Simultaneous Participation of Demand Response Aggregators in Ancillary Services and Demand Response Exchange Markets

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***Abstract*—Participation of consumers in Demand Response (DR) programs improves system stability and reliability as well as market efficiency. Retailers and distributors purchase DR to advance business and system reliability, respectively. Meanwhile, large consumers, Distribution System Operators (DSOs), Load Service Entities (LSEs), and DR aggregators sell DR to increase their own profits. In this context, DR aggregators are key elements of power systems that enhance the participation of consumers in electricity markets. These market participants can negotiate their aggregated DR with other market players in Demand Response eXchange (DRX) markets, and participate in the energy and ancillary service markets. Hence, this paper proposes a stochastic model to optimize the performance of a DR aggregator to take part in the day-ahead energy, ancillary services and intraday DRX markets. In order to mitigate the negative impacts of uncertainties, Conditional Value at Risk (CVaR) is also incorporated to the proposed model. Numerical studies indicate that the proposed model for DR aggregator can arrange its offering/bidding strategies to participate in the mentioned markets simultaneously.**

***Index Terms*—Ancillary services, day-ahead market, demand response exchange, DR aggregator, intraday market.**

I. NOMENCLATURE

Indices

i	index of consumers
s	index of scenarios
t	index of time interval

Parameters and variables

a_i ,b_i	SFE coefficients of consumer i
am_i , bm_i	coefficients of marginal cost function of consumer i
Act_{ts}^{Res}	activated quantity of reserve at time t in scenario s
B_s	profit in scenario s
D_t	total demand at time t
DR_i	purchased DR from consumer i
P_{ts}^{bal}	offer to balancing market at time t in scenario s
P_{ts}^{DA}	day-ahead offer at time t in scenario s
P_{ts}^{I}	offer to intraday market at time t in scenario s
P_{ts}^{Res}	offer to spinning reserve at time t in scenario s
RDR	required DR
r_t^+ , r_t^-	positive and negative imbalance ratios with day-ahead market price
α	confidence level
β	weighting factor for tradeoff between profit and risk
π_{ts}^{bal}	balancing price at time t in scenario s
π_{ts}^{DA}	day-ahead energy price at time t in scenario s
π^{DRX}	clearing price of DRX market
π_{ts}^{I}	intraday energy market price at time t in scenario s
π_{ts}^{Res}	spinning reserve price at time t in scenario s
π_t^{tariff}	consumers' tariff
ξ	value-at-risk
η_s	auxiliary variable to calculate CVaR
θ_i	coefficient of consumer's willingness to participate in DRPs
Δ_{ts}^+ , Δ_{ts}^-	positive and negative deviations at time t in scenario s
Δ_{ts}	total deviation from actual amount at time t in scenario s
$\rho^{Response}$	availability of DR aggregator to deliver energy
ρ_s	occurrence probability of scenario s

II. INTRODUCTION

Demand Response (DR) is a key element of power systems to enhance market efficiency, price stability, and system reliability [1]-[3]. In order to increase the participation level of consumers in Demand Response Programs (DRPs) and support these effective resources to flourish, regulatory bodies have been laying the groundwork for crafting the right policies and incentive mechanisms [4]-[6]. On this basis, Ref. [7] considers DR as a tradable good, and consequently, a Demand Response eXchange (DRX) market is introduced. As indicated in [7], this market can overcome the problem of partial scheduling of DR in comparison with conventional DR scheduling that technically, financially and socially yields suboptimal solutions.

In the aforementioned market, retailers and distributors purchase DR to advance business and system reliability, respectively. Meanwhile, large consumers, Distribution System Operators (DSOs), Load Service Entities (LSEs) and DR aggregators sell DR to increase their own profits [7], [8].

Although some reports have been addressed in the literature regarding the demand-side players who bid in electricity markets [4], [5], [9], DR aggregation has not been studied in these reports.

