Robust Probabilistic Load Flow in Microgrids considering Wind Generation, Photovoltaics and Plug-in Hybrid Electric Vehicles

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***Abstract*—The power demand uncertainties and intrinsic intermittent characteristics of wind and photovoltaic (PV) distributed energy resources (DERs) make the conventional load flow methods inefficient in active distribution networks (ADNs) and microgrids. Some statistical tools such as Monte Carlo simulation (MCS) are always a reliable solution. However, statistical tools are time-consuming and rather useless in large power systems. In this paper, a new method is proposed for robust probabilistic load flow (PLF) in microgrids and ADNs, including renewable energy resources (RERs), based on singular value decomposition (SVD) unscented Kalman filtering. The probability density functions (PDFs) and cumulative distribution functions (CDFs) for some of the ADN variables are compared with the other reported PLF methods for different test systems and the results validate the robustness, efficiency and accuracy of the proposed method.**

***Keywords*—Distribution networks, microgrids, distributed energy resources, Probabilistic load flow.**

I. Nomenclature

$[P]_{abc}$, $[Q]_{abc}$	Three phase real/reactive powers vector, p.u.
$[P]_{abc}$, $[Q]_{abc}$	Three phase real and reactive powers, p.u.
$[V]_{abc}$, $[\delta]_{abc}$	Magnitude/phase angle of three-phase voltage vector, p.u. rad.,
G_{ING} , G_{STC}	Irradiance of the sun/irradiance at standard test condition, W/m^2
$H_k^i\, y_k^i$, , K_k^i	i^{th} column of H_k , y_k , K_k
k	Maximum power temperature coefficient
N	Number of Buses
N_{WT}, N_{PV}	Number of WT in a wind farm and number of PV units
P_D, Q_D	Vector of load active / reactive powers, p.u.
P_{DER}, Q_{DER}	Vector of DERs active/reactive powers, p.u.
PG_{DC} , PD_{DC}	Real power generation/demand at DC bus
P_i , Q_i	Net injected active/reactive powers to i^{th} bus, p.u.
P_L	Load Power, p.u.
P_{PV}	Output power of PV module at irradiance G_{ING}, MW
P_{rs}	Rated power of PV unit, MW
P_{slack}, Q_{slack}	Slack bus generator active/reactive powers, p.u
P_{STC}	Rated power of PV generation at standard test condition, MW
$P_{WT}(v)$, $Pr,_{WT}$	Power generated by WT at wind speed v, Real power of WT, MW
r	Solar irradiation, W/m^2
R_c	Certain radiation point, set to 150 W/m^2
R_{STD}	solar radiation in the standard radiation, set to 1000 W/m^2
T_r	PV cell temperature, °C
v	Wind speed, m/s
$V_{cut\text{-}in}$, $V_{cut\text{-}out}$	Low and high cut speed of WT, m/s
V_i , δ_i	Voltage magnitude/phase angle at i^{th} bus, p.u., rad
$V_i^{min\text{-}PQ}$, $V_i^{min\text{-}PQ}$	Minimum / maximum limit of voltage at P-Q bus i, p.u
v_k, w_k	Process and measurement noise
v_r	Rated speed of WT, m/s
x_k, u_k, y_k	State I/O vectors ($x_k \in \Re^n, u_k \in \Re^r, y_k \in \Re^p$)
x_k^i , P_k^i	i^{th} circle of the state vector x_k and state covariance matrix P_k
YB	Elements of the Y_{BUS} matrix corresp. to DC buses
YB_{abc}	Elementary matrices of the three-phase Y_{BUS} matrix
Y_{ij} , θ_{ij}	Magnitude of the ij^{th} element of Y_{Bus} matrix
α , ρ	Shape parameter/scale parameter of Weibull PDF
α_β, β_β	Beta distribution shape factor, W/m^2
ε_ℓ^j	Error index of random variable j for the statistical characteristic ℓ.
μ, σ	Mean value/standard deviation of the normal distribution

II. Introduction

Grid complexity is increasing with the growth of DERs such as with technological innovations for photovoltaic (PV) power plants, wind generation (WG) farms, and unpredictable loads such as plug-in hybrid electric vehicles (PHEVs) [1-4].

With limited control, little peak power curtailment or reactive power control, and low capacity factors, DERs could adversely affect power quality, generation efficiency or impact existing infrastructure capacity limits and reliability [4, 5].

