Stochastic Mathematical Model for High Penetration of Renewable Energy Sources in Distribution Systems

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***Abstract*—The integration of renewable energy sources (RESs) cannot be postponed given the several techno-economic and environmental factors. However, the intermittent nature of RESs brings several challenges because such sources introduce significant operational variability and uncertainty in the system. Framed in this context a new stochastic mathematical planning model is proposed considering the placement and sizing of energy storage systems and reactive power sources that altogether enable high penetration of RESs. The model employs a linearized AC model, balancing well accuracy with computational burden. The model is tested on a standard distribution network system. Numerical results show that the simultaneous integration of energy storage systems and reactive power sources largely leads to a substantially increased penetration of RESs in distribution systems. Furthermore, dramatic reductions in losses, system costs and emissions are observed. Results analysis also reveal the positive contribution of such a planning on the overall voltage stability in the system.**

***Index Terms*—Distributed generation, energy storage systems, reactive power support, renewable energy sources, stochastic programming, variability and uncertainty**

I. NOMENCLATURE

A. Sets/Indices

t/Ω^t	Index/set of time stages
$g/\Omega^g/\Omega^{DG}$	Index/set of DGs
i/Ω^i	Index/set of buses
es/Ω^{es}	Index/set of energy storages
c/Ω^c	Index/set of capacitor banks
s/Ω^s	Index/set of scenarios
w/Ω^w	Index/set of snapshots
$\varsigma/\Omega^{\varsigma}$	Index/set of substations
k/Ω^{ℓ}	Index/set of branches

B. Parameters

$IC_{g,i}, IC_k, IC_{es,i}, IC_{c,i}$	Investment cost of DG, line, energy storage system and capacitor banks, respectively
$LT_g, LT_k, LT_{es}, LT_c$	Lifetimes of DG, line, energy storage system and capacitor banks, respectively
MC_g^N, MC_g^E	Maintenance costs of new and existing DGs per year
MC_k^N, MC_k^E	Maintenance costs of new and existing branch k per year
MC_c, MC_{es}	Maintenance cost of capacitor bank and energy storage system per year
ρ_s, π_w	Probability of scenario s and weight (in hours) of snapshot group w
$OC_{g,i,s,w,t}^N, OC_{g,i,s,w,t}^E$	Cost of unit energy production by new and existing DGs
$\gamma_{es,i,s,w,t}^{dch}$	Cost of energy discharged from storage system
$\sigma_{\varsigma,s,w,t}$	Price of electricity purchased from upstream
$\upsilon_{s,w,t}$	Penalty for unserved power
$\lambda_{s,w,t}^{CO_2e}$	Price of emissions (€/tons of CO_2 equivalent—€/tCO_2e)
$ER_g^N, ER_g^E, ER_{\varsigma}^{SS}$	Emission rates of new and existing DGs, and energy purchased at sub-stations, respectively
g_k, b_k, S_k^{max}	Conductance, susceptance and flow limit of branch k
MP_k, MQ_k	Big-M parameters associated to active and reactive power flows through link k, respectively
α_l, β_l	Slopes of linear segments
L	Total number of linear segments
$P_{es,i}^{ch,max}, P_{es,i}^{dch,max}$	Charging and discharging power limits of a storage system
$\eta_{ch,es}, \eta_{dch,es}$	Charging and discharging efficiencies of a storage system
$E_{es,i}^{min}, E_{es,i}^{max}$	Energy storage limits
Q_c^0	Rating of minimum capacitor bank
N_i, N_{SS}	Number of buses and substations, respectively

C. Variables

$u_{g,i,t}, u_{k,t}$	Utilization variables of existing DG and lines
$P_{g,i,s,w,t}^N, P_{g,i,s,w,t}^E$	Active power produced by new and existing DGs
$Q_{g,i,s,w,t}^N, Q_{g,i,s,w,t}^E$	Reactive power produced by new and existing DGs
$P_{\varsigma,s,w,t}^{SS}, Q_{\varsigma,s,w,t}^{SS}$	Active and reactive power imported from grid (upstream)
$Q_{c,i,s,w,t}$	Reactive power injected by capacitor bank at node i
$\delta_{i,s,w,t}$	Unserved power at node i
P_k, Q_k, θ_k	Active and reactive power flows, and voltage angle difference of link k, respectively.
V_i, V_j	Voltage magnitudes at nodes i and j
$p_{k,s,w,t,l}, q_{k,s,w,t,l}$	Step variables used in linearization of quadratic flows
PL_k, QL_k	Active and reactive power losses, respectively
$P_{es,i,s,w,t}^{ch}, P_{es,i,s,w,t}^{dch}$	Power charged to and discharged from storage system

