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Cost-efficient Deployment of Storage Unit in Residential Energy Systems

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***Abstract*—With the mushrooming of distributed renewable generation, energy storage unit (ESU) is becoming increasingly important in residential energy systems. This letter proposes a fractional programming model to determine the optimal power and energy capacities of residential ESUs. The objective function maximizes the ratio between the reduced electricity tariff and the investment cost of ESU, ensuring the minimal payback time. A decomposition algorithm is developed to solve the fractional program based on convex optimization; the subproblem is a dual convex quadratic program which provides cutting planes, and the master problem comes down to a linear program after variable transformations. Compared to the widely used cost-minimum method, the proposed model is cost-efficient: it enjoys a higher rate of return which is usually welcomed by small consumers.**

***Index Terms*—cost-efficient investment, residential energy system, return on investment, sizing energy storage unit**

I. Introduction

THE penetration of renewable energy resources at demand side, such as rooftop photovoltaic panels, has witnessed rapid growth during the past decade. With the development of advanced metering and communication technologies, new pricing schemes emerge, for example, real-time pricing [1] and time-and-level-of-use pricing [2]. The new pricing policies encourage consumers to adjust their usage and reshape the load profile. To make full use of distributed renewable generation with limited controllability and time-varying price signals, energy storage unit (ESU) is in great need.

Since the capital cost of ESU is still relatively high compared to the daily electricity tariffs, deploying ESU is a long-term investment, and the capacity of the ESU should be carefully optimized. ESU sizing is a classical topic. Although various technical constraints have been considered and different optimization models formulated in existing works, a similar method is used to compromise the long-term investment cost and the short-term operational cost: a special weight coefficient is employed to aggregate the two costs into a single objective function to be minimized. The weight coefficients could be interpreted as net present values [3], annualized discounting cost [4], and other discounting factors [5]. Nonetheless, the accurate weight coefficient is not always easy to obtain and may depend on the subjective attitude of the decision-maker.

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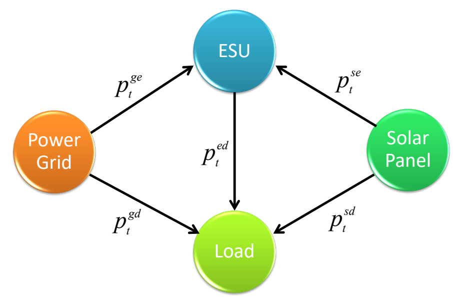

Fig. 1. Configuration of the residential energy system.

This letter proposes an alternative optimization paradigm that coordinates long-term and short-term costs without any manually supplied discounting parameters. The contribution of this work is twofold.

1) A cost-efficient optimization method is proposed to size the energy storage unit in residential energy systems. The ratio between the reduced short-term operation cost and the long-term investment cost is minimized, giving rise to a fractional program. Such a criterion ensures the shortest time of cost recovery, which is desired by small consumers with limited financial capability. Compared to the widely used cost-minimum approach, the proposed method does not require a discounting factor, and enjoys a higher rate of return, i.e., the investment is used more efficiently.

2) A decomposition algorithm is developed to solve the proposed fractional programming model, with the non-closed form optimal value function of the operation problem in the objective. Based on the convexity of the operation problem and pseudo-concavity of the fractional program, the fractional storage sizing problem is decomposed into a master problem and a subproblem. The subproblem generates cutting planes to approximate the optimal value function via solving a convex quadratic program (QP), and the master problem solves the fractional program as a linear program (LP) through an exact variable transformation. Such an algorithm overcomes the computational challenge brought by the non-convexity of the original fractional program.

II. Mathematical Model

The configuration of the residential energy system is shown in Fig. 1. The dynamic operation in periods $t = 1, \cdots, T$ with a step size of Δ_t is modeled. The load in period t is p_t^d;

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solar power p_t^V and import power p_t^g from the grid are used either to supply load or charge the ESU; ESU can also serve some load. The power flow variables and directions inside the system are depicted in Fig. 1, yielding:

$$p_t^g = p_t^{ge} + p_t^{gd},\ \forall t \tag{1}$$

$$p_t^V = p_t^{se} + p_t^{sd},\ \forall t \tag{2}$$

$$p_t^d = p_t^{gd} + p_t^{sd} + p_t^{ed},\ \forall t \tag{3}$$

The charging power of ESU is

$$p_t^{ec} = p_t^{ge} + p_t^{se},\ \forall t \tag{4}$$

and the operation constraints of ESU include

$$0 \le p_t^{ec} \le p_m^e,\ 0 \le p_t^{ed} \le p_m^e,\ \forall t \tag{5}$$

$$E_t = E_{t-1} + \eta_c p_t^{ec} \Delta_t - p_t^{ed} \Delta_t / \eta_d,\ \forall t \tag{6}$$

