Wind Power Forecasting Error Distributions and Probabilistic Load Dispatch

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***Abstract*—Among renewable power sources, wind energy is the most promising technology; however, the inter-temporal uncertainty of this source makes impossible its massive integration. Forecasting of wind generation is a key factor for the economical operation of the power system. Thus, the error related to this process is typically modeled by means of a determined probability distribution to be later incorporated to the unit scheduling and load dispatch optimization procedures. In this paper, wind power forecasting error has been modeled by using Weibull and Lévy α-stable probability distributions and incorporated to the economic dispatch problem in order to probabilistically describe power production and generating cost. The proposed methodology is illustrated by analyzing a case study composed by 13 conventional generators; the obtained results are compared with Monte Carlo Simulation approach for evaluating and testing the capabilities of the proposed model, observing reasonable accuracy on the estimated results.**

***Index Terms*— Economic dispatch, Forecasting error, Lévy α-stable distribution, Weibull distribution, Wind power.**

I. Introduction

Nowadays, sustainability of human activities and economic growth is a preoccupying and widely discussed topic. Thus, availability of energetic resources, environmental issues, and socio-economic development are going to define the power-generation mix in the future. As a representative example, it is expected that in United States more than 60% of power requirements being supplied by means of natural gas between 2025 and 2040, while most of the remainder is going to be supplied by renewable sources [1], hence optimal operation of power systems provided of renewable resources becomes a relevant problem for research purposes [2].

Currently, representation of stochastic behavior of renewable generation using probability theory has become a widely suggested technique to deal with uncertainty incorporated by renewable generation on load dispatch and unit scheduling problems. Specifically, two trends have been suggested in the literature: one based on stochastic analysis using synthetic scenario generation, and another one based on the incorporation of renewable power through a probability distribution on the operation problem. Results obtained from the first trend strongly depends on the scenarios under analysis and the technique used to generate and reduce them [3]; besides of this, only a limited number of situations can be considered due to the increment of the computational burden as the number of scenarios increases.

Regarding the second trend, many of the methodologies proposed under this framework have not been tested and proved, so that the obtained results are not provided of enough reliability to be implemented in realistic situations. Actually, this is a topic under development and several approaches have been presented in the literature; in [4], an economic load dispatch model able to consider wind generation as the combination of a Weibull distribution and a simplified power curve for the wind farm was proposed. In the optimization problem, overestimation and underestimation of forecasted wind power generation thorough penalty costs and spinning reserve provision were considered. Dependence of optimal power dispatch as a function of wind speed distribution factors, penalty factors, and reserve costs were analyzed by means of numerical analysis.

In [5] and [6], two different optimization models were developed based on here-and-now and wait-and-see concepts. Methodology used is similar to that presented in [4]; however, its stochastic model was expressed as a constraint in the optimization problem. Considering similar assumptions; in [7], a closed-form solution of load dispatch problem expressed through incomplete gamma function was derived and used to evaluate the impact of wind generation on oxides of nitrogen (NO_x) emissions. Alternatively, forecasting error of wind generation could be modeled by using a Beta distribution. Thus, following a similar methodology as that presented in [5-7], a dispatch model that incorporates wind generation as a Beta distribution in order to mitigate the effects of greenhouse gas emissions was presented in [8] and other recent method based on Beta distribution is shown in [9], respectively. To avoid the complexity of stochastic models; in [10], load dispatch was analyzed during a short time interval; hence, wind generation is represented through the mean wind speed and turbulence intensity.

This paper focus on developing a technique to solve load dispatch problem considering wind generation as a probability distribution in order to describe output power of conventional generators, wind power production, and generating costs from a probabilistic viewpoint. A pair of different probability distributions to represent forecasting error such as Weibull and Lévy α-stable is incorporated to probabilistic load dispatch optimization model. The paper is organized as follow: Section II describes representation of forecasting error and solution of load dispatch problem, while Section III briefly illustrates the performance of the proposed methodology through the analysis of a case study; and then, final conclusions and remarks are presented in Section IV.

II. PROBABILISTIC LOAD DISPATCH

In this section, the methodology employed to model wind power forecasting error as a discretized probability distribution and its corresponding inclusion on probabilistic load dispatch problem are briefly described.