$D_{i,s,w,t}$, $Q_{i,s,w,t}$ Active and reactive power demand at node
$PL_{ς,s,w,t}$, $QL_{ς,s,w,t}$ Active and reactive losses at substation ς
$I^{ch}_{es,i,s,w,t}$, $I^{dch}_{es,i,s,w,t}$ Charge-discharge indicator variables
$E_{es,i,s,w,t}$ Stored energy

D. Functions

$InvC^{DG}_t$, MC^{DG}_t, EC_t NPV investment/maintenance/expected energy cost of DGs, respectively
$InvC^{LN}_t$, MC^{LN}_t NPV investment/maintenance cost of a distribution line
$InvC^{ES}_t$, MC^{ES}_t, EC^E_t NPV investment/maintenance/expected energy cost of an energy storage system, respectively
$InvC^{CAP}_t$, MC^{Cap}_t NPV investment/maintenance cost of capacitor banks
EC^{SS}_t Expected cost of energy purchased from upstream
$ENSC_t$ Expected cost of unserved power
$EmiC^{DG}_t$ Expected emission cost of power production using DG
$EmiC^{SS}_t$ Expected emission cost of purchased power
$EmiC^{N}_t$, $EmiC^{E}_t$ Expected emission cost of power production using new and existing DGs, respectively

II. Introduction

A. Motivation, Aims and Background

The integration of renewable energy sources (RESs) into the electricity system plays a key role in solving issues related to environmental sustainability. The world-level deployment of RESs in energy systems is likely to maintain the increasing trend. However, a large-scale integration of RESs comes with a set of challenges due to their nature, namely variability and uncertainty. Because of this, RES integration requires an adaptation process of existing infrastructures, whether at the level of transmission and distribution, that can result in an improved flexibility in the network system, in general. From the perspective of stability, reliability and energy quality, the integration of RES-based Distributed Generations (DGs) creates several technical challenges. This is because of the significant variability and operational uncertainty added to the system, which makes control, planning and operation more complicated.

In particular, a higher penetration of RESs in distributed systems can result in the degradation of power quality, particularly in cases of slightly meshed networks [1] or microgrids. Therefore, the flexibility of distribution systems is expected to be increased by means of smart-grid technologies, enabling the system to accommodate large scale variable RES power. From this perspective, Energy Storage Systems (ESSs) and reactive power sources can play vital roles. For example, such technologies can considerably enhance the flexibility and improve voltage profiles of the system, thereby increasing chances of accommodating large-scale RES power. The use of ESS will be one possible way to mitigate the concerns mentioned above.

The DG allocation and sizing subject have attracted special interest from researchers in recent years. Several techniques that investigate the DG impact in the power system are presented and analyzed in [2] and [3]. There is an important set of issues related to DG integration that should be taken into account, in particular, the issues of DGs capacity and placement since each node has its optimal integration level, and network topology. Surpassing such a limit may undermine overall system performance such as increased losses [4], [5].

In recent years, several methods have been proposed to address the target problem (i.e. planning and operation of distribution systems, and issues related to DG allocation). These can be generally classified as numerical [6]–[11], analytical [12], [13] and heuristic [14]–[23] methods. Despite the significant number of publications, a coordinated approach of distribution systems planning from the perspective of maximizing variable RES power is not adequately addressed in the literature. The nature of such a problem means the prospect of stochastic programming should be employed.

From this context, the presented work develops an integrated planning model that simultaneously finds the optimal locations and sizes of DGs (particularly, focusing on wind and solar), energy storage systems and capacitor banks. Optimal deployment of such technologies should inherently meet the goal of maximizing renewable power integration and utilization in the system. The entire optimization model is formulated as a stochastic mixed integer linear programming (SMILP). In addition, instead of the customary DC network models, a linearized AC model is used to better capture the physical characteristics of the network system.

B. Contributions and Paper Organization

The main contributions of this work are as follows:

- A multi-stage stochastic optimization model, which integrates smart-grid enabling technologies to support large-scale RESs. The model employs a linearized AC network model which balances well the accuracy with the computational burden.
- A thorough quantitative analysis regarding the impacts of integrated planning on the overall system performance.

The rest is organized as follows. Section III presents a description of the developed mathematical model. Numerical results are discussed in Section IV. The final section concludes this paper.

III. Mathematical Formulation

A. Objective Function

In this work, the sum of Net Present Value (NPV) of relevant cost terms form the main objective function, as in (1), which is to be minimized. The individual cost terms are defined as follows.

