

# Shared Energy Storage and Direct Load Control for Improved Flexibility of Distribution System Operation

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**Abstract**— In this study, an optimization problem is proposed for improving the flexibility of a distribution system including shared energy storage systems (ESSs) and residential consumers with elastic heating, ventilation and air conditioning (HVAC) units. To this end, a direct compressor control mechanism (DCCM) is used for meeting the load reduction target of load serving entities (LSEs) by controlling the HVACs, and an optimal management strategy is presented for exploiting the benefits of shared ESSs during critical peak periods. Furthermore, an additional objective is included in the problem structure, which leads to a bi-level optimization problem, in order to reduce the possible consumers' discomfort caused by frequent HVAC interruptions. The effectiveness of the proposed bi-level optimization problem is demonstrated by comparing various performance metrics with those of two benchmark methods.

**Index Terms**—Demand response; direct load control; heating, ventilation and air conditioning (HVAC) units; shared energy storage.

## I. NOMENCLATURE

The sets, indices, constants, parameters and decision variables used in this study are listed in Tables I-III. Also, the abbreviations are defined where they first appear.

TABLE I. SETS & INDICES

$B_l^{ij}$	Line set where $i$ and $j$ are the sending and receiving buses.
$B_h^i$	House set which is connected to bus $i$ .
$e$	Structural element index (set).
$h$	House index (set).
$i$	Bus index (set).
$l$	Line index (set).
$t$	Time period index (set).

TABLE II. CONSTANTS & PARAMETERS

$B_l$	Susceptance of line $l$ [pu].
$c_a$	Air thermal capacity [kJ/kg·°C].
$CE_i^{SS}$	Charging efficiency of the ESS unit at bus $i$ .
$COP_{i,h}$	Performance coefficient of HVAC in house $h$ connected to bus $i$ .
$DE_i^{SS}$	Discharging efficiency of the ESS unit at bus $i$ .
$K_{i,h}$	Rated power of the HVAC in house $h$ connected to bus $i$ [pu].
$M_{i,h}$	Air mass in house $h$ connected to bus $i$ [kg].
$M_{i,l}^F$	The parameter showing whether bus $i$ is the receiving or sending end of line $l$ .
$M_{i,l}^L$	The binary parameter showing whether bus $i$ is the sending end of line $l$ .
$M_{i,l}^W$	The parameter for bus $i$ and line $l$ obtained from the transpose of the matrix composed of $M_{i,l}^F$ values.
$N^h$	House number.
$P_{i,t}^{L,other}$	Inflexible demand of bus $i$ in period $t$ [pu].
$Q_{i,t}^R$	Reactive power demand of bus $i$ in period $t$ [pu].
$R_{i,h}^{eq}$	Thermal resistance of household $h$ connected to bus $i$ [h·°C/J].
$R_i^{SS,ch}$	Charging rate limit of the ESS unit connected to bus $i$ [pu].
$R_i^{SS,dis}$	Discharging rate limit of the ESS unit connected to bus $i$ [pu].

$R_l$	Resistance of line $l$ [pu].
$SOE_i^{SS,int}$	Initial SOE of the ESS unit connected to bus $i$ [pu].
$SOE_i^{SS,max}$	Maximum SOE of the ESS unit connected to bus $i$ [pu].
$SOE_i^{SS,min}$	Minimum SOE of the ESS unit connected to bus $i$ [pu].
$T_{i,h}^1$	Initial indoor temperature of household $h$ connected to bus $i$ [°C].
$T_t^a$	Ambient temperature in period $t$ [°C].
$T_{i,h,t}^{des}$	User-selected temperature in house $h$ connected to bus $i$ in period $t$ [°C].
$t_1$	Starting time of the DR event.
$t_2$	Ending time of the DR event.
$V_{i,h}$	Volume of house $h$ connected to bus $i$ [m <sup>3</sup> ].
$V_{max}$	Maximum allowed voltage level for buses [pu].
$V_{min}$	Minimum allowed voltage level for buses [pu].
$X_l$	Reactance of line $l$ [pu].
$\delta_{air}$	Air density [kg/m <sup>3</sup> ].
$\Delta T$	Time granularity [h].

