

Interval based Robust Chance Constrained Allocation of Demand Response Programs in Wind Integrated Power Systems

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Abstract: This paper presents an interval based robust chance constrained (IBRCC) optimization model for allocating demand response program (DRP) to effective buses of the power systems considering wind uncertainty and equipment failures. In the proposed formulation, an interval based robust approach is applied to evaluate the highest uncertainty spectrum of the wind power generation that the power system can tolerate. Accordingly, to cope with the uncertainty sources, chance based constraints are implemented. In the proposed IBRCC optimization framework, the level of the optimal solutions robustness is probabilistically maximized subject to a set of operational constraints. Besides, to facilitate the massive integration of uncertain wind generation and to mitigate congestion in the transmission grid, an efficient allocation and scheduling scheme of demand response programs is proposed. The proposed model is evaluated on the IEEE 24 bus system.

Index Terms— Demand response, wind uncertainty, robust chance constrained optimization.

1. Nomenclature

– *Indices*

$(\cdot)_{\omega}^{\pm}$ Variable related to continuous (\pm) and discrete (ω) uncertainties.
 ω Discrete uncertainty, random unit/line outage.
 \pm Continuous uncertainty, ‘-’ and ‘+’ related to the lower and upper limits of CU, respectively.
 g/w Index for generating unit and wind farm.
 n, m, b Indices of buses.
 k Index of transmission lines.
 t, t' Indices of time.
 \bar{t} Index of peak time
 \wedge Given variables.
 All variables and constants include subscript ω and t refer to scenario ω and hour t .

– *Parameters*

P_g^{\max} / P_g^{\min} max/min power generation of unit g .
 C_g / \tilde{C}_g Cost of base case /uncertainty condition of unit g .
 C_n / \tilde{C}_n Cost of DRP for base case /uncertainty condition of demand n .
 \bar{C} Cost threshold related to the response uncertainties.
 π_{ω} Probability of scenario ω of DU.

C_n^{lol} Value of lost load for demand n .
 $P_{f,wt}$ Wind power forecast for wind farm w .
 $\Delta SF_{k,b}$ Shift factors for line k due to bus b .
 SFI_b Shift factors index.
 B_{mm} Susceptance of line $k(n, m)$.
 λ Number of buses to DRP.
 Θ_{DU} Number of DU.
 $\bar{\beta} / \underline{\beta}$ Max/min radius of continues uncertainty.
 $\bar{R}_g / \underline{R}_g$ Up/down ramping limit of unit g .
 D_{nt}^f Load forecast at bus n .
 $\bar{D}_{nt} / \underline{D}_{nt}$ Max/min demand response of demand n .
 $\bar{R}_n / \underline{R}_n$ Up/down ramping limit of demand n .
 \bar{E}_n Maximum energy adjustment of demand n in the operating horizon.
 $uc_{g,\omega} / uc_{k,\omega}$ Contingency state of (unit g)/(line k).
 $L\bar{C}_n$ Maximum involuntarily load curtailment
 $\bar{D}_{nt} / \underline{D}_{nt}$ Max/min load changed by DRP.
 $\bar{\theta}_n$ Max limit for angle bus n .
 \bar{P}_k Max limit for power flow of line k .
 UT_g / DT_g Minimum on/off hours of a thermal unit g
 η Risk tolerance level.
 M A large number.

– *Variables*

z_{ω} Binary variable that indicates constraint related to scenario ω if it is imposed or not.
 β Radius of continue uncertainty (%).

v_{gt} / w_{gt}	Startup/shutdown binary variable for thermal unit g at the time period t .
u_{gt}	Unit status.
$\theta_n^{(c)}$	Phase angle of bus n .
P_{kt}	Power flow in line k .
$P_{\omega,wt}^{\pm}$	Uncertain wind power generation w .
$P_{gt}^{(c)}$	Power generation of unit g .
$\bar{\Delta}R_{nt} / \underline{\Delta}R_{nt}$	Pick up/ drop down rate of DRP for demand n .
$\bar{\Delta}r_{\omega,gt}^{\pm} / \underline{\Delta}r_{\omega,gt}^{\pm}$	Up/down spinning reserve by unit g .
$\bar{\Delta}r_{\omega,nt}^{\pm} / \underline{\Delta}r_{\omega,nt}^{\pm}$	Up/down spinning reserve by DRP for demand n .
$LC_{\omega,nt}^{\pm}$	Involuntarily load curtailment of demand n .
D_{nt}	Load scheduled by DRP for demand n .
DR_{nt}	DRP for demand n .
$\aleph_{(c)}$	Slack variable.
$\mathfrak{S}_{(c)}$	Power mismatch in subproblem I and II.
$\kappa_{(c)}$	Dual variables.

2. Introduction

Integration of wind energy generation (WEG) resource will increase the intermittency and uncertainty of power system operations. Due to massive penetrations of WEGs, the uncertain operation of power systems would necessitate more ramping and flexibility capabilities for the system to continuously keep balance between supply and demand. In this condition, the commitment of expensive generation units for providing more system ramping capability are also likely to be increased, which will impose economic and technical dilemmas for the power system operations particularly in congested transmission grids. Accordingly, the wind uncertainty and transmission congestion are the main challenges in the power system operations. To cope with these challenges, extra flexibility resources are needed in the power systems.

One of the promising economic solutions is demand response program (DRP) in power systems that can facilitate massive integration of WEGs. The main aim of DRP is to encourage the customers for participation in some programs that lead to modify the demand curves' pattern. Accordingly, the customers who participate in DRP can play an important role in reducing the effect of wind uncertainty on power system operations [1]. The DRPs are applied based on the various policies. Usually they have been deployed to cover the uncertainty of high penetration of renewable energy sources [1] and [2]. Nevertheless, with the recent attentions to the DRP [2], [3] and [4], the flexible demands can smooth the integration of WEGs.

A common approach to managing the uncertain WEGs is to concurrently incorporate both the flexible supply (ramping capability) and demand (DRP) sides [5]. Consequently, the DRPs will reduce demand at the peak hours (and low wind generation periods) and/or shift demand to off peak hours (and high wind generation periods) leading to the decrease in generation's costs and accordingly relieve the network congestion [6]. The DRPs have a high capability to facilitate congestion management and consequently make the grid side flexibility available [7].

The DRP by moderating the system congestion has the capability to acquire further available transfer capability (ATC) to deliver more power generation from wind farms to meet the customers' demands. Besides in [8] and [9], the effects of DRPs on the load curve's characteristics improvement and CM have been studied. It should be noted that in [8] and [9], the wind uncertainty has not been taken into account and, DRP has not been considered as a tool for uncertainty management. However, the above mentioned goals (uncertainty and congestion management) could be achieved only if DRPs are well allocated in the effective buses of the system. Accordingly, the best locations (optimal buses) and scheduling for applying DRPs should be specified by independent power system operators.

In this paper, a new technique to allocate DRPs has been developed which on the basis of the shift factors method (SFM), and DC power flow (DCPF). The SFM shows how power flow in the congested transmission lines will be changed if the injection at load buses are changed by one MW. Once that the SFM for all of the load buses based on a procedure is calculated, the effective buses for implementing DRPs will be specified. The proposed procedure will be explained in Section IV.