A. Discretized Forecasting Error Distribution

Many probability distributions have been suggested in the literature to accurately representing wind power forecasting error for a wide range of power production. In general sense, probability distributions for wind power forecasting error are fitted by considering persistent forecast method. In [11], after applying a statistical methodology based on the kurtosis as the reference parameter, several one-year time series obtained from two different locations, measured with time intervals of 10 and 15 minutes were analyzed, showing the capabilities of Beta distribution.

In [12], information obtained from Electric Reliability Council of Texas of the year 2009, measurements in ten different wind farms were analyzed demonstrating the capabilities of Cauchy-Lorentz distribution to probabilistically represent forecasting error. Motivated by the effects of forecasting error on trading energy in electricity markets; in [13], using Western Wind Resources data sets with a time interval of 10 minutes, mixed probability distribution was suggested. Versatile distribution was proposed in [14], due to its flexibility to represent forecasting error at different time scales and error magnitudes; as well as, its capability to be analytically represented in terms of density and cumulative distribution functions. Recently, from the analysis of data provided by Belgian Transmission System Operator for the years 2012 and 2013, Lévy α-stable distribution has been suggested in [15].

As an effort to improve the proposed probabilistic optimization model presented in [9], Weibull and Lévy α-stable distributions have been incorporated to load dispatch problem; for this purpose, a discretization process of probability distribution under analysis is required; thus, the methodology used in this paper was initially proposed by Barbiero [16]. In this methodology, probability distribution of interest is discretized in several intervals selected in accordance with the accuracy level required; using a Gaussian distribution as a reference, and discretization of the interval $(-\tau,\ \tau)$ is carried out by means of (1),

$$\emptyset_l = -\tau - \frac{2\tau}{L-1} + \left(\frac{2\tau}{L-1}\right) l; \quad l = 1, \ldots, L, \tag{1}$$

where l is the index for each discretization interval, L is the maximum number of intervals, τ is a parameter to define extreme values of reference probability distribution, and $\emptyset_l$ is the value of corresponding discretization interval of reference distribution. In order to obtain the value of each interval that corresponds to the probability distribution to be discretized, in this study Weibull and Lévy α-stable distributions, a transformation process presented in (2) and (3) is applied,

$$\varphi_l = F_G(\emptyset_l); \quad l = 1, \ldots, L, \tag{2}$$

$$DFE_l^t = F_E^{-1}(\varphi_l); \quad l = 1, \ldots, L, \tag{3}$$

where $F_G(\cdot)$ is the cumulative distribution function (CDF) of a normalized Gaussian distribution, φ_l is an intermediate variable with uniform probability distribution function (PDF).

Furthermore, $\mathrm{DFE}_\mathrm{l}^\mathrm{t}$ is the interval l of discretized forecasting error at time t, and $\mathrm{F}_\mathrm{E}^{-1}(\cdot)$ is the inverse CDF of the probabilistic variable to be discretized (Weibull and Lévy α-stable distributions). During the probabilistic transformation, cumulative probabilities estimated in (2) are evaluated in the inverse CDF of the probability distribution to be discretized for each interval (l) and time step (t). Then, the central value of each $\mathrm{L}-1$ disjoint (ω_l) are estimated by using (4),

$$\omega_l = \frac{DFE_l^t + DFE_{l+1}^t}{2}; \quad l = 1, \ldots, L-1. \tag{4}$$

Finally, the corresponding probabilities of each discretized interval are estimated according to (5)-(7),

$$P_r\{FE^t = DFE_1^t\} = F_E(\omega_1); \quad l = 1, \tag{5}$$

$$P_r\{FE^t = DFE_l^t\} = F_E(\omega_l) - F_E(\omega_{l-1}); \ l = 2, \ldots, L-1, \tag{6}$$

$$P_r\{FE^t = DFE_l^t\} = 1 - F_E(\omega_{L-1}); \quad l = L, \tag{7}$$

where $\mathrm{P_r}\{\cdot\}$ is the probability of occurrence of a determined event, in our case $\mathrm{FE^t} = \mathrm{DFE_l^t}$; whereas, $\mathrm{FE^t}$ is the probabilistic variable to represent forecasting error. Equations (5)-(7) describe the probability that forecasted power, represented by the variable $\mathrm{FE^t}$ be equal to the corresponding discretized value $\mathrm{DFE_l^t}$; $\mathrm{l} = 1, \ldots, \mathrm{L}$.