TABLE III. DECISION VARIABLES

$P_{i,h,t}^{AC}$	Actual HVAC power consumption of house $h$ connected to bus $i$ in period $t$ [pu].
$P_{i,t}^G$	Available power at bus $i$ in period $t$ [pu].
$P_{i,t}^L$	Total bus load $i$ in period $t$ [pu].
$P_{i,t}^S$	Local transformer supply at bus $i$ in period $t$ [pu].
$P_{i,t}^{SS,ch}$	Charging power of the ESS unit connected to bus $i$ in period $t$ [pu].
$P_{i,t}^{SS,dis}$	Discharging power of the ESS unit connected to bus $i$ in period $t$ [pu].
$P_{l,t}^{loss}$	Active power loss of line $l$ in period $t$ [pu].
$\hat{P}_{l,t}^{loss}$	Model variable representing the active power loss of line $l$ in period $t$ [pu].
$P_{l,t}^r$	Active power flow at receiving end of line $l$ in period $t$ [pu].
$Q_{l,t}^{loss}$	Reactive power loss of line $l$ in period $t$ [pu].
$Q_{i,t}^G$	Reactive power generated/consumed at bus $i$ in period $t$ [pu].
$Q_{l,t}^R$	Reactive power flow at receiving end of line $l$ in period $t$ [pu].
$SOE_i^{SS}$	SOE of the ESS unit connected to bus $i$ in period $t$ [pu].
$T_{i,h,t}^{dec}$	Indoor temperature decrease regarding the user-selected temperature in house $h$ connected to bus $i$ in period $t$ [°C].
$T_{i,h,t}^{in}$	Indoor temperature of house $h$ connected to bus $i$ in period $t$ [°C].
$T_{i,h,t}^{inc}$	Indoor temperature increase regarding the user-selected temperature in house $h$ connected to bus $i$ in period $t$ [°C].
$u_{i,h,t}$	Binary variable for the HVAC status of house $h$ connected to bus $i$ in period $t$ .
$u_{i,t}^{ch}$	Binary variable showing if the ESS connected to bus $i$ is charging in period $t$ .
$V_{i,t}$	Voltage magnitude at bus $i$ in period $t$ [pu].
$W_{i,t}$	Square of the voltage magnitude at bus $i$ in period $t$ [pu].
$W_{r,t}$	Square of the voltage magnitude at receiving bus $r$ ( $r \in i$ ) in period $t$ [pu].
$\lambda_{i,h,t}^x$	Lagrange multipliers of the problem constraints, where $x = \{1,2,3,4\}$ .

## II. INTRODUCTION

The amount of electrical energy consumed by residential end users has recently increased to almost 40% of the total energy consumed, particularly in developed and developing countries. In order to manage such a huge amount of energy with relatively stochastic and unpredictable characteristics compared to that of commercial and industrial end-users, active participation of consumers through demand response (DR) programs has gained importance at the last years in addition to the investments and services in generation side [1]. With a similar objective, energy storage systems have found wide application areas at the residential level [2].

Storage systems used within the residential premises can provide various benefits to both load serving entities (LSEs) and end users. As for LSEs, the storage systems can support the grid during peak demand periods to prevent any mismatch between generation and consumption that might lead to deviations in voltage and frequency. Regarding the end users, these systems can enable to increase the utilization ratio of renewable sources by storing the excess energy. Recently, high-capacity storage systems, which are generally used for multiple households and generally located in a dedicated area considering cost and space limitations, have been introduced in both literature and real-world applications for further exploitation of the potential of storage systems for energy and cost savings [3]. These systems, called shared energy storage systems (ESS), substantially decrease the initial, operational, maintenance and replacement costs of individual behind-the-meter storage systems. The potential benefits of shared ESS units and their contributions were investigated in the literature in terms of several aspects. Their impacts on the exploitation of renewable energy sources were considered in [4,5]. Their contribution to the household energy management was investigated in [6] and their optimal allocation in a microgrid was examined in [7]. Besides, an optimization method was presented in [8] for the purpose of minimizing the energy cost of a cluster of residential end users.

However, none of these studies has considered the benefits of combining the flexibility from shared ESS units and direct load control (DLC)-based DR programs.

In this study, an optimal management algorithm is proposed for reducing peak load demands during DR event periods by using a two-phase procedure. In the first phase, a DLC strategy, called direct compressor control mechanism (DCCM), is used for exploiting the flexibility from residential Heating, Ventilation and Air Conditioning (HVAC) units. Among the other available appliances that can be used for DLC programs, HVAC units are chosen due to their very high power capacity compared to other appliances and the fact that the thermal inertia of the structures of the premises limits the deterioration in the end-users' comfort level caused by power interruptions [9]. In the latter, ESS units connected at different low voltage (LV) buses are considered for providing additional flexibility to the distribution system operation. Moreover, another objective is taken into account in this study to minimize the discomfort that might be caused by the interruption of the HVAC units.

Compared to the available literature on the similar topics, this study considers residential consumers with elastic appliances and multiple shared ESSs together for examining the potential of these units on DR programs, which is a problem that has not been studied in the literature to the best of the authors' knowledge. The load reductions required during the peak demand periods are primarily satisfied by

controlling the operation of HVACs and then the energy stored in the ESSs is used for supplying energy to the system during critical periods. Furthermore, the minimization of the consumers' discomfort is achieved through a bi-level optimization model in which the upper-level problem represents the loss-minimization problem and the lower level represents the consumer discomfort-minimization problem. Besides, the equations of AC power flow between the components of distribution system are represented by a second order conic programming method as the considered system covers a wide area.