Eq. (2) presents the total investment costs, which corresponds to the first term in Eq. (1), considering a perpetual planning horizon as in [24], where the investment is prorated in annual installments throughout the lifetime of the installed component. New investments are possible in feeders, ESSs, DGs and capacitor banks, whose corresponding costs are computed according to Eqs. (7)—(10). Eq. (3) represents the total cost of energy in the system, which is the second term in (1), again assuming a perpetual planning horizon. This is given by the sum of individual maintenance costs of new and existing feeders, ESSs, DGs and capacitor banks, at each stage, and also the maintenance costs of the last planning stage, according to (11)—(14). Eq. (4) presents the total cost of energy in the system, which corresponds to the third term in (1), considering a perpetual planning horizon, according to Eqs. (15)—(17). And, this is given by the sum of costs of power supplied by the new and existing DGs, ESSs and imported power.

It should be noted that the total cost of energy in the system are conditioned by the energy costs of the last planning stage. Eq. (5) presents the total cost of unserved energy in the system, the fourth term in (1), which is computed using Eq. (18). Finally, Eq. (6) refers to the total emission costs in the system, the fifth term in (1), which is determined using Eqs. (19)—(22). This includes the sum of emission costs pertaining to the existing and new DGs and power imported through the substations.

$$Minimize\ TC = InvC + MC + EC + ENSC + ImiC \quad (1)$$

$$TInvC = \underbrace{\sum_{t\epsilon\Omega^t} \frac{(1+r)^{-t}}{r} (InvC_t^{DG} + InvC_t^{LN} + InvC_t^{ES} + InvC_t^{CAP})}_{NPV\ of\ investment\ cost} \quad (2)$$

$$TMC = \underbrace{\sum_{t\epsilon\Omega^t} (1+r)^{-t} \left(MC_t^{DG} + MC_t^{LN} + MC_t^{ES} + MC_t^{Cap}\right)}_{NPV\ of\ maintenance\ costs} + \underbrace{\frac{(1+r)^{-T}}{r} (MC_T^{DG} + MC_T^{LN} + MC_T^{ES} + MC_T^{Cap})}_{NPV\ maintenance\ costs\ incured\ after\ stage\ T} \quad (3)$$

$$TEC = \underbrace{\sum_{t\epsilon\Omega^t} (1+r)^{-t} \ (EC_t^{DG} + EC_t^{ES} + EC_t^{SS})}_{NPV\ of\ operation\ costs} + \underbrace{\frac{(1+r)^{-T}}{r} (EC_T^{DG} + EC_T^{ES} + EC_T^{SS})}_{NPV\ operation\ costs\ incured\ after\ stage\ T} \quad (4)$$

$$TENSC = \underbrace{\sum_{t\epsilon\Omega^t} (1+r)^{-t}\ ENSC_t}_{NPV\ of\ reliability\ costs} + \underbrace{\frac{(1+r)^{-T}}{r} ENSC_T}_{NPV\ relia.\ costs\ incured\ after\ stage\ T} \quad (5)$$

$$TEmiC = \underbrace{\sum_{t\epsilon\Omega^t} (1+r)^{-t}\ (EmiC_t^{DG} + EmiC_t^{SS})}_{NPV\ emission\ costs} + \underbrace{\frac{(1+r)^{-T}}{r} (EmiC_T^{DG} + EmiC_T^{SS})}_{NPV\ emission\ costs\ incured\ after\ stage\ T} \quad (6)$$

$$InvC_t^{DG} = \sum_{g\epsilon\Omega^g} \sum_{i\epsilon\Omega^i} \frac{r(1+r)^{LT_g}}{(1+r)^{LT_g}-1} IC_{g,i} (x_{g,i,t} - x_{g,i,t-1})\ ; where\ x_{g,i,0} = 0 \quad (7)$$

$$InvC_t^{LN} = \sum_{k\epsilon\Omega^\ell} \frac{r(1+r)^{LT_k}}{(1+r)^{LT_k}-1} IC_k (x_{k,t} - x_{k,t-1})\ ;\ where\ x_{k,0} = 0 \quad (8)$$

$$InvC_t^{ES} = \sum_{es\epsilon\Omega^{es}} \sum_{i\epsilon\Omega^i} \frac{r(1+r)^{LT_{es}}}{(1+r)^{LT_{es}}-1} IC_{es} (x_{es,i,t} - x_{es,i,t-1})\ \ ; where\ x_{es,i,0} = 0 \quad (9)$$