B. Probabilistic Economic Dispatch

Ramp constraints of conventional generators are important limitation for integration of wind generation, because these restrictions reduce the flexibility of the power system to accommodate wind generation. These constraints combined to the inter-temporal correlation of wind speed and wind power directly influences economic dispatch problem. Typically, this characteristic is incorporated by simulating correlated scenarios of wind power, which makes the solution obtained dependent on the scenarios employed. To avoid this inconvenient, in this paper has used an analytical formulation based on discrete probability theory. The approach used considers the changes on output power of each generator between the previous time instant $\mathrm{t}-1$ and the current moment t. In this work, power generation of thermal units is modeled as discretized probability distributions for all generators at any time step t; in this sense, discretized PDF of power production of thermal generators at time $\mathrm{t}-1$ is assumed to be known.

It is important to note that computational complexity related to the incorporation of discretized PDF on dispatch problem depends on the number of units and the number of intervals of discretized distribution according to a potential law; in other words, the number of combinations to be analyzed equals the number of generators elevated to the number of bins used to discretize PDF of thermal power production, which makes the problem mathematically intractable. In order to avoid this problem, a limited amount of cases for power production is considered each other determined by using the quantile concept. For a determined generator $\mathrm{k}\ (\mathrm{k} = 1, \ldots, \mathrm{K})$; let $\mathrm{F}_{\mathrm{P,k}}^{-1}(\bullet)$ be inverse CDF of power generation at $\mathrm{t}-1$ for unit k; then, it is defined $\eta_\mathrm{m}\ (\mathrm{m} = 1, \ldots, \mathrm{M})$ as a value in the interval $[\eta^{\min}, \eta^{\max}]$ with $\eta^{\min} = \xi$ and $\eta^{\max} = 1 - \xi$, being ξ significance level; so that, the corresponding power generation at $t-1$ to be considered in load dispatch model and estimated by using (8):

$$P_{k,m}^{t-1} = F_{P,k}^{-1}(\eta_m); \quad m = 1, \dots, M; \quad k = 1, \dots, K, \qquad (8)$$

where variable m is the index for the combination of power production at $t-1$ being considered, and M is the amount of combinations, which is arbitrarily determined. It could be understood by analyzing an small example, assuming $M = 3$ and $\xi = 0.01$; then, $\eta^{min} = 0.01$ and $\eta^{max} = 0.99$. Hence, this interval is swept by using and step equal to 0.49 obtaining $\eta_1 = 0.01$, $\eta_2 = 0.5$, and $\eta_3 = 0.99$. Finally, power values to be considered on the optimization process is defined as $P_{k,1}^{t-1} = F_{P,k}^{-1}(0.01)$, $P_{k,2}^{t-1} = F_{P,k}^{-1}(0.5)$, and $P_{k,3}^{t-1} = F_{P,k}^{-1}(0.99)$.

For the computational implementation, all this information could be stored in a table with K rows, being K the amount of generators, and M columns; then, each column is considered as a possible starting point for each unit at $t-1$. As this is a probabilistically based simplification, a weighting factor (Ω_m) is associated with each column of the aforementioned table by means of (9),

$$\Omega_m = \frac{\prod_k \left(P_r\{P_k^{t-1} = P_{k,m}^{t-1}\}\right)}{\sum_m \prod_k \left(P_r\{P_k^{t-1} = P_{k,m}^{t-1}\}\right)}; \quad m = 1, \dots, M. \qquad (9)$$

Once the generating conditions at $\mathrm{t}-1$ have been defined, probabilistic load dispatch described in (10)-(15) can be solved. For each combination of indices l and m, optimization problem is solved; then, discretized probability distributions of conventional power generation, wind power curtailment, and generating cost can be built by considering power production at $t-1$ and wind generation as mutually exclusive events; so that, weighting factors of power production and discretized probabilities of forecasting error can be multiplied [9].