In [10], the DRP is utilized to increase the available transfer capacity (ATC) in a power system to enhance the system ATC. However, in [10], the best location of the buses for DRP has not been determined.

In the all above mentioned researches [1] [8], the DRP is implemented at all buses to manage transmission congestion and improve the load curve characteristics, managing wind uncertainty, or reduce operation cost.

But, none of them has introduced a procedure for the implementing DRPs at effective buses to enhance the utilization of the existing power system in hosting significant uncertainty WEGs while satisfying the other aims, i.e., the cost reduction and reliability improvement. If DRP is not implemented in effective buses, it may worsen the congestion and consequently increase the generation costs. For instance, implementing DRP on the buses with positive values of shift factors will increase the costs.

It is noteworthy that repeated implementations of DRP at all system's load buses lead to adverse consequences on power system security and reliability and may necessitate more automated systems. That is, it is not allowed to frequently change some load buses. Accordingly, it is very essential to allocate DRPs to the effective buses. Finally, more investigations on the implementation of DRPs for uncertainties and congestion management have been studied in [5]. Nevertheless, in order to develop an advanced uncertainty management approach, different uncertainties can be classified as continuous uncertainties (CUs) and discrete uncertainties (DUs).

The first category of uncertainties, CUs, belongs to the usually known continuous intervals, and occur in high frequency for the short time period. For instance, the uncertainty of WEG is a kind of CUs. In contrary, DUs are kind of low frequency and long term with a discrete nature. The outages of generating units and transmission lines fall into this category.

The power system operations under CUs, with known upper and lower bounds of the uncertainty range, have been explored by means of some approaches like standard robust

optimization approach (SROA) [11] and our proposed interval based robust approach (IBRA) in this paper which relied on the uncertainty boundary instead of probability density function (PDF). The decision making in the interval based approaches is a kind of conservative strategy based on the worst cases realizations of the uncertainty sources. In [11], a two stage SROA based has been utilized for the day ahead scheduling problem. Also, in [12], security constrained unit commitment problem has been solved using a SROA while the short term uncertainty of wind power generation and load forecast as CUs have been included.

Unlike the SROA [11]–[12] that requires to specify the uncertainty range, the proposed IBRA in this paper tries to find the highest possible unknown variation range of the CUs that the system can tolerate in a reasonable operational cost. That is, the optimal robust risk averse operation of the system based on the preferences of the operators and planners will specify the target variation range of the sources of CUs.

On the other hand, implementing the proposed interval based robust approach for each optimization model is very simple and tractable, and also, does not add the complexity of the existing problem, but, implementing the standard robust optimization for each optimization model is very difficult and in some cases is intractable which is the main *disadvantage* of the standard robust optimization method.

Obviously, to tolerate a wider target uncertainty range, it is required to adopt a more conservative design or operation of the system. The proposed IBRA is well suited for *unstructured* CUs while it needs less information about the uncertainty set. Undoubtedly, IBRA can be more applicable for *structured* CUs as well. The main disadvantage of IBRA is that it cannot characterize the DUs in power system operation while it is DUs are not kinds of interval based structure.

The other methods for addressing the impacts of DUs on the power system operation are probabilistic approaches, e.g., stochastic approach (SA) [13] and chance constrained approach (CCA) [14]. The SA is mainly based on a set of generated scenarios to characterize the probable realizations of DUs. Usually, the SA needs to generate a higher number of scenarios to have a more precise characterization of the uncertainties which may lead to high computational time and costly solution [13]. Unlike SA, in the CCA, the constraints are satisfied based on the predefined probability levels. That is, based on this approach the constraints are kinds of deterministic constraints with respect to the scenario dependent constraints in SA [14]. Accordingly, the CCA is suitable for simulating DUs such as random outages of transmission lines and units.

But, the main disadvantages of the CCA are as follows: (i) CCA is dependent on accurate PDF models, (ii) in CCA the PDF models of the CUs must be pre-determined (iii) to apply CCA for CUs, a rather large number of scenarios should be generated that results in high computational burden [14]. To deal with the above shortages of IBRA and CCA approaches, in this paper, an innovative approach is proposed based on integrated optimization framework of IBRA and CCA model called IBRCC approach to take advantages of both.

Another approach in the literature to handle the uncertainty is distributionally-robust chance constrained (DRCC) optimization [15]–[20]. DRCC is employed to model the continuous uncertainty instead of the discrete uncertainty. In addition, the DRCC model can control the robustness and conservativeness using a coefficient. The model of DRCC is a convex programming problem which is efficient and can be solved very fast using available software. The DRCC approach in some cases is similar to our proposed IBRCC approach *however*, this approach needs the mean and variance of an uncertain parameter to model CUs by DRCC, while the proposed IBRCC requires more limited information about the uncertain parameter [17].

Considering the above addressed existing literature and identifying the state of art challenges, the novelties of the presented research are summarized as follow:

- (i) In this paper, the DRP model is constructed based on an integrated optimization structure of IBRA and CCA to facilitate the large scale wind energy integration. In the resulting robust scheme, the IBRA is implemented to maximize the variation range of wind uncertainty toleration for the system, while the CCA is applied to manage DUs (i.e., random outages of unit and line failures) according to CUs' condition.
- (ii) Here, a new technique based on SFM and DC power flow is proposed for determining the effective buses to allocate the DRPs. Also, the effects of DRPs on the largest variation range of wind uncertainty are investigated.

3. Problem Formulation

3.1. Assumptions

For the sake of clarity, the main modeling assumptions have been summarized as follows:

- Continues uncertainty pertains solely to wind generation and discrete uncertainty relates to unit and line failures. However, the proposed model is capable of considering demand uncertainty as well. The discrete uncertainty of equipment failure is characterized by a set of possible scenarios according to known forced outage rates (FOR).
- Linear cost and utility functions of generating units and DRPs are considered, respectively.
- The power factor of all wind farms is considered to be 1.
- A DC power flow model is used.

3.2. The Basic Mathematical Formulation

The original DRP allocation problem of the 24 hour scheduling period is a kind of two stage optimization to minimize a cost objective function.

The first stage part, i.e., *here and now*, refers to the offered generation cost plus start up cost and the utility function of the DRP at the base case, i.e., C_{bt} . While, the second stage part, i.e., *wait and see*, pertains to the cost of power amendments for generation units and DRP and involuntary load curtailment in response to the different uncertainty sources.