$$InvC_t^{CAP} = \sum_{c\epsilon\Omega^c} \sum_{i\epsilon\Omega^i} \frac{r(1+r)^{LT_c}}{(1+r)^{LT_c}-1} IC_c (x_{c,i,t} - x_{c,i,t-1})\ ; where\ x_{c,i,0} = 0 \quad (10)$$

$$MC_t^{DG} = \sum_{g\epsilon\Omega^g} \sum_{i\epsilon\Omega^i} MC_g^N\, x_{g,i,t} + \sum_{g\epsilon\Omega^g} \sum_{i\epsilon\Omega^i} MC_g^E\, u_{g,i,t} \quad (11)$$

$$MC_t^{LN} = \sum_{k\epsilon\Omega^{e\ell}} MC_k^E\, u_{k,t} + \sum_{k\epsilon\Omega^{n\ell}} MC_k^N x_{k,t} \quad (12)$$

$$MC_t^{ES} = \sum_{es\epsilon\Omega^{es}} \sum_{i\epsilon\Omega^i} MC_{es} x_{es,i,t} \quad (13)$$

$$MC_t^{Cap} = \sum_{c\epsilon\Omega^c} \sum_{i\epsilon\Omega^i} MC_c x_{c,i,t} \quad (14)$$

$$EC_t^{DG} = \sum_{s\epsilon\Omega^s} \rho_s \sum_{w\epsilon\Omega^w} \pi_w \sum_{g\epsilon\Omega^g} \sum_{i\epsilon\Omega^i} (OC_{g,i,s,w,t}^N P_{g,i,s,w,t}^N + OC_{g,i,s,w,t}^E\, P_{g,i,s,w,t}^E) \quad (15)$$

$$EC_t^{ES} = \sum_{s\epsilon\Omega^s} \rho_s \sum_{w\epsilon\Omega^w} \pi_w \sum_{c\epsilon\Omega^c} \sum_{i\epsilon\Omega^i} \gamma_{es,i,s,w,t}^{dch} P_{es,i,s,w,t}^{dch} \quad (16)$$

$$EC_t^{SS} = \sum_{s\epsilon\Omega^s} \rho_s \sum_{w\epsilon\Omega^w} \pi_w \sum_{\varsigma,\Omega^\varsigma} \sigma_{\varsigma,s,w,t} P_{\varsigma,s,w,t}^{SS} \quad (17)$$

$$ENSC_t = \sum_{s\epsilon\Omega^s} \rho_s \sum_{w\epsilon\Omega^w} \sum_{i\epsilon\Omega^i} \pi_w \upsilon_{s,w,t} \delta_{i,s,w,t} \quad (18)$$

$$EmiC_t^{DG} = EmiC_t^N + EmiC_t^E \quad (19)$$

$$EmiC_t^N = \sum_{s\epsilon\Omega^s} \rho_s \sum_{w\epsilon\Omega^w} \pi_w \sum_{g\epsilon\Omega^g} \sum_{i\epsilon\Omega^i} \lambda_{s,w,t}^{CO_2e} ER_g^N P_{g,i,s,w,t}^N \quad (20)$$

$$EmiC_t^E = \sum_{s\epsilon\Omega^s} \rho_s \sum_{w\epsilon\Omega^w} \pi_w \sum_{g\epsilon\Omega^g} \sum_{i\epsilon\Omega^i} \lambda_{s,w,t}^{CO_2e} ER_g^E P_{g,i,s,w,t}^E \quad (21)$$

$$EmiC_t^{SS} = \sum_{s\epsilon\Omega^s} \rho_s \sum_{w\epsilon\Omega^w} \pi_w \sum_{\varsigma\epsilon\Omega^\varsigma} \lambda_{s,w,t}^{CO_2e} ER_\varsigma^{SS} P_{\varsigma,s,w,t}^{SS} \quad (22)$$

B. Constraints

As stated earlier, this proposed optimization is based on a linearized AC network model, used in transmission expansion planning problem [25] but adapted here to serve the intended purpose i.e. the extensive analysis related to maximizing RES penetration in distribution systems. Eqs. (23) and (24) represent the linearized active and reactive power flow equations in new lines. And, the flow limit is given by Eqs. (25). Similarly, the flow equations and the corresponding limits in existing lines can be formulated by replacing the branch investment variable with the utilization variable.