$$\theta_l^m = \sum_k \left\{\lambda_k + \mu_k\left(P_{k,m}^t\right) + \sigma_k\left(P_{k,m}^t\right)^2\right\} + VOLL(ENS_m^t), \quad (10)$$

$$\sum_k P_{k,m}^t + WG_l^t = L^t, \qquad (11)$$

$$P_{k,m}^t - P_{k,m}^{t-1} \le RU_k, \qquad (12)$$

$$P_{k,m}^{t-1} - P_{k,m}^t \le RD_k, \qquad (13)$$

$$P_k^{min} \le P_k^t \le P_k^{max}, \qquad (14)$$

$$0 \le WG_l^t \le DFE_l^t. \qquad (15)$$

In the optimization problem, λ_k, μ_k, and σ_k are parameters to describe fuel consumption cost, $VOLL$ is value of lost load, ENS_m^t is the energy not supplied obtained from the analysis of the interval m, WG_l^t is wind power generation consumed by the system when the value of forecasted generation (DFE_l^t) of interval l occurs; RU_k and RD_k are ramp-up and ramp-down limits of unit k, respectively; while, P_k^{min} and P_k^{max} are minimum and maximum output power, respectively; while L^t is the load demand at time t.

III. CASE STUDY

The methodology described in Section II is illustrated by analyzing a case study of 13 thermal units ($K = 13$), whose data have been taken from [17] without valve point. As stated before, Weibull and Lévy α-stable distributions to model forecasting error have been considered, and the obtained results were compared with Monte Carlo Simulation (MCS) approach. Discretized PDF of each unit k at $t-1$ was obtained by MCS approach; thus, three time steps, namely, $t-2$, $t-1$, and t were considered. Then, power production of each unit at $t-2$ was assumed to be deterministic; while at $t-1$ and $t-2$, discretized PDF of power generation were obtained by considering random values of wind power generation. Finally, results obtained at $t-1$ were used as input to the proposed methodology (probabilistic variable $F_{P,k}$) and results at t were used as a comparison point.

Load demand to be supplied is assumed to be 2600 MW. The number of combinations of power generation at $t-1$ of thermal generators considered was adjusted to 3 ($M = 3$) in order to reduce the computational burden. Significance level of probabilistic analysis was adjusted to 0.01 ($\xi = 0.01$). MCS method and proposed approach were both implemented in MATLAB language.

A. Weibull distribution

In this sub-section Weibull distribution is analyzed by applying the methodology aforementioned in Section II, in order to determine load dispatch under uncertain conditions related to wind power forecasting error. This distribution is characterized by means of two parameters: scale (ρ) and shape (ε) factors, respectively [5-7]. In this case study, $\rho = 350$ MW and $\varepsilon = 2.5$ have been considered.

Regarding the discretization process described in Sub-section II-A, it was carried out by considering power values between 0 MW and 800 MW divided in 150 intervals; while, discretized PDF of dispatched wind power generation (WG^t) was built by considering 200 intervals. As stated before, MCS approach was used as a comparison point with 2500 trials.

Fig. 1 shows discretized PDF obtained from the proposed and MCS approaches for power production of unit 9. As can be observed, variations on wind generation as a consequence of forecasting error results are compensated by this generator in order to operate the system at minimum generating cost. Besides of this, a reasonable performance of the proposed method is appreciated.

Figs. 2 and 3 show the behavior of generating cost and dispatched wind generation, respectively. In Fig. 3, it is possible observe how all available wind generation is consumed by the system, which clearly influences the PDF of generating cost (Fig. 2). Moreover a reasonable agreement between PDF obtained from proposed method and MCS approach is observed. Finally, Table I shows the comparison of expected value obtained from MCS approach and proposed methodology of power production and generating cost. As can be observed, results offered by proposed method have good accuracy.

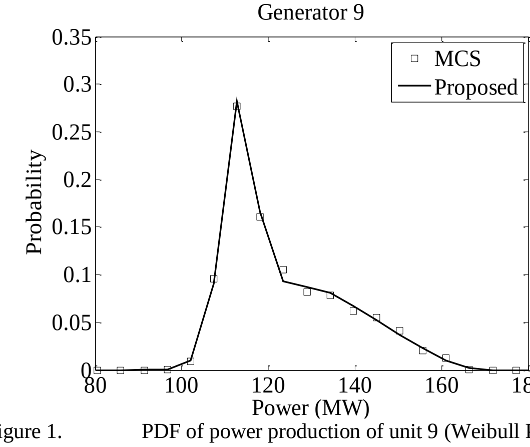

Figure 1. PDF of power production of unit 9 (Weibull PDF).