The objective function and first stage constraints are as follows:

$$\min TC = \sum_t \left\{ \begin{array}{l} \overbrace{\sum_g (C_g \cdot P_{gt} + C_g^{SU} \cdot v_{gt}) + \sum_n (C_n \cdot (\bar{\Delta}R_{nt} + \underline{\Delta}R_{nt}))}^{C_{b,t}} \\ + \sum_\omega \pi_\omega \left(\overbrace{\sum_g \tilde{C}_g \cdot \left(\frac{\Delta r_{\omega,gt}^\pm}{\pm \Delta r_{\omega,gt}^\pm} \right)}^{C_{\omega,t}^\pm} + \sum_n \left(\tilde{C}_n \cdot \left(\frac{\Delta r_{\omega,nt}^\pm + \bar{\Delta}r_{\omega,nt}^\pm}{\pm \Delta r_{\omega,nt}^\pm} \right) \right) \right) \end{array} \right\} \quad (1)$$

$$\sum_{g(n)} P_{gt} + \sum_{w(n)} P_{f,wt} - \sum_{k(n,m)} P_{kt} + \sum_{k(m,n)} P_{kt} = D_{nt} \quad (2)$$

$$P_{kt} = B_{nm} \cdot (\theta_{nt} - \theta_{mt}) \quad (3)$$

$$-\bar{P}_k \leq P_{kt} \leq \bar{P}_k \quad (4)$$

$$-\bar{\theta}_n \leq \theta_{nt} \leq \bar{\theta}_n \quad (5)$$

$$v_{g,t} - w_{g,t} = u_{g,t} - u_{g,t-1} \quad (6)$$

$$\sum_{t'=t-UT_g+1}^t v_{g,t'} \leq u_{g,t}, \quad \forall g, t \in \{UT_g, \dots, T\} \quad (7)$$

$$\sum_{t'=t-DT_g+1}^t w_{g,t'} \leq 1 - u_{g,t}, \quad \forall g, t \in \{DT_g, \dots, T\} \quad (8)$$

$$P_{gt} \cdot u_{gt} \leq P_{gt} \leq \bar{P}_{gt} \cdot u_{gt} \quad (9)$$

$$\underline{R}_g \leq P_{g,t} - P_{g,t-1} \leq \bar{R}_g \quad (10)$$

$$\begin{cases} D_{nt} = D_{nt}^f - DR_{nt} \\ \underline{D}_{nt} \leq D_{nt} \leq \bar{D}_{nt} \end{cases} \quad (11)$$

$$DR_{nt} \cdot ud_{nt} \leq DR_{nt} \leq \bar{DR}_{nt} \cdot ud_{nt} \quad (12)$$

$$\bar{\Delta}R_{nt} - \underline{\Delta}R_{nt} = DR_{nt} \quad (13)$$

$$\underline{R}_n \leq DR_{n,t} - DR_{n,t-1} \leq \bar{R}_n \quad (14)$$

$$\sum_t (DR_{n,t}) \leq \bar{E}_n \quad (15)$$

$$\sum_n (ud_{nt}) \leq \lambda \quad \forall t = \bar{t} \quad (16)$$

In the above DRP allocation problem, the total cost (TC) of the power system as the objective function (1) is minimized over the operation horizon. The TC in (1) includes two main cost terms: first stage and second stage cost functions. Besides, main first stage constraints include the power balance at each bus prior to CUs and DUs realization (2), the power flow of each line as a function of the angle difference of the interconnected buses (3), line flow and voltage angle limits (4) and (5), startup and shutdown binary constraint (6), minimum on/off time limits (7) and (8), min/max limits of generating units, (9), hourly ramping up and down limits of generating units (10). To manage the uncertainties, the DRPs can be utilized through a couple of programs, vis., load reduction (LR) and load shifting (LS). In the LR program, the DRP would curtail the load in different hours of the operation horizon without shifting its consumption to other hours, i.e., $DR_{nt} > 0$. However, in the LS program, the load demand is shifted from peak hours to off peak hours, i.e., $DR_{nt} < 0$. In (11), both kinds of DRPs can be taken place. Noted that, the DR_{nt} can take a negative or positive values during the whole operation horizon. The DR_{nt} is positive when the load is shifted out from bus n at time t , and negative when the load is shifted to the bus n at time t . Also, limits on demand schedule are given in this constraint. Limits on DRP is given in (12). The binary variable ud_{nt} represents the status of DRP schedule. In (13), the absolute value of DR_{nt} (or $|DR_{nt}|$)

has been denoted with two positive variables $\bar{\Delta}R_{nt}$ and $\underline{\Delta}R_{nt}$ to evaluate the cost of DRP in TC , (1).

Pick up/drop down rate limits of DRP have been modeled for the first stage condition in (14). The limit on the acceptable energy changes of the load during the system operation is imposed by (15). In this paper, to determine the number of effective load buses for implementing DRPs, two main approaches have been recommended. The first one is to determine these buses by pre specified value, λ , as shown in (16).

The second approach is to apply the SFM as discussed later in Section IV. Alternatively, the number of load buses for implementing DRPs in a real power system must be restricted due to the technical and security considerations of power systems. Therefore, the maximum number of effective load buses at peak time has been determined by (16). It should be noted that, the transmission congestion is mostly occurred in peak hours, accordingly, the effective load buses for the 24 hour scheduling horizon is determined by (16), i.e., \bar{t} .

The second stage constraints are associated with the operation conditions in the real time as follows:

$$\begin{aligned} & \sum_{g(n)} P_{\omega,gt}^\pm + \sum_{w(n)} P_{\omega,wt}^\pm - \sum_{k(n,m)} P_{\omega,kt} \cdot uc_{k,\omega} \\ & + \sum_{k(m,n)} P_{\omega,kt} \cdot uc_{k,\omega} = D_{nt}^\pm - LC_{\omega,nt}^\pm \end{aligned} \quad (17)$$

$$P_{\omega,kt}^\pm = B_{nm} (\theta_{\omega,nt}^\pm - \theta_{\omega,mt}^\pm) \quad (18)$$

$$-\bar{P}_k \leq P_{\omega,kt}^\pm \leq \bar{P}_k \quad (19)$$

$$-\bar{\theta}_n \leq \theta_{\omega,nt}^\pm \leq \bar{\theta}_n \quad (20)$$

$$\begin{cases} P_{\omega,gt}^\pm = (P_{gt} + \bar{\Delta}r_{\omega,gt}^\pm - \underline{\Delta}r_{\omega,gt}^\pm) \cdot uc_{g,\omega} \\ P_{\omega,gt}^\pm \cdot u_{gt} \leq P_{\omega,gt}^\pm \leq \bar{P}_{gt} \cdot u_{gt} \end{cases} \quad (21)$$

$$\begin{cases} D_{\omega,nt}^\pm = D_{nt} + \bar{\Delta}r_{\omega,nt}^\pm - \underline{\Delta}r_{\omega,nt}^\pm \\ \underline{D}_{nt} \leq D_{\omega,nt}^\pm \leq \bar{D}_{nt} \end{cases} \quad (22)$$

$$0 \leq LC_{\omega,nt}^\pm \leq \bar{LC}_n \quad (23)$$

$$0 \leq \bar{\Delta}r_{\omega,nt}^\pm / \underline{\Delta}r_{\omega,nt}^\pm \leq \bar{R}_{nt} \cdot ud_{nt} / \underline{\Delta}R_{nt} \cdot ud_{nt} \quad (24)$$

$$0 \leq \bar{\Delta}r_{\omega,gt}^\pm / \underline{\Delta}r_{\omega,gt}^\pm \leq \bar{R}_g \cdot uc_{g,\omega} \cdot u_{gt} / \underline{R}_g \cdot uc_{g,\omega} \cdot u_{gt} \quad (25)$$

In constraints (17) – (25), the signs ‘ \pm ’ relate to CUs, i.e., wind uncertainty while ‘ $-$ ’ and ‘ $+$ ’ refer to the *lower* and *upper* limits of the uncertainty range of wind generation, respectively, and ω refers to the possible realization scenarios of DUs, i.e., outage units/lines. Constraint (17) indicates nodal balance at the second stage, wherein the spinning reserves, provided by generation units and DRP, and involuntary load curtailments (LCs) will compensate both CUs and DUs.