$$\left|P_{k,s,w,t} - \{V_{nom}(\Delta V_{i,s,w,t} - \Delta V_{j,s,w,t}) g_k - V_{nom}^2 b_k \theta_{k,s,w,t}\ )\}\right| \le MP_k(1 - x_{k,t}) \quad (23)$$

$$\left|Q_{k,s,w,t} - \{-V_{nom}(\Delta V_{i,s,w,t} - \Delta V_{j,s,w,t}) b_k - V_{nom}^2 g_k \theta_{k,s,w,t}\}\ \right| \le MQ_k(1 - x_{k,t}) \quad (24)$$

$$P_{k,s,w,t}^2 + Q_{k,s,w,t}^2 \le x_{k,t} (S_k^{max})^2 \quad (25)$$

where $\Delta V^{min} \le \Delta V_{i,s,w,t} \le \Delta V^{max}$.

Eq. (25) contains quadratic terms, which are linearized via a piecewise approach, as in (26)—(29), by considering a sufficiently large number of segments, L.

$$P_{k,s,w,t}^2 \approx \sum_{l=1}^{L} \alpha_{k,l} p_{k,s,w,t,l} \quad (26)$$

$$Q_{k,s,w,t}^2 \approx \sum_{l=1}^{L} \beta_{k,l} q_{k,s,w,t,l} \quad (27)$$

$$P_{k,s,w,t}^+ + P_{k,s,w,t}^- = \sum_{l=1}^{L} p_{k,s,w,t,l} \quad (28)$$

$$Q_{k,s,w,t}^+ + Q_{k,s,w,t}^- = \sum_{l=1}^{L} q_{k,s,w,t,l} \quad (29)$$

where $p_{k,s,w,t,l} \le P_k^{max}/L$ and $q_{k,s,w,t,l} \le Q_k^{max}/L$.

Eqs. (30) and (31) refer to the active and reactive power losses in line k, respectively. The active and reactive power balance equations are given by (32) and (33), respectively.

$$PL_{k,s,w,t} = r_k\{P_{k,s,w,t}^2 + Q_{k,s,w,t}^2\}/V_{nom}^2 \quad (30)$$

$$QL_{k,s,w,t} = x_k\{P_{k,s,w,t}^2 + Q_{k,s,w,t}^2\}/V_{nom}^2 \quad (31)$$

$$\sum_{g\epsilon\Omega^{DG}} (P_{g,i,s,w,t}^E + P_{g,i,s,w,t}^N) + P_{\varsigma,s,w,t}^{SS} + \sum_{in,k\epsilon i} P_{k,s,w,t} - \sum_{out,k\epsilon i} P_{k,s,w,t} + \ \delta_{i,s,w,t} = D_{s,w,t}^i + PL_{\varsigma,s,w,t} + \sum_{k\epsilon i} \tfrac{1}{2} PL_{k,s,w,t}\ ;\ \forall\varsigma, \forall\varsigma\epsilon i \quad (32)$$

$$\sum_{g\epsilon\Omega^{DG}} (Q_{g,i,s,w,t}^E + Q_{g,i,s,w,t}^N) + \sum_{c\epsilon\Omega^c} Q_{i,s,w,t}^c + Q_{\varsigma,s,w,t}^{SS} + \sum_{in,k\epsilon i} Q_{k,s,w,t} - \sum_{out,k\epsilon i} Q_{k,s,w,t} = Q_{s,w,t}^i + QL_{\varsigma,s,w,t} + \sum_{in,k\epsilon i} \tfrac{1}{2} QL_{k,s,w,t} + \sum_{out,k\epsilon i} \tfrac{1}{2} QL_{k,s,w,t}\ ;\ \forall\varsigma, \forall\varsigma\epsilon i \quad (33)$$

Inequalities (34) and (35) represent the active and reactive power capacity limits of new generators, respectively. The constraints for existing generators have similar forms, and should be included in the model.

$$P_{g,i,s,w,t}^{N,min} x_{g,i,t} \le P_{g,i,s,w,t}^N \le P_{g,i,s,w,t}^{N,max} x_{g,i,t} \quad (34)$$

$$Q_{g,i,s,w,t}^{N,min} x_{g,i,t} \le Q_{g,i,s,w,t}^N \le Q_{g,i,s,w,t}^{N,max} x_{g,i,t} \quad (35)$$

It should be noted that, for wind and solar PV generators, the upper bound $P_{g,i,s,w,t}^{max}$ should be equal to the actual power production level at a given hour, which in turn depends on the level of primary energy source (wind speed or solar radiation).