To the best knowledge of the authors, the simultaneous participation of DR aggregators in the day-ahead energy, ancillary services and DRX markets has not been reported in the literature.

Due to the high level of uncertainty related to the nature of renewable energies, participation in markets that are closer to the market closure time has economic benefits for market participants. Therefore, participation in intraday markets where the market players are able to update their initial offers/bids is beneficial [10]-[11]. On the other hand, due to the mentioned uncertainty, implementation of the balancing and ancillary service markets is critical to supply the spinning reserve requirement. Hence, the DR aggregator can take part in the balancing and ancillary service markets to supply a part of the spinning reserve and the regulation requirement [12].

In this paper, the optimal offering/bidding strategy of a DR aggregator in the day-ahead energy, intraday DRX and ancillary services markets is addressed. To this end, the consumers' behavior is considered using Supply Function Equilibrium (SFE), in contrast to most previous reports that have used constant DR demand curves [7], [13], [14]. Furthermore, Conditional Value at Risk (CVaR) is incorporated in the proposed model to minimize the negative impacts of uncertainties related to consumers' behavior.

The remainder of this paper is organized as follows. Section III presents the model of DR aggregators for participation in the day-ahead energy, intraday DRX and ancillary service markets. In section IV, numerical results are presented. Section V is devoted to some concluding remarks.

III. MODELING THE DR AGGREGATOR

The DR aggregator maximizes its profit by taking part in the day-ahead energy, balancing and ancillary service markets as well as intraday DRX market. To this end, the DR aggregator contributes in the DRX market as a DR seller.

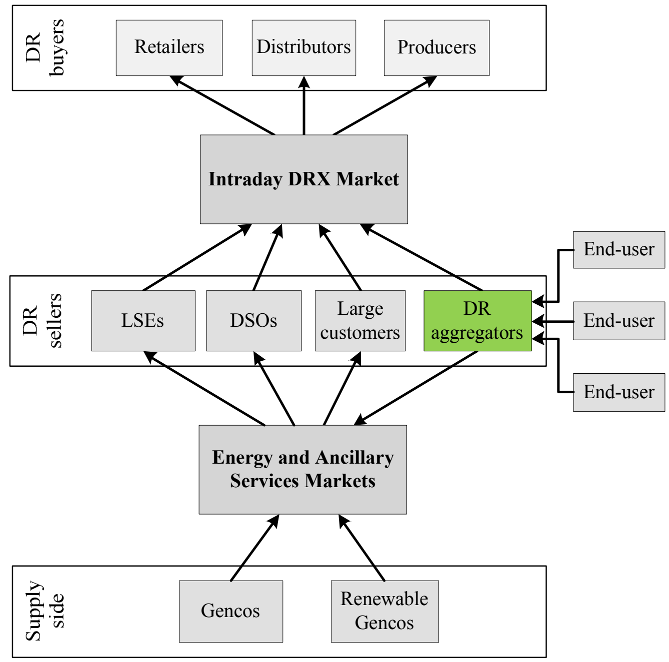

Fig. 1. Participation of DR aggregator in the electricity markets.

Fig. 1 schematically shows the DR aggregator's participation in the electricity markets.

A. Uncertainty Characterization

This paper considers two sources of uncertainty related to market price and the amount of activated reserve by a system operator. In order to consider their impacts, these sources of uncertainty have been characterized as stochastic processes, and the problem has been solved using a stochastic programming approach. The uncertainties are represented in the following manner.