Constraints (18) – (20) represent the response of equations (3) – (5) to CUs (\pm) and DUs (ω). The output of generating units and the actual loads at the uncertainty conditions are defined by (21) and (22), respectively. Besides, limits on power generating units and demand schedule in uncertainty conditions are given in (21) and (22), respectively. The LC limit is applied by (23). Constraints (24) and (25) stand for the maximum up and down spinning reserves provided by DRPs and generating units, respectively.

3.3. Proposed Interval based Robust Formulation

As mentioned, in the DRP allocation problem, the core idea behind the IBRA is to deal with the CUs of wind generations by means of maximizing their highest tolerable variation range while taking system technical considerations into account. Accordingly, the IBRA model is written as follows:

$$\max \beta \quad (26)$$

$$P_{\omega,wt}^{\pm} = (1 \pm \beta) \cdot P_{f,wt}, \quad \forall \beta \in [\underline{\beta}, \bar{\beta}] \quad (27)$$

$$TC \leq \bar{C}, \quad \forall \beta \in [\underline{\beta}, \bar{\beta}] \quad (28)$$

$$(2)-(25) \quad (29)$$

The objective function (26) should maximize the variation range of the CU of wind generation subject to the sets of constraints (27) – (29). In (27), $(1 + \beta) \cdot P_{f,wt}$ and $(1 - \beta) \cdot P_{f,wt}$ are related to the upper and lower variation range of WPG, respectively. Constraint (28) indicates the maximum pre specified threshold, \bar{C} , of the total cost (TC). Constraint (27) includes the DRP allocation constraints (2) – (25) in the IBRA model that should be satisfied for the whole range of $[\underline{\beta}, \bar{\beta}]$.

Noted that, at first, the model (1) to (16) is solved for the forecasted wind power generation to obtain the minimum cost threshold \bar{C} , then the objective function (26) should maximize the size of the variation range of CUs. The cost threshold \bar{C} , is used in constraint (28). The \bar{C} in equation (28) indicates that the cost of the power system operation must not exceed the cost threshold for any realization of uncertainty. The choice of the cost threshold depends on decision makers' conservation level. Compared to a risk-taker, a risk-averse decision maker would be willing to pay more in order to keep system remain reliable with respect to large disturbance, so his cost threshold would be higher.

3.4. Interval based Robust Chance Constrained Model

While in the IBRA model, (random outages of units and transmission lines) cannot be handled, the constraints (27) and (28) should be enforced to the optimization problem for each realization of DUs. Nevertheless, this condition could be too restrictive, as a result, they are permitted to be violated under some intolerable extreme conditions. Therefore, in this paper an interval based robust chance constrained (IBRCC) model is proposed to probabilistically satisfy the constraints (27) and (28) by a pre-defined probability. Consequently, the IBRA formulation should be amended as follows:

$$\Pr \left\{ \begin{array}{l} P_{\omega,wt}^{\pm} = (1 \pm \beta) \cdot P_{f,wt} \\ TC \leq \bar{C} \end{array} \right\} \geq 1 - \eta \quad (30)$$

The constraint (30) namely is a chance constraint, which implies that the constraints (27) and (28) should be satisfied with a probability larger than or equal to $1 - \eta$, and η indicates the risk acceptance level. Indeed, the chance constraint (30) relaxes the optimization problem to mitigate the infeasibility resulting from a set of constraints and numerous DUs. That is, this approach considers a set of probable scenarios, $\Lambda_{\omega} = \{uc_{g_1, \omega_1}, \mathbf{K}, uc_{g_n, \omega_n}, uc_{k_{n+1}, \omega_{n+1}}, \mathbf{K}, uc_{k_2, \omega_{2n}}\}$, for

DUs' realizations. Subsequently, the optimization problem can be formulated using a novel bilinear model of the chance constrained approach. All in all, the formulation of IBRA can be rewritten as the following formulation of IBRCC:

$$\max \beta \quad (31)$$

$$(P_{\omega,wt}^{\pm} - (1 \pm \beta) \cdot P_{f,wt}) \cdot z_{\omega} = 0 \quad (32)$$

$$(TC - \bar{C}) \cdot z_{\omega} \leq 0 \quad (33)$$

$$\sum_{\omega} \pi_{\omega} \cdot (1 - z_{\omega}) \leq \eta \quad (34)$$

$$(2)-(25) \quad (35)$$

Chance constraint (30) is substituted by the nonlinear constraint (32) and (33). The binary variable z_{ω} is applied to specify the constraints (32) and (33) for the scenario ω is satisfied or not. Once $z_{\omega} = 0$, constraints (32) and (33) is ignored. Otherwise, the whole constraints that are related to scenario ω must be satisfied. To limit the number of partially satisfied scenarios, their aggregate probability has been constrained by (34), wherein the probability of scenario ω is denoted by π_{ω} .

3.5. Linearization of IBRCC Formulation

It should be noted that (32) and (33) are nonlinear constraints since they have multiplications of z_{ω} and $P_{\omega,wt}^{\pm}$, β and TC , accordingly, (30) – (35) is in a mixed integer nonlinear programming (MINLP) form. Solving such a problem, within the available timeframe, is beyond existing computational capabilities. To overcome the nonlinearity of these constraints, here, a “big M” technique is used. Consequently, the IBRCC model can be formulated with a set of linear constraints as follows:

$$\begin{cases} (-M(1 - z_{\omega}) + (1 \pm \beta) \cdot P_{f,wt}) \leq P_{\omega,wt}^{\pm} \\ (M(1 - z_{\omega}) + (1 \pm \beta) \cdot P_{f,wt}) \geq P_{\omega,wt}^{\pm} \end{cases} \quad (36)$$

$$TC - \bar{C} - M \cdot (1 - z_{\omega}) \leq 0 \quad (37)$$

where, M is a sufficiently large number. The constraints (32) and (33) can be substituted by the linear constraints of (36) and (37) that are controlled by z_{ω} . Once $z_{\omega} = 1$, these constraints must be satisfied. Once $z_{\omega} = 0$, they can be ignored due to the big M.

4. Solution Approach

To solve the proposed DRP allocation problem based on IBRCC approach, a procedure is presented as follows. Firstly, to allocate DRPs to the effective buses, a technique based on the SFM is presented. Then, a decomposition algorithm is presented to solve the optimization problem of the DRP allocation.