The lower bound $P_{g,i,s,w,t}^{max}$ in this case is simply set to zero. Inequalities (36) and (37) model the reactive power support capabilities of DGs. The inequality in (38) ensures that the reactive power injected to the system from the reactive power sources is bounded between 0 and the upper limit.

$$\begin{aligned} -\tan\left(\cos^{-1}(pf_g)\right) * P_{g,i,s,w,t}^{E} \leq Q_{g,i,s,w,t}^{E} \\ \leq \tan(\cos^{-1}(pf_g)) * P_{g,i,s,w,t}^{E} \end{aligned} \tag{36}$$

$$\begin{aligned} -\tan\left(\cos^{-1}(pf_g)\right) * P_{g,i,s,w,t}^{N} \leq Q_{g,i,s,w,t}^{N} \\ \leq \tan(\cos^{-1}(pf_g)) * P_{g,i,s,w,t}^{N} \end{aligned} \tag{37}$$

$$0 \leq Q_{i,s,w,t}^{c} \leq x_{c,i,t} Q_c^0 \tag{38}$$

In addition to the above constraints, the bulk energy storage model in [26] is employed here.

IV. Numerical Results and Discussions

The proposed planning model is tested on a standard radial distribution system, as shown in Fig. 1. This system has 33 buses and 37 lines including the five tie switches. The nominal voltage of this system is 12.66 kV while the active and the reactive power demand are 3.715 MW and 2.3 MVAr, respectively. This demand is assumed to grow by 5% every year. The current flow capacity of each feeder in the thicker section along nodes 1 and 10 is assumed to be 400 A while all other branches including the tie lines have a maximum current carrying capacity of 200 A. The investment cost of feeders is assumed to be proportional to the impedance with a proportionality constant of 10,000 €/Ω.

A bulk ESS with a rated capacity of 1 MW and 5 MWh is considered for investment, and the capital cost of each ESS is 1.0 M€. A 1 MW wind or solar type DG can be installed in any node of the system, and both DG types are assumed to have a reactive power support capability. The corresponding capital costs are 1.06 and 1.2 M€, respectively. Nodal voltages are allowed to deviate from the nominal value up to a maximum value of 5% (in absolute terms). Other assumptions are described as follows: $Q_c^0 = 0.1\, MVAr$; $LT_g = LT_k = 25\, years$; $LT_c = LT_{es} = 15\, years$; $T = 3\, years$; $r = 7\%$; ΔV^{min}; $L = 5$; $IC_c = €25/kVAr$; $pf_g = 0.95$; $pf_{substation} = 0.85$; $ER_{\varsigma}^{SS} = 0.4\, tCo_2/MWh$; $\upsilon_{s,w,t} = 3000\, €/MWh$.

Table I shows the results of investments in DGs. As revealed in this table, only wind type DGs are selected for investment. This is because of the normally higher capacity factor of wind power generation than that of solar power, as is the case in the current case study. Interestingly, a total of 5 MW wind power is integrated in the distribution network system within the three years planning horizon. Put into perspective with the active power demand in the base case (3.715 MW), this is tremendous. All this covers about 59% of the demand throughout the horizon. This invalidates the wide-spread perception and practice of limiting RES penetration in the order of 25% in the interest of maintaining grid stability and power quality. The results here clearly show that a significant amount of variable energy sources can be integrated and operated seamlessly even at distribution levels, and that their side-effects such as voltage rise issues can be adequately alleviated by deploying relatively inexpensive technologies such as capacitor banks. In other words, the reactive power requirement in the system is mainly met by producing it locally via investment in reactive power sources. But the RES type DGs and the substation can also inject some reactive power to the system.

Table II shows the optimal solution of the capacitor bank placement and sizing problem in the considered system. As can be seen in this table, the total capacity of reactive power sources (capacitor banks) installed in the system is 1.6 MVAr. This amounts to nearly 70% of the total reactive power demand in the base case. Such distributed placement of capacitor banks is critically important to maintain system stability by keeping the nodal voltages within the allowable range. Needless to mention, the optimal location of capacitor banks coincide with the nodes featuring high demand for reactive power, as can be inferred from Table II.

Note that majority of the investments are done in the first stage (year). And, this may be because of lack of constraints related to investment planning such as logistical and budget constraints. In addition to the above investment outcomes, a 1 MW ESS is optimally installed at node 29. Having a closer look at the expanded system, the ESS seems to be strategically placed in close proximity to the wind power generators. Generally, the integration of this storage medium also immensely contributes to the stability of the system by reducing the impact of increased fluctuations as a result of the intermittent power sources, in this case, the wind power generation integrated in the system. Apart from the contributions of ESS to system stability, nearly 6% of the energy consumption throughout the investment period is met by discharging the energy stored (mainly from wind) in ESS.