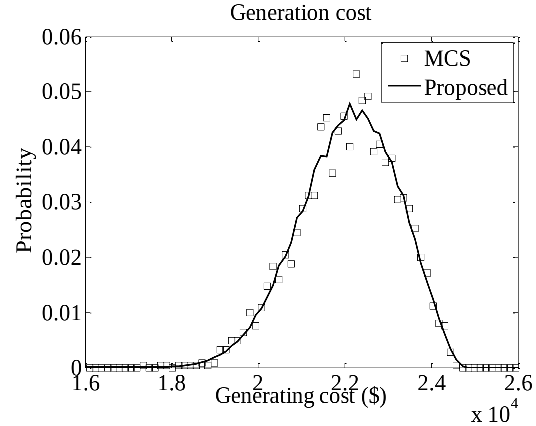

Figure 2. PDF of generating cost (Weibull PDF).

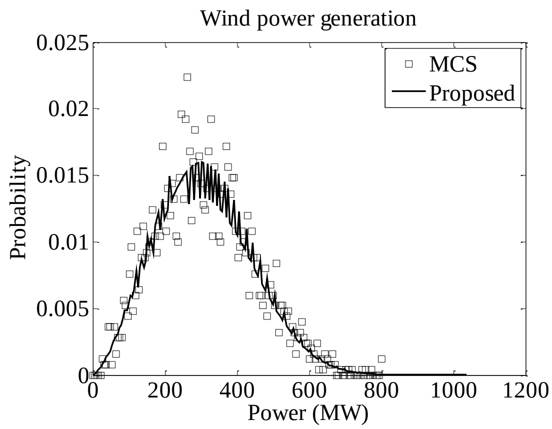

Figure 3. PDF of wind generation (Weibull PDF).

TABLE I. POWER GENERATION AND COST (WEIBULL PDF)

k	MCS	Proposed
1	669.102819	668.479624
2	345.915705	345.217516
3	345.911409	345.215761
4	123.852886	123.600089
5	123.857181	123.600041
6	123.857181	123.606209
7	123.857181	123.619028
8	123.857181	123.611949
9	123.859329	123.634523
10	37.583893	37.583898
11	37.583893	37.583898
12	53.691275	53.691275
13	53.691275	53.691275
$E\{WG^t\}$	307.307383	310.516006
$E\{\theta^t\}$	22097.003651	22068.248662

B. *Lévy α-stable Distribution*

In this Sub-section, incorporation of Lévy α-stable distribution on load dispatch problem is analyzed. This distribution is mathematically represented by using four parameters: index of stability (α), skewness factor (β), scale factor (γ), and location factor (δ), respectively [15]. Specifically, in this case study are considered $\alpha = 1.5$, $\beta = 0.8$, $\gamma = 10$, and $\delta = 350$ to model forecasting error. MCS approach was carried out by considering 10000 trials, while parameters related to discretization process were adjusted by assuming power values between 0 MW and 1000 MW divided in 150 intervals.

Furthermore, discretized PDF of dispatched wind power generation (WG^t) was built by considering 200 intervals. Fig. 4 shows discretized PDF of output power of generators 3. As wind generation has important uncertainty degree, it is possible observing how these units respond with accordance to forecasting error PDF. Besides of this, good agreement between both methods is observed. Figs. 5 and 6 and 7 present discretized PDF of generating cost and wind generation, respectively. From these results, it is possible observe how PDF of wind power forecasting error directly influences PDF of generating cost.

