The change of packages is shown in Figure 13. We observe that the variation of packages can be divided into five regions, noted as Regions 1$\sim$5. Detailed analyses can be found in Appendix D.

1) **Region 1:** When $\bar{U}$ is less than \$60, the optimal solution remains the same as the benchmark case. This outcome is because under asymmetric information, a consumer obtains a positive IR. In addition, when the IR is larger than $\bar{U}$, the optimal solution remains unchanged.

2) **Region 2:** When $\bar{U}$ is between \$60 and \$71, the IR of the Type-H consumer in the benchmark case is smaller than $\bar{U}$, and that of the Type-H consumer still has motivation to mimic Type-L. The binding constraints and the equivalent problem are shown in Appendix D. We find in Figure 13 that the optimal package for Type-L gradually rises with increasing $\bar{U}$, while the optimal package for Type-H decreases because of the response of energy prices.

3) **Region 3:** When $\bar{U}$ is between \$71 and \$98.5, neither type of consumer has incentive to mimic the other type, and the optimal contracts are the same as those under symmetric information.

4) **Region 4:** When $\bar{U}$ continues to increase to between \$98.5 and \$124, the Type-L

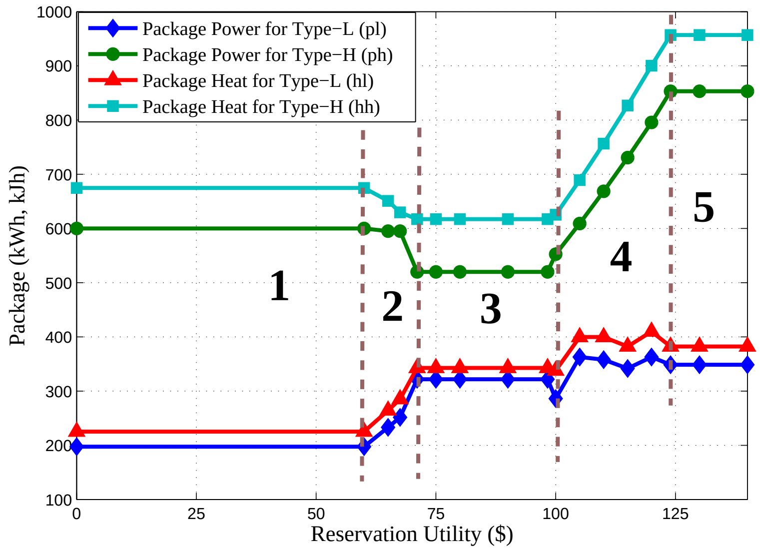

Figure 13: Packages under different $\bar{U}$.

consumer has incentive to mimic Type-H, and the situation is opposite to that in **Region 2**. The binding constraints and equivalent problem are shown in Appendix D. At this time, the retailer tries to distort upward the package for Type-H in order to reduce the IR of Type-L. However, the increase of package for Type-H may increase the retailer's costs. Therefore, we find in Figure 13 that the optimal packages for both consumer types are higher than those in **Region 3**.

5) **Region 5:** If $\bar{U}$ is larger than \$124, the situation becomes the opposite of that in **Region 1**. While $\bar{U}$ is increasing, the optimal solution remains the same.

6. Conclusions

In this paper, a retailer's contract design model for energy mix markets under asymmetric information is presented. A "retailer-consumer" framework is proposed to depict the bilateral relationship between a retailer and consumers in an asymmetric information game by package contracts. The energy prices are determined by solving the market clearing problems. Despite of the complexity of the game, we show the model size can be significantly reduced by identifying redundant constraints, and a separating equilibrium exists under mild conditions. The example and case studies disclose some interesting phenomena in the multi-energy market under asymmetric information.

1. The optimal contract strategy is distorted owing to asymmetric information; the package for a consumer with lower preference is distorted downward in order to reduce the information rent of a consumer with higher preference.
2. Among the three risk types of retailers, the market distortion is smallest for the risk-averse type and largest for the risk-preferent type. The risk-neutral type is in between.
3. Several factors influence the optimal contract:
   - The larger the difference between consumer types is, the more severely the multi-energy market is distorted.
   - The retailer tends to pay more attention to consumers with higher probability of occurrence.
   - All market participants benefit from the power-to-heat efficiency enhancement.
   - The reservation utility level may switch the binding constraints and thereby change the optimal contract.