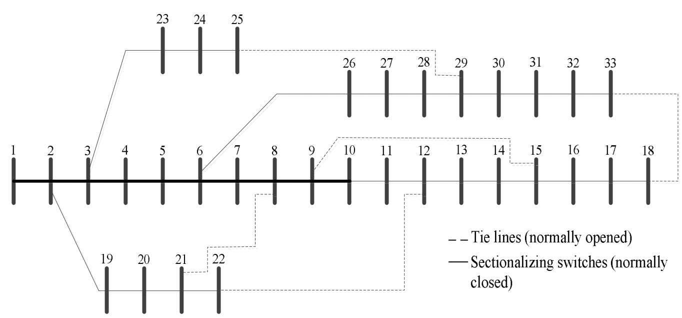

Fig. 1. A standard IEEE 33-bus test system

TABLE I. Optimal Location and Size of DGs

DG Type	Node	Installed capacity (MW)		
		Stage 1	Stage 2	Stage 3
Wind	16	1	0	0
Wind	21	0	0	1
Wind	24	1	0	0
Wind	29	1	0	0
Wind	31	0	1	0

TABLE II. Optimal Location and Size of Capacitor Banks

Node	Installed Capacity (MVAr)	
	Stage 1	Stage 2
4	0.1	0
6	0.1	0
7	0.1	0
8	0.1	0
12	0.1	0
14	0.1	0
23	0	0.1
24	0	0.1
25	0.2	0
30	0.5	0
33	0.1	0

TABLE III. ENERGY BALANCE IN THE SYSTEM

Cases	Generation side				Demand side		
	Discharge	Imported	Wind	Total	Losses	Charge	Demand
Invest case	4713	27871	47318	79902	1337	6131	72434
Base case	0	79903	0	79903	7468	0	72434

TABLE IV. SIMULATION RESULTS OF RELEVANT COST VARIABLES

Stage (Year)	System Cost Breakdown (M€)					Total Cost (M€)
	Investment	Maintenance	Unserved Energy	Emission	Operation	
1	3.668	0.119	0.098	0.116	0.727	
2	2.206	0.161	0.000	0.144	0.599	
3	1.056	2.506	0.000	2.297	8.069	21.766
Sub-total	6.931	2.786	0.098	2.557	9.395	
Base case	0.000	1.073	0.000	9.005	19.224	29.303

This means the ESS and the wind type DGs, combined, cover nearly 65% of the energy consumption in the considered system. The remaining balance (i.e. 35%) is covered by the energy imported via the substation at node 1. The overall energy balance is depicted in Table III.

In practical sense, the performance of the system with and without investments is analyzed by monitoring relevant indicators such as costs, losses and voltage profiles. In this regard, Table IV compares the system costs in the base case (i.e. the do-nothing scenario) and the investment case. It can be observed that, compared to the base case, the system-wide investment planning leads to an approximately 26% reduction in overall cost. This is significant by any standard. The level of emission reduction this results in is dramatic, with a net reduction of over 70% from 1,402,410 to just 414,292 tons of equivalent CO_2 emissions. This is equivalent to over 51% reduction in emission costs (see Table IV).

From the voltage stability perspective, the integrated expansion planning leads to a substantially improved voltage profiles in the system regardless of the operational situations, as shown in Fig. 2. It is evident to see in this figure that the nodal voltages largely stay close to the nominal value. In other words, the absolute voltage deviations remain very close to zero. To appreciate these improvements, the base case scenario (where no investments are made) is solved by relaxing the absolute voltage deviation limits from 5% of the nominal value used in this paper to 10%. Note that this is essentially required to avoid infeasibility in the solution process because, in the base case, it is impractical to expect the voltage to fall within 5% of the nominal voltage.

The voltage profiles corresponding to various operational situations (snapshots) in the base case are illustrated in Fig. 3. We can observe that the bus voltages (particularly, at nodes 7 through 18, and 27 through 33) largely fall outside the commonly acceptable and desired range in distribution networks (i.e. within ±5% range). In contrast, Fig. 2 indicates that the integration of DGs, ESSs and reactive power sources not only solves the voltage sag problems in such nodes but also leads to substantially improved controllability of bus voltages in the considered system. This is another key indicator of the extensive contributions of integrated planning, as proposed in this paper. In Fig. 2, the voltage "hikes" and "sags" between nodes 13 and 18 could be attributed to the weather conditions windy (high wind power production) and calm (low wind power production).