$$\alpha_l E_m \le E_t \le \alpha_h E_m, \forall t,\ E_0 = E_T = \alpha_l E_m \tag{7}$$

where p_m^e and E_m are power and energy capacities of ESU in kW and kWh, respectively; η_c/η_d is the charging/discharging efficiency; α_l/α_h is the minimum/maximum energy ratio; E_t is the amount of electrical energy stored in ESU at the end of period t. Constraints (5)-(7) stipulate charging/discharging power limits, storage dynamics, as well as energy limits and boundary conditions, respectively.

The time-and-level-of-use electricity price is [2]

$$\lambda_t = \lambda_t^0 + \xi p_t^e / 2, \tag{8}$$

where the base price $\lambda_t^0 = \lambda_l$ or λ_h during valley/peak hours. ξ is a constant, and the second term in (8) helps to prevent an excessive rise in demand when the base price drops down.

In summary, the daily economic operation of the residential energy system gives rise to a convex QP:

$$\begin{aligned} \min \quad & \lambda_l \sum_{t \in T_L} p_t^e \Delta_t + \lambda_h \sum_{t \in T_H} p_t^e \Delta_t + \frac{\xi}{2} \sum_{t=1}^{T} \left(p_t^e\right)^2 \Delta_t \\ \text{s.t.} \quad & (1)-(7),\ \text{variable lower and upper bounds} \end{aligned} \tag{9}$$

For notation brevity, problem (9) is written in a matrix form

$$\bar{v}(\theta) = \min \left\{ \bar{x}^\top \bar{Q} \bar{x}/2 + \bar{c}^\top \bar{x} \middle| \bar{A}\bar{x} \ge \bar{b} + \bar{B}\theta \right\} \tag{10}$$

where $\theta = [p_m^e, E_m]$ denotes capacity parameters; decision variable $\bar{x}$ includes power flow variables and ESU operating strategies; $\bar{Q}$, $\bar{A}$, $\bar{B}$, $\bar{b}$, and $\bar{c}$ are constant coefficients. For any given θ, the optimum is $\bar{v}(\theta)$.

When the uncertainty of solar generation and load is taken into account, the constraint right-hand term $\bar{b}$ is unknown. We select S typical days with probabilities ρ_s and construct scenarios $\bar{b}_s$, $s = 1:S$. Problem (10) in scenario s becomes

$$\bar{v}_s(\theta) = \min_{\bar{x}_s} \left\{ \bar{x}_s^\top \bar{Q} \bar{x}_s/2 + \bar{c}^\top \bar{x}_s \middle| \bar{A}\bar{x}_s \ge \bar{b}_s + \bar{B}\theta \right\} \tag{11}$$

The stochastic operation problem can be cast as

$$v_{av}(\theta) = \min \sum_{s=1}^{S} \rho_s \bar{v}_s(\theta) \tag{12}$$

Problem (12) can be written in a more compact form as

$$v_{av}(\theta) = \min \left\{ x^\top Q x/2 + c^\top x \middle| Ax \ge b + B\theta \right\} \tag{13}$$

where $x = [\bar{x}_1^\top, \cdots, \bar{x}_S^\top]^\top$; Q, A, B, b, and c aggregate the coefficients in each scenario.

III. Cost-Efficient Storage Sizing

In this section, we propose a new formulation for sizing the ESU and develop a decomposition algorithm to solve it.

A. Formulation of Cost-efficient Storage Sizing

The investment cost is an affine function in θ

$$C_{\text{invest}} = \kappa_p p_m^e + \kappa_e E_m + \kappa_0 = \kappa^\top \theta + \kappa_0 \tag{14}$$

where κ_0 is the fixed cost of deploying the ESU, representing the transportation cost and installation cost of facilities; κ_e is the unit capacity cost of battery array; κ_p is the unit capacity cost of power electronics convertors.

The average cost without/with ESU is $v_{av}(0)/v_{av}(\theta)$. Let $v_{av}^0 = v_{av}(0)$, $v_{av}^0 \ge v_{av}(\theta)$ must hold for $\theta > 0$. The cost-efficient sizing model aims to maximize the ratio between the reduced operation cost and the investment cost, giving rise to

$$\max_{\theta \ge 0} \ \frac{v_{av}^0 - v_{av}(\theta)}{\kappa^\top \theta + \kappa_0} \tag{15}$$

When $\theta = 0$, the objective value is equal to 0; when θ is large enough, $v_{av}(\theta)$ is a constant, because the excessive capacity is not used. In such a circumstance, the objective value is also very small due to the large investment cost C_{invest}, so the maximum exists. Suppose the optimum is σ^*, its multiplicative inverse $1/\sigma^*$ interprets the minimum payback time.