Renewable energy resources (RERs) are often probabilistic, with regional, seasonal and daily variations, complicating operational grid control, and requiring increased power system reserve]6-10]. On the one hand, the correlation among WG and PV sites influences the power system reliability and security. On the other hand, the reliability criterion power system affects the loads and this issue will also lead to another correlation among loads and RERs.

Moreover, the state of the system has a high degree of uncertainty since the elements of power systems are not 100% reliable [9, 11]. Thus, deterministic load flow (DLF) algorithms may not be efficient and could not exactly disclose states of the system. To handle with this dilemma, probabilistic load flow (PLF) is introduced [12].

On account of the necessary extremely large computational effort, it is practically impossible or at least cumbersome to perform load flow for every probable (possible) combination of the loads, generations, and network topology. Accordingly, the current researches focus to elaborate PLF methods with an acceptable level of accuracy and reasonable computation burden. Several methods have been proposed for quantifying uncertainties in the power system [11-15].

PLF methods are mainly classified into three main categories: simulation, approximation [13,14] and analytical methods [12]. Monte Carlo simulation (MCS) is commonly recognized as a system-dimension independent method and can provide accurate results. It calculates the load flow equations for a large number of iterations to follow the most possibilities that random variables attribute to reach a PDF or CDF for output random variables. However, it needs massive extremely time-consuming computation burden which makes it unattractive for real-time applications.

Combined the concept of Cumulants and Gram–Charlier expansion theory was used to determine the PDF of transmission line flows for the PLF analysis [12]. This method is straight-forward; however, it may have some cumbersome in the systems with the high number of correlations. Generally, point estimate method (PEM) [13], and as a special case, two-point estimate method (2PEM) , 2n+1 PEM and fast point estimate method (FPEM) which are all computationally rapid and are simply applied to PLF problem using the knowledge of first few statistical moments of inputs [14].

However, PEMs do not provide the PDF for the results, and they hold a low accuracy to provide higher statistical moments. Also, their computation time (CT) is relatively high which makes them impractical in large-scale networks and they are not efficient for ADNs with correlated RERs [6]. On the contrary, analytical methods are based on the mathematical formulations and complicated algorithms which may need mathematical simplifications and assumptions in the load flow nonlinear equations that may decrease accuracy.

Nataf transformation [15], and nonlinear transformation of mean and covariance and unscented transformation (UT) have been proposed. However, the accuracy depends on the ability of optimization algorithm to escape from the local minimum and trap the problem to the global optimum point [16].

In this work, a new PLF method is proposed, using a SVD-based unscented Kalman filter (UKF) for microgrids and ADNs including correlated WG/PV/PHEV units. The load demand and generated power of RERs are modeled by PDFs considering Weibull and normal distribution for wind speed and solar irradiation. The proposed method is compared with the MCS, and approximate with analytical and heuristic PLF algorithms for a test microgrid system to validate the proficiency and robustness. The rest of this work is organized as follows. The basic models for loads and RERs are described in Section III. Sections IV, and V elaborate the formulations of the PLF problem and SVD/UKF-based algorithm, respectively. The simulation results are in Section VI, while Section VII presents the main conclusions.

III. Modelling of Microgrids Uncertain Components

A. Probabilistic Load Modelling

With the statistical analysis of the changes, the load can be described by a PDF. Several models such as Weibull, Beta, and normal PDF are proposed. In this work, the load is modeled with normal distribution function as]16]:

$$f(P_L)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(P_L-\mu)^2}{2\sigma^2}\right) \tag{1}$$

B. Probabilistic Model of DER Units

The output generation of the RERs is a function of the availability of the resources, failure rate, mean time between failures (MTBF), and mean time to repair (MTTR) [10].

1) Probabilistic Model of WG

The Weibull distribution is used to represent the PDF of wind speed variations as]17]:

$$f_v(v)=\begin{cases}\dfrac{\rho}{\alpha}\times\left(\dfrac{v}{\rho}\right)^{\rho-1}\times\exp\left(\left(\dfrac{v}{\alpha}\right)\times\rho\right) & v\geq 0\\ 0 & otherwise\end{cases} \tag{2}$$

where the real power generated by wind turbine (WT) is:

$$P_{WT\ v}(v)=\begin{cases}N_{WT}\,P_{r,WT}\times\left(\dfrac{v-v_{cut-in}}{v_r-v_{cut-in}}\right) & v_{cut-in}\leq v\leq v_r\\ N_{WT}\,P_{r,WT} & v_r\leq v\leq v_{cut-out}\\ 0 & otherwise\end{cases} \tag{3}$$