The storage system may be contributing less to offset such fluctuations and enhance voltage stability since it is located a bit far away from these nodes (i.e. node 29). This argument is supported by the fact that the voltages between nodes 25 and 33 (where the ESS is located) have less variability, when subjected to different operational situations, in comparison to those between nodes 25 and 33. Despite these variabilities, the bus voltages largely remain within the allowable range and in most cases close to the nominal voltage. The average voltage profile (see the marked solid line) strengthens this statement. The impact of the planning work on network losses is also astoundingly significant, as shown in Fig. 4. On average, the level of losses reduction is nearly 83% i.e. from about 0.3 to 0.05 MW. In energy terms, this is equivalent to reducing energy losses from 7468 to 1337 MWh within the entire horizon i.e. the three-year time span.

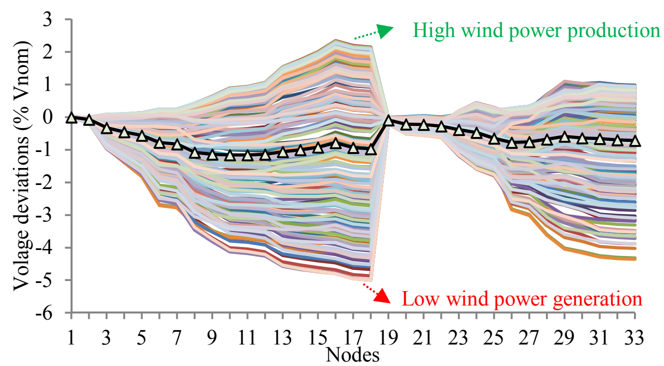

Fig. 2. Average voltage profile of each snapshot after investments. The marked, solid black line refers to the average of the average voltage profiles.

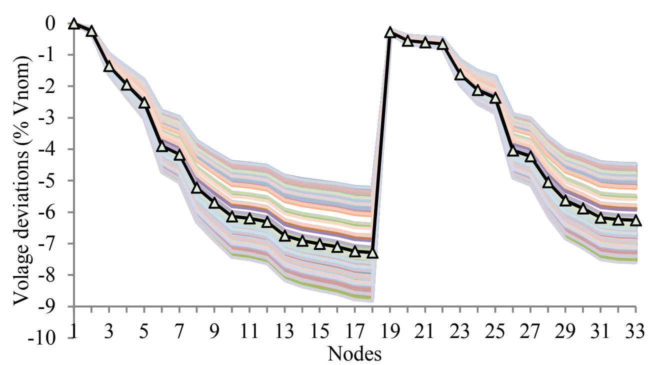

Fig. 3. Average voltage profile of each snapshot in the base case. The marked, solid black line refers to the average of the average voltage profiles.

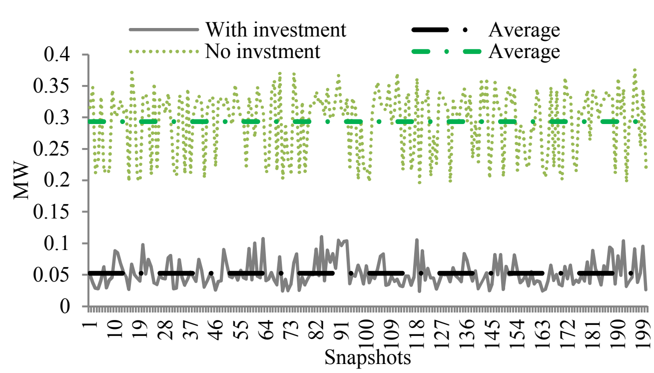

Fig. 4. Comparisons of network loss profiles corresponding to the investment and the base case scenarios.

V. CONCLUSIONS

This paper has proposed a new stochastic optimization model to support the expansion planning of distribution network systems in a coordinated manner. The model considers the most relevant costs terms, and is based on a linearized AC model, which adequately captures the physical characteristics of such network systems. The proposed model is tested on a standard test system. Numerical results show that the simultaneous integration of energy storage systems and reactive power sources largely enables a substantially increased penetration (more than 59%) of variable power sources in the considered system. This system, where the peak demand is about 3.7 MW, is found out to be capable of supporting a staggeringly 5 MW wind power. Furthermore, this paper has performed an extensive analysis concerning the impacts of such a coordinated planning on network losses, system costs, emissions and overall system performance. Generally, results analysis shows average reductions of 25%, 83% and 70% in costs, losses and emissions, respectively. The voltage profile in the system is also considerably improved, which is one of the key indicators of the system's healthy operation.

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