The result in this paper can help reduce the negative impact of asymmetric information on power system, promote the profit of retailer and finally attract investment in retailers' business. The proposed model and algorithm can also provide reference for solving other asymmetric information related problems. Even though there are no mature multi-energy markets yet, in view of the progress of integrated energy systems in both academic and engineering fields, the design of multi-energy market and business mode that allocates energy resource in the optimal way is evidently an attractive direction. The proposed "retailer-consumer" framework is one of such attempts and could also serve as a fundamental tool for retailer's decision making in a multi-energy market. Although this work is somewhat rudimentary, it provides some interesting insights and observations, shedding new light on better understanding and design of future multi-energy markets.

Appendix A. Retailer's pricing problem with multiple classes of consumers

If the retailer offers electricity to more classes of load, such as office buildings and steel mills, the problem can be generalized as follows. Assume there are m classes of load and denote $\mathscr{M} := \{1, 2, \cdots, m\}$ as the set of classes. For class $k \in \mathscr{M}$, there are n_k types of consumers and denote $\mathscr{N}_k := \{1, 2, \cdots, n_k\}$. The utility function of consumer type i in class k is $A_{ik}U_k(p_{ik,d}, h_{ik,d})$. Here, the function $U_k(.), \forall k \in \mathscr{M}$ is common knowledge and the utility coefficient A_{ik} is private knowledge that is known only to the consumer type i in class k. The retailer can distinguish between consumers of different classes by different $U_k(.)$, that is, he can distinguish an office building from a steel mill. Nevertheless, he cannot exactly distinguish consumers within each class (consumers with the same $U_k(.)$ but different A_{ik}), that is, he cannot distinguish office building i from office building j. The distribution of consumers in class k is denoted by $\mathscr{P}_k := \{\pi_{ik}\ (i \in \mathscr{N})\}$, and $\sum_{i\in\mathscr{N}} \pi_{ik} = 1$. θ_k is the proportion of class k and $\sum_{k\in\mathscr{M}} \theta_k = 1$. In practice, π_{ik} and θ_k can be obtained through survey or statistics. Then, the problem can be modeled as (A.1).

$$\begin{aligned}
\max_{S_{ik}, p_{ik,d}, h_{ik,d}} f(\Xi) \quad & = \sum_{k\in\mathscr{M}} \theta_k \sum_{i\in\mathscr{N}_k} \pi_{ik} \Gamma(S_{ik} - \lambda_w^p p_{ik,d} - \lambda_v^h h_{ik,d}) && \text{(A.1a)} \\
\text{s.t.} \quad & A_{ik}U(p_{ik,d}, h_{ik,d}) - S_{ik} \geq 0, \forall i \in \mathscr{N}, k \in \mathscr{M} && \text{(A.1b)} \\
& i = \arg\max_{j\in\mathscr{N}_k} \{A_{ik}U(p_{jk,d}, h_{jk,d}) - S_{jk}\}, \forall k \in \mathscr{M} && \text{(A.1c)} \\
& d_w^p = \textstyle\sum_k \theta_k \sum_i \pi_{ik} p_{ik,d},\ d_v^h = \sum_k \theta_k \sum_i \pi_{ik} h_{ik,d} && \text{(A.1d)} \\
& \lambda_w^p = \varphi_p(d_w^p, d_v^h), \lambda_v^h = \varphi_h(d_w^p, d_v^h) && \text{(A.1e)}
\end{aligned}$$

The objective function (A.1a) and constraints (A.1b), (A.1d), and (A.1e) are similar to problem (4). The incentive compatibility constraint (A.1c) means that in each class k, choosing contract $\mathscr{C}_{ik}$ is the best choice for consumer type i. Following similar procedures in this study, it can be easily proved that all the lemmas and theorems still hold.

Appendix B. Proofs of Lemmas 1$\sim$3

proof of Lemma 1. We only need to prove that the constraints involve nonadjacent types can be implied by the constraints involve adjacent types.