2) Probability Model of PV

Besides the availability of the RERs, failure rate, MTBF, and MTTR, the power generated by PV unit depends on solar irradiation and temperature [10]. The solar irradiation can be modeled with a Beta distribution function as [16]:

$$f(r:\alpha_\beta,\beta_\beta)=\frac{\Gamma(\alpha_\beta+\beta_\beta)}{\Gamma(\alpha_\beta)\Gamma(\beta_\beta)}r^{\alpha_\beta-1}(1-r)\beta_\beta \tag{4}$$

The output power of PV system is expressed as [19]:

$$P_{PV}(r)=\begin{cases}N_{PV}\,P_{rs}\times\left(\dfrac{r^2}{R_{STD}\,R_C}\right) & 0\leq r\leq R_C\\ N_{PV}\,P_{rs}\,\dfrac{r}{R_{STD}} & R_C\leq r\leq R_{STD}\\ N_{PV}\,P_{rs} & r\geq R_{STD}\end{cases} \tag{5}$$

Normal distribution can be also used to model irradiance and air temperature. The module power is calculated [20]:

$$P_{PV} = N_{PV} P_{STC} \times \frac{G_{ING}}{G_{STC}} \times \left(1 + k\left(T_c - T_r\right)\right) \tag{6}$$

where in standard condition, G_{ING} = 1000 W/m^2, T_r = 25 ºC

C. Demand/Supply Probabilistic Model of PHEVs

Considering multiple PHEVs being (dis)charged as a stochastic phenomenon, the demand/supply model of plug-in PHEVs' is expressed. The queuing theory described in [21] is used for EVs' demand/supply of *c* PHEVs being charged at charging station.

In this work, for multiple PHEV probabilistic modeling is used and a probabilistic distribution is presented for PHEVs appearance at charging stations. In addition, PHEVs are considered to follow binomial distribution indicating that consume energy from the grid 90% of times and deliver their surplus charge to the grid 10% of times.

IV. Formulation of Probabilistic Load Flow

A. DPF Formulation

The load flow problem is implemented on the basis of the equations of (7) and (8) with inequality constraint considering maximum and minimum allowable limits for bus voltages at P-Q buses, and reactive power production of generating units at P-V buses as [22-27]:

$$\begin{aligned} P_i &= [PG_{abc}]_i - [PD_{abc}]_i = \sum_{j=1}^{NAC} |V_i||V_j||Y_{ij}|\cos\left(\delta_i - \delta_k - \theta_{ij}\right) = \\ &\operatorname{Re}\left\{[V_{abc}]_i \left(\sum_{j=1}^{NAC} [YB_{abc}]_{ij} [V_{abc}]_j\right)^*\right\} \end{aligned} \tag{7}$$

$$\begin{aligned} Q_i &= [QG_{abc}]_i - [QD_{abc}]_i = \sum_{j=1}^{NAC} |V_i||V_j||Y_{ij}|\sin\left(\delta_i - \delta_k - \theta_{ij}\right) = \\ &\operatorname{Im}\left\{[V_{abc}]_i \left(\sum_{j=1}^{NAC} [YB_{abc}]_{ij} [V_{abc}]_j\right)^*\right\} \end{aligned} \tag{8}$$

$$V_i^{\min-PQ} \le V_i^{PQ} \le V_i^{\max-PQ} \tag{9}$$

$$Q_i^{\min-PV} \le Q_i^{PV} \le Q_i^{\max-PV} \tag{10}$$

These equations are used in the three-phase AC load flow. In the microgrids that have both AC and DC sections, when DC network sections are accommodated, the power balance equation is used as follow [26]:

$$P_{DC,i} = [PG_{DC}]_i - [PD_{DC}]_i = V_{DC,i} \sum_{j=1}^{NDC} \left(YB_{ij} . V_{DC,j}\right) \tag{11}$$

$$\begin{bmatrix} (PG-PD)_{AC} \\ (QG-QD)_{AC} \\ \cdots \\ (PG-PD)_{DC} \end{bmatrix} = \begin{bmatrix} V_{AC} \\ V_{AC} \\ \cdots \\ V_{DC} \end{bmatrix} \left(\begin{bmatrix} YBUS_{AC} & \vdots & \\ \cdots & \cdots & \cdots \\ & \vdots & YBUS_{DC} \end{bmatrix} \begin{bmatrix} V_{AC} \\ V_{AC} \\ \cdots \\ V_{DC} \end{bmatrix} \right)^* \tag{12}$$