The effectiveness of the proposed bi-level optimization problem is examined for different test cases and the capability of shared ESS units for supporting the use of HVAC units during peak load demands while still achieving a substantial decrease in the amount of energy drawn from the distribution system is evaluated. In order to measure the performance of the proposed approach, several benchmark approaches such as the case in which DR program is focused on only peak load demand reduction and the case in which HVAC units are controlled based solely on end users' preferences are considered. The remaining parts of the paper are organized as follows: the methodology and formulation of the proposed optimization model are provided in Section III. Afterwards, the benefits of the proposed methodology are assessed through the case studies in Section IV. Finally, Section V makes some concluding remarks.

## III. METHODOLOGY

General scheme of the distributed system considered in this study is shown in Fig. 1. As seen from Fig. 1, it is assumed that the distribution system is composed of a certain number of LV buses which are connected to a medium-voltage (MV) bus.

Each LV bus supplies energy to the residential and commercial consumers as well as to a shared ESS unit through a MV/LV transformer. As stated in the previous section, the residential HVAC units are considered as flexible loads in this study and all the other residential and commercial loads form the inelastic load demand.

The proposed methodology has two different main objectives. The first objective is to decrease the peak load demand in power system by controlling the operation of both HVAC units and shared ESS units, and the second one is to minimize the discomfort level of the consumers during these distribution system interactions. To this end, the mentioned problem can be formulated as a bi-level optimization problem, as given by (1)-(24).

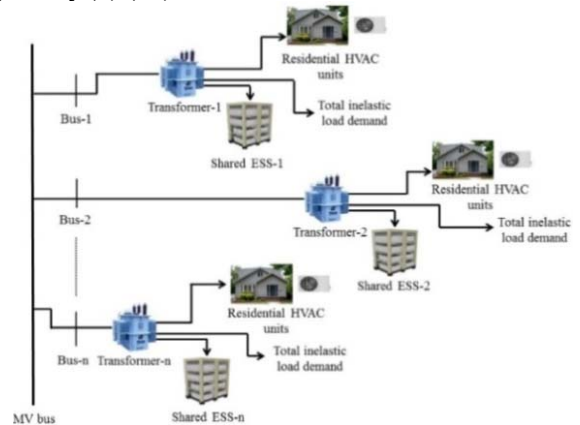


Figure 1. General scheme of the considered distribution system including different types of load and shared ESS unit.

$$\text{Minimize } L = \sum_{t_1}^{t_2} \sum_i P_{i,t}^S \quad (1)$$

subject to:

$$P_{i,t}^G - P_{i,t}^L = \sum_{l \in B_i^j} (M_{i,l}^F \cdot P_{l,t}^r + M_{i,l}^L \cdot P_{l,t}^{loss}) \quad \forall i, t \quad (2)$$

$$= \sum_{l \in B_i^j} (M_{i,l}^F \cdot Q_{l,t}^r + M_{i,l}^L \cdot Q_{l,t}^{loss} - B_l \cdot M_{l,i}^W \cdot W_{i,t}) \quad \forall i, t \quad (3)$$

$$P_{i,t}^G = P_{i,t}^{SS,dis} + P_{i,t}^S \quad \forall i, t \quad (4)$$

$$P_{i,t}^L = P_{i,t}^{L,other} + P_{i,t}^{SS,ch} + \sum_{h \in B_i^k} P_{h,t}^{AC} \quad \forall i, t \quad (5)$$

$$W_{i,t} = V_{i,t}^2 \quad \forall i, t \quad (6)$$

$$P_{i,t}^{loss} = 2 \cdot R_l \cdot \hat{P}_{l,t}^{loss} \quad \forall l, t \quad (7)$$

$$X_l \cdot P_{i,t}^{loss} - R_l \cdot Q_{l,t}^{loss} = 0 \quad \forall l, t \quad (8)$$

$$\sum_i (M_{l,i}^W \cdot W_{i,t}) - 2 \cdot (R_l \cdot P_{l,t}^r + X_l \cdot Q_{l,t}^r) = R_l \cdot P_{l,t}^{loss} + X_l \cdot Q_{l,t}^{loss} \quad \forall l, t \quad (9)$$

$$2 \cdot \hat{P}_{l,t}^{loss} \cdot W_{r,t} \geq P_{l,t}^r{}^2 + Q_{l,t}^r{}^2 \quad \forall l, t \quad (10)$$

$$V_{min}^2 \leq W_{i,t} \leq V_{max}^2 \quad \forall i, t \quad (11)$$

$$0 \leq P_{i,t}^{SS,ch} \leq R_i^{SS,ch} \cdot u_{i,t}^{ch} \quad \forall i, t \quad (12)$$

$$0 \leq P_{i,t}^{SS,dis} \leq R_i^{SS,dis} \cdot (1 - u_{i,t}^{ch}) \quad \forall i, t \quad (13)$$