Two difficulties prevent problem (15) from being solved directly. One is the lack of an explicit expression for the value function $v_{av}(\theta)$; the other is the non-convexity of the fractional objective function. Furthermore, although the dimension of θ is low, problem (13) entailing massive historical weather and load data is non-trivial. The direct search method may not be a good option, because the repeated evaluation of function $v_{av}(\theta)$ is time-consuming.

B. Approximating the Optimal Value Function

To solve problem (15), we first discuss the approximation of value function $v_{av}(\theta)$. Write out the dual of QP (13) [6]

$$\begin{aligned} v_{av}(\theta) = \max_{\mu,\nu} \quad & -\mu^\top Q \mu/2 + (b + B\theta)^\top \nu \\ \text{s.t.} \quad & A^\top \nu - Q\mu = c,\ \nu \ge 0 \end{aligned} \tag{16}$$

The optimum of (16) is equal to $v_{av}(\theta)$ due to strong duality [6]. From (16), $v_{av}(\theta)$ is the pointwise maximum of infinitely many affine functions in θ. As pointwise maximum preserves convexity [7], $v_{av}(\theta)$ is a convex function in θ. Such a convex property plays an extremely important role in solving fractional program (15). As $-v_{av}(\theta)$ is concave, so is the numerator; the denominator is strictly positive and linear, and the feasible region is a polyhedron, so fractional program (15) is pseudo-concave [8], implying that any stationary point is the global maximum. Therefore, a local algorithm can be used to solve problem (15), once $v_{av}(\theta)$ can be approximated.

Given the convexity of $v_{av}(\theta)$, if we have enough sampling points θ_i, and (μ_i, ν_i) are the corresponding dual optimal solutions of (16), then $v_{av}(\theta)$ in (15) can be replaced by a scalar variable ζ while adding the following cutting planes

$$\zeta \ge -\mu_i^\top Q \mu_i/2 + (b + B\theta)^\top \nu_i,\ \forall i \tag{17}$$

in the constraints [7]. The graph of $v_{av}(\theta)$ and the cutting plane are tangent at θ_i. So $v_{av}(\theta)$ can be approximated by (17) around θ_i. Actually, we can update the sampling points θ_i dynamically via iteration, which will be discussed later.

C. Solving Storage Sizing Problem as a Linear Program

Once $v_{av}(\theta)$ has been approximated by cutting planes, we obtain the following problem

$$\begin{aligned} \max_{\theta,\zeta} \quad & \frac{v_{av}^0 - \zeta}{\kappa^\top \theta + \kappa_0} \\ \text{s.t.} \quad & \zeta \ge m_i + n_i^\top \theta, \forall i \\ & \theta \ge 0 \end{aligned} \tag{18}$$

where

$$m_i = b^\top v_i - \mu_i^\top Q \mu_i / 2, \ n_i = B^\top v_i \tag{19}$$

This problem can be solved by a local algorithm, thanks to the pseudo-concavity mentioned above. Nonetheless, it can be converted to an LP. Solving LPs is more tractable and robust than directly solving a non-convex optimization problem using a general-purpose nonlinear programming solver. To this end, define new variables

$$\bar{\theta} = \frac{\theta}{\kappa^\top \theta + \kappa_0}, \bar{\zeta} = \frac{\zeta}{\kappa^\top \theta + \kappa_0}, z = \frac{1}{\kappa^\top \theta + \kappa_0} \tag{20}$$

Then, the following relations hold

$$\kappa^\top \bar{\theta} + \kappa_0 z = 1, \ \theta = \bar{\theta}/z, \ \zeta = \bar{\zeta}/z \tag{21}$$

Because $\kappa^\top \theta + \kappa_0 > 0$, $0 < z < +\infty$, variable transformation (20) is invertible. On this account, linear fractional program (18) can be transformed to a linear program

$$\begin{aligned} \max_{\bar{\theta},\bar{\zeta},z} \quad & v_{\alpha v}^0 z - \bar{\zeta} \\ \text{s.t.} \quad & \bar{\zeta} \ge m_i z + n_i^\top \bar{\theta}, \forall i \\ & \kappa^\top \bar{\theta} + \kappa_0 z = 1 \\ & z \ge 0, \ \bar{\theta} \ge 0 \end{aligned} \tag{22}$$

The transition from (18) to (22) incurs no approximation, and the two problems have the same optimal value. The optimal sizing strategy θ can be recovered from the transformation in (21), based on the optimal solution $(\bar{\theta}, \bar{\zeta}, z)$ of problem (22).