B. Probabilistic Load Flow

Due to the abovementioned uncertainties in the microgrid system, the main goal of the PLF analysis is to express the state of the microgrid as a function of uncertain input variables, where input vector X consists of load, network conditions, states of generating units and power generated:

$$Y = f(X) \tag{13}$$

$$X = \begin{bmatrix} P_D & Q_D & P_{DER} & Q_{DER} & \cdots \end{bmatrix}^T \tag{14}$$

$$Y = \begin{bmatrix} V & \delta & P_{slack} & Q_{slack} & \cdots \end{bmatrix}^T \tag{15}$$

C. Correlation Between Uncertain Variables

Correlation of wind speeds at nearby wind farms leads with a variation in power generation and thus the power transmitted through the lines and loads in nearby buses might be severely affected by these wind farms [17].

This concept is mathematically expressed by correlation coefficient matrix as:

$$\rho_{x,y} = \frac{\operatorname{cov}(x,y)}{\sigma_x \sigma_y} = \frac{E\left[(x-\mu_x)(y-\mu_y)\right]}{\sigma_x \sigma_y}, x, y = 1,2,\ldots,n \tag{16}$$

For the perfect positive and negative linear relationships, the correlation coefficients are +1 and -1, respectively, and for other cases, it has values between -1 and -1 [28].

V. Proposed PLF Method

A. Improved Unscented Transformation

MSC method can reach accurate results; however, it is computationally-intensive and thus, it is not recommended for online applications. The 2PEM method decomposes (13) into several sub-problems by taking only two deterministic values of each uncertain variable located on the two sides of its mean value. So, keeping the other variables at their mean values, the DPF is run twice for each uncertain variable.

Then, each set of selected sample points is mapped by PLF equations to obtain the transformed sample points. Since the 2PEM assumes that all input variables are independent, it is not suitable to cope with correlated variables. Assuming X as a vector of n-dimensional random variables with mean m and covariance P_x, the random variable Y relates to X through a nonlinear function f similar to (13), where f can be a set of nonlinear functions. To obtain the statistics of Y, a matrix χ of $2L$+1 sigma vectors χ_i (with weights W_i):

$$\begin{aligned} \chi_0 &= m \\ \chi_i &= m + \left(\sqrt{(L+\lambda)P_x}\right)_i \quad i = 1,\cdots,L \\ \chi_i &= m + \left(\sqrt{(L+\lambda)P_x}\right)_{i-L} \quad i = L+1,\cdots,2L \end{aligned} \tag{17}$$

$$\begin{aligned} W_0^{(m)} &= \lambda/(L+\lambda) \\ W_0^{(c)} &= \lambda/(L+\lambda) + \left(1 - \alpha^2 + \beta\right) \\ W_0^{(m)} &= W_0^{(c)} = 1/\left(2(L+\lambda)\right) i = 1,\cdots,2L \end{aligned} \tag{18}$$

where $\lambda = \alpha^2(L+\kappa) - L$ is a scaling parameter. α determines the spread of the S*igma* points around m and it is usually set to a small positive value (e.g., 10^{-3}). κ is a secondary scaling parameter which is usually set to 0, and β is used to incorporate prior knowledge of the distribution of (for Gaussian distributions, $\beta = 2$ is optimal). The sigma vectors are propagated through the nonlinear function (13) as:

$$y_i = f(\chi_i) \; i = 0,\cdots,2L \tag{19}$$

$$\bar{y} \approx \sum_{i=0}^{2L} W_i^{(m)} y_i \tag{20}$$

$$P_y \approx \sum_{i-0}^{2L} W_i^{(c)} \left\{y_i - \bar{y}\right\}\left\{y_i - \bar{y}\right\}^T \tag{21}$$

B. Unscented Kalman Filter Algorithm (UKF)

For nonlinear system with addictive noises, it is considered:

$$\begin{cases} x_k = f(x_{k-1}, u_k) + v_k \\ y_k = Hx_k + w_k \end{cases} \tag{22}$$

$$\begin{aligned} &E[v_k] = 0, E[w_k] = 0, \\ &E[w_k w_j^T] = R_k \delta_{kj}, \\ &E[v_k v_j^T] = Q_k \delta_{kj}, \end{aligned} \tag{23}$$

Here, v_k, w_k and x_k are not correlative to each other. Nominal UKF algorithm for the associated noisy nonlinear system is described as follows.