$$SOE_{i,t}^{SS} = SOE_{i,(t-1)}^{SS} + \left( CE_{i,t}^{SS} \cdot P_{i,t}^{SS,ch} - \frac{P_{i,t}^{SS,dis}}{DE_{i,t}^{SS}} \right) \cdot \Delta T \quad \forall i, t \quad (14)$$

$$SOE_{i,t}^{SS,min} \leq SOE_{i,t}^{SS} \leq SOE_{i,t}^{SS,max} \quad \forall i, t \quad (15)$$

$$SOE_{i,1}^{SS} = SOE_{i,1}^{SS,ini} \quad \forall i \quad (16)$$

$$T_{i,h,t}^{in} = \left( 1 - \frac{\Delta T}{1000 \cdot M_{i,h} \cdot c_a \cdot R_{i,h}^{eq}} \right) \cdot T_{i,h,(t-1)}^{in} + \frac{\Delta T}{1000 \cdot M_{i,h} \cdot c_a \cdot R_{i,h}^{eq}} \cdot T_{t-1}^a \quad (17)$$

$$- \frac{COP_{i,h} \cdot \Delta T}{0.000277 \cdot M_{i,h} \cdot c_a} \cdot P_{i,h,t}^{AC} \quad \forall i, h, t > 1 \quad (18)$$

$$T_{i,h,1}^{in} = T_{i,h}^1 \quad \forall i, h \quad (18)$$

$$P_{i,h,t}^{AC} = u_{i,h,t} \cdot K_{i,h} \quad \forall i, h, t \quad (19)$$

$$T_{i,h,t}^{in} \quad \forall i, h, t \in \underset{T_{i,h,t}^{inc}, T_{i,h,t}^{dec}, T_{i,h,t}^{in}}{\operatorname{argmin}} \left\{ \frac{1}{Nh} \cdot \sum_i \sum_h \sum_t (T_{i,h,t}^{inc} + T_{i,h,t}^{dec}) \right\} \quad (20)$$

subject to:

$$T_{i,h,t}^{in} \leq T_{i,h,t}^{des} + T_{i,h,t}^{inc} \quad \forall i, h, t : \lambda_{i,h,t}^1 \quad (21)$$

$$-T_{i,h,t}^{in} \leq T_{i,h,t}^{dec} - T_{i,h,t}^{des} \quad \forall i, h, t : \lambda_{i,h,t}^2 \quad (22)$$

$$-T_{i,h,t}^{dec} \leq 0 \quad \forall i, h, t : \lambda_{i,h,t}^3 \quad (23)$$

$$-T_{i,h,t}^{inc} \leq 0 \quad \forall i, h, t : \lambda_{i,h,t}^4 \quad (24)$$

As indicated by (1), the first objective of the proposed optimization problem is to minimize the energy drawn from the grid by all the buses during the DR event.

The equations defining the active and reactive power balances are described by (2) and (3), respectively. It should be noted that  $M_{i,l}^F$  is defined as a parameter that is 1 if bus  $i$  is the receiving end of line  $l$  and -1 if it is the sending end.

Otherwise, this parameter is considered 0. Similarly, the parameter  $M_{i,l}^L$  is considered 1 if bus  $i$  is the sending end of line  $l$ . Constraint (4) decomposes the power at each bus into two power values: the discharging power of the shared ESS and the injection of the LV transformer. Similarly, the total system load is decomposed by (5) into inflexible load demand, the discharging power of the shared ESS and the HVAC load. The square of the voltage magnitude that is used in (3) is given by (6).

Constraints (7)-(10), which are derived from [10] with related modifications, constitute the AC power flow equations based on second order conic programming. The voltage at each bus is limited between predefined lower and upper limits as indicated by (11).

The constraints (12-16) represent the equations describing the operation of the shared ESS unit. The limits of the charging and discharging rates of the shared ESS are defined by (12) and (13) where  $u_{i,t}^{ch}$  is 1 when shared ESS is charging and is 0 otherwise. Constraint (14) describes the variation of the SOE of the shared ESS over time while (15) describes the maximum and minimum SOE levels. Lastly, the initial SOE of the shared ESS is set using (16).