D. The Decomposition Algorithm

Finally, a decomposition algorithm is developed. The master problem gives the optimal sizing strategy θ; the subproblem generates the cutting plane according to (16) and (17). The set of cutting planes are updated dynamically in the iterative procedure. The flowchart is given in Algorithm 1.

Convergence With more cutting planes added into problem (22), the feasible region shrinks, and the optimal value σ^* generated in step 2 is a decreasing sequence. Furthermore, the objective function is bounded, so Algorithm 1 must converge.

Efficiency The impact of sizing strategy θ on the operation cost is reflected by dual variables. Cutting plane (17) decomposes the large-scale operation problem in the dual form (16) and the master LP (22). Both problems can be readily solved, so Algorithm 1 is generally efficient.

Algorithm 1

1: Initiation: error tolerance $\varepsilon > 0$; σ_{past} is a big number. Solve problem (16) at sampled points θ_s; the optimizers are (μ_s, ν_s), $\forall s$; initiate the cutting plane set Γ.
2: Solve master problem (22) with the current Γ. The optimal solution is θ^*, and the optimal value is σ^*.
3: If $(\sigma_{\text{past}} - \sigma^*)/\sigma^* < \varepsilon$, report solution θ^* and terminate.
4: Solve subproblem (16) at θ^*; the optimizer is (μ^*, ν^*); create a cut according to (17); update the cutting plane set Γ and $\sigma_{\text{past}} = \sigma^*$; go to step 2.

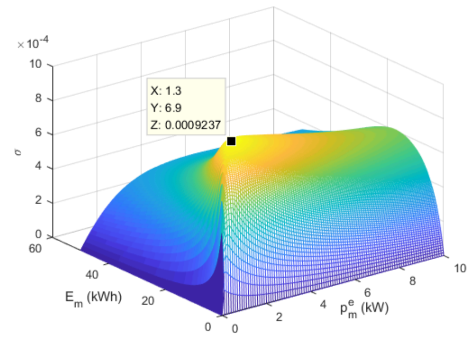

Fig. 2. The objective of problem (15) as a function of capacity parameters.

IV. Case Studies

The demand of a villa in Xining, Qinghai province of China and the solar radiation data at the same place are used in our tests. We select 56 typical days, two weeks in each season, and build the operation problem (13). In the time-and-level-of-use pricing scheme, λ_l, λ_h, and ξ are 0.5¥/kWh, 1.0¥/kWh, and 0.1¥/kWh^2, respectively, where ¥ denotes CNY. Investment parameters κ_p, κ_e, and κ_0 are 1000¥/kW, 800¥/kWh, and 1000¥, respectively. In Algorithm 1, 5×5 samples in $[0.5, 3]$kW$\times[3, 8]$kWh are chosen to generate initial cuts, and the convergence tolerance is set as $\varepsilon = 10^{-6}$.

First, we choose $\Delta t = 1$hour which is a typical setting. The algorithm converges in 4 iterations, reporting $p_m^e = 1.31$kW and $E_m = 6.95$kWh with the minimal payback time of 1082 days. For validation, the objective function of (15) is plotted in θ-plane with a resolution of 0.1×0.1. The optimal sizing strategy is found at $p_m^e = 1.3$kW and $E_m = 6.9$kWh with the same payback time. Nonetheless, this process is time-consuming, as evaluating $v_{av}(\theta)$ at each point entails solving problem (15), which is large in size. As pointed out in [9] and [10], the operation of a small residential energy system may desire a time step less than an hour, so we also test $\Delta t = 15$ minutes. The proposed method gives $p_m^e = 1.16$kW and $E_m = 7.10$kWh with the minimal payback time of 1111 days. We observed that with a finer time resolution, the value of v_{av}^0 decreases because the system operation becomes more flexible; as a result, the cost reduction brought by energy storage drops slightly, and thus the payback time is longer.