1) Initialization

$$\begin{cases} \bar{x}_0 = E[x_0] \\ P_0 = E[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T] \end{cases} \tag{24}$$

$$\begin{cases} w_0^m = \dfrac{\lambda}{n+\lambda} \\ w_0^c = \dfrac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta) \\ w_i^m = w_i^c = \dfrac{1}{2(n+\lambda)}, \ i = 1, \cdots, 2n. \end{cases} \tag{25}$$

where $\lambda = n(\alpha^2 - 1)$ and α is a constant to control the *Sigma* point distribution, and β is nonnegative constant.

2) Computation of the Sigma Points

$$\chi_{k-1} = \left[\bar{x}_{k-1}, \bar{x}_{k-1} + \sqrt{(n+\lambda)P_{k-1}}, \bar{x}_{k-1} - \sqrt{(n+\lambda)P_{k-1}}\right] \tag{26}$$

3) Time Update

$$\begin{cases} \chi^*_{k|k-1} = f(\chi_{k-1}, u_k) \\ \hat{x}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \chi^*_{i,k|k-1} \\ P_{k|k-1} = \sum_{i=0}^{2n} w_i^c (\chi^*_{i,k|k-1} - \hat{x}_{k|k-1})(\chi^*_{i,k|k-1} - \hat{x}_{k|k-1})^T + Q \\ \gamma_{k|k-1} = h(\chi^*_{k|k-1}) \\ \hat{y}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \gamma_{i,k|k-1} \end{cases} \tag{27}$$

4) Measurement Update

$$\begin{cases} P_k^0 = P_{k,k-1} \\ x_k^0 = x_{k,k-1} \\ x_k^j = x_k^{j-1} + K_k^j (y_k^i - H_k^i x_k^{i-1}) \\ K_k^j = P_k^{i-1} H_k^{i\,T} (H_k^i P_k^{i-1} H_k^{i\,T} + R_k^i)^{-1} \\ P_k^i = (I - K_k^j H_k^j) H_k^{i-1}; i = 1, 2 \cdots, m \\ x_k = x_k^m \\ P_k = P_k^m \end{cases} \tag{28}$$

C. SVD-Based UKF

Given the real symmetrical $n \times n$ matrix $M, MMT \geq 0$, it must be a symmetrical matrix and its eigenvalues are all non-negative. To this end:

$$\sigma_i^2 = \sqrt{\lambda_i (M^T M)} \tag{29}$$

$$M = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^T \tag{30}$$

$$\Sigma = diag(\sigma_1, \sigma_2, \cdots, \sigma_r) \tag{31}$$

$$M = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} U^T = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} U^T U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} U^T \tag{32}$$

D. Proposed Algorithm

From the above lemma (expressed by (33)), the state covariance matrix P is decomposed to choose the *Sigma* points, which approximate the posterior mean and covariance of the Gaussian random variable, propagating mean and covariance through a nonlinear transformation.

Replacing the equation (26) of UKF algorithm, the new measurement update is:

$$\begin{cases} [U, D, V] = f_{SVD}(P_{k-1}) \\ C = U\left(\sqrt{(n+\lambda)D}\right)U^T \\ \chi_{k-1} = [\hat{x}_{k-1}, \hat{x}_{k-1} - C, \hat{x}_{k-1} + C] \end{cases} \tag{33}$$

where $D = diag(\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2)$ and σ_i is the *ith* singular value of P (including zero), and $f_{SVD}(.)$ is the function to get eigenvectors and singular values of the matrix (.).

Finally, each iteration for each sample point, Newton-Raphson load flow algorithm is run with a modified structure based on load flow techniques based on radial basis function neural networks as [23-27].