With respect to the building, a model based on thermal resistance is used in this study as the energy consumption of HVAC units generally depends on the heat exchange between the house and ambient, and the thermodynamic properties of both building structure and air, as expressed by (17). Constraint (18) is used for setting the initial indoor temperature. The parameters in (17), which are the building thermal resistance and air mass, can be calculated using the relevant equations given in [11]. Lastly, the HVAC power consumption of houses in each time interval  $t$  is described by (19) where  $u_{i,h,t}$  is 1 when the HVAC unit is ON and 0 if it is OFF. As explained in the previous section, it is considered in this study that the LSE has the capability of directly manipulating the status of HVAC units with the objective of achieving the desired load demand reduction. For this purpose, the operation of the HVAC unit can be controlled within the predefined limits that are generally determined with respect to the requirement of LSEs and preferences of the consumers and that are specified in the contract with the LSE. In addition to this objective, minimization of the discomfort of consumers due to prolonged HVAC interruptions is also aimed in this study. The mentioned problem can be cast as the minimization of the average temperature deviations with respect to the desired temperature over time and represented by (20). This lower-level optimization problem is subject to constraints that force the indoor temperature to remain as much as close to the desired temperature value, as expressed by (21) and (22). Besides, (23) and (24) are the non-negativity constraints for the decision variable.

$$\begin{aligned} \Lambda = & \frac{1}{Nh} \cdot \sum_i \sum_h \sum_t (T_{i,h,t}^{inc} + T_{i,h,t}^{dec}) \\ & + \sum_i \sum_h \sum_t \lambda_{i,h,t}^1 \cdot (T_{i,h,t}^{in} - T_{i,h,t}^{des} - T_{i,h,t}^{inc}) \\ & + \sum_i \sum_h \sum_t \lambda_{i,h,t}^2 \cdot (-T_{i,h,t}^{in} - T_{i,h,t}^{dec} + T_{i,h,t}^{des}) \\ & + \sum_i \sum_h \sum_t (-\lambda_{i,h,t}^3 \cdot T_{i,h,t}^{dec}) \\ & + \sum_i \sum_h \sum_t (-\lambda_{i,h,t}^4 \cdot T_{i,h,t}^{inc}) \end{aligned} \quad (25)$$

In order to solve the bi-level optimization problem, the second optimization problem is replaced by its Karush-Kuhn-Tucker (KKT) optimality conditions. It should be noted that these conditions are both necessary and sufficient in this case. To obtain the KKT conditions, first, the Lagrangian of the problem is defined as in (25). The variables  $\lambda_{i,h,t}^\xi, \xi = 1,2,3,4$  are the Lagrange multipliers and expressed with (21)-(24).

The stationarity conditions are given by (26)-(28).

$$\frac{\partial \Lambda}{\partial T_{i,h,t}^{inc}} = 0 \rightarrow \frac{1}{Nh} - \lambda_{i,h,t}^1 - \lambda_{i,h,t}^4 = 0 \quad \forall i, h, t \quad (26)$$

$$\frac{\partial \Lambda}{\partial T_{i,h,t}^{dec}} = 0 \rightarrow \frac{1}{Nh} - \lambda_{i,h,t}^2 - \lambda_{i,h,t}^3 = 0 \quad \forall i, h, t \quad (27)$$

$$\frac{\partial \Lambda}{\partial T_{i,h,t}^{in}} = 0 \rightarrow \lambda_{i,h,t}^1 - \lambda_{i,h,t}^2 = 0 \quad \forall i, h, t \quad (28)$$

The complementary slackness conditions are defined by (29)-(32). These conditions, which are non-linear constraints, can be linearized using a big-M formulation as in [12].

$$\lambda_{i,h,t}^1 \cdot (T_{i,h,t}^{in} - T_{i,h,t}^{des} - T_{i,h,t}^{inc}) = 0 \quad \forall i, h, t \quad (29)$$

$$\lambda_{i,h,t}^2 \cdot (-T_{i,h,t}^{in} - T_{i,h,t}^{dec} + T_{i,h,t}^{des}) = 0 \quad \forall i, h, t \quad (30)$$

$$\lambda_{i,h,t}^3 \cdot T_{i,h,t}^{dec} = 0 \quad \forall i, h, t \quad (31)$$

$$\lambda_{i,h,t}^4 \cdot T_{i,h,t}^{inc} = 0 \quad \forall i, h, t \quad (32)$$

Finally, the Lagrange multipliers must be non-negative in order to guarantee dual feasibility as shown in (33) and also (21)-(24) of the original problem should be also enforced for primal feasibility.

$$\lambda_{i,h,t}^1, \lambda_{i,h,t}^2, \lambda_{i,h,t}^3, \lambda_{i,h,t}^4 \geq 0 \quad \forall i, h, t \quad (33)$$

The optimization problem can be now re-cast as a second order conic programming problem. The objective function given by (1) is minimized subject to constraints (2)-(19) and (21)-(24). Moreover, the constraints required to approximate the non-linear expressions (26)-(33) are considered in the optimization problem.

#### IV. RESULTS

##### A. Input Data

In this study, a distribution system consisting of 8 LV buses connected to a MV bus is considered. The parameters of the lines, which are expressed in terms of per unit (pu) values, are given in Table IV.