4.1. Determining effective buses for DRP

The number of load buses to applying DRP for a large scale power system in the real world is limited due to reliability issues and computational burden (resulting from binary variables). Consequently, to specify the candidate buses, the

SFM can be implemented in the proposed DRP allocation problem. According to the SFM method, the sensitivities of the congested line flows (e.g., lines with loading over 98% of their nominal rating) to the changes of power injection in the load buses. The DRP at these buses is capable of increasing the contribution of low cost generating units, e.g., WEGs, by releasing the ATC of the congested lines. Indeed, the SFM demonstrates how the power flow through the line will change if the injection at the bus is changed by one MW. Based on this definition, suppose that we want to calculate the shift factor for line k (m, n) due to one MW injection at bus b . Accordingly, the SFM is calculated as follows:

$$\begin{aligned} n &\rightarrow [\mathbf{M}] = \begin{bmatrix} B_{nn} & \mathbf{L} & B_{nn} \\ \mathbf{M} & \mathbf{O} & \mathbf{M} \\ B_{mn} & \mathbf{L} & B_{mn} \end{bmatrix} \begin{bmatrix} \theta_n \\ \theta_m \\ \theta_n \end{bmatrix} \\ b &\rightarrow 1 \\ m &\rightarrow [\mathbf{M}] \end{aligned} \quad (38)$$

By solving (38), we get θ_n and θ_m . Then based on (39), we calculate the $\Delta SF_{k,b}$.

$$\Delta SF_{k,b} = B_{nm} \cdot (\theta_n - \theta_m), \forall k(n, m) \quad (39)$$

Finally, to evaluate the influence of injection at the bus b on the congested line flows, an SFI_b index, (40), is considered.

$$SFI_b = \sum_{k \in K^*} \Delta SF_{k,b} \quad (40)$$

The set of congested lines are shown by K^* in (40) comprising all fully congested and nearly congested lines. The set $\{SFI_b, b \in B^*\}$ which includes load bus candidates are ranked ascendingly. Noted that if the value of SFI_b index is comparatively low (with a negative value), it shows that the changing the power at load bus b can efficiently reduce the power flow in the congested line k for $k \in K^*$. Consequently, this load bus can be considered as an appropriate choice for the DRP.

4.2. Decomposition algorithm

The MILP formulation presented for IBRCC model costs high computational burden to solve even in small size test systems with the reduced number of scenarios to characterize DUs. To accelerate the solution algorithm, a decomposition approach is adopted to solve the proposed IBRCC optimization problem. The implemented decomposition procedure includes a master problem (MP) and two subproblems I and II, named SP I and II, respectively, as detailed in the following subsections.

MP: The MP is to maximize the objective function (31) with respect to the constraints (6) – (16), (21) – (25), (34), (36) – (37), (41) – (42) and feasibility cuts, without the grid security constraints.

$$\sum_g P_{gt} + \sum_w P_{f,wt} = \sum_n D_{nt} \quad (41)$$

$$\sum_g P_{\omega,gt}^{\pm} + \sum_w P_{\omega,wt}^{\pm} = \sum_n (D_{\omega,nt}^{\pm} - LC_{\omega,nt}^{\pm}) \quad (42)$$

SP I: According to the optimal solution in MP, the grid feasibility check for each hour in the base case (without contingencies) is executed by minimizing the objective function (43) subject to the constraints (44) to (46).

$$\mathfrak{F}_t = \sum_n (\aleph_{1,nt} + \aleph_{2,nt}) \quad (43)$$

$$\begin{aligned} \sum_{g(n)} \hat{P}_{gt} + \sum_{w(n)} P_{f,wt} - \sum_{k(n,m)} P_{kt} + \sum_{k(m,n)} P_{kt} \\ = D_{nt}^f - D\hat{R}_{nt} + \aleph_{1,nt} - \aleph_{2,nt} \end{aligned} \quad (44)$$

$$P_{gt} = \hat{P}_{gt} : \kappa_{1,gt}, u_{gt} = \hat{u}_{gt} : \kappa_{2,gt}, DR_{nt} = D\hat{R}_{nt} : \kappa_{3,nt} \quad (45)$$

$$(2)-(5) \quad (46)$$

where $\aleph_{1,nt}$ and $\aleph_{2,nt}$ are surplus and deficit dummy variables of bus power mismatch in the constraint (44), respectively. In constraint (45), the values of some decision variables are fixed according to the results of the MP. Besides, the dual variables of the complicating variables $\{P_{gt}, u_{gt}, DR_{nt}\}$ are defined by $\{\kappa_{1,gt}, \kappa_{2,gt}, \kappa_{3,nt}\}$, respectively. Constraint (46) includes additional constraints of the base case. In (44), the power mismatch should be minimized. According to the value of the power mismatch in (44), in the case of alleviating the pre specified threshold, a related feasibility cut (47) is performed and augmented in the MP.

$$\begin{aligned} \hat{\mathfrak{F}}_t + \sum_g (\kappa_{1,gt} \cdot (P_{gt} - \hat{P}_{gt}) + \kappa_{2,gt} \cdot (u_{gt} - \hat{u}_{gt})) \\ + \sum_g \kappa_{3,nt} \cdot (DR_{nt} - D\hat{R}_{nt}) \leq 0 \end{aligned} \quad (47)$$

SP II: The SP II for CUs, \pm , and DU, ω , and time interval t is formulated as follows:

$$\mathfrak{F}_{\omega,t}^{\pm} = \sum_n (\aleph_{1,\omega,nt}^{\pm} + \aleph_{2,\omega,nt}^{\pm}) \quad (48)$$

$$\begin{aligned} \sum_{g(n)} P_{\omega,gt}^{\pm} + \sum_{w(n)} ((1 \pm \hat{\beta}) \cdot P_{f,wt}) - \sum_{k(n,m)} P_{\omega,kt} \cdot uc_{k,\omega} \\ + \sum_{k(m,n)} P_{\omega,kt} \cdot uc_{k,\omega} = D_{nt}^{\pm} - LC_{\omega,nt}^{\pm} + \aleph_{1,\omega,nt}^{\pm} - \aleph_{2,\omega,nt}^{\pm} \end{aligned} \quad (49)$$

$$\begin{cases} P_{gt} = \hat{P}_{gt} : \kappa_{1,\omega,gt}^{\pm}, u_{gt} = \hat{u}_{gt} : \kappa_{2,\omega,gt}^{\pm} \\ DR_{nt} = D\hat{R}_{nt} : \kappa_{3,\omega,nt}^{\pm}, \beta = \hat{\beta} : \kappa_{4,\omega}^{\pm} \end{cases} \quad (50)$$

$$(18)-(25) \quad (51)$$

$$\begin{aligned} \hat{\mathfrak{F}}_{\omega,t}^{\pm} + \sum_g \pi_{\omega} \cdot (\kappa_{1,\omega,gt}^{\pm} \cdot (P_{gt} - \hat{P}_{gt}) + \kappa_{2,\omega,gt}^{\pm} \cdot (u_{gt} - \hat{u}_{gt})) \\ + \sum_g \pi_{\omega} \kappa_{3,\omega,nt}^{\pm} \cdot (DR_{nt} - D\hat{R}_{nt}) + \sum_g \pi_{\omega} \kappa_{4,\omega}^{\pm} \cdot (\beta - \hat{\beta}) - M \cdot (1 - z_{\omega}) \leq 0 \end{aligned} \quad (52)$$

The objective function (48) checks possible violations of the MP solution for the CUs and DUs. Constraint (49) is similar to (44). The values of the complicating variables in (50) are fixed to the values achieved by the MP. Constraint (51) refers to the additional constraints of the second stage formulations. Finally, a feasibility cut (52) will be generated and augmented in the MP for the cases with the larger values for the objective function (48) with respect to the pre specified threshold. In the feasibility cut (52), z_{ω} is the same binary variable defined in (36) – (37). Unlike the feasibility cuts in SP I, the binary indicator variable z_{ω} is modulated in the feasibility cuts of SP II. Once $z_{\omega} = 1$, the feasibility cut (52) is converted to a trivial constraint, that is, this constraint is deactivated in the MP for the scenario ω . But, once $z_{\omega} = 0$, it is enforced to the MP. Finally, as illustrated in Fig. 1, the proposed solution algorithm is as following steps:

Step I: The proposed DRP problem based on IBRCC approach with $ud_{nt} = 0$ and $IT=0$ (IT is a dummy numerator) is solved by the decomposition algorithm to find the set K^* consisting of all congested lines.