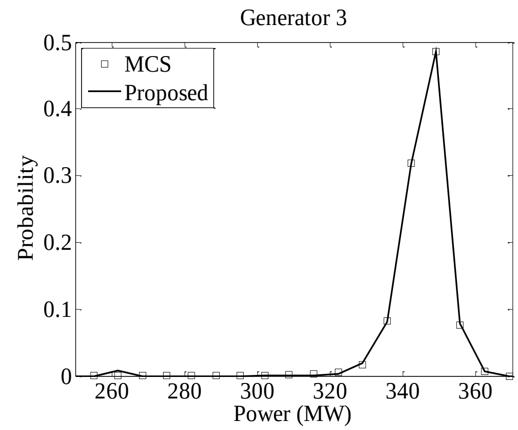

Figure 4. PDF of power production of unit 3 (Lévy α-stable PDF).

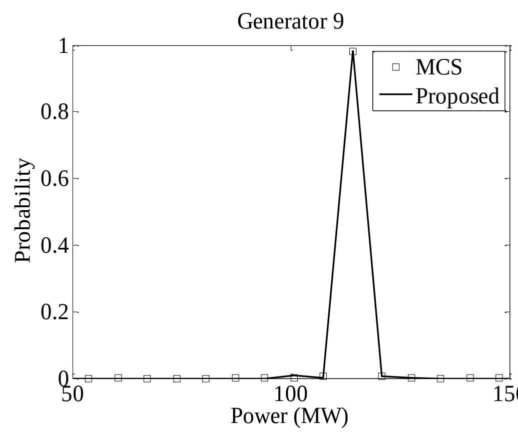

Figure 5. PDF of power production of unit 9 (Lévy α-stable PDF).

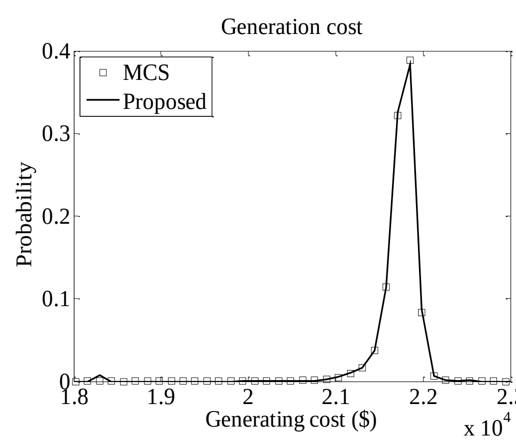

Figure 6. PDF of generating cost (Lévy α-stable PDF).

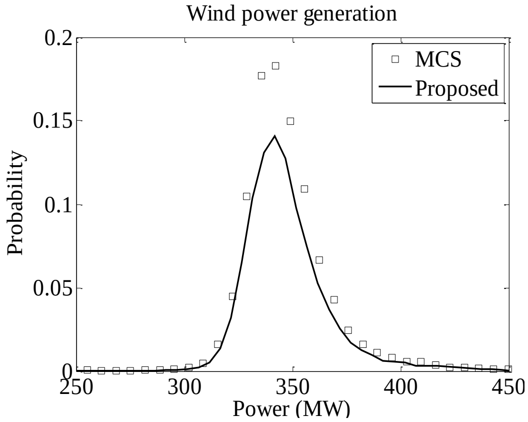

Figure 7. PDF of wind generation (Lévy α-stable PDF).

In Table II, is presented the corresponding comparison of expected value between both methods, where the accuracy level of the obtained results can be easily evaluated; observing the reasonable good capabilities of the proposed methodology.

TABLE II. POWER GENERATION AND COST (LÉVY ALPHA-STABLE)

k	MCS	Proposed
1	676.110738	675.902543
2	345.322819	345.267810
3	345.322819	345.267810
4	114.089262	114.060298
5	114.091275	114.062163
6	114.089933	114.062163
7	114.091275	114.062163
8	114.089262	114.060298
9	114.089933	114.060298
10	40.276510	40.268456
11	40.276510	40.268456
12	53.697987	53.691275
13	53.697987	53.691275
$E\{WG^t\}$	349.367785	350.287439
$E\{\theta^t\}$	21731.134779	21726.008146

IV. CONCLUSIONS

Motivated by the growing integration of wind power generation for power production, probabilistic approaches for optimal load dispatch are becoming a necessity in order to operate power systems in an economical and reasonable way. In this sense, this paper introduced a probabilistic approach to determine PDF of power generation and total cost using discrete probability theory. In the proposed approach, Weibull and Lévy α-stable distributions to model wind power forecasting error have been analyzed by comparison with MCS approach, observing good agreement between both methodologies.