In order to calculate these values, a base voltage of 20 kV, power of 1 MVA and impedance of 400  $\Omega$  are adopted. Besides, the parameters needed to calculate the thermal interactions between the ambient and houses are chosen as  $\delta_{air} = 1.225 \text{ kg/m}^3$  and  $c_a = 1.01 \text{ kJ/kg}^\circ\text{C}$ , which are the standard values, for all the households. For brevity, it is also assumed that all the houses have the same building structure with the parameters shown in Table V and have a HVAC unit with a power rating of 1.7 kW and a coefficient-of-performance (COP) of 1.5.

In order to obtain different HVAC consumption patterns, a different initial indoor temperature ( $T_{i,h}^1$ ) around the desired temperature ( $T_{h,t}^{des}$ ) is considered for each house. For simplicity,  $T_{h,t}^{des}$  is assumed to be  $20^\circ\text{C}$  and the allowed values for the decrease and increase of indoor temperature ( $T_{h,t}^{in}$ ) from the  $T_{h,t}^{des}$  value, i.e.,  $T_{i,h}^{dec,allowed}$  and  $T_{i,h}^{inc,allowed}$  are chosen as  $4^\circ\text{C}$  for the households. Lastly, the specifications of the shared ESS units considered in this study are provided in Table VI. As seen, the minimum and maximum state-of-energy (SOE) values are limited so as to avoid deep discharge and overcharge.

For the DR events, a typical summer day with high ambient temperature values as shown in Fig. 2 is considered. It is assumed that all the houses accept to be involved in a DR event between 12pm and 4pm; however, only the period between 1pm and 4pm is considered for potential power reductions as this period has relatively higher power values.

##### B. Simulation and Results

Three different cases are considered in this study in order to evaluate the effectiveness of the proposed approach.

- Case-1: Both shared ESS and DR program unavailable.
- Case-2: Shared ESS unavailable and DR program available.
- Case-3: Both shared ESS and DR program available.

In the first case, it is assumed that the residential HVAC units are controlled only depending on the  $T_{h,t}^{des}$  values defined by the consumers, which leads to the maximum consumer comfort. In other words, the status of the HVAC units is changed only when the  $T_{h,t}^{in}$  value increases or decreases above/below a certain temperature. In order to realize this case, the objective function shown in (1) is not considered and only the constraints related to the power exchanges between the system components, (2)-(11), and the constraints related to the HVAC energy consumption, (17)-(19), are taken into account.

In the second case, the objective function given by (1) together with the constraints (2)-(11), (17)-(19) and (21-24) are considered with the objective of obtaining a power reduction in the peak load demand during the DR event. Lastly, incorporating also the shared ESS-related constraints (12)-(16) and the constraints of the lower-level problem (29)-(36) aiming to increase the consumer comfort compared to the second case, the benefits of the proposed approach is examined in the third case. It should be noted that a time granularity of 5 min is used for all the cases and the simulations are carried out using GAMS 24.1.3 and MOSEK solver. The power consumed by the HVAC units in all the households during the contracted DR period for three cases is shown in Fig. 3. It is seen from Fig. 3 that the HVAC units are operated almost at each 5-min time step in Case 1 to satisfy the desired temperature defined by the consumers. In the second case, the HVAC units are only used before the DR event period for cooling down the indoor temperature to prevent any power consumption during the DR period. As a difference in the third case, some of the HVACs are also used during the DR period and all of them are used after 3pm not to affect consumer comfort too much, which are imposed by the constraints of lower-level problem.

As mentioned in the previous section, there is also a high number of inelastic residential and commercial loads in the system. Considering these load demands together with the HVAC demands as well as the resulting line losses, the total system load demand is depicted in Fig. 4 for three cases. As seen from Fig. 4, the power values in Case 2 are slightly lower than those in Case 1 during the peak load demand period due to the power reductions achieved by the control of HVAC units. In addition to the benefits of the DR program, the discharging of the shared ESS units also enables a substantial decrease, particularly during the period between 2pm and 2:30pm that has the highest power consumption during the test period.

It should be noted that the difference between the power values of Case 2 and Case 3 at the beginning of the test period

is due to the charging of the shared ESSs for the purpose of later use.

In order to evaluate the benefits of shared ESS units in more detail, their charging and discharging operations are shown in Fig. 5. The shared ESS units are fully charged just before the DR event and discharged when the load demands reach to their maximum values, which is enabled by the proposed optimization problem.

The resulting temperature variations in the houses during the considered test period for three cases are shown in Figs. 6-8. For a clearer representation, the box plots for each case are provided instead of the temperature variations of the 240 houses (i.e., 8 LV buses each of which is connected to 30 houses). In Figs. 6-8, the middle line in the boxes shows the median of the temperature values for the test day. The lower and the upper lines of the box are the first and third quartiles, respectively. Also, the lower and the upper end of the vertical lines show the minimum and maximum temperature values, respectively.