Step II: Equation (38) – (40) is solved to obtain set B^* or priority list based on the ranking of the SFI_b index, (40), that is, the priority list consisting of all load buses with their ranking values is specified to be employed for DRP. Accordingly, the binary variables ud_{nt} for effective buses $n \in B^*$ with the higher values of SFI_n are fixed to one and other buses with low value fixed to zero. Besides, after solving this step, the IT is set to 1 to stop the Step II.

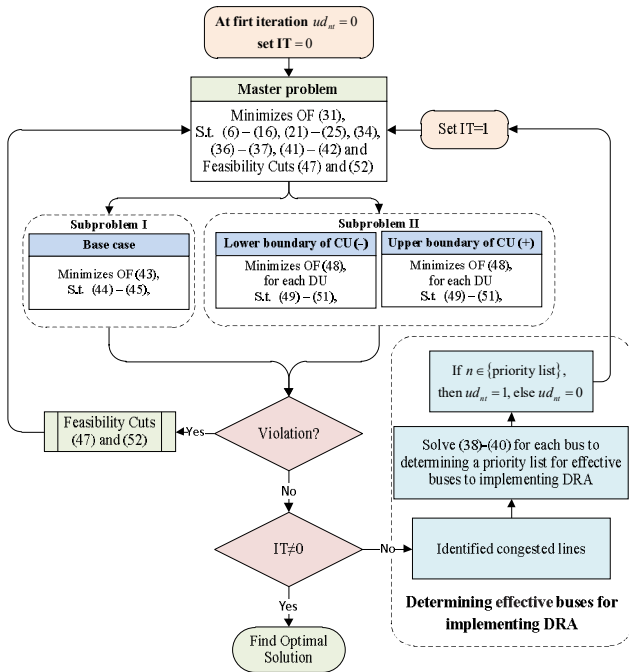


Fig. 1. Flowchart of the proposed solution strategy.

Step III: The proposed DRP allocation problem considering the candidate effective load buses is solved by the proposed decomposition approach to find the optimal β value and the scheduling of the generating units. Then the process will be stopped.

5. Case Studies and Discussion

The proposed DRP allocation problem using IBRCC approach has been tested on the single area IEEE Reliability Test System (RTS) [21]. The RTS system contains 24 buses, 32 generators, 3 wind farms and 38 transmission lines [21]. There are 3 geographically dispersed wind farms, at buses 7, 11 and 24. The wind power output profile of the wind farms located at buses 7, 11 and 24 follows the similar patterns for the forecasted wind generation depicted in Fig. 2, which are scaled by factors of 2.5, 1 and 2, respectively. Also, the total daily load forecast has been given in Fig. 2. Besides, to have better elaborations of the results, some characteristics of the RTS system have been changed as detailed in [22].

Using the modified test system is a common action in previous studies [1] and [5]. In addition, the capacity limits of some transmission lines have been reduced to 175 MW to impose congestion in the grid for simulation purposes.

In this paper CPLEX 12.3 in GAMS 25 software has been used to solve the proposed formulations. The proposed model is executed on a Windows based personal computer with Intel Core i7 7700k 4.20 GHz and 16 GB RAM.

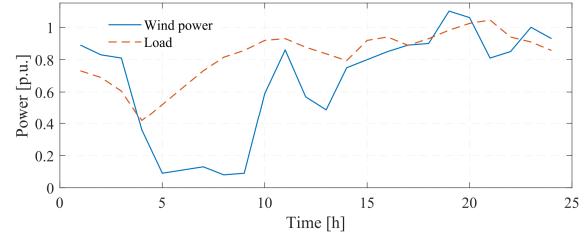


Fig. 2. Daily system's wind power and load curve

The following 3 cases are investigated on this test system:

A. Performance Evaluation of IBRCC: To better illustrate the operation results of the IBRCC problem, i.e., (1) (25), firstly, both uncertainty sources, CUs and DUs, have been ignored in the problem by setting $\lambda = 0$, that resulted in $TC = \$5425087$ which determines the desired cost threshold level for the IBRCC computation. The optimal scheduling of the generating units, in the base case by solving the problem (1) (25) is given in Table I. Noted that this scheduling of units is taken as a reference for the purpose of comparison. Here, for the IBRCC without DRP, the parameters are set by risk tolerance level $\eta = 0.05$ and the number of buses that are allowed to participate in the DRPs is $\lambda = 0$. Also, the cost threshold limit is set as $\bar{c} = \$5425087 \times 1.2$. Accordingly, the robustness level (β) has been maximized by the IBRCC model without DRP for different number of DUs (or Θ_{DU}) as reported in Table II. As shown in this table, the β value without DUs, $\Theta_{DU} = 0$, is 0.062, which is taken as a baseline for the comparison. At this condition, the power flows of transmission lines 7 (3-24), 11 (7-8) and 38 (21-22) reach to their capacity limits. It is noted that the transmission congestion can be worsened when the wind uncertainty is high and consequently the wind power generation may not be fully absorbed by the system. For example, with the higher generated power by the wind farm installed in bus 7, the power flow of the connected line to bus 7 is increased, i.e., line 11 (7-8). Accordingly, the power flowing through line 11 is reached to its limit, consequently, the power grid cannot fully absorb the generation. To decrease the power flow of this line, one possible option is to turn off the available online units 10 and 11 at this bus. As seen in Table I, units 10 and 11 in off peak hour (h4) have been turned off. Another issue, the transmission system congestion can be worsened when DUs and critical CUs occur simultaneously. In this situation, to enhance the wind generation absorption and increase β value, two options are available: starting new fast ramping units as a technical solution and/or relaxing some critical DUs in the computational side with some risks. In this regard, as shown in Table I, at (off) peak hours the hydro units 24 29 (with high power ramp rate) are committed to cover more wind uncertainty and increase the robustness level with the same β value. Furthermore, as described before, some of DUs in the proposed IBRA

problem may tighten the feasible region of the optimization solution space and increase the probability of the problem infeasibility. Here, these kinds of DUs are named critical DUs. To overcome this issue, the constraints pertained to DUs have been modeled as the chance constraints to relax the critical DUs in the solution procedure. This fact has been shown in Table II, as the number of DUs, Θ_{DU} , in IBRA problem is increased, the β value decreases and consequently the problem solution become infeasible, i.e., for $\Theta_{DU} = 12$. But, as can be seen in Table II, the IBRCC approach for $\Theta_{DU} = 20$ is feasible and the β value is 0.039. This is because of the fact that the critical DUs are not relaxed in IBRA, as a result, it imposes more limitations on some constraints, i.e., (27) and (28). Moreover, as shown in Fig. 3, the β value for the IBRCC approach without implementing DRP ($\lambda = 0$ and $\eta = 0.12$) is 0.028. That is that for $\beta > 0.028$, the IBRCC approach without DRP becomes infeasible.