The impacts of the peak price λ_h, the coefficient κ_e, and the coefficient κ_p are investigated. Results are summarized in Tables I-III. For the same reason, the payback times with $\Delta t = 15$ minutes are slightly longer than those with $\Delta t = 1$

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TABLE I
IMPACT OF PEAK PRICE

λ_h (¥/kWh)		0.7	0.9	1.1	1.3	1.5
$\Delta t = $ 1hour	p_m^e (kW)	1.39	1.30	1.30	1.32	1.31
	E_m (kWh)	6.71	6.93	6.91	7.16	7.30
	T_P (days)	1327	1017	819	685	588
$\Delta t = $ 15min	p_m^e (kW)	1.19	1.14	1.23	1.20	1.28
	E_m (kWh)	6.76	6.99	7.59	7.36	7.73
	T_P (days)	1378	1042	832	692	593

TABLE II
IMPACT OF PARAMETER κ_e

κ_e (¥/kWh)		1300	1100	900	700	500
$\Delta t = $ 1hour	p_m^e (kW)	1.36	1.32	1.36	1.32	1.27
	E_m (kWh)	5.46	6.20	6.65	7.28	7.77
	T_P (days)	1542	1362	1177	986	789
$\Delta t = $ 15min	p_m^e (kW)	1.12	1.13	1.15	1.24	1.31
	E_m (kWh)	5.88	6.14	6.64	7.70	8.39
	T_P (days)	1599	1406	1210	1009	804

TABLE III
IMPACT OF PARAMETER κ_p

κ_p (¥/kW)		1300	1100	900	700	500
$\Delta t = $ 1hour	p_m^e (kW)	1.23	1.31	1.37	1.44	1.64
	E_m (kWh)	6.83	6.95	6.86	6.59	6.29
	T_P (days)	1133	1100	1064	1026	983
$\Delta t = $ 15min	p_m^e (kW)	1.10	1.19	1.21	1.29	1.42
	E_m (kWh)	6.98	6.70	7.24	7.02	7.06
	T_P (days)	1159	1127	1094	1059	1021

hour. It can be observed that the peak price λ_h significantly influences the payback time, but has little impact on storage sizing strategies. This is because of the cost-efficient criterion. When λ_h is higher, deploying a larger ESU can save more operation cost, but the investment cost grows at the same time, which may deteriorate the ratio in (15). Coefficient κ_e has tiny impact on the optimal power capacity p_m^e, yet notably affects the energy capacity E_m and payback time. The power capacity p_m^e exhibits a negative correlation with κ_p, whose impact on E_m and payback time T_P is not as significant as κ_e and λ_h.

Finally, the proposes cost-efficient model is compared with the widely used cost-minimum model. Recall the notations in the compact form (13), the cost-minimum model is cast as

$$\begin{aligned} \min \quad & \kappa^\top \theta + \kappa_0 + T_{lf} \cdot \left(\frac{x^\top Q x}{2} + c^\top x \right) \\ \text{s.t.} \quad & \theta \geq 0, \; Ax \geq b + B\theta \end{aligned} \tag{23}$$

where T_{lf} (in days) is the lifespan of the facilities. The cost-minimum model (23) endeavours to minimize the sum of the investment cost and the total operation cost during the service period. Problem (23) is a convex QP and is solved by CPLEX.

The cost-minimum method and the cost-efficient method are compared in terms of cost and profit; results are shown in Table IV. The cost-efficient model is independent of T_{lf}, so the sizing strategy and investment cost remain the same in the three instances. To reduce the total operation cost, which is $T_{lf} \cdot v_{av}(\theta)$, the cost-minimum method always suggests to build a larger ESU, leading to a higher investment cost C_{invest} as well as net profit defined as $T_{lf} \cdot [v_{av}^0 - v_{av}(\theta)]$. In contrast, although the proposed cost-efficient model recommends a smaller ESU and thus the operation cost is higher, the consumer can enjoy a higher rate of return, which is the ratio of net profit and C_{invest}, due to the lower investment cost. This feature is usually desired by small consumers who pursue fast cost recovery and a higher return on investment.

TABLE IV
COMPARISON OF COST-MINIMUM AND COST-EFFICIENT MODELS

Cost/Profit (¥)		T_{lf} (days) 1500	1800	2000
Cost-minimum	Investment cost	13727	16832	17861
	Operation cost	19480	19961	21082
	Total payment	33207	36793	38943
	Net Profit	3737	7540	10315
	Rate of return	27.2%	44.8%	57.8%
Cost-efficient	Investment cost	7926	7926	7926
	Operation cost	26255	31506	35006
	Total payment	34181	39432	42932
	Net Profit	2763	4901	6326
	Rate of return	34.9%	61.8%	79.8%

V. CONCLUSIONS

This letter proposes a cost-efficient optimization framework for storage sizing in residential energy systems. The ratio between cost reduction and investment is maximized, ensuring the minimum time of cost recovery. A decomposition algorithm is developed to solve the fractional programming model based on QP and LP solvers. Compared to the cost-minimum approach, the proposed cost-efficient approach requires a much lower budget at the beginning and leads to a higher rate of return, despite a slightly higher total payment in the long run.

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