1. Market Price Uncertainty

Each DR aggregator should forecast the market price to be able to take part in the electricity markets. This paper considers three types of market prices, namely, day-ahead energy, balancing and reserve prices. To this end, market prices are regarded as log-normal probability distributions [15], then the roulette wheel mechanism is employed to generate scenarios [16]. Note that this scenario generation mechanism uses the probability distribution functions of the individual input random variables. Different realizations of the wind power generation are modeled by usage of the scenario generation process based on roulette wheel mechanism. At first, the distribution function is divided into some class intervals. Moreover, each interval is associated with a probability. Subsequently, according to different intervals and their probabilities obtained by probability distribution function, roulette wheel mechanism is applied to generate scenarios for each hour. Eq. (1) presents the probability distribution function of market prices.

$$f_{Pr}(Pr,\mu,\sigma)=\frac{1}{Pr\,\sigma\sqrt{2\pi}}\exp\left[-\frac{\left(\ln Pr-\mu\right)^2}{2\sigma^2}\right] \qquad (1)$$

where, μ and σ denote the mean and the standard-variation values, respectively.

2. Uncertainty of Activated Reserve

The amount of reserve activated by a system operator, Act_{ts}^{Res}, is considered to be uniformly distributed whose probability distribution function is presented in (2).

$$f_{Res}(x) = \frac{1}{P_{ts}^{Res}} \quad , \quad 0 \le x \le P_{ts}^{Res} \tag{2}$$

Concerning Eq. (2), the behavior of various system operators in calling the DR aggregator are realized by the roulette wheel mechanism [16].

B. Risk Management

In order to manage the risk of DR aggregators arising from the uncertain behavior of consumers, CVaR, a well-known risk assessment technique, is employed. CVaR is formulated as in (3)-(5):

$$B_s = \xi - \frac{1}{1-\alpha}\sum_{s=1}^{S_N} \rho_s \eta_s \tag{3}$$

$$-B_s + \xi - \eta_s \le 0 \tag{4}$$

$$\eta_s \ge 0 \tag{5}$$

This formulation guaranties the CVaR to be unique. Note that the parameter α is typically assigned within the interval of 0.90 to 0.99. In this paper, it is set to 0.95.

C. Modeling the Consumers' Supply Function

In order to model the consumers' behavior, a new model of consumers' supply function is proposed. Therefore, a general model of DRX market price is formulated as in (6) based on the consumers' willingness to participate in DRPs, θ.

$$\pi^{DRX} = a_i . DR_i + b_i .(1-\theta_i) \tag{6}$$

An increase in θ can reduce the DR costs because the consumer can have more enthusiasm to take part in DRPs. In this paper, different values of θ are considered to reflect different types of consumers. There should be a balance between the supplied and the demanded DR [7]; therefore, the required DR and the DRX market price are given as in (7) and (8), respectively.

$$RDR = \sum_{i=1}^{I} DR_i = \sum_{i=1}^{I} \frac{\pi^{DRX} - b_i(1-\theta_i)}{a_i} \tag{7}$$

$$\pi^{DRX} = \left[RDR + \sum_{i=1}^{I} \frac{b_i(1-\theta_i)}{a_i} \right] \Big/ \sum_{i=1}^{I} (\frac{1}{a_i}) \tag{8}$$

The profit of consumers is a function of the traded DR and the DRX market price as presented in (9).

$$pf_i = \pi^{DRX} \times DR_i - \cos t(DR_i) \tag{9}$$

In order to determine the consumers' cost function, a data mining process is required to be carried out in several energy sectors. According to an investigation in this context, a quadratic form is proposed for the consumers' cost function in [17]. Hence, substituting $\theta=0$ in Eq. (9) gives:

$$\begin{aligned} pf_i &= \pi^{DRX} (\frac{\pi^{DRX} - b_i}{a_i}) \\ &\quad - \left[\frac{am_i}{2} \left(\frac{\pi^{DRX} - b_i}{a_i} \right)^2 + bm_i \left(\frac{\pi^{DRX} - b_i}{a_i} \right) \right] \end{aligned} \tag{10}$$

Eq. (10) represents the profit function of consumers. On this basis, the SFE model can be employed by each consumer to suggest its offers to the DR aggregator [18]. Thus, sellers can offer their a_i and b_i to improve the profit; consequently, the price of DRX is computed by (11).