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REFERENCES

[1] J. J. Conti, P. D. Holtberg, J. R. Diefenderfer, S. A. Napolitano, A. M. Schaal, J. T. Turnure, L. D. Westfall, “Annual energy outlook 2015 with projections to 2040,” *U. S. Energy Information Administration*, Apr. 2015.

[2] P. Higgins, A. M. Foley, R. Douglas, K. Li, “Impact of offshore wind power forecast error in a carbon constraint electricity market,” *Energy*, vol. 76, pp. 187-197, Nov. 2014.

[3] E. M. Constantinescu, V. M. Zavala, M. Rocklin, S. Lee, M. Anitescu, “A computational framework for uncertainty quantification and stochastic optimization in unit commitment with wind power generation,” *IEEE Trans. Power Syst.*, vol. 26, pp. 431-441, Feb. 2011.

[4] J. Hetzer, D. C. Yu and K. Bhattarai, “An economic dispatch model incorporating wind power,” *IEEE Trans. Energy Convers.*, vol. 23, pp. 603-611, June 2008.

[5] X. Liu and W. Xu, “Economic load dispatch constrained by wind power variability: A here-and-now approach,” *IEEE Trans. Sustain. Energy*, vol. 1, pp. 2-9, Apr. 2010.

[6] X. Liu, “Economic load dispatch constrained by wind power availability: A wait-and-see approach,” *IEEE Trans. Smart Grid*, vol. 1, pp. 347-355, Dec. 2010.

[7] X. Liu and W. Xu, “Minimum emission dispatch constrained by stochastic wind power availability and cost,” *IEEE Trans. Power Syst.*, vol. 25, pp. 1705-1713, Aug. 2010.

[8] X. Liu, “Impact of Beta distributed wind power on emission mitigation dispatch,” *in: Proceedings of Power and Energy Society General Meeting*, IEEE-Press, pp. 1-6, July 2010.

[9] G. J. Osório, J. M. Lujano-Rojas, J. C. O. Matias, J. P. S. Catalão, “A probabilistic approach to solve the economic dispatch problem with intermittent renewable energy sources,” *Energy*, vol. 82, pp. 949-959, Mar. 2015.

[10] S. Roy, “Inclusion of short duration wind variations in economic load dispatch,” *IEEE Trans. Sustain. Energy*, vol. 3, pp. 265-273, Apr. 2012.

[11] H. Bludszuweit, J. A. Domínguez-Navarro, A. Llombart, “Statistical analysis of wind power forecast error,” *IEEE Trans. Power Syst.*, vol. 23, pp. 983-991, Aug. 2008.

[12] B.-M. Hodge, M. Milligan, “Wind power forecasting error distributions over multiple timescales,” *in: Proceedings of Power and Energy Society General Meeting*, IEEE-Press, pp. 1-8, July 2011.

[13] S. Tewari, C. J. Geyer, N. Mohan, “A statistical model for wind power forecast error and its application to the estimation of penalties in liberalized markets,” *IEEE Trans. Power Syst.*, vol. 26, pp. 2031-2039, Nov. 2011.

[14] Z.-S. Zhang, Y.-Z. Sun, D. W. Gao, J. Lin, L. Cheng, “A versatile probability distribution model for wind power forecast errors and its application in economic dispatch,” *IEEE Trans. Power Syst.*, vol. 28, pp. 3114-3125, Aug. 2013.

[15] K. Bruninx, E. Delarue, “A statistical description of the error on wind power forecasts for probabilistic reserve sizing,” *IEEE Trans. Sustain. Energy*, vol. 5, pp. 995-1002, July 2014.

[16] A. Barbiero, “A general discretization procedure for reliability computation in complex stress-strength models,” *Math. Comput. Simulat.*, vol. 82, pp. 1667-1676, May 2012.

[17] L. d. S. Coelho, V. C. Mariani, “Combining of chaotic differential evolution and quadratic programming for economic dispatch optimization with valve-point effect,” *IEEE Trans. Power Syst.*, vol. 21, pp. 989-996, May 2006.