VI. Simulation Results

The code was written in MATLAB environment and then applied to different power systems. It was developed and run on a Pentium IV CPU 2.9 GHz, 1.96 GB of RAM, 100 GB hard disc computer using MATLAB 7.11.0 version. In the proposed algorithm, $\varepsilon = 10^{-6}$ was considered as the maximum allowable tolerance for the difference between the values of voltage, angle and active and reactive powers vectors in two consecutive iterations. The method considers a maximum 300 iterations for each run and if the program does not converge, the PLF has no solution. The proposed SVD/UKF- based algorithm is tested on IEEE 14-bus ADN including WG-based RERs and PHEVs. Also, to show the accuracy of the proposed method, the error index (34) is applied to the probability information of the output random variables to provide a general overview of the PLF solutions

$$\varepsilon_l^j = \left| \frac{100(\ell^j_{MCS} - \ell^j_{Method})}{\ell^j_{MCS}} \right| (\%) \tag{34}$$

To compare the proposed method with the Parzen window (PW) density estimator-based PLF, the proposed improved SVD/UKF-based PLF algorithm is tested on the modified IEEE 14-bus power system consisting of two WG units connected to buses 3 and 4 as method [9]. All loads and generations are modeled with the normal distribution and standard deviation equal to 5% of mean values. The load's sustainability at bus 4 is assumed to be 70% of times which is determined using a binomial distribution. The wind power generation is calculated using (3) and based on the Weibull distribution for the wind speed, it is assumed that the WGs are capable to inject reactive power to the grid, having the leading power factor which is uniformly distributed between 0.8 and 0.85. A PHEV charging station described in]21] is connected to bus 4 with probabilistic charging model the previous assumptions in this regards. The number of uncertain variables is assumed to be 30.

Statistical features of selected uncertain variables of IEEE 14-bus power system are provided in Table I, for both base case (uncorrelated variables), and the correlated variables with the coefficient matrix of (35) used in]8].

$$\rho = \begin{bmatrix} 1 & 0.8 & -0.5 & -0.5 & 0.5 & 0.5 & -0.2 & -0.2 \\ 0.8 & 1 & -0.5 & -0.5 & 0.5 & 0.5 & -0.2 & -0.2 \\ -0.5 & -0.5 & 1 & 0.5 & -0.2 & -0.2 & 0.1 & 0.1 \\ -0.5 & -0.5 & 0.5 & 1 & -0.2 & -0.2 & 0.1 & 0.1 \\ 0.5 & 0.5 & -0.2 & -0.2 & 1 & 0.8 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.2 & -0.2 & 0.8 & 1 & -0.5 & -0.5 \\ -0.2 & -0.2 & 0.1 & 0.1 & -0.5 & -0.5 & 1 & 0.5 \\ -0.2 & -0.2 & 0.1 & 0.1 & -0.5 & -0.5 & 0.5 & 1 \end{bmatrix} \quad (35)$$

Moreover, it was considering 8 correlated variables namely active/reactive powers of WG at bus 3 ($P_{wf\text{-}3}$, $Q_{wf\text{-}3}$), active/reactive powers of bus 3 (P_3, Q_3), active/reactive powers of WG at bus 4 ($P_{wf\text{-}4}$, $Q_{wf\text{-}4}$), and active/reactive powers of PHEV at bus 4 ($P_{PHEV\text{-}4}$, $Q_{PHEV\text{-}4}$), respectively.

Also, the average error indices of different methods are presented in Table II for mean and STD (defined by (34)) of voltage magnitude (V), phase angles (δ), active/reactive powers of lines/branches (P_{br}, Q_{br}) and active/reactive power generation of power plants (P_g, Q_g) for base case and correlated variables.

These results indicate that the proposed SVD/UKF-based algorithm can provide better accuracy in solving PLF problem for the microgrids including correlated DER units and uncertain loads and generations.

VII. Conclusion

In this paper, a new method that exploits the nonlinear features of SVD-based UKF and for uncertainty quantification was proposed. By using the proposed nonlinear SVD-based transform, the PLF algorithm did not require mapping the random variables to the nonlinear load flow function for all potential points in the statistical variable's space. As a result, the computation time of the proposed algorithm decreased considerably. Finally, by implementing the proposed method in MATLAB programming environment, the efficiency, accuracy and computation speed of the proposed SVD/UKF-based PLF method was demonstrated using an IEEE 14-bus power system, including WG units, PHEV and uncertain loads.