As seen from Fig. 6, the indoor temperature values are kept in a narrow band around the desired temperature value in Case-1. On the contrary, the temperature value is frequently manipulated during the test period in Case-2 as shown in Fig. 7 in order to provide a reduction in the HVAC power consumption. As can be noticed from Fig. 8, the deviations of the temperature values from the desired value in Case-3 are relatively lower than those in Case-3, which is enabled by the lower-level problem.

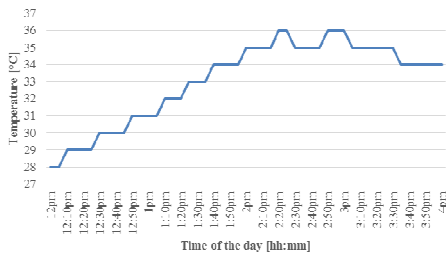


Figure 2. The ambient temperature variation of the considered test day during the DR event period.

TABLE IV. LINE PARAMETERS OF THE DISTRIBUTION SYSTEM

Line	From	To	R [pu]	X [pu]
L1	n2	n1	0.00071	0.00036
L2	n3	n2	0.00017	0.00009
L3	n4	n3	0.00153	0.00051
L4	n5	n4	0.00035	0.00012
L5	n6	n5	0.00133	0.00044
L6	n7	n6	0.00113	0.00041
L7	n8	n7	0.00182	0.00062

TABLE V. STRUCTURAL PARAMETERS OF THE HOUSES

Parameter	Value	Units	Parameter	Value	Units
Length ( $L_1$ )	30	m	Number of windows	6	-
Width ( $L_2$ )	10	m	Window length	1	m
Height ( $L_3$ )	3	m	Window width	1	m
Wall thickness	0.13	m	Windows thickness	0.05	m
Wall thermal coefficient	129	J/h·m·°C	Window thermal coefficient	2780	J/h·m·°C
Roof angle ( $\beta$ )	40	deg			

TABLE VI. PARAMETERS OF THE SHARED ENERGY STORAGE SYSTEM

Parameter	Value	Units	Parameter	Value	Units
$CE_i^{SS}$	0.95	-	$SOE_i^{SS,ini}$	0.12	pu
$DE_i^{SS}$	0.95	-	$SOE_i^{SS,min}$	0.06	pu
$R_i^{SS,ch}$	0.1	pu	$SOE_i^{SS,max}$	0.18	pu
$R_i^{SS,dis}$	0.1	pu			

The benefits of the proposed approach on the consumer comfort can be observed from Fig. 9 in more detail, which shows the average comfort violation in terms of the temperature deviations from the desired value. It is seen from Fig. 9 that the comfort violation has naturally its best results for Case-1 in which no control method is applied to reduce the HVAC power. Regarding the Case-2 and Case-3, the average comfort violation is almost the same excepting the period after 3pm where better results are achieved for Case-3 due to the shared ESS and proposed optimization algorithm.

Finally, the energy supplied by the grid and the average comfort violation are provided for three cases in Table VII for a better comparison. As seen from the first three column of Table VII, the application of DLC-based DR program reduces the energy purchased from the distribution system. Including the shared ESS unit also enables an additional reduction in the consumption values, particularly during the DR event period and peak demand period that is considered the power values over 8000 kW in this study. In terms of the comfort violation, the proposed approach provides a lower value compared to Case-2, even with a higher power reduction.

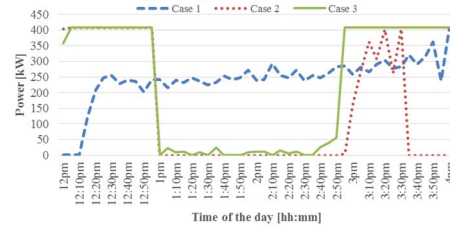


Figure 3. The power consumption of the HVACs during the considered test period for three cases.

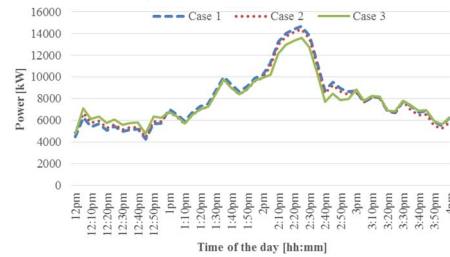


Figure 4. The power supplied by the grid during the considered test period for three cases.

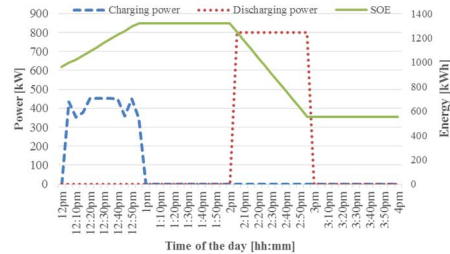


Figure 5. The variations in the power and SOC values of the shared ESS units during the test period for Case-3.