$$\begin{aligned} \pi^{DRX} &= \left[RDR + \sum_{i=1}^{N^{DRS}} \frac{b_i}{a_i} \right] \Big/ \sum_{i=1}^{N^{DRS}} \left(\frac{1}{a_i} \right) \\ &= \left[RDR + \frac{b_i}{a_i} + \sum_{i \ne j}^{N^{DRS}} \frac{b_j}{a_j} \right] \Big/ \sum_{i=1}^{N^{DRS}} \left(\frac{1}{a_i} \right) \end{aligned} \tag{11}$$

D. Mathematical Model of DR Aggregator

The DR aggregator offers to the day-ahead energy and spinning reserve markets. Then, it can modify the energy it offers in the intraday markets. Afterwards, it submits a new offer or updates the previous one in the balancing market. Furthermore, risk aversion is employed by controlling the variations of expected profit by means of the CVaR technique. Based on the aforementioned explanation, the objective function is presented in (12).

$$\begin{aligned} &\text{Max } Expected\ \ Profit = \\ &(1-\beta)\sum_{s=1}^{S_N} \rho_s \sum_{t=1}^{T} \left[\begin{array}{l} \pi_{ts}^{DA} .P_{ts}^{DA} + \pi_{ts}^{Res} .P_{ts}^{Res} + \pi_{ts}^{I} .P_{ts}^{I} \\ -\sum_{d=1}^{ND} \pi_{td}^{DRX} .DR_{td} + \pi_{ts}^{bal} .P_{ts}^{bal} \\ +Act_{ts}^{Res} .\pi_{ts}^{DA} .\rho^{Response} \\ -Act_{ts}^{Res} .\pi_{ts}^{bal} .(1-\rho^{Response}) \\ -D_t .\pi_t^{tariff} + \pi_{ts}^{DA} .r_t^{+} .\Delta_{ts}^{+} - \pi_{ts}^{DA} .r_t^{-} .\Delta_{ts}^{-} \end{array} \right] \\ &+\beta \left(\xi - \frac{1}{1-\alpha}\sum_{s=1}^{S_N} \rho_s .\eta_s \right) \end{aligned} \tag{12}$$

The first line of Eq. (12) refers to the incomes as a result of participating in the day-ahead energy, reserve and intraday markets, respectively. The second line is related to the cost of purchased energy in the intraday DRX market, and the income of balancing market, respectively. The third line shows the income of delivering the activated reserve. The fourth line shows the cost of purchasing energy from the balancing market due to inability to deliver the activated reserve. The first term in the fifth line refers to the cost of responsive demand. The next two terms respectively characterize the positive and the negative deviation of costs in the balancing market. The last line incorporates the risk assessment technique—the CVaR. The following constraints are associated with the objective function:

$$P_{ts}^{DA} + Act_{ts}^{Res} + P_{ts}^{I} - \sum_{d=1}^{ND} DR_{dts} + P_{ts}^{bal} \le D_t \tag{13}$$

$$-\sum_{t=1}^{T}\left[\begin{array}{l}\pi_{ts}^{DA}.P_{ts}^{DA}+\pi_{ts}^{Res}.P_{ts}^{Res}+\pi_{ts}^{I}.P_{ts}^{I}-\sum_{d=1}^{ND}\pi_{td}^{DRX}.DR_{td}\\+\pi_{ts}^{bal}.P_{ts}^{bal}+Act_{ts}^{Res}.\pi_{ts}^{DA}.\rho_{t}^{call}.\rho^{Response}\\-Act_{ts}^{Res}.\pi_{ts}^{bal}.\rho_{t}^{call}.(1-\rho^{Response})-D_{t}.\pi_{t}^{tariff}\\+\pi_{ts}^{DA}.r_{t}^{+}.\Delta_{ts}^{+}-\pi_{ts}^{DA}.r_{t}^{-}.\Delta_{ts}^{-}\end{array}\right]+\xi-\eta_{s}\leq 0 \tag{14}$$