TABLE I: Optimal power schedule of generation units in different cases (in p.u.).

Unit	Base case ($\beta=0, \eta=0, \lambda=0$)		IBRCC ($\beta=0.0057, \eta=0.05, \lambda=0$)		IBRCC ($\beta=0.071, \eta=0.05, \lambda=1$)	
	Off peak (h4)	Peak (h21)	Off Peak (h4)	Peak (h21)	Off peak (h4)	Peak (h21)
1	0.20	0.20	0.00	0.20	0.00	0.20
2	0.20	0.20	0.00	0.20	0.00	0.20
3	0.00	0.76	0.30	0.76	0.45	0.76
4	0.15	0.76	0.15	0.76	0.15	0.76
5	0.20	0.20	0.00	0.20	0.00	0.20
6	0.20	0.20	0.00	0.20	0.00	0.20
7	0.00	0.76	0.15	0.76	0.47	0.76
8	0.00	0.76	0.76	0.76	0.15	0.76
9	0.00	0.00	0.25	0.00	0.00	0.00
10	1.00	0.00	0.00	0.00	0.00	0.00
11	0.37	1.00	0.00	1.00	0.00	0.87
12	1.97	1.97	0.00	1.97	0.00	1.97
13	1.57	1.97	0.69	1.97	0.00	1.97
14	1.97	1.97	0.00	1.97	0.69	1.97
15	0.12	0.12	0.00	0.00	0.00	0.00
16	0.12	0.12	0.00	0.00	0.00	0.00
17	0.12	0.12	0.00	0.00	0.00	0.00
18	0.12	0.12	0.00	0.00	0.00	0.00
19	0.12	0.12	0.00	0.00	0.00	0.00
20	0.54	1.55	0.00	0.00	0.00	0.54
21	0.54	1.55	0.00	0.00	0.00	0.00
22	0.00	0.00	0.00	0.00	0.00	0.00
23	0.00	3.23	1.74	3.93	2.12	3.52
24	0.00	0.00	0.50	0.50	0.50	0.50
25	0.00	0.00	0.50	0.50	0.50	0.50
26	0.00	0.00	0.50	0.50	0.50	0.50
27	0.00	0.00	0.50	0.50	0.50	0.50
28	0.00	0.00	0.50	0.50	0.50	0.50
29	0.00	0.00	0.50	0.50	0.50	0.50
30	0.00	1.55	0.54	1.55	0.54	1.55
31	0.00	1.55	0.54	1.55	0.54	1.55
32	0.00	3.50	1.40	3.50	1.40	3.50

TABLE II: Results of IBRCC

Θ_{DU}	IBRCC		IBRA	
	β	Time [sec]	β	Time [sec]
0	0.062	5	0.062	5
3	0.054	9	0.025	10
6	0.051	10	0.0032	40
12	0.044	16	infeasible	>500
20	0.039	24	infeasible	>500

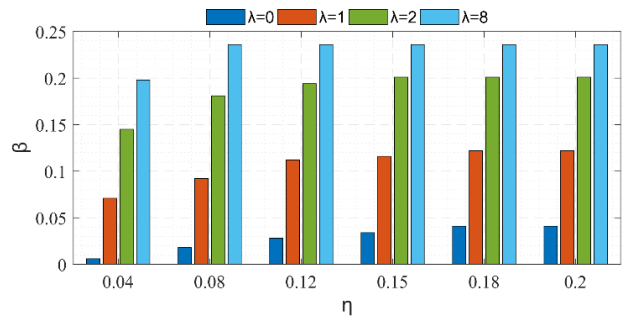


Fig. 3. The β value in IBRCC approach under different λ value.

Also, for $\beta > 0.028$, the security constraints have some limitations on power flowing through lines 7, 11 and 38 (in particular, through the line 11 connected to the bus 7). In this condition, by implementing the DRP at bus 7, as expected the power flowing through line 11 is improved, because DRP can manage wind power injection at bus 7 and reduce power flow through line 7 by reducing the load at peak hours. Similarly, the implementation of DRP will result in an analogous behavior at peak load (hour 21), however, the power dispatch of unit 11 located at bus 7 is decreased by 13%. Therefore, the β value has been improved by 83.92%, (improved from 0.028 to 0.112), as observed in Fig. 3. Furthermore, it can be inferred from Fig. 3 that increasing the number of candidates for effective buses, λ , to implement DRP, will increase the β value, but their impact would be diminished. Moreover, the following observations can be concluded from Fig. 3:

- Selecting one effective bus ($\lambda = 1$) for implementing DRP has more impact on the increase of β value than selecting two or more candidate buses for DRP.
- The β value is increased as η changed from 0.04 to 0.2, indicating higher robustness against CUs. However, the rate of increment is decreased in the IBRCC approach with (without) DRP. This statement approves that a higher value of η causes a more relaxation over DUs, thus, transmission system congestion can be much more reduced leading to more possibility for finding operation solution to handle the short term wind uncertainty or to increase β value more.
- Increasing the value of η will result in widening the budget limit. That is, the critical DUs will increase the probability of violating the budget limit in this condition. Accordingly, more operational constraints related to critical DUs can be ignored in our IBRCC approach with (without) DRP, which allows us to allocate more resources to maximize our capacity in handling the wind uncertainty. For example, when η (for $\lambda = 8$) is higher than 0.04, all of the critical DUs are relaxed in the problem, that is, there is no transmission congestion, thus, the β value remains at 0.236. This intuition is confirmed by the illustrated numerical results in Fig. 3.

B. Comparison performance of IBRCC and IBRA: Here, we set $\lambda = 0$ and $\eta = 0.05$ to compare the performances of IBRA and IBRCC models under a different number of DUs settings, i.e., Θ_{DU} . The results of this comparison can be found in Table II. From Table II, the following remarks can be inferred:

- First, it is observed that once $\Theta_{DU} = 0$, the obtained β value by the IBRA and IBRCC are the same, i.e., $\beta =$

0.062. This is rational because when $\Theta_{DU} = 0$, the chance constraint in IBRCC is discarded. However, for $\Theta_{DU} = 3$, the IBRA is feasible and the β value for this approach is 0.025. But, the β value for IBRA has been reduced more with respect to IBRCC approach, while in the IBRCC, the critical DUs have been relaxed. Moreover, as Θ_{DU} increases, the β value for these two approaches decreases and for IBRA this value has much more change.