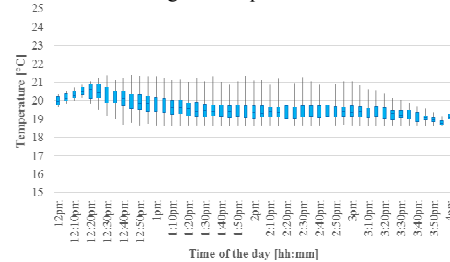


Figure 6. The variations of the indoor temperature of all the households for Case-1.



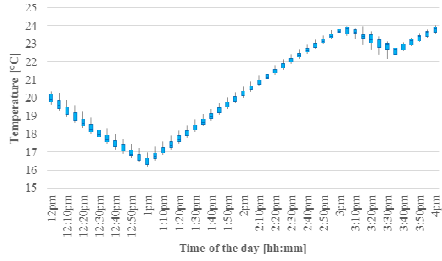


Figure 7. The variations of the indoor temperature of all the households for Case-2.

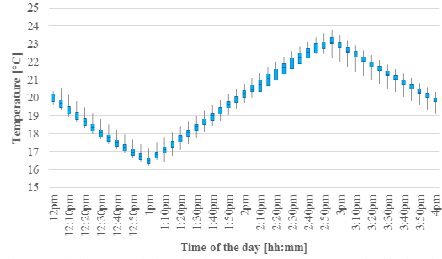


Figure 8. The variations of the indoor temperature of all the households for Case-3.

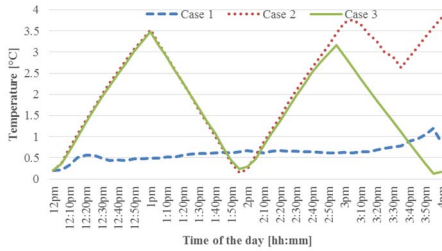


Figure 9. The average comfort violation during the considered test period for three cases.

TABLE VII. OVERVIEW OF THE OBTAINED RESULTS

Case	Energy supplied by the grid [kWh]			Comfort violation [°C/5 min]		
	During test period	During DR event	During peak demand	Min	Max	Average
Case-1	4071.74	3412.65	2023.01	0.21	1.19	0.62
Case-2	4016.03	3326.58	1972.62	0.14	3.81	2.22
Case-3	4011.34	3274.64	1887.40	0.12	3.48	1.67

The simulation studies have validated that the proposed approach has the capability of achieving a higher consumer comfort with a lower power consumption, particularly during peak load periods. In addition to the mentioned benefits, the proposed algorithm is also very effective in preventing the load recovery effect as the lower temperature values at the end of the DR event in Case-3 imply that no significant power will be drawn from the system just after the DR event contrary to the condition in Case-2. It should be noted that the benefits the proposed approach might be significantly high if the energy prices are taken into account as the prices in peak periods are double or even triple the normal rates and drawing energy in off-peak periods does not necessarily increase the total cost of the energy used. Besides, the benefits of the proposed approach might be higher if the shared ESS units are charged during the night times when the energy prices are typically relatively lower and/or using the excess power generation of renewable energy sources.

## V. CONCLUSIONS

A bi-level optimization problem aiming to improve the use of residential HVAC units and shared ESSs for distribution system flexibility was proposed in this study, which in the upper level the peak load demand is minimized and in the lower level the resulting reduction in the discomfort level of

consumers is minimized. The optimization problem was first converted into a single-level problem by replacing the lower-level problem with its KKT conditions and then the nonlinearities in the problem structure were linearized. The results have shown that the proposed DCCM-based DR method had the capability of supporting the power system operation, and including the ESS units had further increased the benefits while reducing the deterioration of the consumers at the same time. For a comparison, two different cases in which no shared ESS is used were considered. In the first case, no optimization strategy was applied while in the second only upper level problem was taken into account. In terms of percentage improvements in power consumption during peak demand, the proposed optimization problem achieved a decrement of 6.7% compared to the first case, and a decrement of 4.3% compared to the second case. Moreover, including the ESSs enabled a reduction of 24.8% in the comfort violation of consumers compared to the single-level problem. As a future direction, the effects of the thermostat set-point control mechanism (TSCM), which is another widely-used method in DR programs, can be considered for the control of HVAC units.

## ACKNOWLEDGMENT

This work was mainly supported by The Scientific and Technological Research Council of Turkey (TUBITAK) under Project Grant Numbers 116E115 and 117E527. J.P.S. Catalão also acknowledges the support by FEDER funds through COMPETE 2020 and by Portuguese funds through FCT, under Projects SAICT-PAC/0004/2015 - POCI-01-0145-FEDER-016434, POCI-01-0145-FEDER-006961, UID/EEA/50014/2013, UID/CEC/50021/2013, UID/EMS/00151/2013, and 02/SAICT/2017 - POCI-01-0145-FEDER-029803, and also funding from the EU 7th Framework Programme FP7/2007-2013 under GA no. 309048.

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