$$0\leq P_{ts}^{DA}\leq P^{\max} \tag{15}$$

$$0\leq P_{ts}^{bal}\leq P^{\max} \tag{16}$$

$$0\leq P_{ts}^{Res}\leq P^{\max} \tag{17}$$

$$0\leq P_{ts}^{I}\leq P^{\max} \tag{18}$$

$$\Delta_{ts}=P_{t}^{Act}-P_{ts}^{DA} \tag{19}$$

$$\Delta_{ts}=\Delta_{ts}^{+}-\Delta_{ts}^{-} \tag{20}$$

Eq. (13) presents the total capacity of the DR aggregator in energy, reserve and DRX markets. Eq. (14) ensures the problem to have a unique solution when incorporating the CVaR technique. Inequalities (15)-(18) limit the offers of DR aggregator in the electricity markets. The energy deviation from the actual amount is calculated by (19) and (20).

IV. NUMERICAL STUDIES

In this paper, three clusters are considered for DR sellers. The willingness coefficient, θ, associated to each cluster is presented in Table I. The load profile is extracted from a real-world system [19]. The peak demand of the test system is considered to be 100 MW. Market prices for day-ahead energy, spinning reserve, intraday and balance markets are extracted from hourly data of the Iberian electricity market in February 2010 [20]. Moreover, DR aggregator's response probability is considered 0.05.

DR sellers compete with each other to sell to the DRX market. SFE pair (a_i , b_i) for sellers are calculated by the approach presented in section III.C. Fig. 2 illustrates the amount of DR traded by each seller. According to this figure, the amount of DR associated with each seller depends on the willingness coefficient. Based on this, only Sellers 1 and 2 can permanently supply DR, while the third seller can only have benefit in peak periods. It should be noted that the DR trade is reduced in off-peak periods.

In order to investigate the impacts of risk-taking on the performance of DR aggregator, β is set to 0 and 1 to reflect a risk averse and a risk taker, respectively. The impact of risk-taking on the profit of DR aggregator is presented in Table II. The results reveal that the DR aggregator's profit decreases as a consequence of increasing the risk coefficient, but the violation of the profit decreases.

On this basis, the risk-averse DR aggregators can decide to alleviate the violation risk and consequently, the decision causes a reduction in the profit. Conversely, risk-taker aggregators prefer more profit that causes more violation probability from the profit.

The impact of implementing various electricity markets on the profit of DR aggregator is also presented in Table II. The participation level of consumers in DRPs is considered 20%. According to the results of Table II, the implementation of intraday DRX markets and ancillary services increases the profit of DR aggregators. On this basis, participation in the ancillary services can raise the DR aggregator's profit by a margin of up to 20%. Taking part in intraday DRX market can also increase the profit up to 10%. In this paper, of the level of DR participation is varied to study its impact on the aggregator's profit. As can be observed in Table III, the intraday DRX can motivate the aggregator to offer to the day-ahead energy market since compensation opportunity is available through the intraday market mechanism. As indicated in Table III, the increase in profit is saturated at about 20% of the participation level of DRPs. Because, this amount of DRPs is sufficient for the DR aggregator to update its initial offers.

The participation of DR aggregator in the electricity market is illustrated in Fig. 3. As can be seen from Fig. 3, the DR aggregator participates in the spinning reserve market more than the day-ahead energy market. Moreover, the aggregator offers to the DRX market more than the balancing market. Terms of the aggregator's profit are presented in Table IV. In this table, β is assumed zero while the participation level of DR is considered 0.2.