- The solution of the IBRCC approach is always feasible under different Θ_{DU} , but for IBRA is not. For example, for $\Theta_{DU} > 6$, the IBRA solution become infeasible. This is because in the IBRA, the security constraints related to critical DUs cannot be relaxed in the solution process.
- As the number of Θ_{DU} increases, the solution time in both approaches is increased. But, in IBRA, the computational time increases significantly once the number of Θ_{DU} increases, i.e., especially for $\Theta_{DU} > 6$. Nevertheless, when $\Theta_{DU} > 10$, the IBRA fails to be feasible. In contrast, the IBRCC approach reveals an incredible capability to solve the problem successfully in a reasonable solution time, i.e., 16 Sec, compared to IBRA, i.e., more than 500 Sec.

These results verify that the proposed IBRCC approach have a computational efficiency to effectively handle the critical DUs as compared to the IBRA, particularly once the number of DUs (or Θ_{DU} value) is high.

C. DRP Allocation based on SFM: The computational efficiency of the SFM to determine the candidate buses for implementing DRP is verified in comparison with DRP allocation problem using the simple strategy of per selection of λ for (16). For the base case, the lines with the numbers 7, 11 and 38 are the congested lines at the hour 21. Therefore, the effective buses would be determined based on the sensitivity of all the congested line flows to the load buses injection are calculated as explained in Section IV. The candidate buses for different values of λ have been determined in Table III. These buses, in Table III, have the most negative amount of $\Delta SF_{k,b}$ at the critical hour (h21), that is, they have the most effects on decreasing the power flow in the congested lines, so, implementing DRP on these buses will increase the β value. In Table III, the SFM does not outperform the pre-selected λ in (16) from optimality point of view, i.e., β value. Indeed, finding best candidate buses based on the pre-selected λ value in (16) always reveals similar or better values for β . However, the difference between the obtained results of two approaches is negligible. When $\lambda < 4$, both approaches result in the same solution. The worst case occurs at $\lambda = 8$, where the difference is 0.02%. Also, in Table III, once $\lambda < 2$, both of them have the same results but SFM executes the problem so faster by 95.62% and once $\lambda > 4$, only SFM computational time. For instance, for $\lambda = 8$, the SFM and pre-selected λ approaches find the optimal solution in 15 and 90 seconds, respectively. However, as the value of the λ is increased, the computational time of the DRP problem considerably increases, while the speed performance of the SFM is still reasonable. Indeed, this considerable increase in the computational burden of the pre-selected λ method is caused by the increased number of binary variables that should be handled by the DRP problem based on IBRCC approach. However in the SFM, these binary variables are

pre-selected before the solution procedure based on the can reach to the optimal solution with a reasonable mentioned approach in Section IV. Accordingly, the SFM is capable of handling the large scale problems to explore the optimal solutions with the cost of reasonable computational burden.

5.1. Modified IEEE 118-Bus system

Case studies in a larger scale system are performed to show the effectiveness of the proposed IBRCC model and to validate the application of the proposed method in the real system operation. The modified IEEE-118 bus system has 54 thermal units, 186 branches, and 91 load buses. The parameters of generators, transmission network, and load profiles are given in [23]. However, the line flow limits for a few lines are reduced to 100 MW to enforce the system congestion in the simulations. There are 3 geographically dispersed WPG at buses 30, 48, and 96. The WPG capacity is 1200 MW. The percentage of hourly load system is the same as the previous six-bus system where the peak demand is 3733.07 MW at hour 17.

Table III: Solution performance of different approaches

λ	Proposed SFM			Pre-selected λ for (16)		
	Bus	β	time [sec]	Bus	β	time [sec]
1	7	0.074	<15	7	0.074	5
2	7,14	0.147		7,14	0.147	10
4	1,3,7,14	0.201		2,3,7,14	0.203	25
8	1,2,6,7,8,10,13,14	0.202		1,3,6,7,8,10,13,14	0.203	90

Table IV: Results of IBRCC in IEEE 118-Bus system

Θ_{DU}	IBRCC		IBRA	
	β	Time [sec]	β	Time [sec]
0	0.093	20	0.093	20
10	0.078	90	0.042	397
20	0.052	150	infeasible	>1000
40	0.034	190	infeasible	>1000

Table V: Solution performance of different approaches in IEEE 118-Bus system

λ	Proposed SFM			Pre-selected λ for (16)		
	Bus	β	time [sec]	Bus	β	time [sec]
1	48	0.085	40	48	0.085	100
5	17,46,48,82,96	0.123	55	17,46,48,82,96	0.124	220
10	17,18,46,48,49,80,82,96,94,95	0.198	62	17,45,46,48,80,82,96,93,95	0.199	440

The installed WPG is 32.14% of the system peak demand. The wind output profile of the WPG units follows the same pattern as that of the six-bus system. The robustness level (β) has been maximized by the IBRCC and IBRA models without DRP for different numbers of DUs (or Θ_{DU}) as reported in Table IV. As shown in Table IV, the β value for both models is same in the case $\Theta_{DU} = 0$, because, in this situation, the IBRCC model and IBRA models are the same. Also, as shown in Table IV, by increasing Θ_{DU} , the β value for both models have been decreased. Also, as shown in Table IV, when Θ_{DU} in the IBRA problem is increased, the β value has been decreased and accordingly the problem solution has become infeasible, for instance in $\Theta_{DU} = 18$. However, the IBRCC approach for $\Theta_{DU} = 20$ is feasible and the β value is 0.052. On the other hand, as the number of Θ_{DU} increases, the solution time in both approaches is increased. But, for the IBRA, the computational time increases significantly once the number of scenarios

increases specially for a number of scenarios above 10. Nevertheless, the IBRCC model reveals an incredible capability to solve the problem successfully in a reasonable solution time, i.e., 90 sec, compared to IBRA model with more than 397 sec to reach optimal result.

As shown in Table V, similar to the pervious test system, when λ value are increased, the computational time of the DRP problem significantly increases, but the computational time of the SFM is still reasonable. For example, for $\lambda = 10$, the SFM and pre-selected λ approaches find the optimal solution in 20 and 440 seconds, respectively while the difference between the obtained results of two approaches is negligible.

6. Conclusions

In this paper, a DRP allocation problem based on an interval based robust chance constrained model was proposed to manage the CUs and DUs, simultaneously. This model was formulated by means of combining the interval based robust optimization and chance constrained optimization formulation to control the wind uncertainty and stochastic behavior of random outages of network equipment (generators and transmission lines) subject to the probabilistic system security constraints. The proposed IBRCC approach has taken the advantages of both robust and chance constrained uncertainty modeling mechanisms. Also, in this paper, the DRP allocation, as a powerful uncertainty management tool for CUs and DUs, has been implemented to determine most effective candidate load buses in the proposed model. Accordingly, a new approach was presented based on the SFM. In the proposed approach, the effective buses to apply DRPs were determined. By the proposed IBRCC approach, the robustness level of DRP problem and the value of β were increased significantly, and consequently the robustness performance was improved. The proposed IBRCC approach could be solved efficiently by our proposed decomposition strategy framework.

Further work is underway to establish an uncertainty model that determines acceptable uncertainty variation range during different hours. Also, our proposed approach always faces the challenges on its over conservatism, due to its objective function of maximizing the worst-case uncertainty. To address this issue, we are going to apply the asymmetrical approach in our future work.

7. Acknowledgment

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