From (9a), $\forall i \in \mathscr{N}\backslash\{n\}$, we have

$$A_iU(p_{i,d},h_{i,d}) - S_i \geq A_iU(p_{i+1,d},h_{i+1,d}) - S_{i+1} \tag{A.1a}$$

and

$$A_{i+1}U(p_{i+1,d},h_{i+1,d}) - S_{i+1} \geq A_{i+1}U(p_{i+2,d},h_{i+2,d}) - S_{i+2} \tag{A.1b}$$

Inequality (A.1b) gives

$$A_{i+1}\left(U(p_{i+1,d},h_{i+1,d}) - U(p_{i+2,d},h_{i+2,d})\right) \geq S_{i+1} - S_{i+2}$$

By Assumption A1 (which implies $A_i > A_{i+1}$) and substituting into (A.1a), we have

$$A_iU(p_{i,d},h_{i,d}) - S_i \geq A_iU(p_{i+2,d},h_{i+2,d}) - S_{i+2} \tag{A.2}$$

Following the similar line by using (9a) and (9b) repeatedly, all other constraints for nonadjacent types are satisfied, provided all the constraints for adjacent types are satisfied. This completes the proof. □

proof of Lemma 2. Assume the constraints in (4b) for $i = n$ and constraints in (9a) for $i \in \mathscr{N}\backslash\{n\}$ are satisfied. Then for $i = n-1$, we have

$$\begin{aligned} & A_{n-1}U(p_{n-1,d},h_{n-1,d}) - S_{n-1} \\ \geq\ & A_{n-1}U(p_{n,d},h_{n,d}) - S_n \\ \geq\ & A_nU(p_{n,d},h_{n,d}) - S_n \geq 0 \end{aligned} \tag{A.3}$$

The first inequality is due to condition 1). The second is due to Assumption A1. The last is due to condition 2).

Inequality (A.3) indicates that constraint in (4b) for $i = n-1$ is satisfied under the condition 1) and 2). Following the same line, it is easy to see all the constraints in (4b) for $i = 1, 2, \cdots, (n-1)$ are satisfied, which completes the proof. □

proof of Lemma 3. We first prove assertion 1). To this end, we assume that (9a) is not binding for i for the sake of contradiction. Define

$$\Delta_i \quad := \quad A_i U(p^*_{i,d}, h^*_{i,d}) - S^*_i - A_i U(p^*_{i+1,d}, h^*_{i+1,d}) + S^*_{i+1}$$

Then there must be $\Delta_i > 0$. We can use Δ_i to construct another solution, denoted by Ξ', following the rules that

$$\Xi' \quad := \quad \begin{cases} (p^*_{j,d}, h^*_{j,d}, S^*_j + \Delta_i) & : \quad \forall j \leq i \\ (p^*_{j,d}, h^*_{j,d}, S^*_j) & : \quad \forall j > i \end{cases}$$

Because we have $i < n$, so $(p'_{n,d}, h'_{n,d}, S'_n) = (p^*_{n,d}, h^*_{n,d}, S^*_n)$ and constraint (4b) for $i = n$ is still satisfied. For $j < i-1$, $\triangle_i$ is subtracted on both side of the inequalities, (9a) and (9b) is still met. For $j = i$, (9b) is still met for the same reason as the former. According to the definition of $\triangle_i$, the left-side of (9a) is equal to the right-side of (9a). So constraint (9a) for $j = i$ is satisfied. For $j = i+1$, obviously (9a) is met. The left-side of (9b) is the same as in Ξ^* and the right-side of (9b) is reduced by $\triangle_i$. Hence (9b) is satisfied. For $j > i+1$, the contracts in Ξ' are the same as in Ξ^*, implying the constraint satisfaction will not change.

From above, Ξ' is also a feasible solution of the problem and $f(\Xi') > f(\Xi^*)$, which is contradict to the optimality of Ξ^*. Hence (9a) must be binding, which proves assertion 1).

Next we prove assertion 2). Assume the constraint in (4b) for $i = n$ is not binding for the sake of contradiction. Define $\Delta_n := A_n U(p^*_{n,d}, h^*_{n,d}) - S^*_n$. Then there must be $\Delta_n > 0$. Let $\Xi'' := (p^*_{i,d}, h^*_{i,d}, S^*_i + \Delta_n)$ for all $i \in \mathcal{N}$. Then following a similar process as above, we can easily prove that Ξ'' is also a feasible solution of the problem and $f(\Xi'') > f(\Xi^*)$, which is contradict to the optimality of Ξ^*. Hence the constraint in (4b) for $i = n$ must be binding, which proves assertion 2).