TABLE I. Comparison Results Between Proposed SVD/UKF-Based PLF Method and Different Methods for IEEE 14-Bus System

IEEE 14-bus System [9]		**Uncorrelated variables [p.u]**						**Correlated Variables [p.u]**					
Method	**Param.**	$\lvert V_4\rvert$	$\lvert V_5\rvert$	P_{1-2}	P_{g-1}	Q_{4-5}	δ_2 (deg.)	$\lvert V_4\rvert$	$\lvert V_5\rvert$	P_{1-2}	P_{g-1}	Q_{4-5}	δ_2 (deg.)
MCS	μ	1.0240	1.0240	1.2830	1.9150	0.1500	-4.0300	1.0180	1.0200	0.9500	1.4300	0.1800	-2.9100
	σ	0.0040	0.0030	0.1750	0.2600	0.0180	0.5800	0.0100	0.0100	0.1800	0.2470	0.0200	0.5600
2n+1 PEM [13]	μ	1.0240	1.0240	1.2820	1.9140	0.1500	-4.0300	1.0180	1.0300	1.1000	1.4100	0.1800	-2.9000
	σ	0.0030	0.0020	0.1750	0.2600	0.0210	0.5800	0.0030	0.0050	0.2000	0.3000	0.0300	0.6700
2n PEM [13]	μ	1.0240	1.0240	1.2800	1.9200	0.1500	-4.0400	-	-	-	-	-	-
	σ	0.0030	0.0020	0.1760	0.2600	0.0210	0.5800	-	-	-	-	-	-
Diffusion	μ	1.0240	1.0240	1.2850	1.9300	0.1600	-4.2300	1.0170	1.0200	0.9500	1.4300	0.1850	-2.9500
	σ	0.0040	0.0020	0.1810	0.2900	0.0200	0.5900	0.0020	0.0020	0.1710	0.3120	0.0300	0.5400
PW [9]	μ	1.0240	1.0240	1.2700	1.9200	0.1500	-4.0200	1.0170	1.0200	0.9500	1.4260	0.1700	-2.9000
	σ	0.0040	0.0030	0.1850	0.2700	0.0170	0.5950	0.0050	0.0030	0.1800	0.2230	0.0200	0.5500
Proposed SVD/UKF-PLF	μ	1.0241	1.0241	1.2811	1.9150	0.1501	-4.0202	1.0180	1.0201	0.9500	1.4301	0.1801	-2.9002
	σ	0.0040	0.0031	0.1751	0.2620	0.0181	0.5800	0.0101	0.0091	0.1801	0.2410	0.0301	0.5400

TABLE II. Comparison of Average Error Indices between SVD/UKF-PLF and Different Methods for IEEE 14-Bus System

IEEE 14-bus System [9]		**Uncorrelated variables**						**Correlated Variables**					
Method	**Param.**	$\lvert V\rvert$	δ	P_{br}	Q_{br}	P_g	Q_g	$\lvert V\rvert$	δ	P_{br}	Q_{br}	P_g	Q_g
2n+1 PEM [13]	$\overline{\varepsilon_\mu}$	0.0019	0.2279	0.3730	0.8335	0.3370	1.470	0.0020	0.2260	0.3700	0.8530	1.0700	1.4600
	$\overline{\varepsilon_\sigma}$	0.2665	0.1100	0.1410	0.3431	0.0410	0.2520	0.2680	0.1070	0.1200	0.3330	0.8180	-
2n PEM [13]	$\overline{\varepsilon_\mu}$	0.0020	0.2350	0.2530	0.3000	0.2760	0.7000	-	-	-	-	-	-
	$\overline{\varepsilon_\sigma}$	145	40	148	15	29	14.8000	-	-	-	-	-	-
Diffusion	$\overline{\varepsilon_\mu}$	0.0020	0.2000	0.2500	0.7100	0.2400	0.7800	0.0020	0.1540	0.2270	0.7510	0.8410	1.0200
	$\overline{\varepsilon_\sigma}$	0.1570	0.1400	0.1120	0.4100	0.0500	0.1940	0.1540	0.1020	0.1140	0.1230	0.7120	0.2140
PW [9]	$\overline{\varepsilon_\mu}$	0.0020	0.0042	0.0080	0.00240	0.0060	0.0330	0.0020	0.0090	0.0090	0.0210	0.0160	0.0130
	$\overline{\varepsilon_\sigma}$	0.0250	0.0348	0.0310	0.0420	0.0830	0.0860	0.0310	0.0470	0.0350	0.0630	0.0050	0.0250
Proposed SVD/UKF- PLF	$\overline{\varepsilon_\mu}$	0.0020	0.0320	0.0051	0.0030	0.5200	0.0610	0.0021	0.0070	0.0080	0.0200	0.0200	0.0100
	$\overline{\varepsilon_\sigma}$	0.0201	0.0310	0.0290	0.0381	0.0680	0.0710	0.0220	0.0360	0.0310	0.0550	0.0030	0.0302

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