TABLE I
WILLINGNESS COEFFICIENT FOR DR SELLERS

DR sellers	1	2	3
θ	0.9	0.6	0.3

TABLE II
IMPACT OF VARIOUS MARKETS ON DR AGGREGATOR'S PROFIT AND RISK

Risk Level (β)	Electricity Markets			Expected Profit (€)	CVaR (€)
	Day-ahead Energy	Ancillary Services	Intraday DRX		
0	✓			9421.5	8861.7
	✓	✓		11231.1	10783.3
	✓	✓	✓	12356.4	11860.9
1	✓			9159.4	8989.8
	✓	✓		11036.4	10962.0
	✓	✓	✓	12068.3	12047.6

TABLE III
EFFECT OF DR PARTICIPATION LEVEL ON DR AGGREGATOR'S COSTS AND INCOMES

DR participation	0.0	0.1	0.2	0.3	0.4	0.5
Income from day-ahead energy (€)	4475	4978	5438	5676	5719	5726
Income from ancillary services (€)	7735	7179	8331	8659	8729	8729
Cost from DR sellers (€)	0	982	1237	1298	1305	1320
Expected Profit (€)	11231	11886	12356	12850	12944	13032

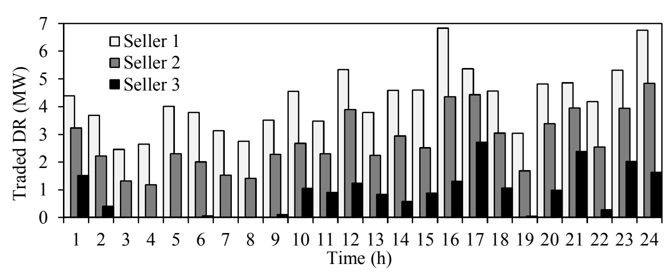

Fig. 2. Traded amount of DR in the DRX market.

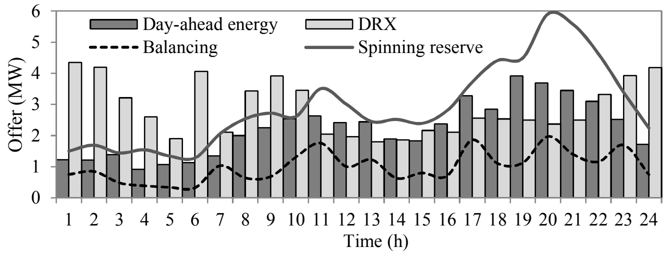

Fig. 3. DR aggregator's offer to the electricity markets.

TABLE IV
TERMS OF COST AND INCOME OF DR AGGREGATOR

Terms of profit	**Value (€)**
Income from day-ahead energy	5437.9
Income from balancing	858.6
Income from spinning reserve	7472.2
Positive imbalance income	277.9
Income from intraday market	2361.9
Cost from DR sellers	1236.6
Negative imbalance cost	498.1
Penalty as a result of not responding	272
Cost of responsive demands	2045.4
Expected profit	12356.4

V. CONCLUSION

This paper studied the effects of DR participation in different electricity markets on DR aggregator's performance. On this basis, the DR aggregator was modeled to participate in day-ahead energy, ancillary services, intraday DRX and balancing markets. Consumers' behavior was considered using an SFE-based technique to optimize their profits while the required DR was considered. Uncertainties related to market prices and the activated quantity of reserve were also considered through the roulette wheel mechanism. In addition, CVaR was employed to control the risk, enabling the DR aggregator to set the required tradeoff between profit and risk. Numerical results revealed that the formation of intraday DRX markets could bring opportunity for DR aggregators. Moreover, taking part in balancing and ancillary services could lead to another major opportunity. Participation in the mentioned markets could not only augment the DR aggregators' profit but also mitigate their risk.

ACKNOWLEDGMENT

The work of M. Shafie-khah, D. Z. Fitiwi and J.P.S. Catalão was supported by FEDER funds through COMPETE and by Portuguese funds through FCT, under FCOMP-01-0124-FEDER-020282 (Ref. PTDC/EEA-EEL/118519/2010) and UID/CEC/50021/2013, and also by the EU Seventh Framework Programme FP7/2007-2013 under grant agreement no. 309048.

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