This completes the proof. □

Appendix C. Proofs of Lemma 4 and Theorem 2

proof of Lemma 4. For a risk-neutral type, we have $\Gamma'(V_i) =$ constant for all $i \in \mathcal{N}$, which means $B_i = B_j$ and $D_i = D_k$ for all $i, k \in \mathcal{N}$. If we have $C_i > C_k$, then from (15a) and (15b), we have $\Delta U_p^{'} = -U_p^{'}(p_{i,d}, h_{i,d}) + U_p^{'}(p_{k,d}, h_{k,d}) > 0$ and $\Delta U_h^{'} = -U_h^{'}(p_{i,d}, h_{i,d}) + U_h^{'}(p_{k,d}, h_{k,d}) > 0$. Denoting $\Delta p := p_{k,d} - p_{i,d}$ and $\Delta h := h_{k,d} - h_{i,d}$, we have

$$\Delta U_p^{'} = U_{pp}^{''}\Delta p + U_{ph}^{''}\Delta h \quad \text{(A.1a)}$$

$$\Delta U_h^{'} = U_{hp}^{''}\Delta p + U_{hh}^{''}\Delta h \quad \text{(A.1b)}$$

Solving these equations yields

$$\Delta p = \frac{\Delta U_p^{'} \cdot U_{hh}^{''} - \Delta U_h^{'} \cdot U_{hp}^{''}}{U_{pp}^{''} \cdot U_{hh}^{''} - U_{hp}^{''} \cdot U_{ph}^{''}} \quad \text{(A.2a)}$$

$$\Delta h = \frac{\Delta U_h^{'} \cdot U_{pp}^{''} - \Delta U_p^{'} \cdot U_{ph}^{''}}{U_{pp}^{''} \cdot U_{hh}^{''} - U_{hp}^{''} \cdot U_{ph}^{''}} \quad \text{(A.2b)}$$

With Assumption A3, we have $U_{hh}^{''} \leq 0$, $U_{hp}^{''} \geq 0$ and

$$U_{pp}^{''} \cdot U_{hh}^{''} - U_{hp}^{''} \cdot U_{ph}^{''} > 0$$

By noting $U_p' > 0$ and $U_h' > 0$, it is easy to see

$$U_p^{'} \cdot U_{hh}^{''} - U_h^{'} \cdot U_{hp}^{''} < 0$$

Hence we have $\Delta p < 0$ and so as Δh, which means $p_{i,d} > p_{k,d}$ and $h_{i,d} > h_{k,d}$. This completes the proof. □

proof of Theorem 2. We prove the theorem from the following two aspects.

Sufficiency: If we have $C_1 > C_2 > ... > C_{n-1} > C_n$, by Lemma 4 we have $p_{1,d} > p_{2,d} > ... > p_{n,d}$ and $h_{1,d} > h_{2,d} > ... > h_{n,d}$. With Assumption A3, (12b) is strictly satisfied, so there is a seperating equilibrium.

Necessity: If a separating equilibrium exists, but we have $C_i \leq C_k$. If $C_i < C_k$, by Lemma 4 a contradiction appears. If $C_i = C_k$, it is obvious that we have $p_{i,d} = p_{k,d}$ and $h_{i,d} = h_{k,d}$, and it is contradict to the existence of a separating equilibrium.

This completes the proof. □

Appendix D. Impact of reservation utility

In this subsection, we analyze the impact of non-zero reservation utility of consumers. To simplify the discussion, we suppose that there are only two types of consumers, i.e., $i = 2$. The retailer is still assumed to be risk-neutral.

We change the reservation utility of consumer 1 from zero to a positive value, $\bar{U} > 0$. Denote $\tilde{\Xi} := \{(\tilde{p}_{i,d}, \tilde{h}_{i,d}, \tilde{S}_i),\ \forall i \in \mathcal{N}\}$ as the retailer's contract strategy resulting in a reservation utility of consumer 1 no less than $\bar{U}$. Then the retailers decision-making problem becomes:

$$
\begin{aligned}
\max_{(p_1,h_1,S_1),(p_2,h_2,S_2)} \quad & \pi_1(S_1 - \lambda_w^p p_1 - \lambda_v^h h_1) \\
& + \pi_2(S_2 - \lambda_w^p p_2 - \lambda_v^h h_2) && \text{(A.1a)} \\
\text{s.t.} \quad & A_1 U(p_{1,d}, h_{1,d}) - S_1 \geq \bar{U} && \text{(A.1b)} \\
& A_2 U(p_{2,d}, h_{2,d}) - S_2 \geq 0 && \text{(A.1c)} \\
& A_1 U(p_{1,d}, h_{1,d}) - S_1 \\
& \geq A_1 U(p_{2,d}, h_{2,d}) - S_2 && \text{(A.1d)} \\
& A_2 U(p_{2,d}, h_{2,d}) - S_2 \\
& \geq A_2 U(p_{1,d}, h_{1,d}) - S_1 && \text{(A.1e)} \\
& d_w^p = \textstyle\sum_i \pi_i p_{i,d},\ d_v^h = \sum_i \pi_i h_{i,d} && \text{(A.1f)} \\
& \lambda_w^p = \varphi_p(d_w^p, d_v^h), \lambda_v^h = \varphi_h(d_w^p, d_v^h) && \text{(A.1g)}
\end{aligned}
$$

With the increase of $\bar{U}$, the solution to the problem exhibits the following five different regimes.

Region 1: When $\bar{U} \leq (A_1 - A_2)U_p(p_{2,d}^*, h_{2,d}^*)$, Ξ^* satisfies all the constraints in (A.1) and the optimal package is still Ξ^*.

Region 2: When $(A_1 - A_2)U_p(p_{2,d}^*, h_{2,d}^*) < \bar{U} \leq (A_1 - A_2)U(\hat{p}_{2,d}, \hat{h}_{2,d})$, (A.1b) is not met for Ξ^*. The binding constraints become (A.1b), (A.1c) and (A.1d). In this situation, problem (A.1) is equivalent to

$$
\begin{aligned}
\min_{S_2,p_2,h_2} \quad & \lambda_w^p p_2 + \lambda_v^h h_2 && \text{(A.2a)} \\
\text{s.t.} \quad & \sqrt{a_p p_2} + \sqrt{a_h h_2} = \frac{\bar{U}}{A_1 - A_2} && \text{(A.2b)}
\end{aligned}
$$

The optimal solution to problem (A.2) can be obtained simply by invoking Cauchy-Inequality.

If the energy prices are fixed, we have $(\tilde{p}_1,\tilde{h}_1) = (\hat{p}_1,\hat{h}_1)$, $(\tilde{p}_2,\tilde{h}_2) > (\hat{p}_2,\hat{h}_2)$. However, if we take into account the regulation of energy prices as in our model, when

p_1, h_1 goes up, the energy prices will change. Hence, the optimal package ($\tilde{p}_2$,$\tilde{h}_2$) will change accordingly. Unfortunately, there is no closed-form expression to describe such a change.

Region 3: When $(A_1 - A_2)U(\hat{p}_{2,d}, \hat{h}_{2,d}) < \bar{U} \leq (A_1 - A_2)U(\hat{p}_{1,d}, \hat{h}_{1,d})$, both types of consumers have no incentive to mimic the other type, the binding constraints change into (A.1b) and (A.1c). The optimal contract strategy is the same as the situation under symmetric information, which means $\hat{\Xi} = \tilde{\Xi}$.

Definition 3 (countervailing incentive)**.** The consumer with lower A_i has the incentive to mimic the consumer with a higher A_i, i.e., A_2 has the incentive to choose the package decided for A_1.

A4. The optimum, $\Xi^{*'} := \{(p^{*'}_{i,d}, h^{*'}_{i,d}, S^{*'}_{i}),\ \ \forall i \in \mathcal{N}\}$, of the retailer's pricing model under countervailing incentive exists.

Region 4: When $(A_1$ - $A_2)U(\hat{p}_{i,d}, \hat{h}_{i,d}) < \bar{U} \leq (A_1 - A_2)U(p^{*'}_{i,d}, h^{*'}_{i,d})$, the situation is opposite to the situation in **Region 2**. Constraint (A.1e) is not fulfilled for $\hat{\Xi}$. The binding constraints are (A.1b), (A.1c) and (A.1e).

Region 5: When $\bar{U} > (A_1 - A_2) \cdot U(p^{*'}_{i,d}, h^{*'}_{i,d})$, the situation is opposite to the situation in **Region 1**. The binding constraints are (A.1b) and (A.1e).

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